

Analytical Solution for Stress Distribution in Cementitious Backfills Considering Stope Inclinations

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ABSTRACT

To enhance stope stability and decrease surface waste disposal, fill is used in underground mines. Moreover, the cementitious fill is used as a structural support element for maximizing the extraction ratio of run-of-mine ore in irregularly shaped orebodies. Assessing the stress distribution within the fill and neighboring rock mass is of great importance to the design of backfilled stopes. Inappropriate design of these stopes can cause the backfill failures, thus resulting in the devastation of stope walls, production delays and even losses in ore dilution form, and security matters. Great effort has been made to acquire the stresses within vertical voids analytically, and there is a clear need for works which states the stress developed in inclined backfilled stopes. In this study, an analytical model is presented to explain the non-uniform distribution of stress in backfill on stope cross-section when the friction force between the footwall and hanging wall are not equal. Considering the force balance of the arc differential element, the stress distribution within mine fill is obtained. This model was compared with the existing model. It has been demonstrated analytically that the existing model underestimates the fill stress when the stope inclination becomes larger. Besides, when the surrounding mass produces a large deformation and squeezes backfill, the primary principle stress arch happens. Consequently, the outcomes of the present study will offer the knowledge needed to develop a cost-effective backfill structure in underground mines that are environmentally friendly, safe and durable.

Keywords: Cementitious backfill, Arching, Minor and major principal stress arch, Analytical model

Stope Eğimlerini Dikkate Alan Çimentolu Dolgularda Gerilme Dağılımı İçin Analitik Çözüm

ÖZET

Stope stabilitesini artırmak ve yüzey atık depolayı azaltmak için maden dolgusu yeraltı madenlerde kullanılır. Ayrıca, çimentolu dolgu, eşit olmayan şekilde eğimli cevher yataklarında tüvanan cevherinin çıkarma oranını arttırmak için yapısal destek elemanı olarak kullanılır. Dolgu ve komşu kaya kütleindeki gerilme dağılımının değerlendirilmesi, dolgulu stope'ların tasarımı için büyük önem taşımaktadır. Bu stope'ların uygun olmayan tasarımı, dolgu yenilmelerine neden olabilir, bu da stope duvarlarının tahrip olmasına, üretim gecikmelerine ve hatta cevher seyrelmesi şeklinde kayıplara ve güvenlik sorunlarına yol açar. Şimdiye kadar dikey açıklıklardaki gerilmeleri analitik olarak elde etmek için yoğun çaba sarf edilmiş olup, eğimli dolgulu stope'larda meydana gelen gerilmeleri ifade eden çalışmalara açık bir ihtiyaç vardır. Bu çalışmada, taban ve tavan duvar arasındaki sürtünme kuvveti eşit olmadığında stope kesitindeki dolgudaki gerilmenin muntazam olmayan dağılımını çözmek için analitik bir model sunulmuştur. Kemer diferansiyel elemanın kuvvet dengesi göz önüne alınarak, dolgudaki gerilme dağılımı elde edilir. Bu model, mevcut model ile karşılaştırıldı. Mevcut modelin, stope eğimi büyüdüğünde dolgu gerilmesini olduğundan az hesapladığı analitik olarak gösterilmiştir. Dahası, çevreleyen kütle büyük bir deformasyon yarattığında ve dolguyu sıkıştırdığında, büyük ana gerilme kemeri meydana gelir. Sonuç olarak, mevcut çalışmanın sonuçları yeraltı madenlerinde çevre dostu, güvenli ve dayanıklı olan uygun maliyetli bir dolgu yapısı geliştirmek için gereken teknik bilgiyi sunacaktır.

Anahtar Kelimeler: Çimentolu dolgu, Kemerlenme, Küçük ve büyük ana stres arki, Analitik model

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1. Introduction

Mine fill is broadly employed in underground mines due to its benefits of efficiently disposing of problematic wastes such as tailings and waste rocks (Aubertin et al., 2003), reducing the environmental impact (Helinski et al., 2007), supporting adjacent rock formations (Belem and Benzaazoua, 2008), improving ore recovery (Jiang et al., 2020), preventing ore losses and dilution (Cao et al., 2020); and supporting as a subsidence control (Koohestani et al., 2020). By re-filling mined-out stopes with waste rocks and tailings, it is possible to avoid the safety hazards linked with the surface storage of wastes and the construction of tailings impoundments (Yilmaz et al., 2013). After it is filled into underground voids or stopes, backfilling can fulfill different functions as follows: it can have absolute self-supporting stability, slow down the deformation of surrounding rock, and decrease the bearing capacity of mining equipment (Fall and Nasir, 2010; Xue et al., 2019). Hence, a reasonable strength design of the filling body is crucial for sustainable mining operations (Yilmaz, 2018). To rationally design the required strength of the fill body, it is critical to understand the internal stress development law after backfilling fully is poured into mined-out stopes (Li et al., 2005, 2010; Yan et al., 2020; Yang et al., 2020).

After stope backfilling, the interaction between fill and surrounding rock presents a complex mechanical response thanks to different stope shapes, original rock stress, and underground mine conditions (Singh et al., 2011; Thompson et al., 2012; El Mkadmi et al., 2014). One of the most important aspects considered for studying the stress distribution in the cementitious fill is the arching effect (Pirapakaran and Sivakugan, 2007; Ting et al., 2011). The main reason for the arch effect is the friction between the filling and the neighboring rock (Sivakugan et al., 2014). The result is that the self-weight of filling may transfer to the surrounding rock mass, making

the stress value at the filling floor significantly lower than the self-weight stress (Falaknaz et al., 2015; Ting et al., 2012). Thus, if the arching effect is not considered, it is then possible to design excessive fill strength, hence increasing the cost of backfill (Widisinghe and Sivakugan, 2015). Besides, the design of fill walls needs to consider the backfill-induced stress, and when considering the arching effect of fill, the need for holding the walls can be decreased mainly due to a drop in the stress value compared to the self-weight stress, hence reducing the cost of filling (Li et al., 2010; Sobhi et al., 2017).

Many researchers have carried out a theoretical analysis on stress distribution within fill under vertical/inclined stope conditions and obtained the vertical stress prediction models of different forms (Brachman and Krushelnitzky, 2005; Li and Aubertin, 2010; Jahanbakshzadeh et al., 2017). Ting et al. (2014) simplified filling stope to a plane strain model and gained a theoretical model of the stress distribution in the fill matrix with different wall inclination. They assumed that the inclination of footwall and hanging wall were equal and had the same shear force without considering the horizontal balance. Moreover, Singh et al. (2011) deduced the solution of the stress distribution inside the filling based on the circular arc differential element by considering the non-equivalence of stress distribution on the cross-section of filling. However, they assumed that shear forces on footwall and hanging wall were equal, and when filled stope had a certain inclination angle, the tangible outcomes of the research carried out by Ting et al. (2014) were not equivalent seemingly.

Considering the limitations mentioned above, in the present paper, numerous shear friction forces will be generated in an interface among footwall, hanging wall and backfill attributable to the effect of inclination angle, and analytical expression of vertical stress within the filling is derived. As a final point, the improved model was compared with the existing model in the

literature. Depending on the recovered results, a significant number of recommendations can be made on backfill performance needed to avoid volatility. Besides, the results address the topics relating to mining and fill implementations.

2. Analytical Model

An analytical model of stress distribution with different shear forces on footwall and hanging wall was expressed in this study. Considering the occurrence condition and mining method of the actual deposit, according to the theoretical model of Singh et al. (2011), the circular arc differential element is shown in Figure 1.

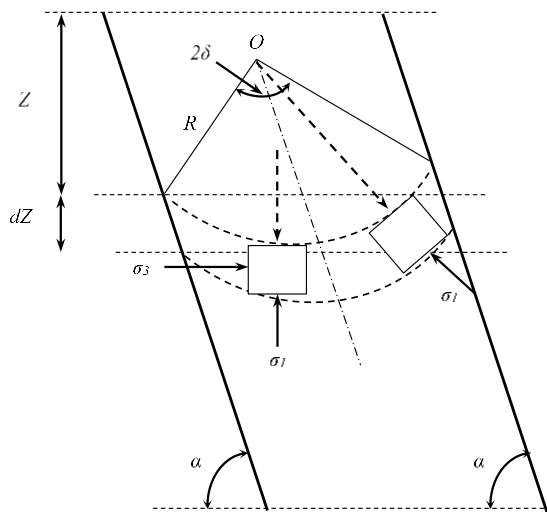


Figure 1. Schematic view of an inclined stope and a minor principal stress arch

When considering the minor stress arch and circular arc differential element of backfill, the theoretical solutions to stress distribution within cementitious backfill (Singh et al., 2011) can be explained as follows:

$$\sigma_1 = \frac{M}{2N} \left(1 - e^{-\frac{2N}{P}z} \right) + \sigma_0 e^{-\frac{2N}{P}z} \tag{1}$$

Where $M = \gamma \delta B \sin^2 \alpha \csc \delta - 2c$

$$N = \left(\frac{1+k}{2} - \frac{1-k}{2} \cos 2\delta \right) \tan \xi.$$

In the above equation, it is assumed that the shear forces on the footwall and hanging wall of the stope are equal. When the stope is in an inclined state, the normal stress on the footwall and hanging wall of the backfill will be different because of the action of fill's dead weight stress, resulting in different possibilities for the shear forces on the footwall and hanging wall (Ting et al. 2014). Hence, based on Singh et al. (2011), this paper further deduces the unequal shear forces in the footwall and hanging wall, as shown in Figure 1, F_{s1} and F_{s2} , respectively. Also, when the deformation of the nearby rock mass increases and filling is squeezed, the fill matrix may have a situation of significant stress arch. Arching is defined by the stress redistribution process where stress is delivered on an area of rocks or fillings, so that it is subjected to lower stresses. Arching happens as a result of stiffness between filling and nearby rock. When filling is more challenging than rock, the equivalent stress arches to filling. Alternatively, when filling is softer than rock, the equivalent stress arches away from filling. Accordingly, in this study, the faces of stress distribution inside the fill material with primary stresses at the arch are derived respectively as displayed in Figures 2 and 3.

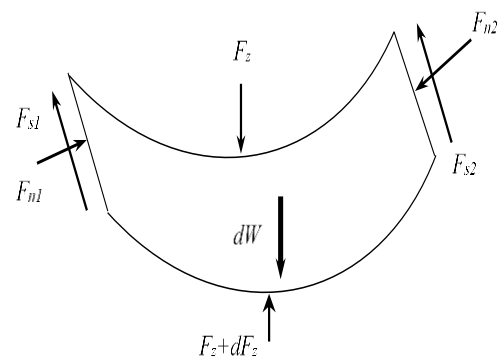


Figure 2. A differential element of arch

Shear force at the interface between backfill and surrounding rock:

$$\begin{aligned} \tau_{s1} &= c + \sigma_{n1} \tan \xi, & F_{s1} &= (c + \sigma_{n1} \tan \xi) \csc \alpha \, dz \\ \tau_{s2} &= c + \sigma_{n2} \tan \xi, & F_{s2} &= (c + \sigma_{n2} \tan \xi) \csc \alpha \, dz \end{aligned} \tag{13}$$

Then,

$$\begin{aligned} & \frac{d\sigma_1}{\left(\gamma \delta B \sin^2 \alpha \cos e \, c \delta - 2c + \frac{2c \tan \xi}{\tan \alpha + \tan \xi} \right) - 2\sigma_1 \left(\frac{1+k}{2} - \frac{1-k}{2} \cos 2\delta \right) \tan \xi \frac{\tan \alpha}{\tan \alpha + \tan \xi}} \\ &= \frac{dz}{\frac{B}{2} \left[(1+k)\delta - \frac{1-k}{2} \sin 2\delta \cos 2\alpha \right] \sin^3 \alpha \csc \delta} \end{aligned} \tag{14}$$

Through derivation, it is found that when the shear forces of the upper and lower discs are unequal, M and N in Singh formula change to the following M' , N' :

$$\begin{aligned} M' &= \gamma \delta B \sin^2 \alpha \cos e \, c \delta - 2c + \frac{2c \tan \xi}{\tan \alpha + \tan \xi}, & N' &= \left(\frac{1+k}{2} - \frac{1-k}{2} \cos 2\delta \right) \tan \xi \frac{\tan \alpha}{\tan \alpha + \tan \xi} \\ p &= \frac{B}{2} \left[(1+k)\delta - \frac{1-k}{2} \sin 2\delta \cos 2\alpha \right] \sin^3 \alpha \csc \delta, & \sigma_1 &= \frac{M'}{2N'} \left(1 - e^{-\frac{2N'}{p} z} \right) + \sigma_0 e^{-\frac{2N'}{p} z} \end{aligned} \tag{15}$$

Figure 4 shows a distribution of stress exerting on stope cross-section in the filled stope, where the friction angle in the filling is 36° and the friction angle in the interface is 24° .

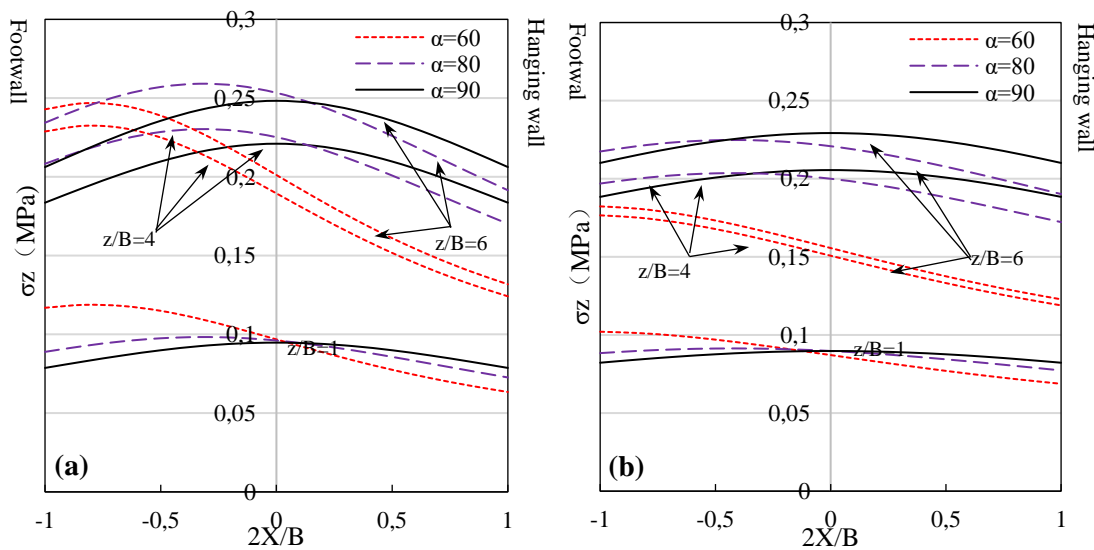


Figure 4. Variation in the vertical stress (σ_z) as a role of a width of filled stope with an inclination of 60° , 80° and 90° : (a) unequal; and (b) equal shear forces on footwall and hanging wall

2.2. Major Principal Stress Arch

When the deformation of surrounding rock is large and squeezing backfill induced an upward relative movement trend relative to surrounding rock, the maximum principal stress direction inside the backfill is horizontal, so the situation of the minor principal stress

arch proposed by Singh et al. (2011) will not occur. Thus, taking Singh's analysis method of the minor principal stress arch as a reference, this study assumes that the stress distribution in the backfill body gives major principal stress arch, and deduces the analytical expression of vertical stress under large principal stress arch as follows:

$$\sigma_3 = \frac{M'}{2N'} \left(1 - e^{-\frac{2N'}{P}z}\right) + \sigma_0 e^{-\frac{2N'}{P}z} \quad (16)$$

Where

$$M' = \gamma \delta B \sin^2 \alpha \cos e c \delta - 2c + \frac{2c \tan \xi}{\tan \alpha + \tan \xi}$$

$$N' = \left(\frac{1+k}{2} + \frac{1-k}{2} \cos 2 \delta\right) \tan \xi \frac{\tan \alpha}{\tan \alpha + \tan \xi}$$

3. Verification of Results

The theoretical model of minor principal stress arch obtained by considering the different shear forces of footwall and hanging wall gained by this method is compared with the solution proposed by Singh et al. (2011), as shown in Figure 5. Fundamental characteristics are presented in Table 1.

Table 1. Input parameters of minor stress arch

Backfill unit weight, γ	N/m ³	
Width of the stope, B	m	6
Inclination, α	°	Var
Cohesion, c	pa	0
Interfacial friction, ξ	°	24
Friction of backfill, φ	°	36

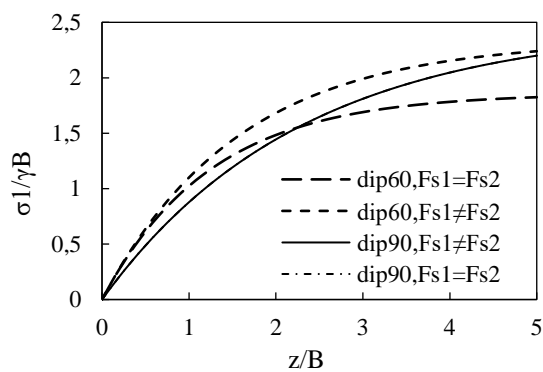


Figure 5. Comparison of vertical stress gained from Singh et al. (2011) solutions and proposed solutions of minor principal arch

From Figure 5, this proposed solution considering the interfacial behavior with diverse shear force was compared to the same shear force offered by Singh et al. (2011) is obtained. When stope angle becomes 90°, two methods are consistent with stress distribution buildup in the backfill, when the stope angle less than 90°, such as the inclination of the wall is 60°. In this study, stress obtained by

this study is much bigger than that of Singh et al. (2011), mainly due to a certain angle of the stope. The backfill subjected to the shear force of the footwall and the hanging wall will be significantly reduced (e.g., the normal stress will be reduced). Accordingly, assuming that footwall and hanging wall have the same shear force, the stress-arching effect of fill will be overestimated (Singh et al. 2011). In this study, the stress distribution within filling when stope with diverse inclination is analyzed considering that the footwall and hanging wall are different, as shown in Figure 6.

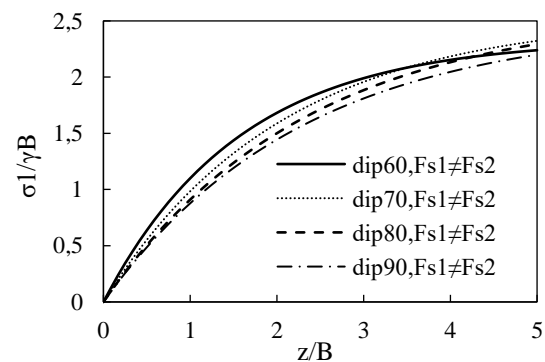


Figure 6. Vertical stress of analytical solutions with depth for different wall inclinations

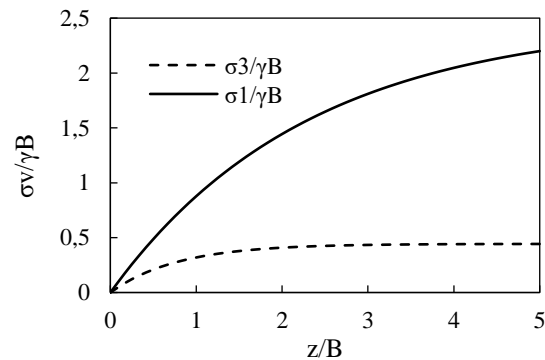


Figure 7. Comparison of vertical stress between the minor and maximum arch of the wall inclination of 90 degree

Figure 7 concludes that the arching effect becomes the smallest when the inclination is near 70°. However, when the inclination angle is less than 70°, then the arching effect gradually decreases. Besides, the arching effect becomes higher when the inclination angle is 90°. This deviates remarkably from the results obtained from the numerical simulations, and the theoretical model gained when considering

horizontal differential units (Li and Aubertin, 2010; Ting et al., 2011; 2014). Nevertheless, they are consistent in trend with the results obtained by Ting et al. (2012) from laboratory testing.

4. Conclusions

Based on the assumption that the differential unit is arc-shaped, as declared earlier by Singh et al. (2011), this study presents the solution to the stress distribution inside the backfilling body. It is deduced assuming that the shear forces of both footwall and hanging wall are different. Also, the form of the main stress arch in the backfilling body is so limited by the convergent deformation of the surrounding rocks when the surrounding rocks undergo a large displacement towards the backfilled stope. Furthermore, the compression of the backfilling body and hence a sizeable main stress arch will look like, and the specific distribution of stresses within quarry's cross-section is gained. Based on the analytical solutions finalized, some findings can be drawn as follows:

- (1) Overlooking the influence of different shear forces between fill and nearby rock underrates the stress distribution in filling.
- (2) According to the effect of wall deformation conditions, there exist minor principal stress and prominent principal stress arch within fill.
- (3) The model proposed in this study can change into Singh's theoretical solution when the shear forces of footwall and hanging wall are equal, indicating that the model proposed in this study is an extension of Singh's model.

Therefore, the analytical solution demonstrated in this study gives some rough parameters to estimate the behavior of the backfill considering stope inclinations during adjacent mining but cannot present the failure configurations of the backfills. Analytical solutions not only assess the durability behavior the backfill free faces, but also can develop a cost-effective fill in underground

mines. As a result, an analytical solution was presented to evaluate stress distribution within inclined filled stopes better.

In addition to the results drawn from this work, further research could offer a better assessment of stress distribution considering the numerical model with more case studies on stope geometry and location regarding adjacent stopes. Field test, observation and instrumentation should be done for the distribution of stress within the multiple backfilled stopes. The fill-rock mass interface should be investigated through numerical and laboratory studies to precisely determine both standart and shear stiffness of the fill-rock mass interface.

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