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Global Existence of Solutions for a Coupled Viscoelastic Plate Equation with Degenerate Damping Terms

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Abstract: We investigate a viscoelastic plate nonlinear system with degenerate damping terms on a bounded domain R^n with Dirichlet boundary conditions. The nonlinearities $f_1(u,v)$ and $f_2(u,v)$ act as a strong source in the system. Under some restriction on the parameters in the system, we prove the global existence result of weak solution.

Keywords: Degenerate damping, Global existence, Plate equation, Viscoelastic equation.

1 Introduction

This article is concerned with the following a coupled viscoelastic plate equation with degenerate damping terms:

$$\begin{cases} u_{tt}+\Delta^2 u-\int_0^t \omega_1(t-s)\Delta^2 u(s)ds+\left(|u|^k+|v|^l\right)|u_t|^{p-1}u_t=f_1(u,v), & (x,t)\in\Omega\times(0,T), \\ v_{tt}+\Delta^2 v-\int_0^t \omega_2(t-s)\Delta^2 v(s)ds+\left(|v|^\theta+|u|^\varrho\right)|v_t|^{q-1}v_t=f_2(u,v), & (x,t)\in\Omega\times(0,T), \\ u=v=\frac{\partial u}{\partial\nu}=\frac{\partial v}{\partial\nu}=0, & (x,t)\in\partial\Omega\times(0,T), \\ u(x,0)=u_0(x),\ u_t(x,0)=u_1(x), & x\in\Omega, \\ v(x,0)=v_0(x),\ v_t(x,0)=v_1(x), & x\in\Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain in R^n $(n\geq 1)$ with smooth boundary $\partial\Omega$, ν is the unit outer normal to $\partial\Omega$. The function $f_i(.,.):R^2\longrightarrow R$ $(i=1,2)$ are source and the function $\omega_i(.):R^+\to R^+$ $(i=1,2)$ are the kernel functions and satisfies some conditions to be specified later.

This type problem that is better suited for describing the transverse displacement u of a thin flickering plate subjected to an internal viscoelastic dissipation. For the last several decades, the mathematical analysis of plate equations has attracted a lot of attention. The other model regarding (1) is the Petrovsky system related to a plate model:

$$u_{tt}-\gamma\Delta u_{tt}+\Delta^2 u-\int_0^t \omega(t-s)\Delta^2 u(s)ds=0.$$

Rivera et al. [1] studied the asymptotic behaviour of solution with the initial and dynamical boundary conditions. Alabau-Boussouira et al. [2] studied exponential and polynomial decay results of solutions of this problem under the memory ω decay exponentially and polynomially, respectively in the case $\gamma=0$ and with $f(u)$ to be source term.

Also, Messaoudi [3] studied the plate equation and proved an existence result and studied global solution in case $m \geq p$. Then, blow-up of solutions with nonpositive initial energy and $m < p$ was obtained.

On the orther hand, the evolution equations with degenerate damping are of much interest in material science and physics. It particularly appears in physics when the friction is modulated by the strains. Our problem has degenerate damping term. The papers [4]-[5]-[6]- [7]-[8] provide other results on problems involving the degenerate damping terms.

The outline of the paper is as follows. In section 2, we introduce some assumptions used later in the proof our results and we mentioned local existence result of Theorem 2. In Section 3, the global solution will be proved.

2 Preliminaries

Throughout this work, we denote $|u|_{s,\Omega} = \|.\|_{H^s(\Omega)}$, $\|.\|_p = \|u\|_{L^p(\Omega)}$ and $\langle u, v\rangle = \langle u, v\rangle_{L^2(\Omega)}$. In addition, the following Sobolev embeddings will used often, and occasionally without mention:

$$\begin{cases} H_0^2(\Omega) \hookrightarrow L^p(\Omega), \text{ for } 2 \leq p < \infty, \ n \leq 4, \\ H_0^2(\Omega) \hookrightarrow L^p(\Omega), \text{ for } 2 \leq p \leq \frac{2n}{n-4}, \ n > 4. \end{cases} \tag{2}$$

To our result, we also use the following assumptions:

$(\mathbf{A1})$ For the nonlinearity in damping, we suppose that $p, q > 0$, in addition

$$\begin{cases} r \geq 3 \text{ if } n = 1, 2, \\ r = 3 \text{ if } n = 3. \end{cases}$$

- $u_0, v_0 \in H_0^2(\Omega)$, $u_1, v_1 \in L^2(\Omega)$.
- $$c_0 \left(|u|^{r+1} + |v|^{r+1} \right) \leq F(u, v) \leq c_1 \left(|u|^{r+1} + |v|^{r+1} \right) \tag{3}$$

for all $(u, v) \in R^2$ and positive constants c_0, c_1. We take $f_1(u, v)$ and $f_2(u, v)$ as follows

$$\begin{array}{c} f_1(u, v) = (r+1)\left[a|u+v|^{r-1}(u+v) + b|u|^{\frac{r-3}{2}} u |v|^{\frac{r+1}{2}}\right], \\ f_2(u, v) = (r+1)\left[a|u+v|^{r-1}(u+v) + b|v|^{\frac{r-3}{2}} v |u|^{\frac{r+1}{2}}\right], \\ uf_1(u, v) + vf_2(u, v) = (r+1)F(u, v) \ \forall (u, v) \in R^2, \end{array} \tag{4}$$

where $a, b > 0$ are constants and $F(u, v) = a|u+v|^{r+1} + 2b|uv|^{\frac{r+1}{2}}$.

$(\mathbf{A2})$ The kernel functions $\omega_i \in C^1$, $(i = 1, 2)$ and satisfy, for $s \geq 0$

$$\omega_i(s) \geq 0, \ \ \omega_i'(s) \leq 0, \ \ \int_0^\infty \omega_i(s) ds < 1. \tag{5}$$

Along this work, we use the following notations:

$$\begin{array}{c} 0 < \rho_i = 1 - \int_0^\infty \omega_i(\tau) d\tau < 1, (i = 1, 2), \\ (\omega_i \diamond \vartheta)(t) = \int_0^t \omega_i(t-\tau) \|\vartheta(t) - \vartheta(\tau)\|_2^2 d\tau, \ (i = 1, 2), \\ \rho = \min\{\rho_1, \rho_2\}. \end{array}$$

Now, we present the definition of a weak solution to problem (1).

Definition 1. *Under the stated assumptions, a pair of functions (u, v) is said to be weak solution of (1) on the interval $[0, T]$ provided, if*

(i) $u, v \in C_w\left([0, T], H_0^2(\Omega)\right)$,

(ii) $u_t, v_t \in C_w\left([0, T], L^2(\Omega)\right)$,

(iii) $(u(0),v(0)) = \left(u^0,v^0\right) \in H_0^2(\Omega)\times H_0^2(\Omega)$,
(iv) $(u_t(0),v_t(0)) = \left(u^1,v^1\right) \in L^2(\Omega)\times L^2(\Omega)$;
(v) *and* (u,v) *satisfies*

$$\begin{aligned}
&\left\langle u'(t),\varphi\right\rangle_{L^2(\Omega)} - \left\langle u^1,\varphi\right\rangle_{L^2(\Omega)} + \int_0^t \langle \Delta u(s),\Delta\varphi\rangle_{L^2(\Omega)}\,ds - \int_0^t \left\langle \int_0^s \omega_1(s-\tau)\Delta u(\tau)d\tau,\Delta\varphi\right\rangle_{L^2(\Omega)} ds \\
&+\int_0^t \left\langle \left(|u|^k+|v|^l\right)\left|u'(s)\right|^{p-1}u'(s),\varphi\right\rangle_{L^2(\Omega)} ds \\
=\;& \int_0^t \langle f_1(u(s),v(s)),\varphi\rangle_{L^2(\Omega)}\,ds, \qquad (6)
\end{aligned}$$

$$\begin{aligned}
&\left\langle v'(t),\vartheta\right\rangle_{L^2(\Omega)} - \left\langle v^1,\vartheta\right\rangle_{L^2(\Omega)} + \int_0^t \langle \Delta v(s),\Delta\vartheta\rangle_{L^2(\Omega)}\,ds - \int_0^t \left\langle \int_0^s \omega_2(s-\tau)\Delta v(\tau)d\tau,\Delta\vartheta\right\rangle_{L^2(\Omega)} ds \\
&+\int_0^t \left\langle \left(|v|^\theta+|u|^\varrho\right)\left|v'(s)\right|^{q-1}v'(s),\vartheta\right\rangle_{L^2(\Omega)} ds \\
=\;& \int_0^t \langle f_2(u(s),v(s)),\vartheta\rangle_{L^2(\Omega)}\,ds, \qquad (7)
\end{aligned}$$

for all test functions $\varphi,\vartheta\in H_0^2(\Omega)$ *and for almost all* $t\in[0,T]$ *.*

Theorem 1. *(Local Weak Solutions). Assume (A1)-(A2) hold. Then, there exists a unique local weak solution* (u,v) *to (*1*) defined on interval* $[0,T_0]$ *for some* $T_0>0$*. Also, the said solution satisfies the energy identity*

$$\begin{aligned}
&E(t)+\frac{1}{2}\int_0^t\left[\omega_1(s)\left\|\Delta u(s)\right\|^2+\omega_2(s)\left\|\Delta v(s)\right\|^2\right]ds-\frac{1}{2}\int_0^t\left[(\omega_1'\diamond\Delta u)(s)+(\omega_2'\diamond\Delta v)(s)\right]ds \\
&+\int_0^t\int_\Omega\left(|u(s)|^k+|v(s)|^l\right)\left|u'(s)\right|^{p+1}dxds+\int_0^t\int_\Omega\left(|v(s)|^\theta+|u(s)|^\varrho\right)\left|v'(s)\right|^{q+1}dxds \\
=\;& E(0), \qquad (8)
\end{aligned}$$

where

$$\begin{aligned}
E(t) \;=\;& \frac{1}{2}\left(\|u_t\|^2+\|v_t\|^2\right)+\frac{1}{2}\left[(1-\int_0^t\omega_1(s)ds)\left\|\Delta u(t)\right\|^2+(1-\int_0^t\omega_2(s)ds)\left\|\Delta v(t)\right\|^2\right] \qquad (9)\\
&+\frac{1}{2}\left[(\omega_1\diamond\Delta u)(t)+(\omega_2\diamond\Delta v)(t)\right]-\int_\Omega F(u(t),v(t))\,dx.
\end{aligned}$$

Proof: We can prove the existence and uniqueness of solutions as well as the regularity through Faedo-Galerkin method (c.f. [3]-[8]- [9]). □

3 Global Solution

Theorem 2. *(Global Weak Solutions). Let (A1), (A2) hold. Assume further that* $n = 1, 2$*; and* $r \leq \min\{k+p, l+p, \theta+q, \varrho+q\}$ *. Then, the local weak solution* (u, v) *given by Theorem 2 is a global solution and* T_0 *may be taken arbitrarily large.*

Proof: Suppose that (u, v) be a weak solution to the problem (1) defined on interval $[0, T]$ as given by Theorem 2. Let $n = 1, 2$,

$$r \leq \min\{k+p, l+p, \theta+q, \varrho+q\}$$

and set

$$E_0(t) = \frac{1}{2}\left(\left\|u'(t)\right\|_2^2 + \left\|v'(t)\right\|_2^2 + \rho\left(\|\Delta u(t)\|_2^2 + \|\Delta v(t)\|_2^2\right)\right)$$

and

$$E_1(t) = E_0(t) + \int_\Omega F(u(t), v(t))\, dx.$$

For all $t \in [0, T]$, we will show that the following inequality holds:

$$\begin{aligned} & E_1(t) + \int_0^t\int_\Omega \left(|u(s)|^k + |v(s)|^l\right)\left|u'(s)\right|^{p+1} dxds + \int_0^t\int_\Omega \left(|v(s)|^\theta + |u(s)|^\varrho\right)\left|v'(s)\right|^{q+1} dxds \\ \leq\ & C_T\left(\left|u^0\right|_{2,\Omega}, \left|v^0\right|_{2,\Omega}, \left|u^1\right|_{0,\Omega}, \left|v^1\right|_{0,\Omega}\right), \end{aligned} \tag{10}$$

where $T > 0$ is arbitrary. By recalling (3) and using Poincare's inequality, we have

$$\begin{aligned} c_0\left(\|u(t)\|_{r+1}^{r+1} + \|v(t)\|_{r+1}^{r+1}\right) &\leq \int_\Omega F(u(t), v(t))\, dx \leq c_1\left(\|u(t)\|_{r+1}^{r+1} + \|v(t)\|_{r+1}^{r+1}\right) \\ &\leq C\left(\|\Delta u(t)\|_2^{r+1} + \|\Delta v(t)\|_2^{r+1}\right). \end{aligned} \tag{11}$$

Thus,

$$E_0(t) \leq E_1(t) \leq C\left(E_0(t) + E_0(t)^{\frac{r+1}{2}}\right), \tag{12}$$

and

$$\|u(t)\|_{r+1}^{r+1} + \|v(t)\|_{r+1}^{r+1} \leq CE_1(t). \tag{13}$$

Moreover, by using (8), we have

$$\begin{aligned} & E_0(t) + \frac{1}{2}\int_0^t \left[\omega_1(s)\|\Delta u(s)\|^2 + \omega_2(s)\|\Delta v(s)\|^2\right] ds - \frac{1}{2}\int_0^t \left[(\omega_1' \diamond \Delta u)(s) + (\omega_2' \diamond \Delta v)(s)\right] ds \\ & + \int_0^t\int_\Omega \left(|u(s)|^k + |v(s)|^l\right)\left|u'(s)\right|^{p+1} dxds + \int_0^t\int_\Omega \left(|v(s)|^\theta + |u(s)|^\varrho\right)\left|v'(s)\right|^{q+1} dxds \\ =\ & E_0(0) + \int_0^t\int_\Omega \frac{\partial}{\partial s} F(u(s), v(s))\, dxds. \end{aligned} \tag{14}$$

After adding the term

$$\int_0^t\int_\Omega \frac{\partial}{\partial s} F(u(s), v(s))\, dxds = \int_\Omega F(u(t), v(t))\, dx - \int_\Omega F\left(u^0, v^0\right) dx$$

to both sides of (14), we get

$$\begin{aligned}
& E_1(t) + \frac{1}{2}\int_0^t \left[\omega_1(s)\left\|\Delta u(s)\right\|^2 + \omega_2(s)\left\|\Delta v(s)\right\|^2\right] ds - \frac{1}{2}\int_0^t \left[(\omega_1' \diamond \Delta u)(s) + (\omega_2' \diamond \Delta v)(s)\right] ds \\
& + \int_0^t\int_\Omega \left(\left|u(s)\right|^k + \left|v(s)\right|^l\right)\left|u'(s)\right|^{p+1} dxds + \int_0^t\int_\Omega \left(\left|v(s)\right|^\theta + \left|u(s)\right|^\varrho\right)\left|v'(s)\right|^{q+1} dxds \\
= \; & E_1(0) + 2\int_0^t\int_\Omega \frac{\partial}{\partial s}F\left(u(s), v(s)\right) dxds. \qquad (15)
\end{aligned}$$

We assume that $W_t = \Omega \times (0, t)$. Our aim is to estimate the last term in (15) therefore we set

$$\begin{aligned}
W_{11} &= \left\{(x,s) \in W_t : \left|u(x,s)\right| \le 1, \left|v(x,s)\right| \le 1\right\}, \\
W_{12} &= \left\{(x,s) \in W_t : \left|u(x,s)\right| \le 1, \left|v(x,s)\right| > 1\right\}, \\
W_{21} &= \left\{(x,s) \in W_t : \left|u(x,s)\right| > 1, \left|v(x,s)\right| \le 1\right\}, \\
W_{22} &= \left\{(x,s) \in W_t : \left|u(x,s)\right| > 1, \left|v(x,s)\right| > 1\right\}. \qquad (16)
\end{aligned}$$

Firstly, we notice that

$$\begin{aligned}
2\int_{W_t}\left|\frac{\partial}{\partial s}F\left(u(s), v(s)\right)\right| dxds &\le 2\int_{W_t}\left(\left|f_1(u,v)\right|\left|u'\right| + \left|f_2(u,v)\right|\left|v'\right|\right) dxds \\
&\le G(t) + H(t), \qquad (17)
\end{aligned}$$

where

$$\begin{aligned}
G(t) &= C\int_{W_t}\left(|u|^r + |v|^r + |u|^{\frac{r-1}{2}}|v|^{\frac{r+1}{2}}\right)\left|u'\right| dxds \\
H(t) &= C\int_{W_t}\left(|u|^r + |v|^r + |v|^{\frac{r-1}{2}}|u|^{\frac{r+1}{2}}\right)\left|v'\right| dxds. \qquad (18)
\end{aligned}$$

We write $G(t) = G_{11} + G_{12} + G_{21} + G_{22}$ and $H(t) = H_{11} + H_{12} + H_{21} + H_{22}$ to estimate $G(t)$ and $H(t)$ where

$$\begin{aligned}
G_{ij}(t) &= C\int_{W_{ij}}\left(|u|^r + |v|^r + |u|^{\frac{r-1}{2}}|v|^{\frac{r+1}{2}}\right)\left|u'\right| dW_{ij} \\
H_{ij}(t) &= C\int_{W_{ij}}\left(|u|^r + |v|^r + |v|^{\frac{r-1}{2}}|u|^{\frac{r+1}{2}}\right)\left|v'\right| dW_{ij}, \;\; i,j = 1,2. \qquad (19)
\end{aligned}$$

Firstly, we get

$$\begin{aligned}
G_{11}(t) &\le C\int_{W_{11}}\left|u'(s)\right| dW_{11} \le \delta\left|W_t\right| + C_\delta\int_{W_{11}}\left|u'(s)\right|^2 dW_{11} \\
&\le \delta\left|W_t\right| + C_\delta\int_0^t E_1(s)\, ds, \qquad (20)
\end{aligned}$$

for some $\delta > 0$. In addition, in this paper $|W_t|$ denotes the Lebesgue measure of W_t. Likewise,

$$H_{11}(t) \le \delta\left|W_t\right| + C_\delta\int_0^t E_1(s)\, ds. \qquad (21)$$

We have

$$G_{22}(t) \leq C \int_{W_{22}} \left(|u(s)|^r + |v(s)|^r + |u(s)|^{\frac{r-1}{2}} |v(s)|^{\frac{r+1}{2}} \right) |u'(s)| \, dW_{22}. \tag{22}$$

Since $u, v > 1$ on W_{22}, the first two terms in $G_{22}(t)$ are estimated in the same way. With $m = \frac{r-p}{p+1}$ and noting that $r - m = \frac{p(r+1)}{p+1}$, then we have

$$\int_{W_{22}} |v(s)|^r |u'(s)| \, dW_{22} \leq \left(\int_{W_{22}} |v(s)|^{r+1} dW_{22} \right)^{\frac{p}{p+1}} \left(\int_{W_{22}} |v(s)|^l |u'(s)|^{p+1} dW_{22} \right)^{\frac{1}{p+1}}, \tag{23}$$

where by using Hölder's inequality and assumption $r \leq l + p$ and $|v| > 1$ on W_{22} in (23).Then, by using Young's inequality, we get

$$C \int_{W_{22}} |v(s)|^r |u'(s)| \, dW_{22} \leq \epsilon \int_{W_t} |v(s)|^l |u'(s)|^{p+1} dxds + C_\epsilon \int_0^t E_1(s) \, ds, \tag{24}$$

where $\epsilon > 0$ that will be chosen later. Likewise, we obtain

$$C \int_{W_{22}} |u(s)|^r |u'(s)| \, dW_{22} \leq \epsilon \int_{W_t} |u(s)|^k |u'(s)|^{p+1} dxds + C_\epsilon \int_0^t E_1(s) \, ds. \tag{25}$$

We use (24)-(25) to estimate the last term in (22) as follows:

$$\begin{aligned} \int_{W_{22}} |u(s)|^{\frac{r-1}{2}} |v(s)|^{\frac{r+1}{2}} |u'(s)| \, dW_{22} \quad \leq \quad & \epsilon \int_{W_t} |u(s)|^k |u'(s)|^{p+1} dxds \\ & + \epsilon \int_{W_t} |v(s)|^l |u'(s)|^{p+1} dxds + 2C_\epsilon \int_0^t E_1(s) \, ds. \end{aligned} \tag{26}$$

Therefore, it follows from (24)-(26) that

$$G_{22}(t) \leq 2\epsilon \int_{W_t} |u(s)|^k |u'(s)|^{p+1} dxds + 2\epsilon \int_{W_t} |v(s)|^l |u'(s)|^{p+1} dxds + 4C_\epsilon \int_0^t E_1(s) \, ds. \tag{27}$$

Likewise, we get

$$H_{22}(t) \leq 2\epsilon \int_{W_t} |v(s)|^\theta |v'(s)|^{q+1} dxds + 2\epsilon \int_{W_t} |u(s)|^\varrho |v'(s)|^{q+1} dxds + 4C_\epsilon \int_0^t E_1(s) \, ds. \tag{28}$$

Now, we estimate G_{ij} for $i \neq j$. By noting that $u \leq 1$ and $v > 1$ on W_{12}, we have

$$G_{12}(t) \leq \delta |W_t| + C_\delta \int_0^t E_1(s) \, ds + C \int_{W_{12}} |v(s)|^r |u'(s)| \, dW_{12}. \tag{29}$$

Again, since $v > 1$ on W_{12}, then by recalling (24), we have

$$G_{12}(t) \leq \delta |W_t| + (C_\delta + C_\epsilon) \int_0^t E_1(s) \, ds + \epsilon \int_{W_t} |v(s)|^l |u'(s)|^{p+1} dxds. \tag{30}$$

We estimate H_{21} by repeating the same steps in (29)-(30) with switching u and v, using $m = \frac{r-p}{p+1}$, and using the fact that $r \leq \theta + p$ and $|u| > 1$ on W_{21}. One easily has

$$H_{21}(t) \leq \delta |W_t| + (C_\delta + C_\epsilon) \int_0^t E_1(s) \, ds + \epsilon \int_{W_t} |v(s)|^\theta |v'(s)|^{q+1} dxds. \tag{31}$$

The estimates for G_{21} and H_{12} are similar and they are omitted. Indeed, we have

$$\begin{aligned} G_{21}(t) &\leq \delta|W_t| + (C_\delta + C_\epsilon)\int_0^t E_1(s)\,ds + \epsilon\int_{W_t}|u(s)|^k\left|u'(s)\right|^{p+1}dxds, \\ H_{12}(t) &\leq \delta|W_t| + (C_\delta + C_\epsilon)\int_0^t E_1(s)\,ds + \epsilon\int_{W_t}|u(s)|^\varrho\left|v'(s)\right|^{q+1}dxds. \end{aligned} \tag{32}$$

By combining (18)-(21), (27)-(28), and (30)-(32), we obtain

$$\begin{aligned} G(t) + H(t) &\leq 6\delta|W_t| + C_{\delta,\epsilon}\int_0^t E_1(s)\,ds + 3\epsilon\int_{W_t}\left(|u(s)|^k + |v(s)|^l\right)\left|u'(s)\right|^{p+1}dxds \\ &\quad + 3\epsilon\int_{W_t}\left(|v(s)|^\theta + |u(s)|^\varrho\right)\left|v'(s)\right|^{q+1}dxds. \end{aligned} \tag{33}$$

By choosing $\epsilon > 0$ small enough and from (15), (16) and (33), we have

$$\begin{aligned} & E_1(t) + \frac{1}{2}\int_0^t\left[\omega_1(s)\|\Delta u(s)\|^2 + \omega_2(s)\|\Delta v(s)\|^2\right]ds - \frac{1}{2}\int_0^t\left[(\omega_1'\diamond\Delta u)(s) + (\omega_2'\diamond\Delta v)(s)\right]ds \\ & + c_\epsilon\int_0^t\int_\Omega\left(|u(s)|^k + |v(s)|^l\right)\left|u'(s)\right|^{p+1}dxds + c_\epsilon\int_0^t\int_\Omega\left(|v(s)|^\theta + |u(s)|^\varrho\right)\left|v'(s)\right|^{q+1}dxds \\ \leq\ & E_1(0) + 6\delta|W_t| + C_{\delta,\epsilon}\int_0^t E_1(s)\,ds, \end{aligned} \tag{34}$$

for some constant $c_\epsilon > 0$. By applying Gronwall's inequality, $E_1(t) \leq (E_1(0) + 6\delta|W_t|)\,e^{Ct}$, where positive constant C. Lastly, (34) yields to the following inequality

$$\begin{aligned} & E_1(t) + \frac{1}{2}\int_0^t\left[\omega_1(s)\|\Delta u(s)\|^2 + \omega_2(s)\|\Delta v(s)\|^2\right]ds - \frac{1}{2}\int_0^t\left[(\omega_1'\diamond\Delta u)(s) + (\omega_2'\diamond\Delta v)(s)\right]ds \\ & + \int_0^t\int_\Omega\left(|u(s)|^k + |v(s)|^l\right)\left|u'(s)\right|^{p+1}dxds + \int_0^t\int_\Omega\left(|v(s)|^\theta + |u(s)|^\varrho\right)\left|v'(s)\right|^{q+1}dxds \\ \leq\ & C_T\left(E_1(0) + 6\delta|W_t|\right), \end{aligned} \tag{35}$$

where (35) is valid for all $0 < t \leq T$, where T is being arbitrary. Hence, the proof is complete. □

The second result establishes an answer to the global existence of weak solutions without the condition $r \leq \min\{k+p, l+p, \theta+q, \varrho+q\}$.

Theorem 3. *(Global Small Solutions). Let (A1), (A2) hold. Assume that* $\Gamma(0) > 0$ *and* $\frac{4^{\frac{r}{2}}c_0^2}{\rho}\left(\frac{E(0)}{\rho}\right)^{\frac{r-1}{2}} < 1$, *where* c_0 *is a positive constant that depends on* r *and* Ω. *Then, the said solution* (u,v) *in Theorem 2 is a global solution and* T_0 *may be taken arbitrarily large.*

Proof: The proof of Theorem 4 will be conclude after we prove the following lemma. For the simplicity, we reminding

$$\begin{aligned} \Gamma(t) &= (1 - \int_0^t\omega_1(s)ds)\|\Delta u(t)\|^2 + (1 - \int_0^t\omega_2(s)ds)\|\Delta v(t)\|^2 \\ &\quad + 2\left[(\omega_1\diamond\Delta u)(t) + (\omega_2\diamond\Delta v)(t)\right] - 4\int_\Omega F(u(t), v(t))\,dx. \end{aligned}$$

□

Lemma 1. *Let (u, v) be the solution to the problem (1) defined on $[0, T]$ established in Theorem 2. Assume further that $\Gamma(0) > 0$ and $\frac{4^{\frac{r}{2}} c_0^2}{\rho}\left(\frac{E(0)}{\rho}\right)^{\frac{r-1}{2}} < 1$, where $c_0 > 0$ is a constant. Then, $\Gamma(t) > 0$ on $[0, T]$ and for all $t \in [0, T]$, the following inequality holds:*

$$
\begin{aligned}
& \frac{1}{4}\left(\left\|u'(t)\right\|^2 + \left\|v'(t)\right\|^2\right) + \frac{\rho}{4}\left(\|\Delta u(t)\|^2 + \|\Delta v(t)\|^2\right) + \int_\Omega F(u(t), v(t))\, dx \\
& + \int_0^t \int_\Omega \left(|u(s)|^k + |v(s)|^l\right) \left|u'(s)\right|^{p+1} dxds + \int_0^t \int_\Omega \left(|v(s)|^\theta + |u(s)|^\varrho\right) \left|v'(s)\right|^{q+1} dxds \\
\leq\ & 2E(0), \qquad (36)
\end{aligned}
$$

where $T > 0$ is arbitrary.

Proof: Similar as the proof in (Theorem 1.6, [9] and Theorem 2.3, [8]) we can prove the lemma. □

4 Conclusion

As far as we know, there is not any global existence results in the literature known for plate viscoelastic equation with degenerate damping terms. Our work extends the works for some viscoelastic plate wave equations treated in the literature to the plate viscoelastic wave equation with degenerate damping terms.

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