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Design of Control Charts for Number of Defects Based on Pythagorean Fuzzy Sets

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Abstract:
Control charts (CCs) are completely useful techniques to monitor the process' stability. If the process is stable and statistically under control, it only displays a variation that is inherent to the process. Thus, usage of them is completely critical for the process' improvements. Two types of Attribute Control Charts (ACCs) are related to the total count of defects that are the designs of the non–measurable characteristics and they are called u and c control charts. If the process related to evaluations about defects includes uncertainty, the traditional ACCs are insufficient to reflect the available data in design stages. The fuzzy set theory (FST) is a tool for representing uncertainty by assigning membership functions. When it comes to decisions made by quality engineers on the basis of their intuition, ordinary fuzzy sets cannot adequately represent the data. In such data, using Pythagorean fuzzy sets (PFSs), an extension of fuzzy sets, is a convenient way to represent not only the uncertainty in the data, but also the hesitations of the quality engineers. Through that, in this paper, the design of control charts for the number of defects based on PFSs has been proposed. An extension of u and c control charts based on PFSs are constructed and the design of these control charts has been detailed. Moreover, a descriptive example is introduced to check the applicability of the proposed method.

Keywords: c control chart, Fuzzy logic, Pythagorean fuzzy sets, u control chart.

1 Introduction

In a competitive market, quality of the products is one of the most important criteria, since it is vital in any industry to reach the most desirable product level. By implementing statistical process control in the production plants, managers are able to improve production quality by decreasing variations based on the unnatural events in the specifications. CCs are useful techniques to monitor these processes' stability and help to improve their efficiency. If the process is stable and statistically under control, it only displays a variation that is inherent to the process. Therefore, while analyzing the observed system, it is crucial that the available data represent the system in the best possible way.

Two types of CCs are related to the total count of defects that are the designs of the non-measurable characteristics and called u and c control charts. The design of these CCs is based on the number of defects in a sample or a product. If the process is monitored based on the number of defects in a sample, c control chart is appropriate to use. In the same way, if the process monitored based on the number of defects per unit then u control chart can be used. Classical rules of statistical control indicate that a process is out of control when one of its samples is outside of the control limits of the chart. In case of situations where the data are uncertain and there is intuitive judgments of the decision makers or experts, the ordinary CCs cannot mathematically represent this environment of uncertainty. For instance, the operators or the observers of the system can be hesitant while measuring these values during the data gathering process. Therefore, new adaptive charts are needed to represent both uncertainty and the hesitancy.

FST is a tool of representing uncertainty by assigning membership functions, which indicates the belonging an element to a fixed set [1]. They are used to extend u and c control charts in many studies to increase the models' data representation and interpretation. Ercan-Teksen and Anagun [2] proposed fuzzy c CCs using ranking methods and interval-valued type-2 trapezoidal fuzzy numbers. Senturk [3] developed the fuzzy c CCs by using triangular fuzzy numbers and applied this chart in a facility producing armature. Shu et al. [4] developed fuzzy u CCs employing triangular fuzzy numbers and ranking methods and they applied this chart in the welding quality process of a company producing a stainless steel table. Aslangiray [5] developed a fuzzy u CC and compared it with the classical u CC for a fabric quality process in an enterprise. Fadaei and Pooya [6] developed fuzzy u CC for variable sample size using triangular fuzzy numbers and they applied this CC for the quality of sewing in a company that produces seat belts. Studies made by [7, 8] present the effects of considering control limits as value ranges with probability distributions associated, achieving better results in comparison to traditional CCs.

Compared to studies on fuzzy CCs in the literature, the use of PFSs in integration with CCs can increase the sensitivity of these charts since PFS allows decision makers to give more precise judgments by providing them with a wider domain to define membership functions [10]. Although u and c control charts have been analyzed based on the fuzzy set theory before, it is the first time in this study that these control charts are derivate by using PFSs that is one of fuzzy extensions and the formulas for center line and control limits are formed according to Pythagorean triangular fuzzy numbers. In this study, we analyzed ACCs based on PFSs to better reflect the opinions of decision makers on the data. It is a powerful way of representing not only uncertainty in the data but also hesitancy of the decision–makers [9]. It is a useful tool to represent attribute information by using scales corresponded with Pythagorean fuzzy numbers (PFNs).

The rest of this paper is organized as follows: In Section 2, ACCs and their application areas are briefly introduced. In Section 3, the proposed methodology and its preliminaries are presented. In Section 4, a descriptive example is conducted to check the applicability of the proposed control chart. The obtained results and future research directions have been discussed into Section 5.

2 Attribute Control Charts

CCs are one of the most used statistical quality control techniques, which enable the controllers can measure the product specifications in the manufacturing process to determine the process is whether or not in the accepted limits [11]. If the measured specifications are outside of the determined limits, CCs notice the system to take necessary precautions to keep the process at the desired level. The CCs can be classified based on the quality characteristics, as measurable on numerical scales and not [12], which can be classified into two groups based on data as “variable” and “attribute”. Two well–known ACCs named u and c control charts are designed to measure the defectives during the manufacturing stages. If the process is deal with the number of defects in a sample, then c control chart is used. Similarly, if the process deals with number of defects per unit, then u control chart is used.

Let m is the number of samples with size n_i. In each sample, D_i represents the total number of non–conformities [13]. Then, the i^{th} sample’s non–conformities u_i is calculated by using Eq. (1).

$$u_i = \frac{D_i}{n_i}. \tag{1}$$

The limits in u control charts are calculated by using Eqs. (2–4) as follows:

$$\bar{u} = \frac{\sum_{i=1}^{k} D_i}{\sum_{i=1}^{k} n_i}, \tag{2}$$

$$UCL = \bar{u} + k\sqrt{\frac{\bar{u}}{n_i}}, \tag{3}$$

$$LCL = \bar{u} - k\sqrt{\frac{\bar{u}}{n_i}} \tag{4}$$

where k is a multiplier chosen to control the likelihood of false alarms.

In the c control charts procedure, similar to the u control charts, let m is the sample size and D_i represents the number of non–conformities of the i^{th} unit [13]. Then, the center line is calculated as in Eq. (5) as follows:

$$\bar{c} = \frac{\sum_{i=1}^{m} D_i}{m}. \tag{5}$$

Moreover, by utilizing (5), the upper and the lower control limits are calculated by using Eq. (6–7).

$$UCL = \bar{c} + k\sqrt{\bar{c}}, \tag{6}$$

$$LCL = \bar{c} - k\sqrt{\bar{c}}, \tag{7}$$

where k is a multiplier chosen to control the likelihood of false alarms.

Based on the SCOPUS database, there are 741 studies, related with the attribute control charts by using the following search pattern:

TITLE-ABS-KEY (*attribute* AND *control* AND *charts*)

Among them, 32 articles related with attribute control charts are reviewed in terms of their application fields. In Figure 1, the application areas are presented.

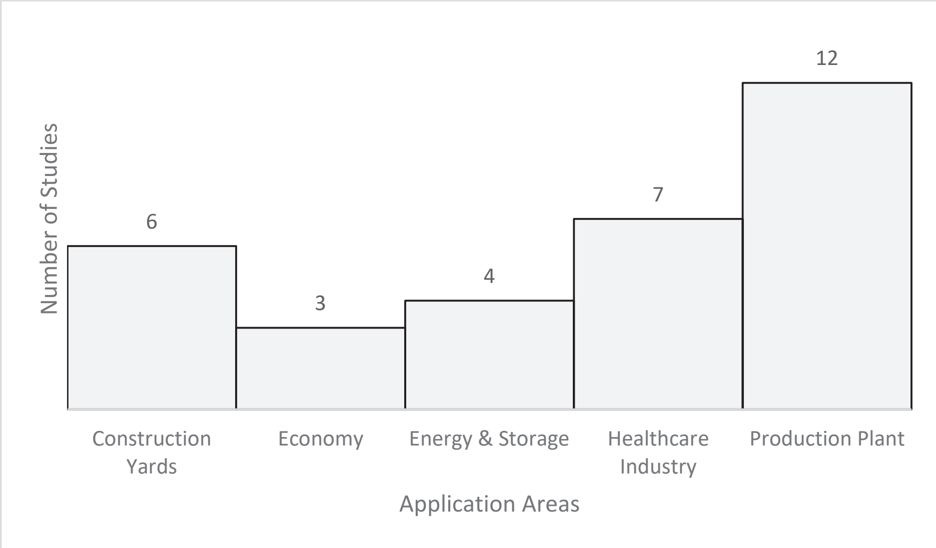

Fig. 1: Application areas of the attribute control charts

As shown in Figure 1, ACCs are widely used in the fields of construction yards, economy, energy & storage, healthcare industry, production plants etc. Among them, in the production plants, ACCs are the one of the most used techniques for the inspection of the system during the production stages for the observation.

In the case that ordinary CCs are incapable of representing uncertainty, they are extended with FST to handle it. FST is a tool, which enables to the representation of uncertainty by assigning membership function to an element by indicating the level of belongingness to a set [1]. In the literature, classical fuzzy sets are used to extend u an c control charts in many studies to increase the models' data representation and interpretation such as [14]-[7]-[15]-[16]- [17]-[18]-[6]-[19]. Since the FST are incapable of representing hesitancy, the models are not practical to include hesitancy during the calculations. In this paper, unlike the literature, the u and c control charts are extended with PFSs to increase their ability to reflect the available data based on the linguistic terms under uncertain environments.

3 Proposed Methodology

In this section, preliminaries for PFSs and details of the proposed methodology are presented.

3.1 Preliminaries: Pythagorean Fuzzy Sets

Yager introduces PFSs to express decision maker's opinions in a larger domain area than the ordinary fuzzy sets by using not only membership functions but also non-membership functions of the elements in a fuzzy set [9]. It can be in many forms such as single valued, interval–valued, triangular, and trapezoidal. During the study, single valued forms are used for the descriptive example. Since the PFSs are the extension of intuitionistic fuzzy sets, they also provide a theoretical basis for managing the knowledge of hesitation by people when evaluating the questions [20]. The mathematical representation of an single valued PFS is given in Eq. (8).

$$\tilde{A} = \left\{x, \left(\mu_{\tilde{A}}, v_{\tilde{A}}\right); x \in X\right\} \tag{8}$$

where $0 \leq \mu_{\tilde{A}}^2 + v_{\tilde{A}}^2 \leq 1$ for every $x \in X$.

By assigning two functions, PFSs enable to represent the hesitancy of the decision makers. The mathematical formula of the hesitancy is given in Eq. (9).

$$\pi = \sqrt{1 - \left(\mu_{\tilde{A}}^2 + v_{\tilde{A}}^2\right)} \tag{9}$$

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When PFSs and intuitionistic fuzzy sets (IFS) are compared, Figure 2 visually represents the domain area.

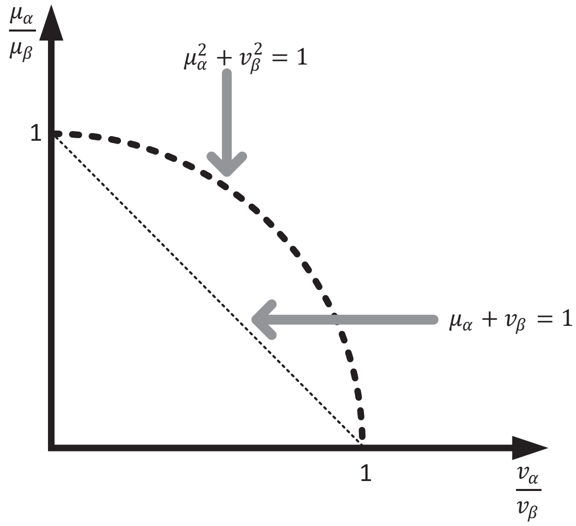

Fig. 2: Visual comparison of the PFS and IFS

Let $\tilde{A} \cong \langle \mu_{\tilde{A}}, \vartheta_{\tilde{A}} \rangle$ and $\tilde{B} \cong \langle \mu_{\tilde{B}}, \vartheta_{\tilde{B}} \rangle$ be PFNs, and $\lambda > 0$. The arithmetical operations of these PFNs are defined in Eq. (10–13) [9].

$$\tilde{A} \oplus \tilde{B} \cong \left\langle \sqrt{\mu_{\tilde{A}}^2 + \mu_{\tilde{B}}^2 - \mu_{\tilde{A}}^2 \mu_{\tilde{B}}^2}, \vartheta_{\tilde{A}} \vartheta_{\tilde{B}} \right\rangle, \tag{10}$$

$$\tilde{A} \otimes \tilde{B} \cong \left\langle \mu_{\tilde{A}} \mu_{\tilde{B}}, \sqrt{\vartheta_{\tilde{A}}^2 + \vartheta_{\tilde{B}}^2 - \vartheta_{\tilde{A}}^2 \vartheta_{\tilde{B}}^2} \right\rangle \tag{11}$$

$$\lambda \tilde{A} = \left(\sqrt{1 - (1 - \mu^2)^\lambda}, \vartheta^\lambda \right) \tag{12}$$

$$\tilde{A}^\lambda = \left(\mu^\lambda, \sqrt{1 - (1 - \vartheta^2)^\lambda} \right) \tag{13}$$

3.2 *Pythagorean fuzzy u control chart*

The proposed methodology consists of the u and c CCs with their Pythagorean fuzzy extensions. The notations of the Pythagorean fuzzy u control chart are given as follows:

Let $\widetilde{U}$ be the sample, which is checked for the number of defectives. The representation of the $\widetilde{U}$ is given in Eq. (14).

$$\widetilde{U} = \left[(a, b, c), \left(a', b, c' \right), (\mu, v) \right] \tag{14}$$

where a is the lower bound, b is the middle value, c is the upper bound, a' is the lower reference point, c' is the upper reference point, μ is the membership function, and v is the non–membership function.

Then the proposed Pythagorean u CC limits are calculated as in Eqs. (15–17).

$$\begin{gathered} C\tilde{L}_U = \left((\bar{a}, \bar{b}, c,), (\bar{a}', \bar{b}, c',); \min(\mu_i); \max(v_i) \right) = \\ \left(\left(\frac{\sum_{i=1}^m a_i}{m}, \frac{\sum_{i=1}^m b_i}{m}, \frac{\sum_{i=1}^m c_i}{m} \right), \left(\frac{\sum_{i=1}^m a_i'}{m}, \frac{\sum_{i=1}^m b_i}{m}, \frac{\sum_{i=1}^m c_i'}{m} \right); \min(\mu_i); \max(v_i) \right) \end{gathered} \tag{15}$$

$$U C\tilde{L}_U = \left((\bar{a} + 3\sqrt{\frac{\bar{a}}{n}}, \bar{b} + 3\sqrt{\frac{\bar{b}}{n}}, \bar{c} + 3\sqrt{\frac{\bar{c}}{n}}), \left(\bar{a}' + 3\sqrt{\frac{\bar{a}'}{n}}, \bar{b} + 3\sqrt{\frac{\bar{b}}{n}}, \bar{c}' + 3\sqrt{\frac{\bar{c}'}{n}} \right); \min(\mu_i); \max(v_i) \right) \tag{16}$$

$$\begin{gathered} L C\tilde{L}_U = ((\max(0, \bar{a} - 3\sqrt{\frac{\bar{c}}{\mathrm{n}}}), \max(0, \bar{b} - 3\sqrt{\frac{\bar{b}}{\mathrm{n}}}), \max(0, \bar{c} - 3\sqrt{\frac{\bar{a}}{\mathrm{n}}})), \\ (\max(0, \bar{a}' - 3\sqrt{\frac{\overline{c'}}{\mathrm{n}}}), \max(0, \bar{b} - 3\sqrt{\frac{\bar{b}}{\mathrm{n}}}), \max(0, \overline{c'} - 3\sqrt{\frac{\bar{a}'}{\mathrm{n}}})); \min(\mu_i); \max(v_i)) \end{gathered} \tag{17}$$

3.3 Pythagorean fuzzy c control chart

The same representations as in Pythagorean fuzzy u CC are transformed to trapezoidal number forms for Pythagorean fuzzy c CC. Then the control limits are calculated by using the Eqs. (18–20) as follows:

$$\tilde{CL_c} = \left(\left(\frac{\sum_{i=1}^{m} a_i}{m}, \frac{\sum_{i=1}^{m} b_i}{m}, \frac{\sum_{i=1}^{m} c_i}{m}\right), \left(\frac{\sum_{i=1}^{m} a_i'}{m}, \frac{\sum_{i=1}^{m} b_i}{m}, \frac{\sum_{i=1}^{m} c_i'}{m}\right); \min\left(\mu_i\right); \max\left(v_i\right)\right) \tag{18}$$

$$U\tilde{C}L_c = \left((\bar{a} + 3\sqrt{\bar{a}}, \bar{b} + 3\sqrt{\bar{b}}, \bar{c} + 3\sqrt{\bar{c}}), \left(\bar{a}' + 3\sqrt{\bar{a}'}, \bar{b} + 3\sqrt{\bar{b}}, \bar{c'} + 3\sqrt{\bar{c'}}\right); \min\left(\mu_i\right); \max\left(v_i\right)\right) \tag{19}$$

$$\begin{aligned} L\tilde{C}L_c = ((\max(0, \bar{a} - 3\sqrt{\bar{c}}), \max(\bar{b} - 3\sqrt{\bar{b}}), \max(\bar{c} - 3\sqrt{\bar{a}})), \\ \max(\left(\bar{a}' - 3\sqrt{\bar{c}'}), \max(\bar{b} - 3\sqrt{\bar{b}}), \max(\bar{c}' - 3\sqrt{\bar{a}'})\right); \min\left(\mu_i\right); \max\left(v_i\right)) \end{aligned} \tag{20}$$

4 A Descriptive Example

An example is created for the proposed Pythagorean fuzzy c attribute chart methodology. During the inspection, an operator assigns the number of non–conformities by using the "Approximately" linguistic terms. Then, the possible range of assigned term is constructed based on the expert judgments by using triangular PFNs and its corresponded membership and non–membership functions.

Let us consider a contract manufacturer for an automobile company who decided to monitor its process with the statistical charts. At the end of the production process, a batch, which consists 30 products is sent to an operator for the quality inspection. The operator, which controls the batch, realizes this process by assigning the below linguistic terms with the determined rules. 21 samples are taken for the analysis. The assigned linguistic terms by the operator with respect to each sample are presented in Table 1.

Sample	Linguistic Term	Corresponded Pythagorean Fuzzy Number
1	Approximately 30	<(25, 30, 35), (20, 30, 40), (0.7, 0.4)>
2	Approximately 56	<(51, 56, 61), (46, 56, 66), (0.7, 0.4)>
3	Approximately 47	<(42, 47, 52), (37, 47, 57), (0.7, 0.4)>
4	Approximately 86	<(81, 86, 91), (76, 86, 96), (0.7, 0.4)>
5	Approximately 44	<(39, 44, 49), (34, 44, 54), (0.7, 0.4)>
6	Approximately 23	<(18, 23, 28), (13, 23, 33), (0.7, 0.4)>
7	Approximately 16	<(11, 16, 21), (6, 16, 26), (0.7, 0.4)>
8	Approximately 64	<(59, 64, 69), (54, 64, 74), (0.7, 0.4)>
9	Approximately 80	<(75, 80, 85), (70, 80, 90), (0.7, 0.4)>
10	Approximately 54	<(49, 54, 59), (44, 54, 64), (0.7, 0.4)>
11	Approximately 73	<(68, 73, 78), (63, 73, 83), (0.7, 0.4)>
12	Approximately 65	<(60, 65, 70), (55, 65, 75), (0.7, 0.4)>
13	Approximately 76	<(71, 76, 81), (66, 76, 86), (0.7, 0.4)>
14	Approximately 69	<(64, 69, 74), (59, 69, 79), (0.7, 0.4)>
15	Approximately 53	<(48, 53, 58), (43, 53, 63), (0.7, 0.4)>
16	Approximately 58	<(53, 58, 63), (48, 58, 68), (0.7, 0.4)>
17	Approximately 30	<(25, 30, 35), (20, 30, 40), (0.7, 0.4)>
18	Approximately 91	<(86, 91, 96), (81, 91, 101), (0.7, 0.4)>
19	Approximately 90	<(85, 90, 95), (80, 90, 100), (0.7, 0.4)>
20	Approximately 36	<(31, 36, 41), (26, 36, 46), (0.7, 0.4)>
21	Approximately 57	<(52, 57, 62), (47, 57, 67), (0.7, 0.4)>

Table 1 Analyze report of the operator after the inspection

By using the Eq. (18) center line is calculated as follows:
$CL =< (52.05, 57.05, 62.05), (47.05, 57.05, 67.05), (0.7, 0.4) >$
By using the center line, upper and lower control limits for Pythagorean fuzzy c attribute chart are obtained as follows:
$UCL =< (73.69, 79.71, 85.68), (67.62, 79.71, 91.61), (0.7, 0.4) >$
$LCL =< (28.42, 34.39, 40.4), (22.48, 34.39, 46.47), (0.7, 0.4) >$
Based on the results, the Pythagorean fuzzy c CC is constructed as in Figure 3 that shows that some of the samples are not inside the control limits.

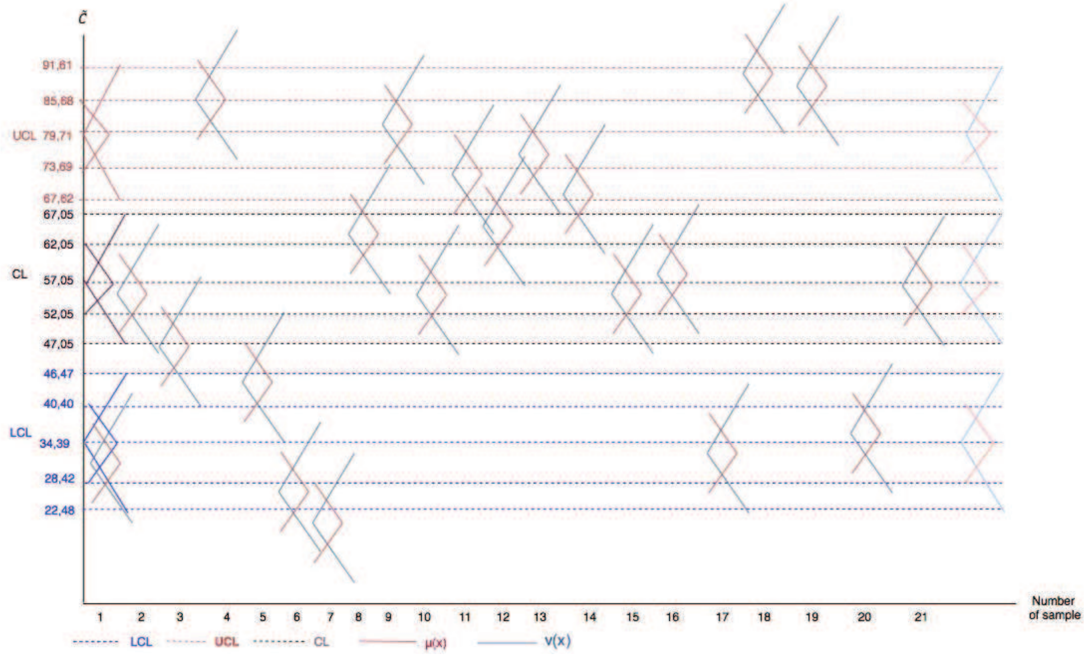

Fig. 3: Constructed Pythagorean fuzzy c control chart

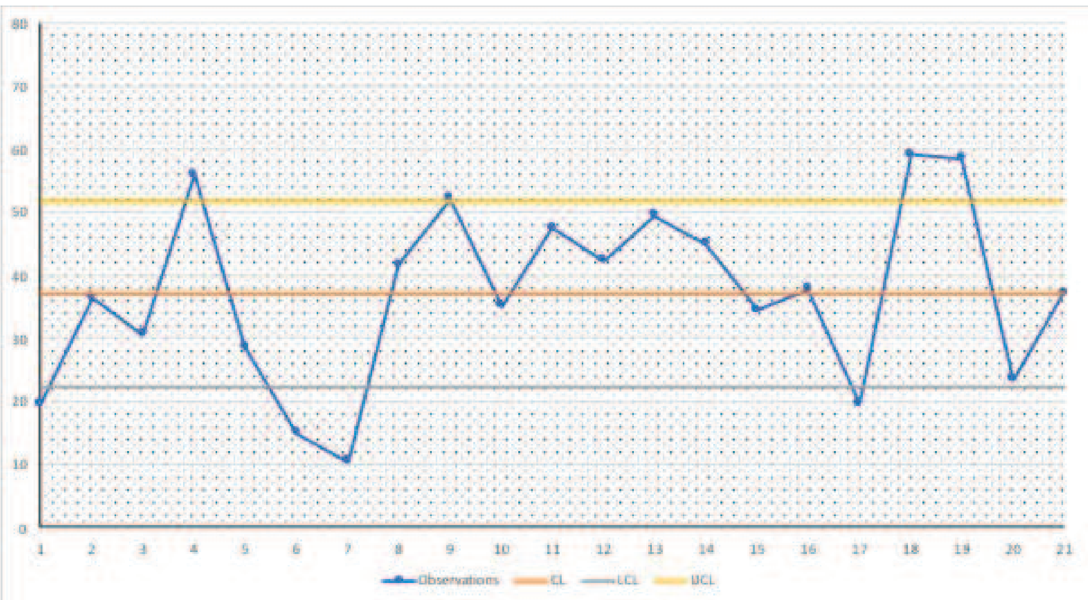

Fig. 4: Constructed c control chart based on defuzzified values

The chart given in Figure 4 is constructed by defuzzifying the obtained values. According to the Figure 4, the process is not under control. Some of the obtained values are located out of the control limits. Both the fuzzy CC obtained using the proposed approach and the CC with the defuzzified values indicate that the process in not under control, but when we further examine these charts, it is possible to say that Figure 4 better represents the sensitivity of the data. For example, although 9^{th} sample is obviously outside the control limits in Figure 4, most of the PFNs is on the upper control limit in Figure 3. Moreover, even though the samples 4^{th}, 18^{th} and 19^{th} are out of control limits in Figure 3, they are not completely outside of the control limits in Figure 4. Therefore, it seems that CCs based on PFSs represent expert judgments more precisely than the CCs with crisp values. This kind of sensitivity in the CCs might be important to analyze and clarify the causes of variations.

5 Conclusions

CCs are completely critical tool to follow variations in a process and to determine whether it is between control limits or not. It is clear that the control procedure includes many uncertainties and the quality characteristics may not be accurately defined. The fuzzy set theory can be successfully used to overcome difficulties of defining and modeling uncertainty in CCs. The extensions of fuzzy sets can improve the ability of CCs by defining uncertainty in different ways. The superiority of PFSs is that they provide experts with a wider range to define membership and non-membership functions, so CCs developed based on PFSs are able to better represent precision of expert judgments. Therefore, in this study, the extensions of u and c control charts based on PFSs are proposed and the design of these control charts has been detailed for the first time. Moreover, the control limits and center lines have been re–formulated based on PFSs. As a result, it is determined that PFSs provide effective results in order to overcome the uncertainty. Based on these findings, it is possible to say that the proposed method presents sensitive outcomes.

For future research, other types of fuzzy sets representing uncertainty in a different way such as IFSs, type-2 fuzzy sets and hesitant fuzzy sets can be used for the comparative purposes. Process capability analysis can be implemented as further study. Moreover, the proposed techniques can be implemented to a real-world problem to further analyze its effectiveness.

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