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## Auto-Tuning by Using Double Extended Kalman-Bucy Filter: An Application to Dc Motor for Controlling Speed

Hakan KIZMAZ<sup>1\*</sup>

### Abstract

In this study, a modified adaptive control algorithm is proposed and investigated. The algorithm consists of a controller, an estimator and an auxiliary model like in model reference adaptive control strategy. PID controller is used to provide controlling. The controller includes adjustable parameters. Traditional PID controller parameters are usually set to fulfil the reference behaviour criterion. In this study, minimum-time criterion is chosen. Extended Kalman-Bucy estimator is employed for estimating controller parameters to make system behave like auxiliary model. The estimator adjusts the controller parameters so that system output can catch the reference input at minimum time. The study may call as the minimization of the settling time problem. The controller and estimator of the system are operated simultaneously. The achievement of the proposed algorithm is proved by simulation results including a simple dc motor model.

**Keywords:** DC motor, Extended Kalman-Bucy Filter, parameter estimation, PID control, adaptive control, speed control

### 1. INTRODUCTION

DC motors have some advantages such as stability, linearity and controllability in point of applicability of various control algorithms. They are also able to demonstrate a high performance [1], [2]. Motion control of DC motors with high accuracy are paid attention manufacturing and industrial applications [3], [4].

Speed control is an important issue for dc and ac motors. Since DC motors could provide an easy

speed or position control, they are used in many field where speed and position control are required such as industrial, robotic applications and home appliances [5]–[7].

The most used feedback control technique, PID (Proportional – Integral - Derivative) controller was defined with a formal mathematical rule by C. Minorsky a Russian American engineer in 1922[8]. PID controllers are mostly employed for industrial motor control applications due to their simplicity and efficiency [9]–[16]. However PID

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control with non-adaptive coefficient may not ensure acceptable high performance [17], [18]. In control engineering, various modern methods have been developed for dc motors, such as sliding mode control [6], [19], model reference adaptive control [20], observer based robust control [21], intelligent control [5], [22]. Each of them with their original characteristics has been implemented effectively. Adaptive control assures predefined output tracking performance and steady-state accuracy with the system uncertainties [3].

Adjusting the PID controller parameters allows the performance criterion to be maximized according to performance criterion such as minimum-time, minimum-energy etc. Although PID parameters could be adjusted manually, there exist some approaches to adjust the parameters. In [23] Ziegler-Nichols proposed a PID tuning formulation, based on time and frequency response of the system. Åström and Hägglund have a work about auto-tuning intending to simplify the applicability of Ziegler-Nichols method by relay method [24]. Schei proposed a technique for the auto-tuning of PID controllers [25]. Robust PID auto-calibration method is demonstrated in [26]. Poulin et al. presented an adaptive PID controller using an explicit identification with iterative parameter estimation [27].

Kalman filter is widely used estimation algorithm [28], [29]. The filter is able to estimate the states [30]–[32] and parameters [33] of the system by using and processing the sensor measurements. Kalman filter is capable of filtering the measurements of any sensor that may include white noise [34]. In case Kalman-Bucy filter estimates the states of any system and also the parameters of the system, the filter is called Extended Kalman-Bucy filter. The filter requires state-space model of the whole system. Including the controllable system and PID controller, can be combined as an only one system so that Extended Kalman-Bucy filter can be employed and estimate controller parameters.

In this study, an adaptive PID control algorithm is demonstrated. The adaptation of the PID controller parameter is provided by Extended

Kalman-Bucy filter with minimum-time optimization criterion. The proposed algorithm uses the Extended Kalman-Bucy filter as a parameter estimator to adjust the PID parameters. To try out the performance of the algorithm, the permanent magnet dc motor model has been used. Section 2 determine how to model of dc motor as a sample model. Section 3 demonstrates how to obtain combined system including the controller and the system in order to estimator can use. Section 4 explains how to apply the filter to the system. Section 5 includes the simulation results. Section 6 discusses the proposed algorithm in the study.

## 2. MODELING DC MOTOR

In this study, the dc motor is thought as a controllable object. To perform the controller system simulation, the mathematical model of the motor is required. The schematic diagram of dc motor is shown in Figure 1.

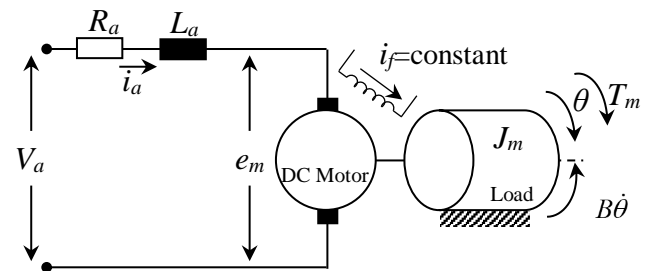


Figure 1. Schematic diagram of dc motor

Describing simulation model of dc motor requires system equations. DC motor model can be composed by following four equations.

The left side of the diagram in Figure 1 presents the armature circuit partition of dc motor. According to Kirchhoff's voltage law,

$$V_a(t) = R_a i_a(t) + L_a \frac{d}{dt} i_a(t) + e_m(t) \quad (1)$$

where  $V_a$  is applied voltage,  $R_a$  the electrical resistance of the armature circuit,  $i_a$  the armature current,  $L_a$  the electrical inductance of the armature circuit,  $e_m$  the electromotive force. The right side of the diagram presents mechanical partition of dc motor. According to Newton law,

$$T_m(t) = J \frac{d}{dt} \omega_r(t) + B\omega_r(t) + T_L(t) \quad (2)$$

where  $T_m$  the motor torque,  $J$  the moment of inertia,  $\omega_r$  the rotor angular speed,  $B$  mechanical system damping ratio or the motor viscous friction constant,  $T_L$  the load moment. The motor torque  $T_m$  has a proportional relation with armature current,  $i_a$ , by a coefficient  $K_a$ .

$$T_m(t) = K_a i_a(t) \quad (3)$$

DC motor electromotive force  $e_m$  is related to the angular speed  $\omega_r$ , by coefficient  $K_b$ .

$$e_m(t) = K_b \omega_r(t) \quad (4)$$

Substituting (3) and (4) into (1) and (2), the useful state space equations is obtained as (5) and (6)

$$\frac{d}{dt} \omega_r(t) = -\frac{B}{J} \omega_r(t) + \frac{K_a}{J} i_a(t) - \frac{1}{J} T_L(t) \quad (5)$$

$$\frac{d}{dt} i_a(t) = -\frac{K_b}{L_a} \omega_r(t) - \frac{R_a}{L_a} i_a(t) + \frac{1}{L_a} V_a(t) \quad (6)$$

Any linear time-invariant system is explained as follows

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (7)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \quad (8)$$

where  $\mathbf{A}$  the system matrix,  $\mathbf{B}$  control input coefficient matrix and  $\mathbf{C}$  observing matrix. (5) and (6) can be reformed like linear time invariant equations (7)-(8). Hence the state-space form of the dc motor system is explained as (9)-(10) [35].

$$\begin{bmatrix} \dot{\omega}_r(t) \\ \dot{i}_a(t) \end{bmatrix} = \begin{bmatrix} -\frac{B}{J} & \frac{K_a}{J} \\ -\frac{K_b}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} \omega_r(t) \\ i_a(t) \end{bmatrix} + \begin{bmatrix} -\frac{1}{J} & 0 \\ 0 & \frac{1}{L_a} \end{bmatrix} \begin{bmatrix} T_L \\ V_a \end{bmatrix} \quad (9)$$

$$y = [1 \quad 0] \begin{bmatrix} \omega_r \\ i_a \end{bmatrix} \quad (10)$$

### 3. COMBINED SYSTEM

PID controllers are mostly used in industrial applications due to simplicity and practicability. Here, PID controller is described for a DC motor model. The input of the DC motor is symbolized as  $V_a$ . It defines the control voltage applying to the motor and PID controller output signal as well. PID controller is then described as follows:

$$V_a = K_p e(t) + K_i \int e(t) dt + K_d \dot{e}(t) \quad (11)$$

$$e(t) = \bar{\omega}_r(t) - \omega_r(t) \quad (12)$$

where  $V_a$  the controller output,  $K_p$  proportional gain,  $K_i$  integral gain,  $K_d$  derivative gain,  $\bar{\omega}_r$  desired or reference angular speed.  $\omega_r$  is actual angular speed.  $e(t)$  is the angular speed error which consists of the difference between the reference input and system output as seen in (12). Figure 2 demonstrates how to apply a PID controller to a dc motor speed control system.  $\omega_{r\_mec}$  is mechanical angular speed in Figure 2 [36]. The measurement of  $\omega_r$  is generally performed by encoders or tachometers in practically. As a mechanical magnitude  $\omega_{r\_mec}$  is transformed into electrical magnitude by speed measuring block.

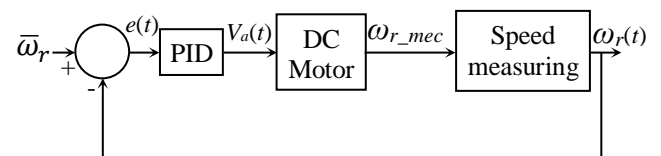


Figure 2. Overall, dc motor control system

If (11) is substituted into (6), (13) is obtained,

$$\frac{d}{dt} i_a = -\frac{K_b}{L_a} \omega_r - \frac{R_a}{L_a} i_a + \frac{1}{L_a} \begin{bmatrix} K_p e(t) + \\ + K_i \int e(t) dt + \\ + K_d \frac{d}{dt} e(t) \end{bmatrix} \quad (13)$$

Substituting (12) into (13) demonstrates the combined control system including PID controller and dc motor. The integral of the error  $\int e(t) dt$  is

deemed as a new state variable of the whole controlled system. Finally combined system, including PID controller and the dc motor, is described by (14)-(15) as a linear time invariant

$$\frac{d}{dt} \begin{bmatrix} \int e(t)dt \\ \omega_r \\ i_a \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & -\frac{B}{J} & \frac{K_a}{J} \\ \frac{K_i}{L_a} & \left[-\frac{K_b}{L_a} - \frac{K_p}{L_a} + \frac{K_d B}{L_a J}\right] & \left[-\frac{R_a}{L_a} - \frac{K_d K_a}{L_a J}\right] \end{bmatrix} \begin{bmatrix} \int e(t)dt \\ \omega_r \\ i_a \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{J} & 0 & 0 \\ \frac{K_d}{L_a} & \frac{1}{J} & \frac{K_p}{L_a} \end{bmatrix} \begin{bmatrix} T_L \\ \bar{\omega}_r \\ \dot{\bar{\omega}}_r \end{bmatrix} \quad (14)$$

$$y = [0 \quad 1 \quad 0] \begin{bmatrix} \int e(t)dt \\ \omega_r \\ i_a \end{bmatrix} \quad (15)$$

#### 4. EXTENDED KALMAN-BUCY FILTER

Richard Bucy and et. al. developed the continuous-time Kalman filter as an unpublished work in John Hopkins Applied Physics Lab. in the late 1950s. Meantime Rudolf Kalman developed the discrete Kalman filter in 1960 [28]. Kalman and Bucy got aware of each other's work in April 1960 and cooperated to the publish of [29]. The work is sometimes called as Kalman-Bucy filter [37].

Kalman-Bucy filter is not widely used in practice owing to inapplicability in digital computers. However, the continuous-time filter provides how to make a state or parameter estimation [34]. In order to estimate the states and parameters of the system simultaneously, the whole system must be extended with parameters and linearized according to states and parameters. This kind of filter is called as Extended Kalman Filter. Here Extended Kalman Filter is going to be employed for estimating the PID parameters,  $K_p$ ,  $K_d$ ,  $K_i$  existing in (14) as new states of the whole system. Beside load moment  $T_L$  is needed to be estimated due to difficulty of measurement. Extended Kalman filter is presented in this study directly. The reader is referred to [28], [34], [37]–[39] for Kalman-Bucy theoretical details.

The continuous-time extended Kalman filter is written as

system. Extended Kalman-Bucy filter needs system state-space model like given below. The filter deems,  $K_p$ ,  $K_i$ ,  $K_d$  coefficients as system unknown parameters and estimate them.

$$\dot{\hat{\mathbf{x}}}' = f(\hat{\mathbf{x}}', \mathbf{u}') + \mathbf{P}(t)\mathbf{H}^T \mathbf{R}^{-1}(\bar{\mathbf{y}} - \mathbf{H}\hat{\mathbf{x}}') \quad (16)$$

$$\dot{\mathbf{P}} = \begin{pmatrix} \mathbf{F}\mathbf{P} + \mathbf{P}\mathbf{F}^T - \\ -\mathbf{P}\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}\mathbf{P} + \mathbf{G}\mathbf{Q}\mathbf{G}^T \end{pmatrix} \quad (17)$$

where  $\hat{\mathbf{x}}'$  estimated states vector,  $\mathbf{H}$  observer output matrix,  $\bar{\mathbf{y}}$  measurable states vector,  $\mathbf{P}$  is error covariance matrix.  $\mathbf{Q}$  and  $\mathbf{R}$  are the system and input white noise variance matrices, respectively.

##### 4.1. Load Moment Estimator

Measuring the load moment of dc motor has difficulties during the dc motor run. Load moment then can be estimated by using estimator. The estimator used here is Extended Kalman-Bucy filter. Unlike the indices in (16) and (17),  $L$  is used here as the load moment indice. The load moment estimator equations are then

$$\dot{\hat{\mathbf{x}}}'_L = f(\hat{\mathbf{x}}'_L, \mathbf{u}'_L) + \mathbf{P}_L \mathbf{H}_L^T \mathbf{R}_L^{-1}(\bar{\mathbf{y}}_L - \mathbf{H}_L \hat{\mathbf{x}}'_L) \quad (18)$$

$$\dot{\mathbf{P}}_L(t) = \begin{pmatrix} \mathbf{F}_L \mathbf{P}_L + \mathbf{P}_L \mathbf{F}_L^T - \\ -\mathbf{P}_L \mathbf{H}_L^T \mathbf{R}_L^{-1} \mathbf{H}_L \mathbf{P}_L + \mathbf{G}_L \mathbf{Q}_L \mathbf{G}_L^T \end{pmatrix} \quad (19)$$

Another estimator is used for estimating the PID controller coefficients. Considering the load moment,  $T_L$ , in (9) as a parameter of the dc motor, demonstrates (20) as system functions to be used for composing load moment estimator equations.

$$\dot{\mathbf{x}}'_L(t) = f(\mathbf{x}'_L, \mathbf{u}'_L) = \begin{bmatrix} \frac{d}{dt} \omega_r(t) = -\frac{B}{J} \omega_r(t) + \frac{K_a}{J} i_a(t) - \frac{T_L(t)}{J} \\ \frac{d}{dt} i_a(t) = -\frac{K_b}{L_a} \omega_r(t) - \frac{R_a}{L_a} i_a(t) + \frac{1}{L_a} V_a(t) \\ \frac{d}{dt} T_L = w_{T_L} \end{bmatrix} \quad (20)$$

$$\mathbf{y}_L = \mathbf{H}_L \mathbf{x}'_L + \mathbf{v}_L \quad (21)$$

where  $\mathbf{x}'_L$  the system state vector,  $w_{T_L}$  the artificial noise term added to the system so that Kalman filter can modify the estimate of  $T_L$  parameter. Moreover  $\mathbf{v}_L(t) \in R^{1 \times 1}$  is output white noise vector. The  $\mathbf{x}'_L$ ,  $\mathbf{u}'_L$  and  $\mathbf{H}_L$  is defined as follows

$$\mathbf{x}'_L = \begin{bmatrix} \omega_r \\ i_a \\ T_L \end{bmatrix}, \mathbf{u}'_L = \begin{bmatrix} V_a(t) \\ w_{T_L} \end{bmatrix}, \mathbf{H}_L = [1 \quad 0 \quad 0]$$

The load moment parameter is considered as the new states of the whole control system. It is required to find the partial derivative matrices, first as follows

$$F_L = \left. \frac{\partial f(\mathbf{x}'_L, \mathbf{u}'_L)}{\partial \mathbf{x}'_L} \right|_{\hat{\mathbf{x}}'_L, \mathbf{u}'_L} = \begin{bmatrix} -\frac{B}{J} & \frac{K_a}{J} & -\frac{1}{J} \\ -\frac{K_b}{L_a} & -\frac{R_a}{L_a} & 0 \\ 0 & 0 & 0 \end{bmatrix}_{\hat{\mathbf{x}}'_L, \mathbf{u}'_L} \quad (22)$$

$$G_L = \left. \frac{\partial f(\mathbf{x}'_L, \mathbf{u}'_L)}{\partial \mathbf{u}'_L} \right|_{\hat{\mathbf{x}}'_L, \mathbf{u}'_L} = \begin{bmatrix} 0 & 0 \\ \frac{1}{L_a} & 0 \\ 0 & 1 \end{bmatrix}_{\hat{\mathbf{x}}'_L, \mathbf{u}'_L} \quad (23)$$

(22) and (23) are substituted into (19) the equations get run in the estimator algorithm.

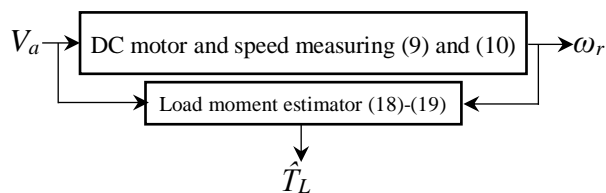


Figure 3. Load moment estimator

### 4.2. PID Coefficients Estimator

The extended form of the system state functions are defined as (24) - (25). Here  $\mathbf{x}'$  the system state vector,  $w_p$ ,  $w_i$ ,  $w_d$  the artificial noise terms added to the system so that Kalman filter can modify the estimate of PID parameters. Moreover  $\mathbf{v}(t) \in R^{2 \times 1}$  is output white noise vector. The  $\mathbf{x}'$ ,  $\mathbf{u}'$  and  $\mathbf{H}$  is defined as follows

$$\hat{\mathbf{x}}' = \left[ \int e(t)dt \quad \omega_r \quad i_a \quad K_p \quad K_i \quad K_d \right]$$

$$\mathbf{u}' = \left[ T_L \quad \bar{\omega}_r \quad \dot{\bar{\omega}}_r \quad w_p \quad w_i \quad w_d \right]^T$$

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The PID parameters are obviously considered as the new states of the whole control system. It is required to find the partial derivative matrices first as (26) - (27). The  $\mathbf{I}$  is identity cubic square matrix. The reason of using Extended Kalman filter is to estimate the controller parameters. However, controller needs a reference rule or criterion. The criterion is chosen as minimum-time criterion. Time-optimized system is used as an auxiliary reference model. The ideal transfer function of reference auxiliary model is expected as a constant. In other words, the settling time of any ideal time-optimized system is supposed to be zero.

$$\dot{\mathbf{x}}'(t) = f(\mathbf{x}', \mathbf{u}') = \begin{bmatrix} \frac{d}{dt} \int e(t) dt = -\omega_r + \bar{\omega}_r \\ \frac{d}{dt} \omega_r = -\frac{B}{J} \omega_r + \frac{K_a}{J} i_a - \frac{1}{J} T_L \\ \frac{d}{dt} i_a = \frac{K_i}{L_a} \int e(t) dt + \left[ -\frac{K_b}{L_a} - \frac{K_p}{L_a} + \frac{K_d B}{L_a J} \right] \omega_r + \left[ -\frac{R_a}{L_a} - \frac{K_d K_a}{L_a J} \right] i_a + \frac{K_d}{L_a} \frac{1}{J} T_L + \frac{K_p}{L_a} \bar{\omega}_r + \frac{K_d}{L_a} \dot{\bar{\omega}}_r \\ \frac{d}{dt} K_p = w_p \\ \frac{d}{dt} K_i = w_i \\ \frac{d}{dt} K_d = w_d \end{bmatrix} \quad (24)$$

$$\mathbf{y} = \mathbf{H}\mathbf{x}' + \mathbf{v} \quad (25)$$

$$\mathbf{F} = \frac{\partial f}{\partial \mathbf{x}'} \bigg|_{\mathbf{x}', \mathbf{u}'} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{B}{J} & \frac{K_a}{J} & 0 & 0 & 0 \\ \frac{K_i}{L_a} \left[ -\frac{K_b}{L_a} - \frac{K_p}{L_a} + \frac{K_d B}{L_a J} \right] & \left[ -\frac{R_a}{L_a} - \frac{K_d K_a}{L_a J} \right] & \left[ -\frac{1}{L_a} \omega_r + \frac{1}{L_a} \bar{\omega}_r \right] & \left[ \frac{1}{L_a} \int e(t) dt \right] & \left[ \frac{1}{L_a} \frac{B}{J} \omega_r - \frac{1}{L_a} \frac{K_a}{J} i_a + \frac{1}{L_a} \frac{1}{J} T_L \right] \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \bigg|_{\mathbf{x}', \mathbf{u}'} \quad (26)$$

$$\mathbf{G} = \frac{\partial f}{\partial \mathbf{u}'} \bigg|_{\mathbf{x}', \mathbf{u}'} = \begin{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{J} & 0 & 0 \end{bmatrix} & [0]_{3 \times 3} \\ \begin{bmatrix} \frac{K_d}{L_a} \frac{1}{J} & \frac{K_p}{L_a} & \frac{K_d}{L_a} \end{bmatrix} & \\ [0]_{3 \times 3} & \mathbf{I}_{3 \times 3} \end{bmatrix} \quad (27)$$

The input of the auxiliary system is reference angular speed and estimated load moment. According to the DC motor state equations (5)-(6) the derivatives of the state variables become zero when the steady state occurs. From (5) and (6), the equilibrium point elements of the system is considered as  $(\bar{\omega}_r, \bar{i}_a)$ . The state variables become on equilibrium point on the steady state. The derivatives of the system state vector then become zero. From (5) and (6)

$$0 = -\frac{B}{J} \bar{\omega}_r(t) + \frac{K_a}{J} \bar{i}_a(t) - \frac{T_L(t)}{J}$$

$$\bar{i}_a(t) = \frac{B \bar{\omega}_r(t) + T_L(t)}{K_a} \quad (28)$$

Hence, equilibrium point or the output vector of the auxiliary system is defined as

$$\bar{\mathbf{y}} = (\bar{\omega}_r, \bar{i}_a) = \left( \bar{\omega}_r, \frac{B \bar{\omega}_r(t) + T_L(t)}{K_a} \right) \quad (29)$$

(29) demonstrates the outputs of ideal reference auxiliary model. The vector is used in Kalman filter algorithm as  $\bar{\mathbf{y}}$  defined before in (16). The controller coefficient estimator takes the outputs of ideal auxiliary model as if they are the measured outputs of system. Thus, the estimator will estimate the controller coefficients based on the behaviour of the reference model. Figure 4 demonstrates the auto-tuning block diagram about how to apply all equations to the control system. The block diagram in the figure includes dc motor model, PID controller, load estimator, auxiliary ideal reference model and Extended Kalman-Bucy controller parameters estimator.

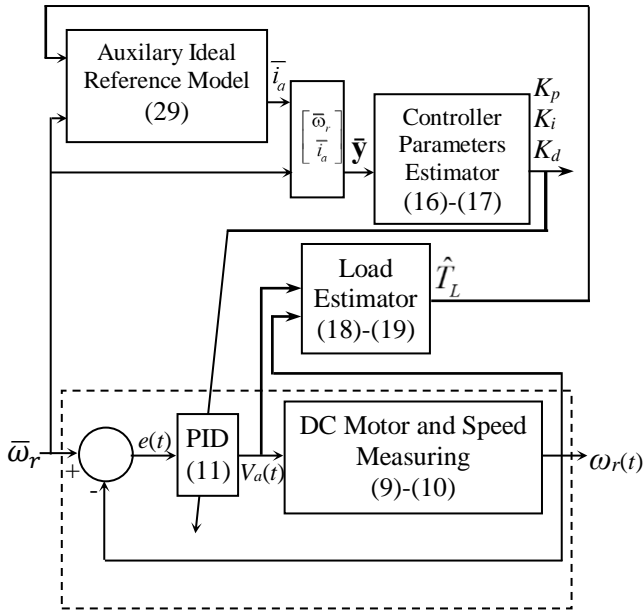


Figure 4. Auto-Tuning block diagram using Extended Kalman-Bucy Filter

### 5. SIMULATION RESULTS

Some illustrative results are presented here related to auto-tuning using Kalman-Bucy algorithm explained in the previous section. Motor model parameters and initial conditions are chosen as below [11].

- Voltage supply=110 V                      Rated current=10 A
- $J=0.002215 \text{ kg}\cdot\text{m}^2$                        $B=0.002953 \text{ N}\cdot\text{m}\cdot\text{s}$
- $K_b=1.28 \text{ V}/(\text{rad}/\text{s})$                        $K_a=1.28 \text{ N}\cdot\text{m}/\text{A}$
- $R_a=11.2 \Omega$                                    $L_a=0.1215 \text{ H}$

Figure 5 demonstrates the step response of the dc motor.  $V_a=1 \text{ V}$  is applied to motor as an input signal. It seems uncontrolled dc motor does not have an appropriate settling time. The speed of dc motor could be controlled by any controller such as traditional PID controller. The adjustment of PID controller parameters determines how the speed of DC motor behaves until the actual speed reaches the reference speed. Ziegler-Nichols [23] Åström-Hägglund [24] intended to find optimal PID parameters by using the time and frequency response of the system. This study intends to optimize PID parameters online, using extended

Kalman-Bucy filter at the minimum settling time of the speed of DC motor.

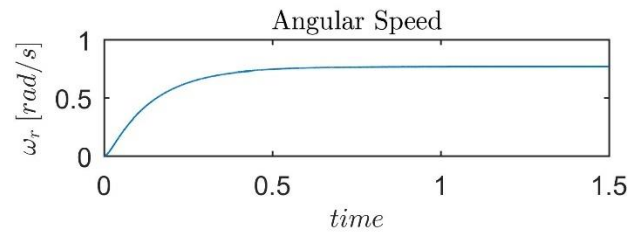


Figure 5. Dc motor step response

DC motor control system in Figure 4 has been simulated in Matlab. The initial conditions and parameters of the Kalman-Bucy filter are given as follows.

$$\hat{\mathbf{x}}'_L(0) = [0 \ 0 \ 0]^T,$$

$$\mathbf{Q}_L = \text{diag}([1 \ 1]),$$

$$\mathbf{R}_L = 1,$$

$$\mathbf{P}_L(0) = \text{diag}([1 \ 1 \ 10^6]),$$

$$\hat{\mathbf{x}}'(0) = [0 \ 0 \ 0 \ 0 \ 0 \ 0]^T,$$

$$\mathbf{Q} = \text{diag}([1 \ 1 \ 1 \ 1 \ 1]), \mathbf{R} = \text{diag}([1 \ 1]),$$

$$\mathbf{P}(0) = \text{diag}([1 \ 1 \ 1 \ 10^6 \ 10^6 \ 10^6])$$

The initial states of all controller parameters are in estimate vector  $\hat{\mathbf{x}}'(0)$  and chosen as zero.

Step response simulation results of the whole control system is demonstrated in Figure 6. The reference speed  $\bar{\omega}_r$  has been chosen as 1 rad/s and applied to the control system at 0.5th second. The actual speed  $\omega_r$  seems to be settled about in 1 second. Angular speed error graph demonstrates the error between reference speed and actual angular speed. Reference speed and load moment is applied to the rotor simultaneously. The load moment has been chosen as 1 Nm. The algorithm estimates the load moment as well about in 0.5 second. The PID parameters are estimated simultaneously once the algorithm runs.



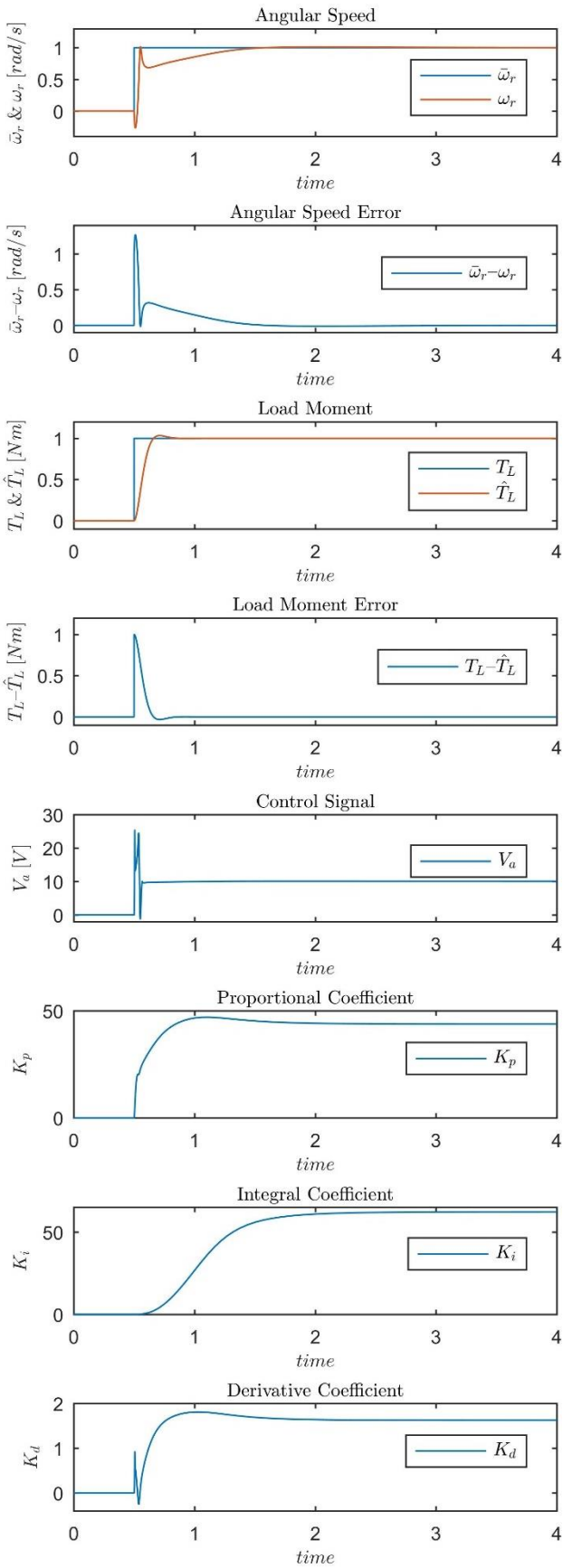


Figure 6. The step response simulation results of dc motor control system for load moment test ( $T_L=1$  Nm)

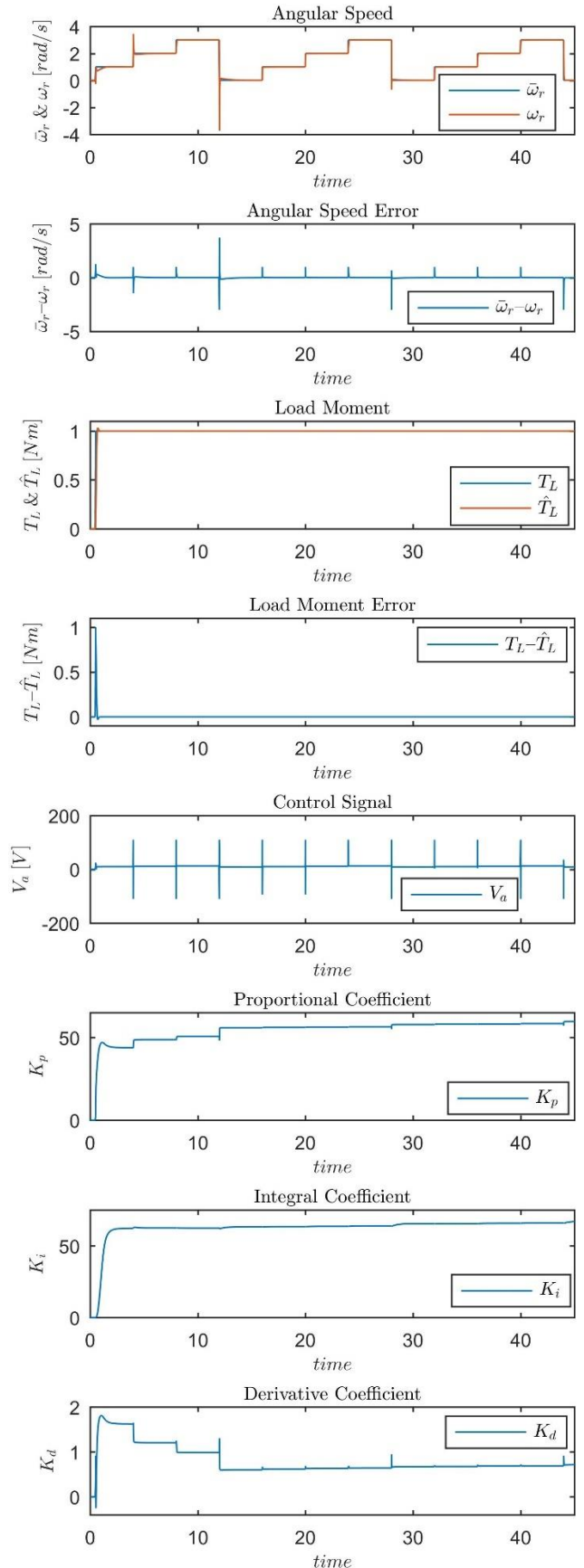


Figure 7. The simulation results with load for varying reference speed in different time intervals

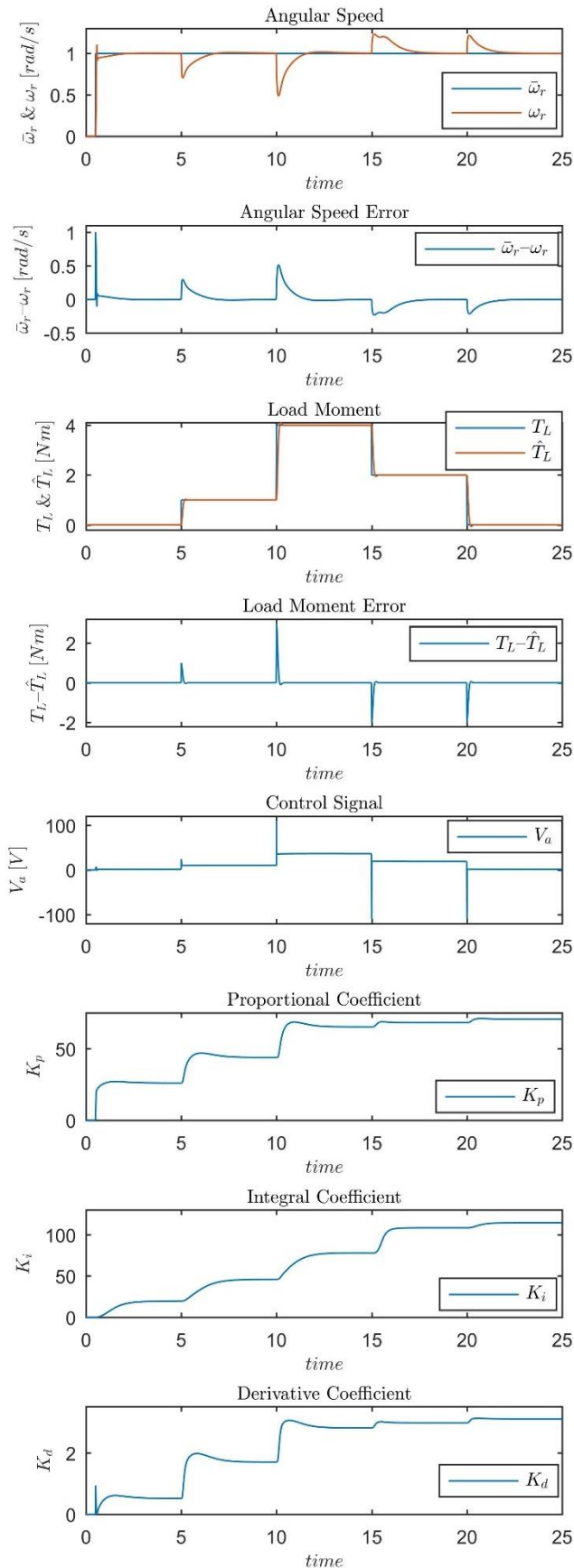


Figure 8. The simulation results with constant reference speed in different time intervals

Figure 7 demonstrates the simulation results that the reference speed  $\bar{\omega}_r$  is taken as different values for different time intervals to examine the robustness of the algorithm. Reference speed  $\bar{\omega}_r = 1$  rad/s is applied to the system at the beginning. The reference speed is then changed for every 4 second. The actual speed  $\omega_r$  seems to be able to converge on the reference speed  $\bar{\omega}_r$ . Angular speed error graph demonstrates that error decreases over time.

Figure 8 includes the step response results of the control system with applying varying load moment to the rotor. The load moment is  $T_L=0$  Nm at the beginning. Load moment is then increased to 1 Nm at the fifth second. Rotor speed seems to be affected and decreased at that time suddenly. However, the speed then increases to reference speed. After load moment increased, the PID parameters are being adjusted by the Extended Kalman-Bucy algorithm as well. According to error graphs, angular speed and load moment errors decrease over time.

## 6. CONCLUSION

Traditional PID controllers do not include adaptive parameters. The parameters are tuned by some traditional auto-tuning methods such as aforementioned above. In order the tune the controller, the step response or frequency response of the system which is desired to be controlled, is required. However, in this study, the PID controller is tuned by Extended Kalman-Bucy filter according to optimal time criterion. The estimator does not need the step or frequency response etc. except the system model. The effectiveness of the whole control algorithm showed up in the illustrative simulation results.

The controller may need high power to reach the reference speed in minimum-time according to optimal-time criterion. In such a situation, the auxiliary reference model can be chosen as a simple first order transfer function instead of a constant model. The controller can imitate the behaviour of chosen first order system. The study guides how to employ Extended-Kalman-Bucy filter to obtain model reference adaptive control system.

In this paper, as a sample model, DC motor, has been used to test the algorithm. Alternatively, any linear or nonlinear system which is desired to be controlled could be chosen to analysis the algorithm. Especially, Expanded Kalman-Bucy filter is useful for non-linear system control due to linearizable. The technique in this paper, can be applied to another type control techniques such as sliding mode control etc. In addition, it is believed that the method will be more useful in systems with short time constant.

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No conflict of interest or common interest has been declared by the author.

### ***Author's Contribution***

The author H.K. contributed 100% to the study about data processing, analysis and interpretation, intellectual/critical arrangements regarding content of the study draft, preparation the work for publication.

### ***The Declaration of Ethics Committee Approval***

This study does not require ethics committee permission or any special permission.

### ***The Declaration of Research and Publication Ethics***

The author of the paper declares that he complies with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that he

does not make any falsification on the data collected. In addition, he declares that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

## **REFERENCES**

- [1] K. V. C. R. V Rajasekhar, "Design and Analysis of DC Motor With PID Controller - A State Space Approach," *ITSI Trans. Electr. Electron. Eng.*, vol. 1, no. 3, pp. 11–14, 2013.
- [2] P. M. Meshram and R. G. Kanojiya, "Tuning of PID Controller using Ziegler-Nichols Method for Speed Control of DC Motor," in *2013 IEEE International Conference on Control Applications (CCA)*, 2012, pp. 117–122.
- [3] Y. Yang, Y. Wang, and P. Jia, "Adaptive robust control with extended disturbance observer for motion control of DC motors," *Electron. Lett.*, vol. 51, no. 22, pp. 1761–1763, 2015, doi: 10.1049/el.2015.1009.
- [4] H. Chu, B. Gao, W. Gu, and H. Chen, "Low-Speed Control for Permanent-Magnet DC Torque Motor Using Observer-Based Nonlinear Triple-Step Controller," *IEEE Trans. Ind. Electron.*, vol. 64, no. 4, pp. 3286–3296, 2017, doi: 10.1109/TIE.2016.2598298.
- [5] O. Prem, "Intelligent Speed Control of DC Servo Motor Drive," pp. 3029–3033, 2018.
- [6] S. Rakhonde and V. Kulkarni, "Sliding mode controller (SMC) governed speed control of DC motor," *2018 3rd IEEE Int. Conf. Recent Trends Electron. Inf. Commun. Technol. RTEICT 2018 - Proc.*, pp. 1657–1662, 2018, doi: 10.1109/RTEICT42901.2018.9012572.
- [7] M. Mallareddy, "Application of Bio

- geography based Fractional order PID controller in DC motor drive speed control,” 2020.
- [8] L. Samir, G. Said, and S. Youcef, “PID Control of DC Servo Motor using a Single Memory Neuron,” no. October, pp. 25–27, 2018.
- [9] W. G. M. Elnaim and S. F. Babiker, “Comparative study on the speed of DC motor using PID and FLC,” in *Proceedings of 2016 Conference of Basic Sciences and Engineering Studies, SGCAC 2016*, 2016, pp. 24–29, doi: 10.1109/SGCAC.2016.7458001.
- [10] Y. Ma, Y. Liu, and C. Wang, “Design of parameters self-tuning fuzzy PID control for DC motor,” 2010, vol. 2, pp. 345–348, doi: 10.1109/ICINDMA.2010.5538300.
- [11] S. K. Suman and V. K. Giri, “Speed Control of DC Motor Using Different Optimization Techniques Based PID Controller,” *IEEE Int. Conf. Eng. Technol.*, vol. 2, no. 7, pp. 6488–6494, 2012.
- [12] P. A. Adewuyi, “DC Motor Speed Control: A Case between PID Controller and Fuzzy Logic Controller,” *Int. J. Multidiscip. Sci. Eng.*, vol. 4, no. 4, pp. 36–40, 2013.
- [13] M. Jaiswal and M. P. H. O. D. Ex, “Speed Control of DC Motor Using Genetic Algorithm Based PID Controller,” vol. 3, no. 7, pp. 247–253, 2013.
- [14] M. D. Amanullah, M. Jain, P. Tiwari, S. Gupta, and G. Kumari, “Optimization of PID Parameter for Position Control of DC-Motor using Multi-Objective Genetic Algorithm,” *Int. J. Innov. Res. Electr. Instrum. Control Eng.*, vol. 2, no. 6, pp. 1644–1650, 2014.
- [15] B. Hekimoglu, “Optimal Tuning of Fractional Order PID Controller for DC Motor Speed Control via Chaotic Atom Search Optimization Algorithm,” *IEEE Access*, vol. 7, pp. 38100–38114, 2019, doi: 10.1109/ACCESS.2019.2905961.
- [16] S. Ekinçi, D. Izci, and B. Hekimoğlu, “PID Speed Control of DC Motor Using Harris Hawks Optimization Algorithm,” *2020 Int. Conf. Electr. Commun. Comput. Eng.*, vol. 2, no. June, pp. 3–8, 2020, doi: 10.1109/ICECCE49384.2020.9179308.
- [17] L. Harnefors, S. E. Saarakkala, and M. Hinkkanen, “Speed control of electrical drives using classical control methods,” *IEEE Trans. Ind. Appl.*, vol. 49, no. 2, pp. 889–898, 2013, doi: 10.1109/TIA.2013.2244194.
- [18] J. Han, “From PID to active disturbance rejection control,” *IEEE Trans. Ind. Electron.*, vol. 56, no. 3, pp. 900–906, 2009, doi: 10.1109/TIE.2008.2011621.
- [19] A. Damiano, G. L. Gatto, I. Marongiu, and A. Pisano, “Second-order sliding-mode control of dc drives,” *IEEE Trans. Ind. Electron.*, vol. 51, no. 2, pp. 364–373, 2004, doi: 10.1109/TIE.2004.825268.
- [20] Y. Shao and J. Li, “Modeling and Switching Tracking Control for a Class of Cart-Pendulum Systems Driven by DC Motor,” *IEEE Access*, vol. 8, pp. 44858–44866, 2020, doi: 10.1109/ACCESS.2020.2978269.
- [21] J. Yao, Z. Jiao, and D. Ma, “Adaptive robust control of dc motors with extended state observer,” *IEEE Trans. Ind. Electron.*, vol. 61, no. 7, pp. 3630–3637, 2014, doi: 10.1109/TIE.2013.2281165.
- [22] S. A. Hamoodi, I. I. Sheet, and R. A. Mohammed, “A Comparison between PID controller and ANN controller for speed control of DC Motor,” *2nd Int. Conf. Electr. Commun. Comput. Power Control Eng. ICECCPCE 2019*, pp. 221–224, 2019, doi: 10.1109/ICECCPCE46549.2019.203777.
- [23] N. Y. R. J.G. Ziegler, N.B. Nichols, “Optimum Settings for Automatic

- Controller,” *Trans. A.S.M.E.*, pp. 759–768, 1942, doi: 10.1115/1.2899060.
- [24] K. J. Åström and T. Hägglund, “Automatic Tuning of Simple Regulators with Specificaiotns on Phase and Amplitude Margins,” *Automatica*, vol. 20, no. 5, pp. 645–651, 1984.
- [25] T. S. Schei, “A method for closed loop automatic tuning of PID controllers,” *Automatica*, vol. 28, no. 3, pp. 587–591, 1992, doi: 10.1016/0005-1098(92)90182-F.
- [26] A. A. Voda and I. D. Landau, “A method for the auto-calibration of PID controllers,” *Automatica*, vol. 31, no. 1, pp. 41–53, 1995, doi: 10.1016/0005-1098(94)00067-S.
- [27] É. Poulin, A. Pomerleau, A. Desbiens, and D. Hodouin, “Development and evaluation of an auto-tuning and adaptive PID controller,” *Automatica*, vol. 32, no. 1, pp. 71–82, 1996, doi: 10.1016/0005-1098(95)00105-0.
- [28] R. E. Kalman, “A new approach to linear filtering and prediction problems,” *Trans. ASME-Journal Basic Eng.*, vol. 82, no. Series D, pp. 35–45, 1960, doi: 10.1115/1.3662552.
- [29] R. E. Kalman and R. S. Bucy, “New results in linear filtering and prediction theory,” *J. Basic Eng.*, vol. 83, no. 1, pp. 95–108, 1961, doi: 10.1115/1.3658902.
- [30] P. Deshpande and A. Deshpande, “Inferential control of DC motor using Kalman Filter,” *2012 2nd Int. Conf. Power, Control Embed. Syst.*, pp. 1–5, 2012, doi: 10.1109/ICPCES.2012.6508056.
- [31] A. Khalid and A. Nawaz, “Sensor less control of DC motor using Kalman filter for low cost CNC machine,” *2014 Int. Conf. Robot. Emerg. Allied Technol. Eng.*, pp. 180–185, 2014, doi: 10.1109/iCREATE.2014.6828362.
- [32] Z. Aydogmus and O. Aydogmus, “A comparison of artificial neural network and extended Kalman filter based sensorless speed estimation,” *Measurement*, vol. 63, pp. 152–158, 2015, doi: 10.1016/j.measurement.2014.12.010.
- [33] A. H. Z. Farnaz, H. S. Sajith, P. J. Binduhewa, M. P. B. Ekanayake, and B. G. L. T. Samaranyake, “Low cost torque estimator for DC servo motors,” *2015 IEEE 10th Int. Conf. Ind. Inf. Syst. ICIIS 2015 - Conf. Proc.*, pp. 187–192, 2016, doi: 10.1109/ICIINFS.2015.7399008.
- [34] G. M. Siouris, *An Engineering Approach to Optimal Control and Estimation Theory*. New York: John Wiley & Sons, 1996.
- [35] P. C. Krause, O. Wasynczuk, and S. D. Sudhoff, *Analysis of Electric Machinery and Drive Systems*, 2nd ed. New York: John Wiley & Sons, 2002.
- [36] A. A. A. El-Gammal and A. A. El-Samahy, “Adaptive Tuning of a PID Speed Controller for DC Motor Drives Using Multi-Objective Particle Swarm Optimization,” in *Uksim 2009: Eleventh International Conference on Computer Modelling and Simulation*, 2009, pp. 398–404.
- [37] D. Simon, *Optimal State Estimation: Kalman,  $H_\infty$ , and Nonlinear Approaches*. New Jersey, 2006.
- [38] R. F. Stengel, *Optimal control and estimation*. New York: Dover Publications, 1994.
- [39] J. L. Crassidis and J. L. Junkins, *Optimal estimation of dynamic systems*. New York: CRC Press, 2004.