I-PD Controller Design Based on Analytical Rules for Stable Processes with Inverse Response

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Abstract— **Inverse response characteristic makes the control of a process more challenging. In this study, simple and analytical expressions have been obtained to evaluate optimum settings of I-PD controllers for controlling open loop stable processes with time delay and a positive zero. Time weighted versions of Integral of Squared Error (ISE) criterion, namely ISTE, IST^2E and IST^3E criteria, which have been proved to be leading to very adequate closed loop responses, have been exploited to obtain mentioned optimum settings. Simulation examples have been considered to for evaluating the effectiveness of obtained tuning rules.**

Index Terms—**PID, I-PD, Stable process, Time delay, Inverse response, Integral performance indices.**

I. INTRODUCTION

IT HAS BEEN reported that more than 95% of controllers in the process control applications are PID (Proportional-Integral-Derivative) type controllers [1]. However, it has also been reported that 75% of all PID based control loops are out of tune. Therefore, researchers are still studying on developing new design methods for calculating the setting parameters of PID controllers.

Design of PID controllers to control inverse response processes is interesting and challenging. Hence, in recent decades, researchers have given many efforts to design PID controllers for inverse response processes. Control of stable processes with inverse response and dead time was given by Luyben [2]. It was stated that the method could give good responses up to dead time value of 3.2 [2]. Luyben suggested an identification approach for modelling integrating processes with inverse response and a design method for tuning a PID controller for controlling it [3]. The response obtained by their method was very sluggish. The use of direct synthesis approach for designing PID controllers to control time delay stable processes with inverse response was studied in [4]. was Pai et al. [5] also used direct synthesis method to obtain analytical expressions for calculating PI/PID controller settings for controlling integrating processes with time delay and inverse response. Simple tuning rules were provided, however, obtained closed loop responses were very oscillatory. Jeng and Lin [6] proposed a Smith-type control structure for controlling both stable and integrating processes with time delay and inverse response. Hamamci [7] suggested design of PID controllers based on graphical optimization for stable, integrating and unstable processes without inverse response. Kaya and Cengiz [8] designed PI/PID controllers using analytical rules for controlling time delay stable processes with inverse response. Authors extended their method to controlling integrating processes with inverse response and time delay, as well. A similar study were later conducted by Irshad and Ali [9], too.

Control of a process becomes more troublesome by inclusion of inverse response characteristic, and the use of conventional PID control, which is the case for the above given references, may lead to unacceptable closed loop responses. Therefore, a PI-PD controller, which has proven to give rise to much better closed loop responses, in the Smith predictor scheme was suggested for controlling stable processes with inverse response [10]. Two difficulties with this method can be expressed. First, the Smith predictor scheme is sensitive to modelling errors and parameter variations. Second, determination of tuning parameters of the forward path controller, PI, includes a trade-off and they must be limited to a value by the designer.

I-PD control has a similar structure to the PI-PD control configuration. PI-PD control configuration has a PI controller on the forward path, whereas I-PD control has an I only controller on the forward path. Having only three tuning parameters to be calculated simplifies the design procedure. Additionally, I-PD controllers can result in very similar closed loop responses that can be achieved with PI-PD controllers.

In this study, simple analytical expressions have been provided to calculate optimum settings of an I-PD controller for improving closed loop responses of stable processes with inverse response and time delay. It is assumed that the process can be identified as a first order plus dead time with inverse response (FOPDT-IR) transfer function. Then, repetitive optimizations in the sense of time weighted integral performance criteria were performed on the error signal of the

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Manuscript received June 01, 2020; accepted July 30, 2020.
DOI: 10.17694/bajece.746352

I-PD control configuration. Thereby, simple analytical expressions have been obtained in terms of parameters of the I-PD controller and the FOPDT model transfer function. Several simulation examples have been given to illustrate the much improved closed loop responses achieved with proposed I-PD controller design method.

The remaining parts of the paper is organized as follows: A very short survey of integral performance criteria is provided in Section II. Section III explains the design of optimum I-PD controllers for controlling FOPDT-IR processes. Simulation examples are supplied in Section IV to show effectiveness of the proposed I-PD controller design method. Finally, conclusions are given in Section V.

II. Integral Performance Criteria

In this paper, the following general form of integral performance criteria is used in the optimization.

$$J_n(\eta)=\int_0^{\infty}[t^n e(\eta,t)]^2\,dt, \qquad n=1,2,3 \tag{1}$$

Here, η stands for the settings of I-PD controller to minimize the performance criteria given in Eq. (1). Selecting various n values, various performance criteria can be obtained. For example, selecting $n=1$ the criteria is named as ISTE, selecting $n=2$ the criteria is named as IST^{2}E, and selecting $n=3$ the criteria is named as IST^{3}E. Increasing the value of n, results in improved closed loop performance in the sense of lower overshoots. Larger n value may slightly slow down the speed of response and increase the settling time.

For minimization of the integral given by Eq. (1), PSO (Particle Swarm Optimization), which is a metaheuristic optimization algorithm, is used [11], [12]. The PSO algorithm works as follows: all particles are randomly distributed in the search space at the beginning. The best position of a particle itself and the best positions of its neighbors are used to update the position at each step. The process is repeated for all particles. The algorithm runs continuously in this manner until the best solution has been obtained [12], [13].

III. I-PD Controller Design

I-PD controller structure used to control open loop stable processes with inverse response is shown in Fig. 1. In the figure, $G(s)$ is the process transfer function to be controlled. I and PD controllers, respectively, are given by $G_{c1}(s)$ and $G_{c2}(s)$.

The following transfer function is used to identify a stable processes with inverse response and time delay:

$$G(s)=\frac{K(-T_0 s+1)e^{-\theta s}}{(Ts+1)} \tag{2}$$

I-PD controller transfer functions, $G_{c1}(s)$ and $G_{c2}(s)$, are assumed to be given by the following ideal transfer functions:

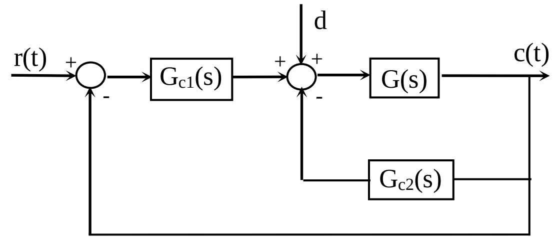

Fig. 1. I-PD Control Structure

$$G_{c1}(s)=\frac{K_c}{T_i s} \tag{3}$$

$$G_{c2}(s)=K_c(1+T_d s) \tag{4}$$

In order to find correlations between the I-PD controller parameters and process transfer function model parameters, $sT=\overline{s}$ normalization is used in Eqs. (2), (3) and (4). 3/3 Pade approximation was used for the normalized time delay. For a specified value of T_0/T, a unit step reference input $r(t)$ was applied to the closed loop system shown in Fig. 1. Then for various values of normalized time delay θ/T, repeated optimizations were carried out by using the PSO algorithm to minimize the error in closed loop system, illustrated in Fig. 1. In the PSO algorithm, number of the particles was selected as 33, and number of iterations for each θ/T value was selected as 300.

For chosen values of T_0/T (0.7 and 1.0), correlations between KK_c, T_i/T and T_d/T and varying values of θ/T (in the range of 0.1 to 1.0 and 1.1 to 2.0) are given in Fig. 2 and Fig. 3 for the ISTE criterion. Tuning parameters obtained from the optimization are shown by asterisk. Then, a curve fitting method has been used to find expressions that fits to obtained tuning parameters. Results of expressions obtained from curve fitting are shown by solid lines. It is observed that very satisfactory fittings have been achieved.

Formulas obtained from the curve fitting method are given below:

$$KK_c=a_1+b_1\left(\frac{\theta}{T}\right)+c_1\left(\frac{T_0}{T}\right)+d_1\left(\frac{\theta}{T}\right)^2+e_1\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) \tag{5}$$

$$\frac{T_i}{T}=a_2+b_2\left(\frac{\theta}{T}\right)+c_2\left(\frac{T_0}{T}\right)+d_2\left(\frac{\theta}{T}\right)^2+e_2\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) \tag{6}$$

$$\frac{T_d}{T}=a_3+b_3\left(\frac{\theta}{T}\right)+c_3\left(\frac{T_0}{T}\right)+d_3\left(\frac{\theta}{T}\right)^2+e_3\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) \tag{7}$$

It has been observed that these expressions can be used for both ranges, that is, $0.1\le\theta/T\le1.0$ and $1.1\le\theta/T\le2.0$, as long as convenient constants are used.

The procedure given above can be repeated for IST^{2}E and IST^{3}E criteria too. Interestingly, it was found that above given expressions can be used for IST^{2}E and IST^{3}E criteria with

 ISSN: 2147-284X

appropriate constants, as well. Constants in these expressions for different integral performance criteria and different normalized time delay ratio ranges are summarized in Table I and Table II.

Therefore, once the model transfer function of open loop stable process with inverse response is known, given by Eq. (2), then normalized θ/T and T_0/T ratios can be found and substituted into Eqs. (5)-(7) to calculate optimum I-PD controller settings by selecting appropriate constants from Table I or Table II.

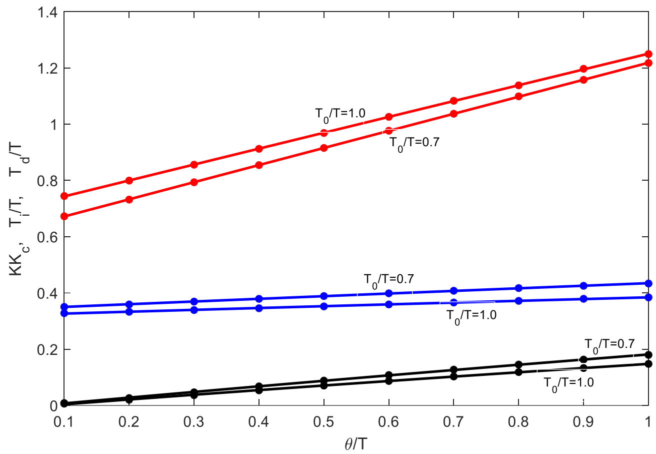

Fig. 2. I-PD parameters over range $0.1 \le \theta/T \le 1.0$ for ISTE criterion (red: T_i/T, blue: KK_c, black: T_d/T)

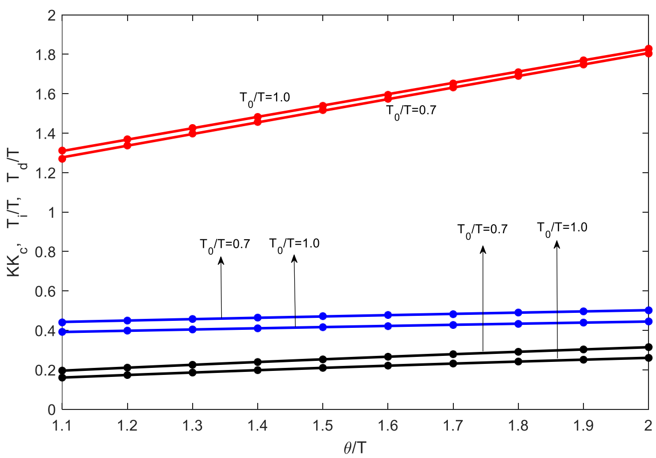

Fig. 3. I-PD parameters over range $1.1 \le \theta/T \le 2.0$ for ISTE criterion (red: T_i/T, blue: KK_c, black: T_d/T)

IV. Simulations And Results

In this section several simulation examples are provided to show the effectiveness of the suggested I-PD controller design method. Irshad and Ali [9], Chien et al. [4] and Jeng and Lin [6] suggested PID controller design methods for controlling stable processes with inverse response and time delay, as well. Therefore, their design methods have been used to evaluate the closed loop performance of suggested I-PD controller design method.

TABLE I

I-PD CONTROLLER PARAMETERS FOR $0.1 \le \theta/T \le 1.0$

		ISTE	IST²E	IST³E
KK_c	a_1	0.38910	0.40550	0.35780
	b_1	0.16670	0.07243	0.04696
	c_1	-0.06931	-0.10900	-0.06851
	d_1	-0.00343	-0.00551	0.02534
	e_1	-0.09871	-0.01586	-0.04792
T_i/T	a_2	0.34300	0.53490	0.52450
	b_2	0.71270	0.57540	0.52200
	c_2	0.25240	0.15130	0.15960
	d_2	-0.00375	0.00148	0.03913
	e_2	-0.14470	-0.05475	-0.08905
T_d/T	a_3	-0.01359	-0.01958	-0.01971
	b_3	0.28810	0.32870	0.37070
	c_3	0.000244	0.00520	0.00269
	d_3	-0.14940	-0.01275	-0.01877
	e_3	-0.11200	-0.13560	-0.15000

TABLE II

I-PD CONTROLLER PARAMETERS FOR $1.1 \le \theta/T \le 2.0$

		ISTE	IST²E	IST³E
KK_c	a_1	0.45520	0.40960	0.34210
	b_1	0.10410	0.07709	0.09369
	c_1	-0.14300	-0.11150	-0.08028
	d_1	-0.00670	-0.00121	-0.00533
	e_1	-0.02445	-0.02727	-0.03791
T_i/T	a_2	0.50700	0.56760	0.49330
	b_2	0.63880	0.55120	0.59280
	c_2	0.15690	0.12810	0.15330
	d_2	-0.00614	0.01095	0.00096
	e_2	-0.04533	-0.04985	-0.08645
T_d/T	a_3	0.02686	0.03825	0.07456
	b_3	0.25310	0.28430	0.28550
	c_3	-0.03932	-0.05992	-0.08782
	d_3	-0.02317	-0.02780	-0.02632
	e_3	-0.07018	-0.06979	-0.06130

Example 1:

Consider an inverse response process transfer function given by

$$G(s) = \frac{0.5848(-0.3546s+1)e^{-0.3567s}}{(0.6302s+1)}.$$

This transfer function was studied by Irshad and Ali [9]. It was derived for an isothermal CSTR exhibiting multiple steady state solutions. For this example comparisons are given only with design method of Irshad and Ali [9] because design methods of Chien et al. [4] and Jeng and Lin [6] requires a

 ISSN: 2147-284X http://dergipark.gov.tr/bajece

second order stable process transfer function with a positive zero and time delay, hence they cannot be used for this case. Irshad and Ali [9] suggested a PI controller which its tuning parameters found from optimum tuning rules. This process transfer function has a normalized time delay ratio of $\theta / T = 0.566$ and $T_0 / T = 0.563$. Using these values in Eqs. (5)-(6), calculated settings of the proposed I-PD controller and PI controller of Irshad and Ali [9] are given in Table III for various integral performance criteria. A unity step input and a disturbance having magnitude of 0.5 were applied to closed loop control system. It was assumed that the disturbance exist in the system at time $t = 10$ s. Obtained closed loop responses for both design methods are illustrated in Fig. 4.

For the I-PD controller, it is observed that there is not much difference in the closed loop performances for ISTE, ISTE2E and IST3E criteria based designs. Thus, in the following examples comparisons will be performed for I-PD controller that is designed based on ISTE criterion. For the PI controller design suggested by Irshad and Ali [9] ISTE criterion results in slightly oscillatory response. Also, PI controller designs suggested by Irshad and Ali [9] lead to larger initial inverse responses.

Closed loop responses for both design methods are depicted in Fig. 5 for +20% change in all parameters (K, T_0, T and θ) of the model transfer function. It is observed that the proposed I-PD provides less overshoot and slightly shorter settling time for both set point tracking and disturbance rejection. Actually, this example verifies the better performance of an I-PD controller than classical PID type controller, because the proposed design method the design method of Irshad and Ali [9] are both rely on the integral performance criteria. Thus, improved response is due to the control structure of I-PD controller.

TABLE III
CONTROLLER PARAMETERS FOR EXAMPLE 1

	Proposed I-PD			PI (Irshad and Ali)		
	ISTE	IST^{2}E	IST^{3}E	ISTE	IST^{2}E	IST^{3}E
K_c	0.704	0.647	0.579	1.270	1.003	0.857
T_i	0.587	0.585	0.563	0.890	0.773	0.718
T_d	0.069	0.077	0.087	-	-	-

Example 2:

Here, the following higher order stable process with inverse response and time delay, which was studied by Jeng and Lin [6], is considered:

$$G(s) = \frac{(-2s+1)e^{-0.5s}}{(2s+1)(s+1)(0.5s+1)}.$$

To identify settings of the I-PD controller, FOPDT-IR model given in Eq. (2) must be determined. For simplicity, approximation method suggested in [14] is used in this study.

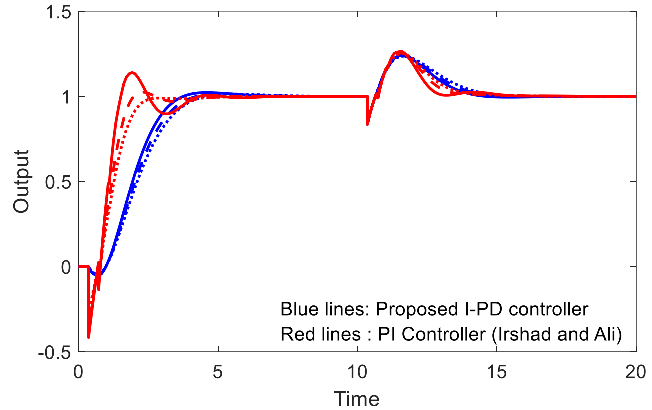

Fig. 4. Closed loop responses for example 1 (solid line: ISTE, dashed line: IST^{2}E, dotted line: IST^{3}E)

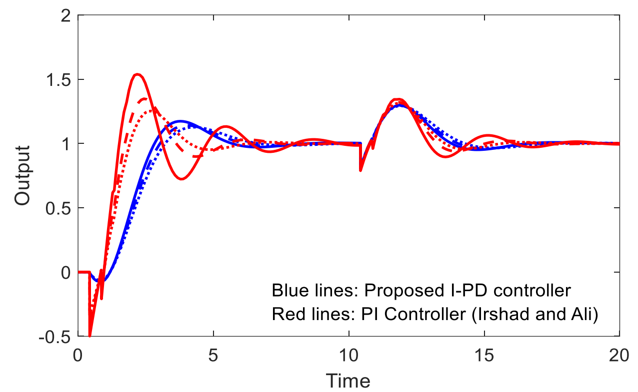

Fig. 5. Closed loop responses for perturbed for example 1 (solid line: ISTE, dashed line: IST^{2}E, dotted line: IST^{3}E)

This approach suggests the use of a first order Taylor series expansion for the smaller poles in order to approximate them. Thus, applying this approach to the two smaller poles, one can obtain the following model:

$$G_m(s) = \frac{(-2s+1)e^{-2s}}{(2s+1)}.$$

This model has a normalized time delay ratio of $\theta / T = 1.0$ and $T_0 / T = 1.0$. Substituting these values in Eqs. (5)-(6) with proper constants selected from Table I, the proposed I-PD controller settings were calculated and given in Table VI together with tuning parameters for other design methods. Additional tuning parameter, λ, of design method suggested by Jeng and Lin [6] has also been supplied in the table.

A unity step input and a disturbance having magnitude of -1 were applied to closed loop control system. It was assumed that the disturbance exist in the system at time $t = 40$ s. Obtained closed loop responses for all design approaches are given in Fig. 6. It is observed from figure that proposed I-PD controller design gives the most satisfactory closed loop responses for both the set point tracking and disturbance rejection in the sense of less overshoot and oscillations. Also, unlike the proposed one, others have large initial inverse responses. Corresponding process inputs are shown Fig. 7, which reveals that the proposed I-PD controller has the smallest control signal magnitude. Design method suggested by Chien et al. [4] requires very large initial control effort.

TABLE VI
CONTROLLER PARAMETERS FOR EXAMPLE 2

Design Methods	K_c	T_i	T_d	λ
Proposed I-PD	0.384	2.501	0.296	-
Irshad and Ali	0.549	3.512	-	-
Jeng and Lin	0.826	4.283	1.011	3.3
Chien et al.	0.596	2.706	1.500	-

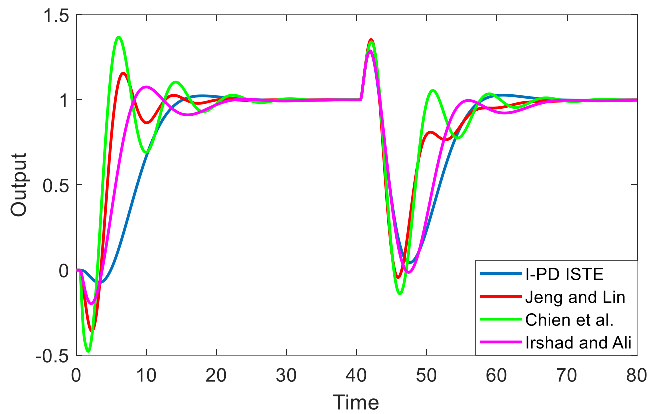

Fig. 6. Output responses for example 2

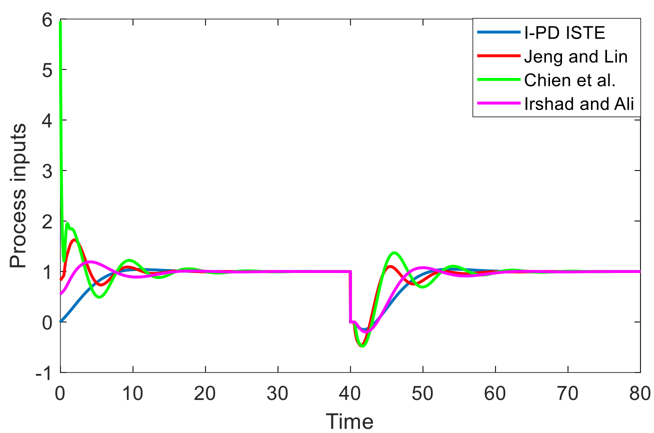

Fig. 7. Process inputs for example 2

Example 3:

Here, a high order stable process with inverse response without time delay given below

$$G(s)=\frac{(-2s+1)}{(s+1)^5}$$

is studied. This process transfer function was also studied by Jeng and Lin [6]. Again, to evaluate setting of the proposed I-PD controller, FOPDT-IR model given in Eq. (2) must be identified. Using the approach explained in example 2, the following model

$$G_m(s)=\frac{(-2s+1)e^{-3s}}{(2s+1)}.$$

which has $\theta/T=1.5$ and $T_0/T=1.0$, can be obtained. Replacing these normalized values into Eqs. (5)-(6) with appropriate constants chosen from Table II, the proposed I-PD controller tuning parameters were evaluated and are provided in Table V. The table gives tuning parameters of design methods used for comparison, as well.

A unity step input and a disturbance having magnitude of -1 were applied to closed loop control system. It was assumed that the disturbance exists in the system at time $t=50$ s. Closed loop responses for all design methods are depicted in Fig. 8. It is seen from the figure that proposed I-PD controller design has much superior performance than others, which all have large overshoots for set point tracking. Additionally, design method suggested by Irshad and Ali [9] has large oscillations for set point tracking and disturbance rejection. Process inputs for all design methods are shown Fig. 9. Similar to example 2, proposed I-PD controller has a smaller control signal magnitude when compared to others.

TABLE V
CONTROLLER PARAMETERS FOR EXAMPLE 3

Design Methods	K_c	T_i	T_d	λ
Proposed I-PD	0.417	3.081	0.420	-
Irshad and Ali	0.632	3.112	-	-
Jeng and Lin	0.795	4.038	1.287	3.7
Chien et al.	0.831	3.681	1.095	-

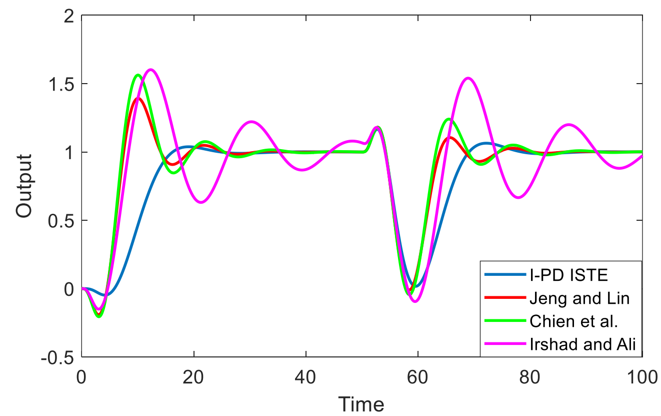

Fig. 8. Output responses for example 3

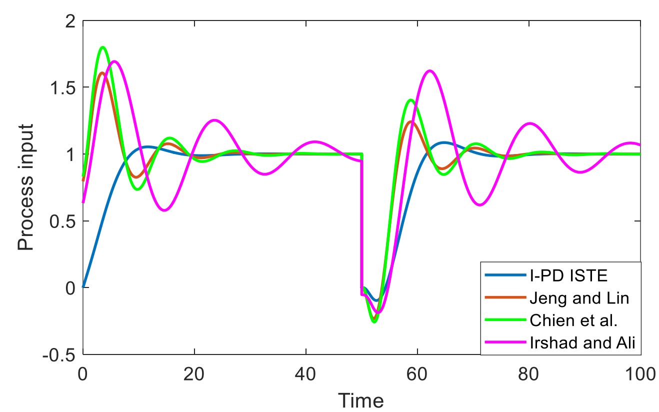

Fig. 9. Process inputs for example 3

V. Conclusions

Analytical expressions enabling one to calculate optimum tuning parameters of an I-PD controller for controlling stable processes with inverse response and dead time have been provided. ISTE, IST^2E and IST^3E criteria were exploited to derive those expressions. As there is not much difference between the closed loop performances obtained from ISTE, IST^2E and IST^3E criteria for the case studied in this paper, but slightly faster response can be achieved with ISTE criterion, for the comparisons ISTE based rules were used. Provided simulation results exhibited the much better performance of the proposed I-PD controller design for both set point tracking and disturbance rejection.

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