



Mamdani Model based ANFIS and Application in Prediction of Soot Emission

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Abstract

This paper proposed Mamdani-based Adaptive Neuro-Fuzzy Inference System (MANFIS). In literature, there are very applications of Sugeno Adaptive Neuro-Fuzzy Inference System (ANFIS) because of simplicity of the Sugeno defuzzification step. Mamdani defuzzification step is linguistic but the Sugeno defuzzification step has constant and linear functions. So, Mamdani parameters training algorithms given in the open literature are not efficient and give worse results when compared to the Sugeno ANFIS. The proposed Mamdani ANFIS is tested for an equation and to predict vehicle soot emission that soot emission is effective at global warming and melting of sea ice in the Arctic. The proposed Mamdani ANFIS is compared to the Sugeno ANFIS for Least Square Estimation method and Gradient Descent method. The training results show that The Mamdani ANFIS consumes less time and needs less epoch number. It is determined that for Gradient Method, the proposed Mamdani has less training error.

Keywords: Mamdani ANFIS, Gradient, Emission Prediction, RMSE, Least Square Estimation.

Mamdani Modeli tabanlı ANFIS ve Kurum Emisyon Tahmininde Uygulanması

Öz

Bu çalışmada Mamdani Bulanık Çıkarım Sisteminin Yapay Sinir Ağları Tabanlı Eğitimi (MANFIS) için bir metod tasarlanmıştır. Sugeno duruluşturma işleminde sabit ve doğrusal fonksiyonlar kullanıldığından ANFIS kullanımı kolay olmuştur. Bu sebeple literatürde Sugeno ANFIS ile alakalı çok çalışma bulunmaktadır. Mamdani duruluşturma aşaması ise üyelik fonksiyonlarını içermektedir. Bu sebeple Mamdani giriş ve çıkış parametrelerinin eğitimi için bazı kabuller yapmak zorunludur. Literatürde, Mamdani eğitimi için yapılan çalışmalarda, Mamdani ANFIS sonuçlarının Sugeno ANFIS'e göre verimsiz olduğu görülmüştür. Geliştirdiğimiz metodu bir denklem ve araçların emisyon değerlerinin tahmini için denedik. Emisyon değerleri küresel ısınma ve kuzey kutbundaki buzulların erimesinde etkili faktörlerdir. Bu çalışmada ki eğitimlerde Sugeno ANFIS ve Mamdani ANFIS algoritmaları En Küçük Kareler ve Gradyan metodları için karşılaştırılmıştır. Karşılaştırma sonuçlarında bu çalışmada geliştirilen Mamdani ANFIS algoritmalarının daha az zamanda ve daha az eğitim adımıyla Sugeno ANFIS'e kıyasla daha iyi sonuçlar verdiği görülmüştür.

Anahtar Kelimeler: Mamdani ANFIS, Gradyan, Emisyon Tahmini, RMSE, En Küçük Kareler.

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1. Introduction

Computational Intelligence methods are very useful to represent nonlinear mathematical models and to solve problems that impossible to solve so they are wisely implemented in engineering and social projects. These methods involve neural networks, gradient methods, genetic algorithms, fuzzy logic, etc. The methods have some advantages and disadvantages to each other and so Computer Intelligence methods are examined as a combination of methodologies. When they are combined, better results can be obtained (Eberhart, 1998). In these studies, Fuzzy Logic is used with Adaptive Neuro-Fuzzy Inference System (ANFIS) training.

Fuzzy Inference System (FIS) are offered a linguistic approximation for complicated problems and systems in 1973 (Zadeh, 1973). The Mamdani model is proposed as a fuzzy approach to engineering control problems (Mamdani & Assilian, 1975). The Mamdani structure consists of fuzzy inputs and outputs, so it was linguistic and it had a humanly structure. Mamdani structure has a defuzzification step to take crisp values as output. Even if Mamdani is well suited to human, it was hard to training and use in engineering problems because of low computer processing power. Afterwards, the Sugeno model is proposed as more convenient to engineering issues (Sugeno, 1985). Sugeno FIS has fuzzy inputs as Mamdani but it does not need the defuzzification step. The Sugeno FIS outputs membership functions (MF) are linear or constant, so Sugeno outputs are crisp values.

FIS can be constructed with expert opinion but this is not possible or effective for every problem. So, ANFIS is proposed for Sugeno structure that it was a hybrid system consisting of Least Square Estimation (LSE) method and Gradient Descent (GD) method (Jang, 1993). The ANFIS is a generally used technique (Şahin, OKTAY, & Konar, 2020) to train fuzzy inference systems that it is explicit and needs less computational processing load when compared to other training methods (Raja & Pahat, 2016). Besides, it gives better results than other fuzzy training methods (Neshat, Adeli, Masoumi, & Sargolzae, 2011).

ANFIS has two processes as forward and backward. The forward process takes input MFs and trains output MFs. The backward process takes output MFs and trains input MFs. So, fuzzy inference system input and output parameters are trained. Forward process training are executed with LSE and Gradient Descent, Backward process training is executed with gradient descent method (Raja & Pahat, 2016).

Sugeno has advantages in engineering problems when compared to Mamdani and so it is easier to train Sugeno. However, Mamdani has a more suited structure to human than Sugeno. So, Mamdani ANFIS models are created and tested in traffic solution problem (Chai, Jia, & Zhang, 2009). The given Mamdani ANFIS model is changed and it is explained in the Mamdani ANFIS section.

The proposed Mamdani ANFIS and ANFIS is tested with a nonlinear equation and Soot Emission prediction. There are very much soot emission prediction studies in the open literature (Özhan, 2020). The used soot emission test data is taken from (Ondes, Bayezit, Poergye, & Hafsi, 2017).

The rest of this paper is organized as follows. In section 2, the Sugeno ANFIS structure is examined as stated in the open

literature. In section 3, the proposed method Mamdani ANFIS (MANFIS) is given in detail. In section 4, the methods are implemented to a nonlinear equation and prediction of emission problem.

2. ANFIS

Adaptive Neuro-Fuzzy Inference System (ANFIS) is a hybrid computational hybrid system that consists of neural networks and fuzzy logic (Rai, Pai, & Rao, 2015). The ANFIS is based on the given Sugeno model that Sugeno Fuzzy Inference System has 5 steps as fuzzification," and/or" method, implication, aggregation, and defuzzification. Sugeno FIS structure is given as "and" operator and Implication is product, aggregation is sum and defuzzification is wtaver. These steps' mathematical equation is given below.

Step 1: Fuzzification:

$$O_i^1 = \mu_{A_i}(x) \quad O_i^1 = \mu_{B_i}(y) \quad (1)$$

Step 2: "and/or" method; ANFIS uses "and-prod" operator:

$$O_i^2 = \mu_{A_i}(x)\mu_{B_i}(y) = w_i \quad (2)$$

Step 3: Implication; ANFIS uses "product" operator:

$$f_i = p_i x_1 + q_i x_2 + r_i$$

Step 4: Aggregation; ANFIS uses "sum" operator:

$$O_i^4 = \sum_{i=1}^n O_i^3 \quad (4)$$

Step 5: Defuzzification; ANFIS uses "wtaver" (weighted average) operator:

$$O_i^5 = \frac{O_i^4}{\sum_{i=1}^n w_i} \quad (5)$$

For the given Sugeno FIS, the ANFIS structure is given in Figure 1. For the given Sugeno; the ANFIS structure is generated as below step by step.

Layer 1: Membership functions generation are formed.

$$O_i^1 = \mu_{A_i}(x) \quad O_i^1 = \mu_{B_i}(y) \quad (6)$$

Layer 2: The and/or method implementation. "prod" method is implemented. "prod" method crosses the input's membership grades.

$$O_i^2 = \mu_{A_i}(x)\mu_{B_i}(y) = w_i \quad (7)$$

Layer 3: In Sugeno FIS third step is implication but it is represented in layer 4. In the last step of Sugeno is "wtaver" defuzzification method are used. Wtaver method is represented in this layer.

$$O_i^3 = \frac{w_i}{\sum_{i=1}^n w_i} \quad (8)$$

Layer 4: The implication step "product" method is implemented. "product" method multiplies the normalized weights and f values.

$$O_i^4 = \bar{w}_i f_i = y_i \quad (9)$$

Layer 5: The aggregation step is represented that all rule output values are gathered.

$$O_i^5 = \sum_{i=1}^n y_i \tag{10}$$

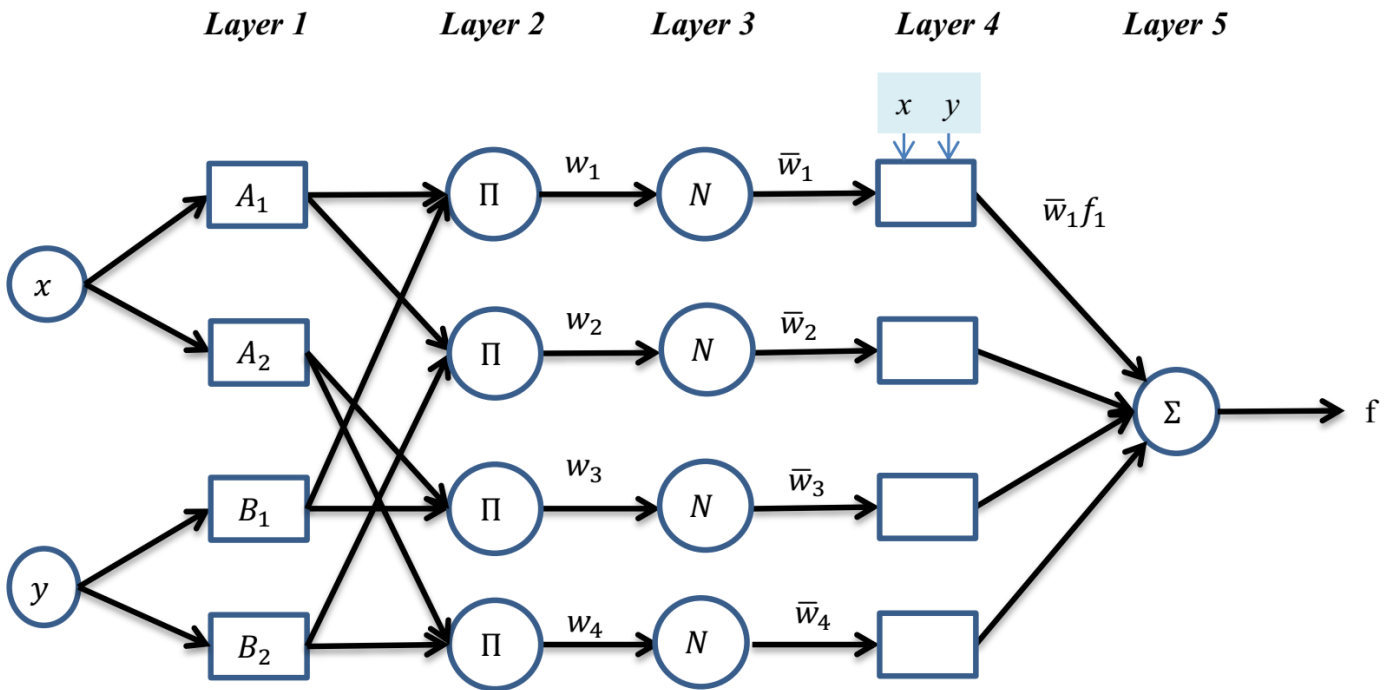


Figure 1. ANFIS Structure

2.1. ANFIS LSE Method

LSE method gives the exact results. The LSE method steps are given below (Wesley Hines, 1997):

Layer 1: Membership functions generated.

$$O_i^1 = \mu_{A_i}(x) \quad O_i^1 = \mu_{B_i}(y) \tag{11}$$

Layer 2: "and-prod" method are used to find weights.

$$O_i^2 = \mu_{A_i}(x)\mu_{B_i}(y) = w_i \tag{12}$$

Layer 3: Weights are normalized.

$$O_i^3 = \frac{w_i}{\sum_{i=1}^n w_i} \tag{13}$$

Layer 4: The rule outputs are calculated.

$$O_i^4 = y_i = \bar{w}_i f_i = \bar{w}_i(p_i x_1 + q_i x_2 + r_i) \tag{14}$$

Layer 5: The aggregation step is represented that all output values are gathered.

$$O_i^5 = \sum_{i=1}^n y_i = \sum_{i=1}^n \bar{w}_i f_i = (\bar{w}_1 x_1)p_1 + (\bar{w}_1 x_2)q_1 + \bar{w}_1 r_1 + (\bar{w}_2 x_1)p_2 + (\bar{w}_2 x_2)q_2 + \bar{w}_2 r_2 \tag{15}$$

The equation 15 must be put into a usable form to implement Least Square algorithm (Wesley Hines, 1997).

$$y = (w_1 x_1)p_1 + (w_1 x_2)q_1 + w_1 r_1 + (w_2 x_1)p_2 + (w_2 x_2)q_2 + w_2 r_2 \tag{16}$$

$$y = [w_1 x_1 \quad w_1 x_2 \quad w_1 \quad w_2 x_1 \quad w_2 x_2 \quad w_2] \begin{bmatrix} p_1 \\ q_1 \\ r_1 \\ p_2 \\ q_2 \\ r_2 \end{bmatrix} = XW \tag{17}$$

"w" is the weights, "x" is the inputs and "W" is the output parameters. Weights and input values are known and so output parameters will be found.

$$y = XW \Rightarrow X^{-1}Y \tag{18}$$

If X is not invertable than pseudoinverse can be used. In this study, pseudoinverse is used.

2.2. ANFIS Gradient Descent Method

In some situations, there is not any convenient inverse of matrices. So, the fuzzy training can be implemented with some methods and Gradient is one of the simplest. The method equation is given below for "y" is measured output values, "y^t" is calculated output values, "lr" is learning rate, and W is the output parameters.

$$E = \frac{1}{2}(y - y^t)^2 \quad y = XW \tag{19}$$

$$\frac{\partial E}{\partial W} = (y - y^t)y' = (y - y^t)X \tag{20}$$

$$W(t + 1) = W(t) - lr \frac{\partial E}{\partial W_i} = W(t + 1) = W(t) - lr(y - y^t)X \tag{21}$$

3. Mamdani ANFIS

Mamdani model inputs and outputs are fuzzy; however, Sugeno outputs are crisp. So, Mamdani has advantages on the Sugeno model: it is heuristical and very compatible to human thought structure (Chai et al., 2009).

Mamdani model operators are chosen as; for and/or operator "and/prod", for implication operator "product", for aggregation operator "sum" and for defuzzification operator "centroid" is used. The MANFIS structure is given in Figure 2. The trained Mamdani structure:

Step 1: Fuzzification:

$$O_i^1 = \mu_{A_i}(x) \quad O_i^1 = \mu_{B_i}(y) \quad (22)$$

Step 2: "and/or" method; MANFIS uses "and-prod" operator:

$$O_i^2 = \mu_{A_i}(x)\mu_{B_i}(y) = w_i \quad (23)$$

Step 3: Implication; MANFIS uses "product" operator where "(area)_i" is area of the consequent MFs:

$$O_i^3 = w_i * (area) = a_i \quad (24)$$

Step 4: Aggregation; MANFIS uses "sum" operator where "z_i" is center of the consequent MFs:

$$O_i^4 = \sum_{i=1}^n a_i z_i \quad (25)$$

Step 5: Defuzzification; MANFIS uses "centroid" operator:

$$O_i^5 = \frac{\sum_{i=1}^n c_i \mu_{A_i}(x)\mu_{B_i}(y)}{\sum_{i=1}^n \mu_{A_i}(x)\mu_{B_i}(y)} \quad (26)$$

when simplified for our Mamdani structure:

$$O_i^5 = \frac{\sum_{i=1}^n a_i z_i}{\sum_{i=1}^n a_i} = \bar{a}_i z_i \quad (27)$$

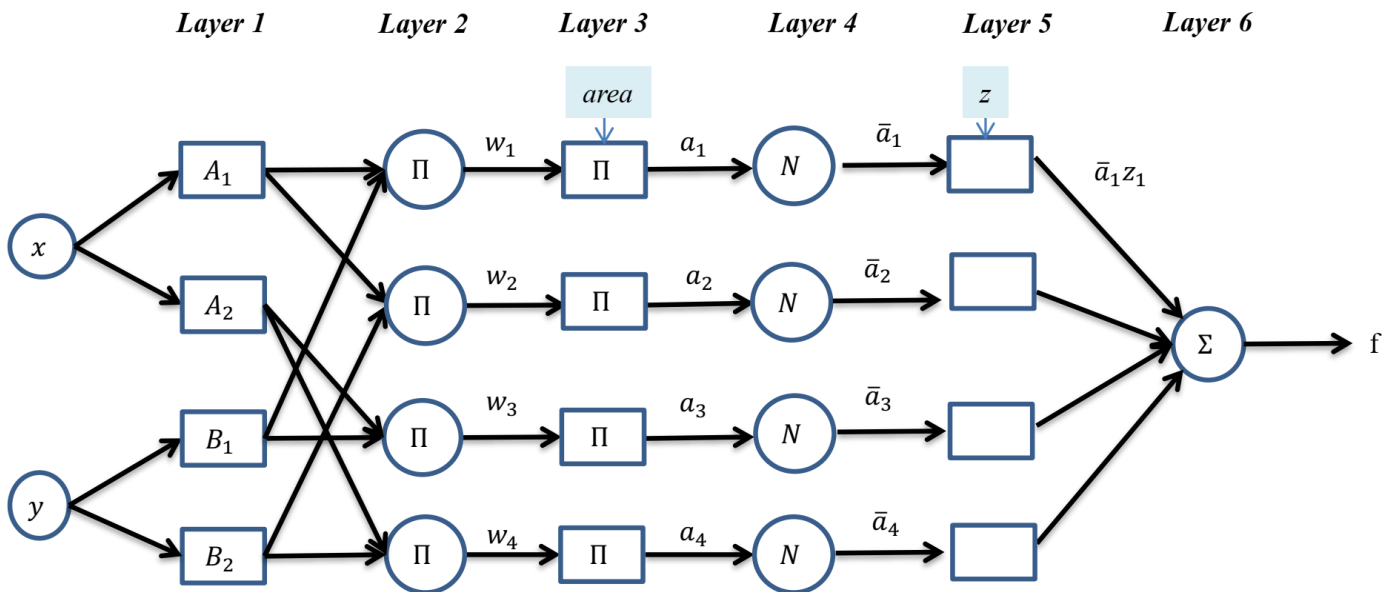


Figure 2. MANFIS Structure

For the given Mamdani; MANFIS structure is generated as below step by step.

Layer 1: Membership functions generation are formed.

$$O_i^1 = \mu_{A_i}(x) \quad O_i^1 = \mu_{B_i}(y) \quad (28)$$

Layer 2: The "and/or" operator implementation. "prod" operator is implemented. "prod" operator multiplies the input's membership grades.

$$O_i^2 = \mu_{A_i}(x)\mu_{B_i}(y) = w_i \quad (29)$$

Layer 3: Implication "prod" operator. \$a_i\$ is the output MFs' area.

$$O_i^3 = w_i * (area) = a_i \quad (30)$$

Layer 4: Aggregation "sum" operator.

$$O_i^4 = \frac{a_i}{w_1 + w_2} = \frac{a_i}{\sum_{i=1}^n a_i} = \bar{a}_i \quad (31)$$

Layer 5: Defuzzification.

$$O_i^5 = y_i = \bar{a}_i z_i \quad (32)$$

Layer 6: Summation.

$$O_i^6 = \sum_{i=1}^n y_i \quad (33)$$

In MANFIS training main goal is to train output membership functions. In this work, output values are trained with Gradient Descent and Least Square Estimation methods. Afterwards, as a second step input membership functions are trained with the neural network.

3.1. Mamdani ANFIS Gradient Descent Method

Gradient method regulated for MANFIS where "y" is measured output values, "\$y^t\$" is calculated output values, "lr" is learning rate and \$W = r_i\$.

$$E = \frac{1}{2}(y - y^t)^2 \quad y = \bar{a}_i z_i \quad (34)$$

$$z(t + 1) = z(t) - lr \frac{\partial E}{\partial z_i} \tag{35}$$

$$\frac{\partial E}{\partial z_i} = (y - y^t) \bar{a}_i \tag{36}$$

$$z(t + 1) = z(t) - lr(y - y^t) \bar{a}_i \tag{37}$$

3.2. Mamdani ANFIS LSE Method

LSE method are very effective method in training that it is used in ANFIS, but it could not be able used in Mamdani. The $[\bar{w}_i \bar{a}_i]$ matrix that was given in (Chai et al., 2009) was a $[1, \text{rulenum}]$ matrix. So, the inverse of $[\bar{w}_i \bar{a}_i]$ matrix is not accurate.

In our Mamdani ANFIS training structure that is given in Figure 2 $[\bar{a}_i]$ matrix will be inverted. The $[\bar{a}_i]$ matrix are $[\text{rulenum}, \text{datanumber}]$. So, the matrix inverse can be calculated as accurate.

The Mamdani ANFIS LSE method steps are given below (Wesley Hines, 1997):

Layer 1: Membership functions generated.

$$O_i^1 = \mu_{A_i}(x) \quad O_i^1 = \mu_{B_i}(y) \tag{38}$$

Layer 2: "and-prod" method is used to find weights.

$$O_i^2 = \mu_{A_i}(x) \mu_{B_i}(y) = w_i \tag{39}$$

Layer 3: Triggered MF areas are calculated.

$$O_i^3 = w_i * (\text{area}) = a_i \tag{40}$$

Layer 4: Areas are normalized.

$$O_i^4 = \frac{a_i}{\sum_{i=1}^n a_i} = \bar{a}_i \tag{41}$$

Layer 5: The rule outputs are calculated.

$$O_i^5 = y_i = \bar{a}_i z_i \tag{42}$$

Layer 6: The aggregation step is represented that all output values are gathered.

$$O_i^6 = \sum_{i=1}^n y_i = a_1 z_1 + a_2 z_2 + a_3 z_3 \dots \tag{43}$$

$$y = [a_1 \quad a_2 \quad a_3] \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = XW \tag{44}$$

"X" is triggered MF areas and "W" is the output parameters. The triggered MF's and areas are known but the output parameters are not known. So the output parameters will be found.

$$y = XW \Rightarrow X^{-1}Y \tag{45}$$

If X is not invertible than pseudoinverse can be used. In this study, pseudoinverse is used.

4. Results and Discussion

In this section, the proposed MANFIS is compared to ANFIS for a nonlinear equation and soot emission prediction. The ANFIS algorithms has two stages as consequent and antecedent trainings. In this study, only consequent parameters are trained to show the effectiveness of the proposed MANFIS method. The antecedent

parameters are chosen as trapezoid membership functions. The scaling is given in Figure 3. Four trapezoid functions are used as input membership functions and they are scaled between zero and two.

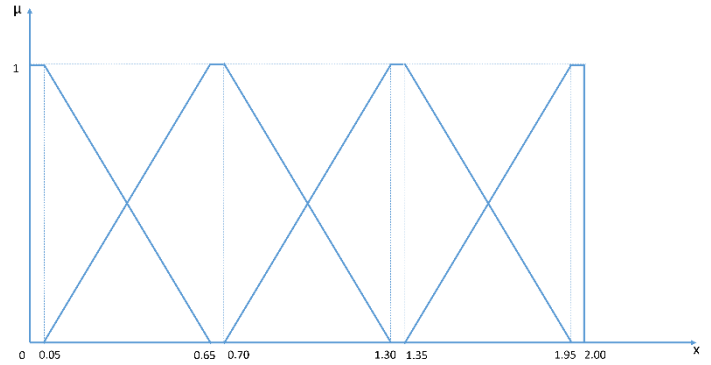


Figure 3. Scaling of antecedent parameters

4.1. Example 1: Modeling for three input nonlinear function

Three input nonlinear function is used to compare the performances of the methods. The used equation is given below (Jang, 1993; Shoorehdeli, Teshnehlab, Sedigh, & Khanesar, 2009). The equation system has three inputs as (x, y, z) and one output. The grid points are taken as $(x \in [1,6], y \in [1,6], z \in [1,6])$. So, there are 216 data pairs. As shown in Figure 3, there are 4 membership functions and so, there are 64 rules. Learning rate is $lr = 10^{-5}$ and for every iteration it changes. If the error becomes smaller the learning is increased by one thousandth, when becomes bigger the learning rate is decreased by one tenth.

$$y = (1 + x^{0.5} + y^{-1} + z^{-1.5})^2 \tag{46}$$

As shown in Table 1, the ANFIS has better results in LSE method. However, the MANFIS has less error for the same iteration number when compared to ANFIS. The gradient method is used for control studies like Neuro-fuzzy control (Öztürk & Özkol, 2021) however, the LSE can not be used. So, the gradient method has priority for evaluation. The training records for gradient descent methods are given in Figures 4 that the Gradient-based Mamdani ANFIS training gives faster reaction than Gradient based ANFIS training.

Table 1. Comparison of ANFIS and Mamdani ANFIS (Ex. 1)

Parameters	ANFIS Gradient	ANFIS LSE	MANFIS Gradient	MANFIS LSE
Training Steps	1000	2	1000	2
Training Error (RMSE)	0.82661	0	0.07688	0.07038
Simulation Time (sec)	2.34027	0.02717	4.75705	1.07010

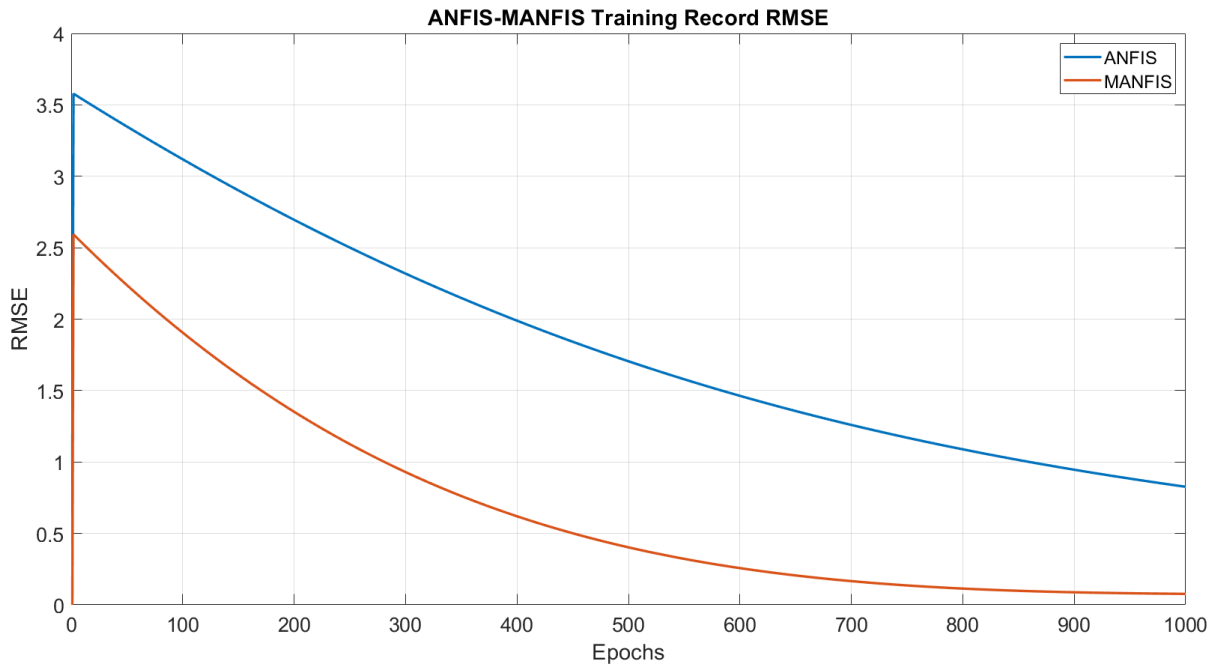


Figure 4. Gradient based ANFIS-MANFIS Training Error (Example 1)

4.2. Example 2: Soot Emission Prediction

In this example, a fuzzy model is trained between input parameters (Torque, Engine Speed, Lambda, EGR Ratio) and output parameter (Soot Emission). The Soot Emission Fuzzy Logic Structure is given in Figure 5. The same membership function and same structure is used in the example 2. ANFIS learning rate $lr = 10^{-5}$ diverged and so learning rate is started from $lr = 10^{-9}$. MANFIS worked properly for very big learning rates like $lr = 10^{-1}$, however, for the same conditions $lr = 10^{-9}$ is used for the both of ANFIS and MANFIS.

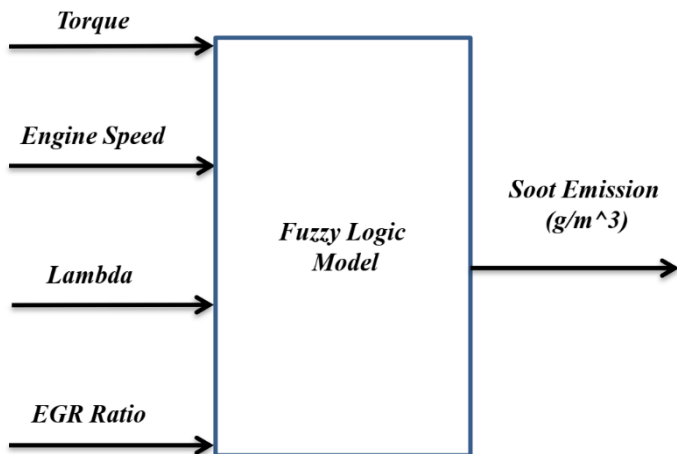


Figure 5. Soot Emission Fuzzy Logic Structure

ANFIS LSE is a convenient training approach for Sugeno and so as seen from the Table 2, the ANFIS results are better at training error. However, MANFIS LSE needs less time when

compared to ANFIS LSE. The main target in proposed training algorithm is the Gradient method. Especially in control studies, the Gradient method is a useful tool. When the Gradient methods compared, MANFIS gradient has less training error and less simulation time when compared to ANFIS gradient. So, this will bring the on-line training results to a much better level.

Table 2. Comparison of ANFIS and Mamdani ANFIS (Ex. 2)

Parameters	ANFIS Gradient	ANFIS LSE	MANFIS Gradient	MANFIS LSE
Training Steps	1000	2	1000	2
Training Error (RMSE)	28.4627	0.14100	1.03293	0.26825
Simulation Time (sec)	376.214	1.09041	162.881	0.08149

As seen in Figure 6, the Mamdani ANFIS error is less when compared to ANFIS. In Figure 7, measured data, ANFIS LSE results, Manfis Gradient results and Manfis LSE results are given. ANFIS Gradient was not included in the drawing because its results are meaningless. As seen in the Figure 7, all three methods can represent real models.

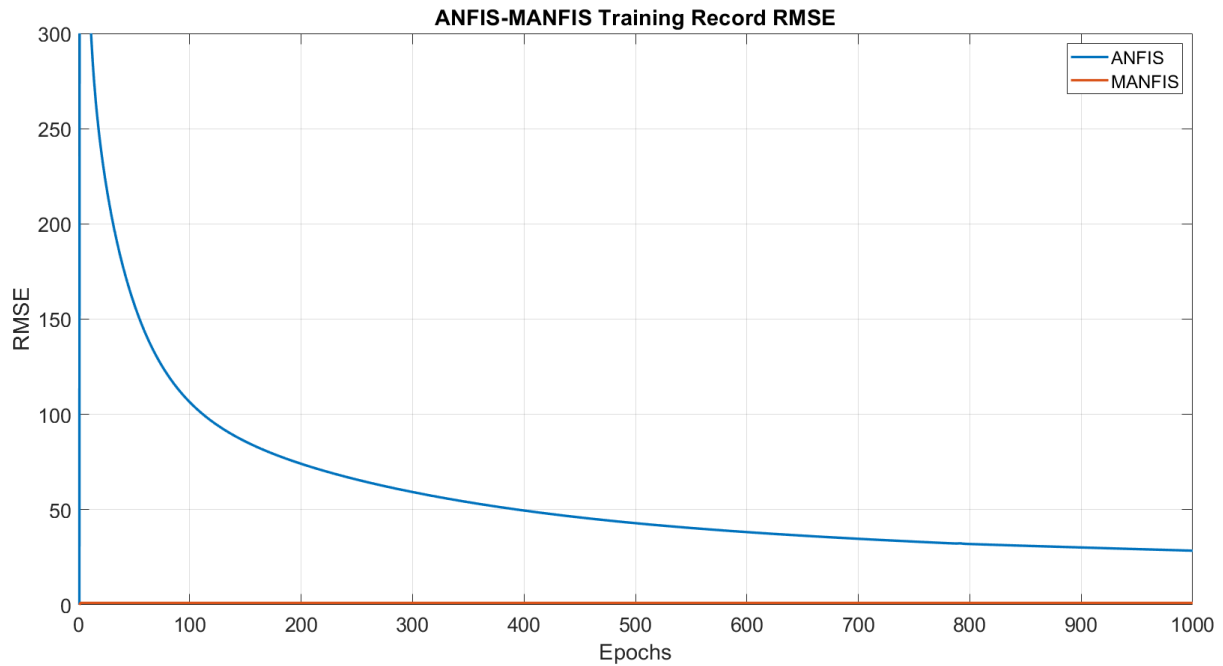


Figure 6. Gradient based ANFIS-MANFIS Training Error (Example 2)

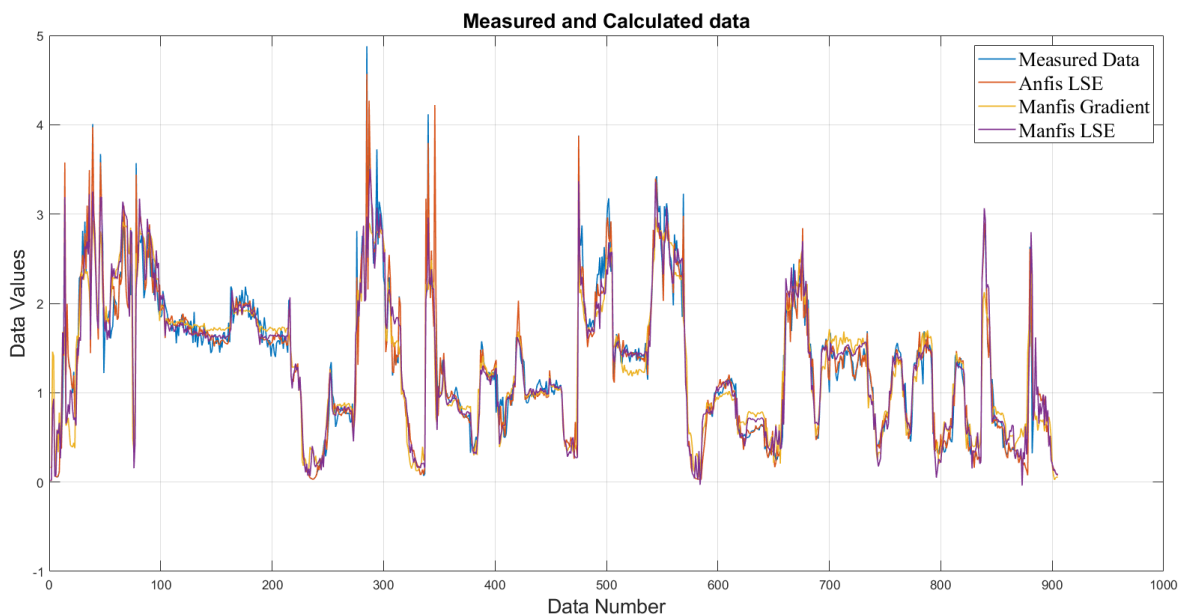


Figure 7. Results of Sugeno and Mamdani Training Algorithms

5. Conclusions and Recommendations

The ANFIS with LSE method is the best for training error and as seen from Figure 7 the three implementations are available to represent a model. However, the LSE method can not be used effectively as Gradient method in Neuro-Fuzzy control studies. The ANFIS and MANFIS structures with Gradient Methods are used in control studies in the continuation of this study and it is seen that MANFIS (especially Gradient method) can be used effectively at modelling any system.

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