



POLİTEKNİK DERGİSİ

JOURNAL of POLYTECHNIC

ISSN: 1302-0900 (PRINT), ISSN: 2147-9429 (ONLINE)

URL: <http://dergipark.org.tr/politeknik>



A linear time pattern based algorithm for n-queens problem

N-vezir problemi için lineer zamanlı örüntü temelli algoritma

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Bu makaleye şu şekilde atıfta bulunabilirsiniz(To cite to this article): Karabulut B., Ergüzen A. and Ünver H. M., “A linear time pattern based algorithm for n-queens problem”, *Politeknik Dergisi*, 25(2): 615-622, (2022).

Erişim linki (To link to this article): <http://dergipark.org.tr/politeknik/archive>

DOI: 10.2339/politeknik.762967

A Linear Time Pattern Based Algorithm for N-Queens Problem

Önemli noktalar (Highlights)

- ❖ *This study proposes a new algorithm for a benchmark problem.*
- ❖ *The proposed algorithm produces solution(s) for all n values.*
- ❖ *The proposed algorithm runs in linear time with $O(n)$ time complexity.*
- ❖ *Even very large n values, it produces solution(s) without using complex calculations or searching.*
- ❖ *This study provide an effective solution to the handled problem.*

Aim

The purpose of this study is to develop an algorithm producing solution(s) in large n values of n -queens problem.

Design & Methodology

A pattern based approach that produces at least one unique solution for all n values ($n > 3$) was detected and it used to create an algorithm.

Originality

The developed algorithm can produce solutions without using complex calculations or searching. In this respect, the proposed method provides efficiency compared to similar studies.

Findings

*The developed algorithm with **$O(n)$** time complexity produces quite faster solution to n -queens problem and even in some values this algorithm produces $(n-1)/2$ unique solutions in linear time.*

Conclusion

In this study, a linear time pattern based algorithm was proposed for the n -queens problem. The developed method provides an important contribution in terms of producing solutions for large values in linear time.

Declaration of Ethical Standards

The authors of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

A Linear Time Pattern Based Algorithm for N-Queens Problem

Araştırma Makalesi / Research Article

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(Geliş/Received : 06.07.2020 ; Kabul/Accepted : 17.12.2020 ; Erken Görünüm/Early View : 18.12.2020)

ABSTRACT

The n-queens problem is the placing of n number of queens on an $n \times n$ chessboard so that no two queens attack each other. This problem is important due to various usage fields such as VLSI testing, traffic control job scheduling, data routing, dead-lock or blockage prevention, digital image processing and parallel memory storage schemes mentioned in the literature. Besides, this problem has been used as a benchmark for developing new artificial intelligence search techniques. However, it is known that backtracking algorithms, one of the most frequently used algorithms to solve this problem, cannot produce all solutions in large n values due to the exponentially growing time complexity. Therefore, various methods have been proposed for producing one or more solutions, not all solutions for each n value. In this study, a pattern based approach that produces at least one unique solution for all n values ($n > 3$) was detected. By using this pattern, a new algorithm that produces at least one unique solution for even very large n values in linear time was developed. The developed algorithm with $O(n)$ time complexity produces quite faster solution to n-queens problem and even in some values, this algorithm produces $(n-1)/2$ unique solutions in linear time.

Keywords: N-queens problem, constraint satisfaction problem, NP-hard, NP-complete, optimization problem.

N-Vezir Problemi için Lineer Zamanlı Örüntü Temelli Algoritma

ÖZ

N-vezir problemi, $n \times n$ boyutundaki bir satranç tahtasına n adet vezirin herhangi iki vezir birbirine saldırmayacak şekilde yerleştirilmesidir. Bu problem literatürde değinilen VLSI testi, trafik kontrol işi planlama, veri yönlendirme, ölümcül kilitlenme ya da tıkanıklık önleme, dijital görüntü işleme ve paralel bellek depolama şemaları gibi çeşitli kullanım alanlarından dolayı önemlidir. Ayrıca bu problem, yeni yapay zekâ arama tekniklerinin geliştirilmesi için bir referans noktası olarak kullanılmaktadır. Fakat bilindiği üzere bu problemin çözümde sıklıkla kullanılan geri-izleme algoritmaları, katlanarak büyüyen zaman karmaşıklığından dolayı büyük n değerleri için tüm çözümleri üretememektedir. Bu nedenle, her bir n değeri için tüm çözümleri bulmak yerine bir veya daha fazla çözüm üretebilmek için çeşitli yöntemler önerilmiştir. Bu çalışmada, tüm n değerleri ($n > 3$) için en az bir tane eşsiz çözüm üreten bir örüntü tespit edilmiştir. Bu örüntü kullanılarak, çok büyük n değerlerinde bile lineer zamanda en az bir tane eşsiz çözüm üreten yeni bir algoritma geliştirilmiştir. $O(n)$ zaman karmaşıklığı ile geliştirilen algoritma, n-vezir problemine oldukça hızlı çözüm üretmektedir ve hatta bazı değerler için lineer zamanda $(n-1)/2$ adet eşsiz çözüm üretmektedir.

Anahtar Kelimeler: N-vezir problemi, kısıt sağlama problemi, NP-hard, NP-complete, optimizasyon problemi.

1. INTRODUCTION

The n-queens problem is the placement of n number of queens on an $n \times n$ chessboard in such a way that no two queens are on the same row, column, or diagonal. Therefore, there can be only one queen on each row, column and diagonal. This problem was initially suggested as 8-queens problem by a chess player Max Bezzel in 1848 [1]. It was later extended to the n-queens problem, with n queens on an $n \times n$ board. The various application fields of this problem, which has been studied more than a century ago, have been emphasized in the literature. Sosic and Gu [2] stated that the n-queens problem has practical applications in VLSI testing and traffic control. Waqas and Bhatti [3] have listed applications of n-queens problem and $(n+1)$ queens problem for real world problem as job/shop scheduling,

data routing, dead-lock or blockage prevention, efficient resource management in computer systems, task assignment in multiprocessors, digital image processing and parallel memory storage schemes. It has also been used in various practical applications. Wang et al. [4] presented pixel decimation technique using the n-queen lattice and presented an application for block-based motion estimation using this novel technique. Bell and Stevens [5], have presented one of the best survey about the n-queens problem and have mentioned applications of the problem in detail.

Many studies have been conducted to offer a solution to the n-queens problem which was suggested initially in 1848. Solutions have been sought for the problem by using various methods including optimization techniques, parallel programming, mathematical approaches, backtracking algorithms and different patterns. Different methods have been applied to solve

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this problem, which has importance for various fields. The studies in the literature can be divided into three categories; finding one solution, finding more than one solutions but not all and finding all solutions. One of the most frequently used method for solving n-queens problem is backtracking search that generate all possible solutions. Murali et al. [6] simulated the N-queens problem using backtracking algorithm. Gldal et al. [7] proposed a hybrid approach by integrating sets and backtracking. They have removed the threatened cells in order to decrease the number of trial and error steps. Due to the exponentially growing time complexity of backtracking search [8], it cannot produce all possible solutions in large n values [9]. For this reason, methods which produce one or more solutions, instead of all solutions for each n value were developed.

N-queens problem is also a combinatorial optimization problem [10] and this problem has been used as a benchmark for developing new AI search techniques [11, 12]. In the literature, there are different studies in which artificial intelligence and computational intelligence techniques have been used to solve the problem. Meng and Wu [13] presented a Hybrid Genetic Algorithm in their study to solve the problem. Khan et al. [14] was proposed a solution for n-queen problem based on Ant Colony Optimization. In their study Turkey and Ahmad [15] have been used Genetic Algorithm to produce solution to the problem. Motameni [16] have used Gravitational Search Algorithm to solve n-queens problem. Maazallahi et al. [17], presented a DNA computing model based on Adleman-Lipton model to solve the n-queens problem. This model provides all solutions and solve problem in a polynomial time complexity. In other studies, Biogeography based optimization [18], particle swarm optimization Amooshahi et al. [19] were used to solve this problem.

However, in recent years more algorithmic studies have been done and algorithms have been proposed for solution of the problem. Abramson and Yung [1], developed a new linear time divide and conquer algorithm to solve n-queens problem and a related problem that is the toroidal problem. Susic and Gu [20], presented a new probabilistic local search algorithm. This algorithm runs in polynomial time and based on a gradient based heuristic. Their algorithm finds a solution for extremely large size n-queens problem. It is stated that previous AI search algorithms can find solution with n up to about 100, this method can find a solution with n up to 500,000. In another study, Susic and Gu [11] developed two gradient-based heuristic based probabilistic local search algorithms called Queen Search 2 (QS2) and Queen Search 3 (QS3). QS2 and QS3 algorithms run in almost linear time and can find a solution for extremely large size n-queens problems. With QS3 algorithm, a solution can be found for 500,000 queens in approximately one and a half minutes. In their subsequent study, Susic and Gu [9] presented a linear time algorithm. With this algorithm, a solution can be found for 3,000,000 queens in approximately 55 seconds.

In a new study they did in the following years, Susic and Gu [21] presented an efficient local search algorithm. This algorithm runs in linear time. They can find a solution for 3,000,000 queens using a workstation computer. Lijo and Jose [22], proposed an algorithm that has less computational complexity compared with backtracking algorithm. Their algorithm based on arithmetic progression and predicts potential candidate solutions. The proposed algorithm has reduced the time complexity by $O(n^3)$.

El-Qawasmeh and Al-Noubani [23], developed an algorithm working in linear time for the problem. Solutions have a modular structure and queens are placed in pre-calculated positions, allowing for producing a fast solution to the problem. They used 3 different strategies in which they divided chessboard into two parts. They have directly placed queens on non-overlapping points beginning from the top of the board for the first half and beginning from the bottom of the board for the second half. They obtained a unique solution for 75% of n values, and used the algorithm developed by Susic for the remaining 25%. They proved their strategies with a software they developed and with mathematical operations. In conclusion, they developed an algorithm that works 100 faster in deterministic linear time in only 75% of n values for $n>3$ compared to Susic's algorithm. Rohith et al. [24], obtained a unique solution in polynomial time for $(n+1)\times(n+1)$ chessboard by expanding $n\times n$ solution using a pattern that they observed.

In this study, a pattern-based algorithm running in linear time for n-queens was developed. A pattern that produces at least one unique solution for every consecutive 6 values where $n>3$ was observed. Besides, this pattern produces $(n-1)/2$ unique solutions in one in every consecutive six values. By using this pattern, a general algorithm was developed. With this algorithm, unique and fast solutions were obtained even for extremely large n values without resorting to complex calculations. The developed algorithm is iterative and it only assigns a position for queens in any n values. Computational complexity of the algorithm is $O(n)$.

2. METHODOLOGY

2.1. Notation for the N-Queens Problem

Any possible solution of the n-queens problem was represented as the n-tuple (q_0, q_1, \dots, q_n) . In this notation, q_i is a column position on which the queen in the i-th row is placed. Figure 1 shows an example of n-tuple notation for $n=4$.

	q_0		
			q_1
q_2			
		q_3	

(2, 4, 1, 3)

Figure 1. Solution to 4-queens problem

2.2. Observed Pattern

The observed pattern produces at least one unique solution for every value where $n > 3$. It produces one unique solution in four, two unique solutions in one, three solutions in three and $(n-1)/2$ unique solutions in four in every consecutive twelve values. In Table 1 solutions of the pattern was presented. Firstly, the n values are divided into two categories according to whether they are even or odd. Then subcategories are determined by mode operation. There are 3 subcategories for both even and odd, so there are 6 subcategories in total. Solution sets were given for 6 subcategories in Table 1.

Shift operation: In this operation an existing solution is handled. In existing solution's set, the last queen is taken first and all other queens are shifted one position to the right. When this operation is applied to an existing solution, a new unique solution is obtained. However, this operation can be performed only $((n-1)/2)-1$ times. So, totally $(n-1)/2$ solutions are obtained. An example was given in Figure 2. As shown in Figure 2.a, solution set of $n=5$ queens is (1, 3, 5, 2, 4). Shift operation can be

2.3. Proposed Algorithm

The proposed algorithm is based on a pattern and does not contain search operation. The positions of queens on the board are predetermined and the solution set is generated directly by the pattern. The algorithm works for $n > 3$.

The main function calls the decision function called *determineSolutionFunction()* if n is greater than 3.

Main Function: *nQueens*

- 1: Set $n =$ get the board size from user
- 2: **if** $n > 3$ **then**
- 3: Call *determineSolutionFunction(n)*;
- 4: **else** quit

Function: *determineSolutionFunction(n)*

- 1: **if** n is even **then**
- 2: **if** $(n+1) \bmod 3$ is equal to zero **then**
- 3: **if** $n/2 \bmod 2$ is equal to zero **then**

Table 1. Solution sets

Residue class		Solution(s)
n is even	$(n+1) \bmod 3 = 0 \ \& \ (n/2) \bmod 2 = 0$ (e.g., $n= 8, 20, 32, 44 \dots$)	Solution 1= (2, 4, 6, ... n, 3, 1, 7, 5, ... (n-1), (n-3)) Solution 2= (4, 6, ...n, 3, 1, 7, 5, ... (n-1), (n-3), 2)
	$(n+1) \bmod 3 = 0 \ \& \ (n/2) \bmod 2 \neq 0$ (e.g., $n= 14, 26, 38, 50 \dots$)	Solution 1 = <i>((n-1), 2, 4, ... n, 3, 1, 7, 5, ... (n-3), (n-5))</i> Solution 2 = <i>(n, 3, 5, ... (n-1), 1, 4, 2, 8, 6, ... (n-2), (n-4))</i> Solution 3 = <i>((n-5), (n-1), 2, 4, ... n, 3, 1, ... (n-7), (n-9), (n-3))</i>
	$(n+1) \bmod 3 \neq 0$ (e.g., $n= 4, 6, 10, 12, 16, 18 \dots$)	Solution 1= (2, 4, ... n, 1, 3, ... (n-1))
n is odd	$n \bmod 3 = 0 \ \& \ (n-1) \bmod 4 = 0$ (e.g., $n= 9, 21, 33, 45 \dots$)	Solution 1 = <i>((n-1), 5, 3, 9, 7, ... n, (n-2), 2, 4, ... (n-3), 1)</i> Solution 2 = <i>(1, (n-1), 5, 3, 9, 7, ... n, (n-2), 2, 4, ... (n-3))</i> Solution 3 = <i>((n-2), 4, 2, 8, 6, ... (n-1), (n-3), 1, 3, ... (n-4), n)</i>
	$n \bmod 3 = 0 \ \& \ (n-1) \bmod 4 \neq 0$ (e.g., $n= 15, 27, 39, 51 \dots$)	Solution 1 = <i>((n-1), 3, 5, 7, ... n, 4, 2, 8, 6, ... (n-3), (n-5), 1)</i> Solution 2 = <i>(3, 5, 7, ... n, 4, 2, 8, 6, ... (n-3), (n-5), 1, (n-1))</i> Solution 3 = <i>((n-1), 3, 5, 7, ... (n-2), 1, 4, 2, 8, 6, ... (n-3), (n-5), n)</i>
	$n \bmod 3 \neq 0$ (e.g., $n= 5, 7, 11, 13, 17, 19 \dots$)	Solution 1 = (1, 3, ... n, 2, 4, ... (n-1)) Solution 2 = ((n-1), 1, 3, ... n, 2, 4, ... (n-3)) (<i>Shift operation</i>) ⋮ Solution $(n-1)/2 = (4, 6, ... (n-1), 1, 3, ... n, 2)$ (<i>Shift operation</i>)

**Non-consecutive numbers in the solution sets typed as bold and italic.
Shift operation: Create new solution by shifting the previous solution one column right.

applied in this value one times and new unique solution set is (4, 1, 3, 5, 2). So, two unique solutions are produced.

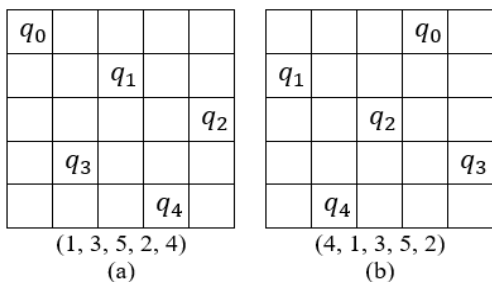


Figure 2. a) 1st solution of 5-queens b) 2nd solution of 5-queens

- 4: Call *Function1(n)*
- 5: **else**
- 6: Call *Function2(n)*
- 7: **else**
- 8: Call *Function3(n)*
- 9: **else**
- 10: **if** $n \bmod 3$ is equal to 0 **then**
- 11: **if** $(n-1) \bmod 4$ is equal to 0 **then**
- 12: Call *Function4(n)*
- 13: **else**
- 14: Call *Function5(n)*
- 15: **else**
- 16: Call *Function6(n)*

In decision function, a solution function is selected according to the mode operation and the program branches to the appropriate function. Six functions; *Funtion1*, *Funtion2*, *Funtion3*, *Funtion4*, *Funtion5* and *Funtion6* are defined for the 6 different subcategories determined in the pattern. By using these functions, solution sets for related n values are produced.

Procedure: *Function1(n)*

```

1: Set  $number=2$ ,  $index=0$ 
2: while  $number$  is smaller than or equal to  $n$  do
3: Set  $solutionArray[index] = number$ 
4: Increment  $index$  by one and  $number$  by two
5: end while
6:
7: Set  $tempIndex = index$ 
8: Set  $number=1$ 
9: while  $number$  is smaller than  $n$  do
10: Set  $solutionArray[index] = number$ 
11: Increment  $index$  by one and  $number$  by two
12: end while
13:
14: while  $tempIndex$  is smaller than  $n-1$  do
15: Increase  $solutionArray[tempIndex]$  by two
16: Decrease  $solutionArray[tempIndex+1]$  by two
17: Increase  $tempIndex$  by two
18: end while
19:
20: Call  $printSolution(solutionArray)$ 
21:
22: Set  $newSolutionArray[n-1] = solutionArray[0]$ 
23: for  $i=0, 1, 2, \dots, n-2$  do
24: Set  $newSolutionArray[i] = solutionArray[i+1]$ 
25: end for
26:
27: Call  $printSolution(newSolutionArray)$ 

```

Procedure: *Function2(n)*

```

1: Set  $number=2$ ,  $index=1$ 
2: Set  $solutionArray[0] = n-1$ 
3: while  $number$  is smaller than or equal to  $n$  do
4: Set  $solutionArray[index] = number$ 
5: Increment  $index$  by one and  $number$  by two
6: end while
7:
8: Set  $tempIndex=index$ 
9: Set  $number=1$ 
10: while  $index$  is smaller than  $n$  do
11: Set  $solutionArray[index] = number$ 
12: Increment  $index$  by one and  $number$  by two
13: end while
14:
15: while  $tempIndex$  is smaller than  $n-1$  do
16: Increase  $solutionArray[tempIndex]$  by two
17: Decrease  $solutionArray[tempIndex+1]$  by two
18: Increase  $tempIndex$  by two
19: end while
20:

```

```

21: Call  $printSolution(solutionArray)$ 
22:
23: Copy  $SolutionArray$  to  $tempSolutionArray$ 
24: for  $i=0, 1, 2, \dots, n-1$  do
25: if  $solutionArray[i]$  is equal to 0 then
26: Set  $solutionArray[i]=1$ 
27: else
28: Increase  $solutionArray[i]$  by one
29: end for
30:
31: Call  $printSolution(solutionArray)$ 
32:
33: Set  $newSolutionArray[0]=tempSolutionArray[n-1]$ 
34: for  $i=0, 1, 2, \dots, n-2$  do
35: Set  $newSolutionArray[i+1]=tempSolutionArray[i]$ 
36: end for
37:
38: Call  $printSolution(newSolutionArray)$ 

```

Procedure: *Function3(n)*

```

1: Set  $number=2$ ,  $index=0$ 
2: while  $number$  is smaller than or equal to  $n$  do
3: Set  $solutionArray[index] = number$ 
4: Increase  $index$  by one and  $number$  by two
5: end while
6:
7: Set  $number=1$ 
8: while  $number$  is smaller than  $n$  do
9: Set  $solutionArray[index] = number$ 
10: Increment  $index$  by one and  $number$  by two
11: end while
12:
13: Call  $printSolution(solutionArray)$ 

```

Procedure: *Function4(n)*

```

1: Set  $number=3$ ,  $index=1$ 
2: Set  $solutionArray[0] = n-1$ ,  $solutionArray[n-1] = 1$ 
3: while  $number$  is smaller than or equal to  $n$  do
4: Set  $solutionArray[index] = number$ 
5: Increase  $index$  by one and  $number$  by two
6: end while
7:
8: Set  $tempIndex=index$ 
9: Set  $number=2$ 
10: while  $number$  is smaller than  $n-2$  do
11: Set  $solutionArray[index] = number$ 
12: Increase  $index$  by one and  $number$  by two
13: end while
14:
15: Set  $index=1$ 
16: while  $index$  is smaller than  $tempIndex$  do
17: Increase  $solutionArray[index]$  by two
18: Decrease  $solutionArray[index+1]$  by two
19: Increase  $index$  by two
20: end while
21:
22: Call  $printSolution(solutionArray)$ 
23:

```

```

24: Set newSolutionArray[0] = solutionArray[n-1]
25: for i=1, 2, 3, ..., n-1 do
26: newSolutionArray[i] = solutionArray[i-1]
27: end for
28:
29: Call printSolution(newSolutionArray)
30:
31: for i = 0, 1, 2, ..., n-1 do
32: if solutionArray[i]-1 is equal to 0 then
33: Set solutionArray[i]=n
34: else
35: Decrease solutionArray[i] by one
36: end for
37:
38: Call printSolution(solutionArray)

```

Procedure: *Function5*(*n*)

```

1: Set number=3, index=1
2: Set solutionArray[0] = n-1,
3: while number is smaller than or equal to n do
4: Set solutionArray[index] = number
5: Increase index by one and number by two
6: end while
7:
8: Set tempIndex=index
9: Set number=2
10: while number is smaller than n-1 do
11: Set solutionArray[index] = number
12: Increase index by one and number by two
13: end while
14:
15: while tempIndex is smaller than n-1 do
16: Increase solutionArray[tempIndex] by two
17: Decrease solutionArray[tempIndex+1] by two
18: Increase tempIndex by two
19: end while
20:
21: solutionArray[n-1] = 1
22: Call printSolution(solutionArray)
23:
24: Set newSolutionArray[n-1] = solutionArray[n]
25: for i = 0, 1, 2, ..., n-2 do
26: Set newSolutionArray[i] = solutionArray[i+1]
27: end for
28:
29: Call printSolution(newSolutionArray)
30: Set tempValue= solutionArray[n/2]
31: Set solutionArray[n/2]= solutionArray[n-1]
32: Set solutionArray[n-1]= tempValue
33:
34: Call printSolution(newSolutionArray)

```

Procedure: *Function6*(*n*)

```

1: Set number=1, index=0
2: while number is smaller than or equal to n do
3: Set solutionArray[index] = number
4: Increase index by one and number by two
5: end while

```

```

6:
7: Set number=2
8: while number is smaller than n do
9: Set solutionArray[index] = number
10: Increase index by one and number by two
11: end while
12:
13: Call printSolution(solutionArray)
14:
15: while newSolutionArray[n-1] is not equal to 2 do
16: Set newSolutionArray[0] = solutionArray[n-1]
17: for i=1, 2, 3, ..., n-1 do
18: Set newSolutionArray[i] = solutionArray[i-1]
19: end for
20:
21: Call printSolution(newSolutionArray)
22: Copy newSolutionArray to solutionArray
23:
24: end while

```

2.4. Control of Solutions

A correct solution requires that no two queens share the same row, column, or diagonal. If the place of the q_i queen is $B[X_1, Y_1]$ and the place of the q_{i+1} is $B[X_2, Y_2]$, then;

- $X_1 \neq X_2$ (not in the same row)
- $Y_1 \neq Y_2$ (not in the same column)
- $|X_1 - X_2| \neq |Y_1 - Y_2|$ (not in the same diagonal)

If all the requirements above are achieved, it means that no two queens threaten each other. The correctness of the solution can be checked whether all requirements for n value queens placed in $n \times n$ board are achieved. To control solution a check function called *solutionCheck* was defined. This function is used to check whether the solutions are correct.

Procedure: *solutionCheck*(*solutionArray*: array[1...*n*] of integer)

```

1: for k=0, 1, 2, ..., n-1 do
2: for i=k+1, k+2, ..., n-1 do
3: if solutionArray[k] is equal to solutionArray[i] or
|k-i| is equal to
|solutionArray[k]-solutionArray[i] then
4: return false
5: end for
6: end for
7: return true

```

3. RESULTS AND DISCUSSION

In this study, a method which runs in linear time and produces at least one unique solution for the solution of n -queens problem has been studied. In the conducted study, a pattern was observed which produce at least one unique solution for $n > 3$ in 6 different categories. This pattern produces different numbers of solutions for each

Table 2. Number of solutions

Number of queens n	Number of solutions
4	1
5	$(n-1)/2=2$
6	1
7	$(n-1)/2=3$
8	2
9	3
10	1
11	$(n-1)/2=5$
12	1
13	$(n-1)/2=6$
14	3
15	3

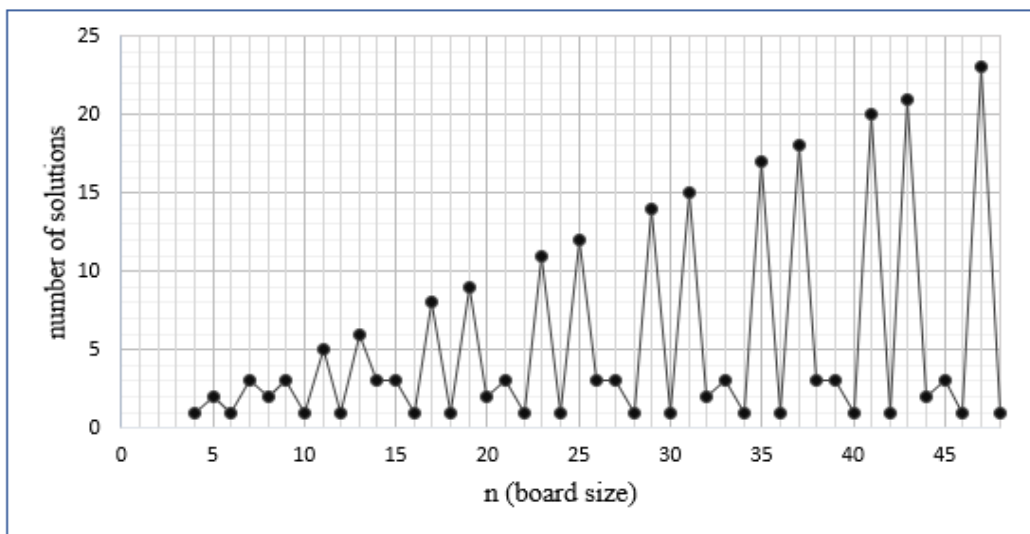


Figure 3. Number of solutions

category. In Table 2, number of solutions obtained with the pattern for related n values was presented. Only for the first 12 numbers, number of solutions are given in Table 2, since these number of solution numbers are repeated every consecutive 12 values. As stated in the table, a fixed number of solutions are obtained at some values. However, in some values $(n-1)/2$ solutions are obtained.

The number of obtained solutions is presented graphically in Figure 3. When the Figure 3 is examined, it is seen that in some values, a fixed number of solutions are obtained, whereas in some values, the number of solutions increases linearly.

A linear time algorithm was developed using the specified pattern. In order to test the developed algorithm, a program was created on the NetBeans programming platform using the Java programming language. The developed program was run on a personal computer with i7 2.40GHz processor. With the help of the program, we have investigated the time needed to obtain a unique solution. The obtained results for some values were given in Table 3.

Various studies have been done to solve this problem. One of the best known of these studies was done by Susic and Gu [22]. Susic and Gu stated that they could find a solution less than 55 seconds using a workstation for 3,000,000 queens in their recent work. The developed method in this study is able to produce at least one unique solution for all values as in the study of Susic and Gu [22]. In addition, as shown in Table 3, our algorithm

Table 3. Test results of proposed algorithm

Number of queens n	Time to find a solution (seconds.milliseconds)
4,000,000	0.15
5,000,000	0.29
6,000,000	0.32
7,000,000	0.28
8,000,000	0.54
9,000,000	0.28
10,000,000	0.28

developed in this study is able to produce solutions in less than 1 second at much larger values of 3,000,000.

In a study conducted in the following years, El-Qawasmeh and Al-Noubani [23] presented an algorithm that can generate a solution 100 times faster than Susic's algorithm. Queens' positions are predetermined in the study and there is no need for any calculation or searching operation. In this way, the researchers have achieved speed increase. However, their algorithm is able to produce a correct solution of 75% rather than the full value of n . They used Susic's algorithm in 25% values that they cannot produce solution. Our developed algorithm predetermines queens' positions similar to the algorithm of El-Qawasmeh and Al-Noubani, and does not require search operation or any computation. Moreover, our algorithm is able to produce a solution of all n values (100%). In addition, in most values (about 83%) can produce more than one unique solution. Even in some values (about 33%), $(n-1)/2$ unique solutions can be produced.

4. CONCLUSION

N-queens problem is an important problem with its applications in various fields and subject to new studies. However, it is also a difficult problem due to the time complexity that grows exponentially. It is known that backtracking algorithms, one of the most frequently used algorithms to solve this problem, cannot produce all solutions in large n values due to this time complexity problem. It is not possible to get all possible solutions for large values. For this reason, the development of methods that can produce one or more solutions but not all for large values has become important.

In this study, a linear time pattern based algorithm was proposed for the n-queens problem. The developed algorithm produce at least one unique solution for all n values and it produces solution(s) quite faster with $O(n)$ time complexity. Besides, for most values (about 67%) it can produce more than one solution. Moreover, for some values (about 33%) it can produce $(n-1)/2$ unique solutions. The developed method provides an important contribution in terms of producing solutions for large values in linear time. In addition, for many values it can produce solutions without using complex calculations or searching. In this respect, the proposed method provides efficiency.

DECLARATION OF ETHICAL STANDARDS

The authors of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

AUTHORS' CONTRIBUTIONS

Bergen KARABULUT: Developed the theoretical framework, designed the algorithm, performed the experiments and analyse the results.

Atila ERGÜZEN: Developed the theoretical framework, designed the algorithm, performed the experiments and analyse the results.

Halil Murat ÜNVER: Developed the theoretical framework, wrote the manuscript.

CONFLICT OF INTEREST

There is no conflict of interest in this study.

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