ALKÜ Fen Bilimleri Dergisi 2021, 3(1): 1-13
e-ISSN: 2667-7814

Original Article

A non-analog Monte Carlo simulation method for slab albedo problem with linear-anisotropic scattering

Bahram R. Maleki[1]*
[1] *Department of Nuclear Engineering, Hacettepe University, Beytepe, Ankara 06800, Turkey*
* *Bahram.maleki@hacettepe.edu.tr*

Abstract

In this study, a non-analog Monte Carlo method is developed to simulate the albedo and transmission factor of an infinite non-multiplying slab medium subjected to a direction-dependent one-speed neutron beam on the left side. In order to obtain more precise results different variance reduction techniques such as forced collision, implicit capture, and Russian-Roulette are taken into consideration. For different incident directions of the neutrons and in the case of both isotropic and linear anisotropic scatterings, the albedo and transmission factor are estimated from the Monte Carlo and compared with the results obtained from the $H_{N=7}$ deterministic method. It is seen that in most cases, the results of both Monte Carlo and $H_{N=7}$ methods are comparable with each other. In some cases, it is also observed that the deterministic method falls short in predicting the albedo and transmission factor, whereas, in contrast, the results of the Monte Carlo are physically meaningful.

Keywords: Anisotropic Scattering, Neutron transport, Monte Carlo, Albedo, Transmission factor

Lineer-anizotropik saçılma ile slab albedo problemi için bir Analog Olmayan Monte Carlo simülasyon yöntemi

Özet

Bu çalışmada, sol taraftan bir anizotropik ve tek hızlı nötron demetine maruz kalan sonsuz çoğaltıcı olmayan bir levha ortamının albedo ve iletim faktörünü simüle etmek için analog olmayan bir Monte Carlo yöntemi geliştirilmiştir. Daha yüksek hassasiyete sahip sonuçlar elde etmek için, simülasyon sırasında zorla çarpışma, örtük yakalama ve Russian-Roulette gibi farklı varyans azaltma teknikleri kullanılmıştır. Nötronların farklı geliş yönleri için, hem izotropik hem de lineer anizotropik saçılımlar durumunda, albedo ve iletim faktörü Monte Carlo'dan tahmin edilip ve $H_{N=7}$ deterministik yönteminden elde edilen sonuçlarla karşılaştırılmıştır. Çoğu durumda Monte Carlo ve $H_{N=7}$ Metotlarının sonuçlarının birbiriyle karşılaştırılabilir olduğu görülmektedir. Bazı durumlarda, deterministik yöntemin albedo ve iletim faktörünü tahmin etmede yetersiz kaldığı, buna karşın Monte Carlo'nun sonuçlarının fiziksel olarak anlamlı olduğu görülmektedir.

Anahtar Kelimeler: Anizotropik saçılma, Nötron transport, Monte Carlo, Albedo, İletim faktörü.

Geliş/Received: 13 Kas 2020/13 Nov 2020
Kabul Ediliş/Accepted: 24 Ara 2020/24 Dec 2020

1. INTRODUCTION

In nuclear reactors, the behavior of the neutron is described by the integro-differential neutron transport equation. To learn the neutronic behavior of the systems such as scalar flux, reaction rate, reactor power, etc. we need to solve either the time-dependent or steady-state forms of the neutron transport equations. Unfortunately, except for some simple cases, the neutron transport equations cannot be solved analytically. The main reason is the high number of variables. It is a seven-dimensional problem: three dimensions in space (x, y, z), two in direction $(\mu = \cos\theta, \varphi)$, one in energy (E) and one in time (t). Both deterministic and stochastic (i.e., Monte Carlo) solution methods are being used to solve and simulate the neutron transport equations, respectively [1-4].

Deterministic solution methods, by discretizing energy, space, and time variables, using the appropriate approximations for the angular dependency, and finally by applying the boundary and initial conditions, a group of mathematical equations are formed and solved numerically. In addition to numerical solutions error, thus obtained results contain truncation and discretizing errors as well. Deterministic methods are faster, but fall short in addressing the nuclear systems with complex geometries, strong anisotropy of neutron scattering, and complicated neutron energy spectrums [1,4].

Monte Carlo method is a stochastic simulation method that is used to solve deterministic problems using randomly generated numbers between zero and one. In Monte Carlo method, without dealing with the integro-differential equations and also without using different approximations, the neutron motions and interaction types are sampled randomly and used to simulate the neutronic behavior of the system. This solution method can be easily applied on the problems with complex geometry and continuous energy. One of the most restriction in utilizing these methods is the significant Central Processing Unit (CPU) time-cost and uncertainty of the results. Nowadays, advances in computational capabilities render Monte Carlo methodology feasible. The imposed variances can be reduced by increasing the particle numbers in the simulation. However, the technical difficulties in using a large number of histories led to the development and use of several variance reduction techniques [4-9].

In the slab albedo problem, a one-speed non-multiplying slab of the thickness of $\tau\ cm$, extended from $z = -a$ to $z = a$ and surrounded by the vacuum, is taken into account. Schematic representation of the problem is presented in Figure 1. The angular dependent neutron beam incident on the left side of the slab and on the other surface there is no neutron entrance. Moreover, the neutron scattering is assumed to be linearly anisotropic [10-13].

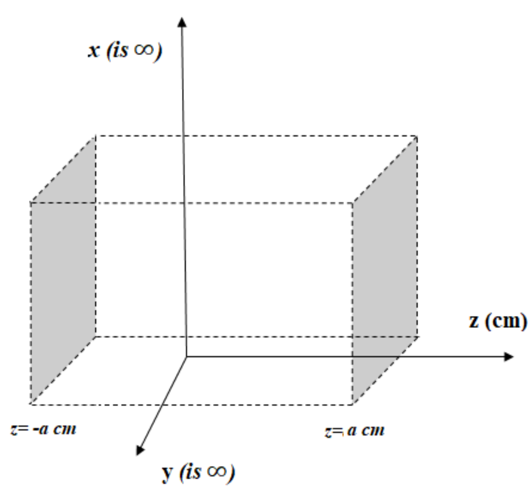

Figure 1. Schematic representation of the non-multiplying infinite slab media.

$$\psi(z=-a,\mu>0)=\mu^{\beta} \tag{1}$$

$$\psi(z=a,\mu<0)=0.0 \tag{2}$$

where μ is direction cosine and is equal to $\vec{\Omega}.\hat{z}$, $\vec{\Omega}$ is the neutron direction, $\psi(z,\mu)=\int_0^{2\pi}\psi(z,\Omega)\,d\varphi$ represents the azimuthally integrated angular flux and β is an integer constant.

The one speed, steady-state and azimuthally integrated neutron transport equation for a homogeneous non-multiplying slab media with linear anisotropic scattering is expressed as follows:

$$\mu\frac{\partial\psi(z,\mu)}{\partial z}+\Sigma_t\,\psi(z,\mu)=\frac{\Sigma_{s_0}}{2}\int_{-1}^{1}[1+3f_1P_1(\mu')P_1(\mu)]\,\psi(z,\mu')\,d\mu' \tag{3}$$

where Σ_t is the neutron total macroscopic cross section, Σ_{s_0} is the total scattering cross section which is also referred to as zeroth moment of the scattering kernel, f_1 is equal to $\Sigma_{s_1}/\Sigma_{s_0}$ and is named as mean scattering cosine, $\Sigma_{s_1}$is the first moment of the scattering kernel that can take both positive and negative values, and $P_1(\mu)=\mu$ is the first order Legendre polynomial. The mean number of secondary neutrons per collision is denoted by c and expressed as the ratio of Σ_{s_0} to Σ_t [14,15].

The scattering kernel that is used in azimuthally integrated neutron transport equation of one-dimensional problems is as follows.

$$\Sigma_s(\mu'\to\mu)=\sum_{l=0}^{\infty}\frac{2l+1}{2}\;\Sigma_{s_l}P_l(\mu')\,P_l(\mu) \tag{4}$$

$\Sigma_s(\mu'\to\mu)$ describes the probability that a neutron with initial direction cosine of μ' undergoes scattering event and takes a new direction with direction cosine of μ.

Different types of deterministic solution methods were presented to solve the azimuthally integrated neutron transport equations with linearly anisotropic scattering in slab geometry [10-13].

For one-speed problems, the Legendre polynomial expansion of the scattering kernel in general form is expressed as shown in Eq. (5), which describes the probability that a neutron with an initial direction of $\vec{\Omega}'$ is scattered into a new direction of $\vec{\Omega}$ [14-18].

$$\Sigma_s\left(\vec{\Omega}'\to\vec{\Omega}\right)=\frac{1}{2\pi}\Sigma_s\left(\vec{\Omega}'.\vec{\Omega}\right)=\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}\;\Sigma_{s_l}\,P_l\left(\vec{\Omega}'.\vec{\Omega}\right) \tag{5}$$

It is a well-known fact that the scattering is rotationally invariant in one-dimensional problems, that is, the probability that a neutron be scattered from direction $\vec{\Omega}'$ to a new direction $\vec{\Omega}$ only depends on the scattering angle Θ_b between $\vec{\Omega}$ and $\vec{\Omega}'$. Therefore, the $\Sigma_s\left(\vec{\Omega}'.\vec{\Omega}\right)$ can be expressed as follows:

$$\Sigma_s(\mu_b)=\sum_{l=0}^{\infty}\frac{2l+1}{2}\;\Sigma_{s_l}\,P_l(\mu_b) \tag{6}$$

where μ_b is called the scattering cosine.

$$\vec{\Omega}'.\vec{\Omega} = \mu_b = cos(\Theta_b) \quad (7)$$

In this manuscript, a novel non-analog Monte Carlo methodology is provided to estimate the albedo and transmission factor of the slab media with linear anisotropic scattering. The obtained results are compared with the results obtained from the H_N deterministic method.

2. NON-ANALOG MONTE CARLO METHOD

To start the Monte Carlo simulation, it is assumed that N_n number of neutrons of the weight of $w_0 = 1$ incident on the left surface of the slab at $z_0 = -a\ cm$. In order to sample the incident neutrons directions, the incoming current boundary condition on the left boundary is taken into consideration.

$$J_{in}(-a) = \int_0^1 \mu\ \psi(-a,\mu)\ d\mu = \frac{1}{(\beta+2)} \quad (8)$$

Thus, $(\beta+2)\ \mu\ \psi(-a,\mu)$ can be considered as the probability density function for the incident neutrons' directions. By calculating the cumulative distribution function and using the inverse transform method the incident direction of the neutrons is sampled as follows:

$$\mu_0 = \xi^{\frac{1}{\beta+2}} \quad (9)$$

where ξ represents a uniformly distributed random number between zero and one.

The azimuthal angle of the incident neutron is randomly sampled as:

$$\varphi_0 = 2\pi\xi \quad (10)$$

Due to the inability of using a large number of histories in the Monte Carlo simulations, killing a neutron due to leakage imposes an additional variance on the simulation results. To overcome this problem, the forced collision variance reduction technique is implemented: $e^{\Sigma_t\, d_s}$ fraction of the neutron weight is killed due to leakage and the remaining weight is forced to do a collision after traveling a path length of $d\ (\epsilon[0\,, ds])$. Here d_s is the minimum distance to surface in the neutron direction. This method causes neutron to live longer and subsequently have more chance to score, that is, the forced collision technique increases sampling of collisions in specified regions.

$$w_{leak} = w_0\ e^{\Sigma_t\, d_s} \quad (11)$$

$$w_{int} = w_0\left(1 - e^{\Sigma_t\, d_s}\right) \quad (12)$$

$$d = -\frac{1}{\Sigma_t} ln\left[\,1 - \xi\left(1 - e^{-\Sigma_t d_s}\right)\right] \quad (13)$$

The new position of the neutron is calculated as:

$$z = z_0 + d\mu_0 \tag{14}$$

In order to minimize the imposed variance due to absorption of a neutron, implicit capture variance reduction technique is implemented. Therefore, Σ_a/Σ_t fraction of the w_{int} is killed due to absorption and the remaining weight undergoes a scattering event.

To sample the new direction of the scattered neutron, first the scattering cosine and its corresponding azimuthal angle are sampled. To sample the scattering cosine the given expression for $\Sigma_s(\mu_b)$ in Eq. (6) is used. Since the $\int_{-1}^{1} \Sigma_s(\mu_b)\, d\mu_b$ is equal to Σ_{s_0}, therefore, $\Sigma_s(\mu_b)/\Sigma_{s_0}$ can be taken as the probability density function for the μ_b.

$$Pdf(\mu_b) = \frac{\Sigma_s(\mu_b)}{\Sigma_{s_0}} = \frac{1}{2} + \frac{3}{2} f_1\mu_b + \cdots + \frac{2N+1}{2}\frac{\Sigma_{s_N}}{\Sigma_{s_0}} P_N(\mu_b) + \cdots \tag{15}$$

The given expression in Eq.(15) is always a positive quantity. In the case of the linearly anisotropic scattering, the two first terms of the series expansion are taken into account. in this case, for $1 < |3f_1| \leq 3$ the $Pdf(\mu_b)$ may become negative, which is an unacceptable condition for the density function, therefore, it cannot be used as a probability density function to sample the scattering cosine. To overcome this difficulty, a sampling method for selecting the scattering angle from the linearly anisotropic distribution function was developed by Coveyou [19-21]. According to this method, the corresponding probability density function for the positive f_1 values is expressed in the form of:

$$Pdf(\mu_b) = \begin{cases} \frac{1}{2}\,(1 + 3\,f_1\mu_b\,) & ,if\ 3f_1 \leq 1 \\ \frac{3}{2}(1-f_1)\left(\frac{1+\mu_b}{2}\right) + \frac{1}{2}(3f_1-1)\delta(\mu_b-1) & ,1 < 3f_1 \leq 3 \end{cases} \tag{16}$$

The density function corresponding to negative f_1 values is obtained by changing the signs of the f_1 and μ_b in Eq. (16).

In this manuscript, we deal with positive f_1 values. Also, we follow a different sampling method from Coveyou's method. In our case, the probability density function is re-written as follows.

$$Pdf(\mu_b) = \begin{cases} \frac{1}{2}\,(1 - 3\,f_1\,) + 3f_1\left(\frac{1+\mu_b}{2}\right) & ,3f_1 \leq 1 \\ \frac{3}{2}(1-f_1)\left(\frac{1+\mu_b}{2}\right) + \left(1 - \frac{3}{2}(1-f_1)\right)\delta(\mu_b-1) & ,1 < 3f_1 \leq 3 \end{cases} \tag{17}$$

For the cases that $3f_1 \leq 1$, a uniformly distributed random number ξ ($\in [0,1]$) is generated. If the ξ be less than $3f_1$ then the $Pdf(\mu_b)$ is taken equal to $\left(\frac{1+\mu_b}{2}\right)$ and used to sample μ_b. Otherwise, $Pdf(\mu_b)$ is taken equal to $\frac{1}{2}$.

$$Pdf(\mu_b) = \frac{1+\mu_b}{2} \quad \rightarrow \quad \mu_b = -1 + 2\sqrt{\xi} \tag{18}$$

$$Pdf(\mu_b) = \frac{1}{2} \quad \rightarrow \quad \mu_b = -1 + 2\xi \tag{19}$$

In contrast, for the case that $1 < 3f_1 \leq 3$, a uniformly distributed random number $\xi (\in [0,1])$ is generated. If the ξ be less than $\frac{3}{2}(1-f_1)$ then the $Pdf(\mu_b)$ is taken equal to $\left(\frac{1+\mu_b}{2}\right)$. Otherwise, $Pdf(\mu_b)$ is taken equal to $\delta(\mu_b - 1)$ and subsequently μ_b becomes equal to $+1$, that is, neutron does not change its direction.

The corresponding azimuthal angle for the scattering cosine is sampled randomly as follows:

$$\Phi_b = 2\pi\xi \tag{20}$$

As shown in Figure 2, Φ_b and Θ_b are the azimuthal and polar angles of the scattered neutron with respect to its initial direction. These angles are used to obtain the actual direction of the scattered neutron.

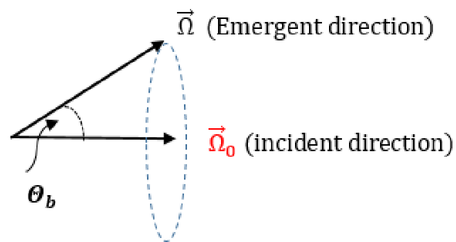

Figure 2. Schematic representation of scattering cosine.

For a particle with initially direction of $\vec{\Omega}_0 = \left(\sqrt{1-\mu_0^2}\cos(\varphi_0), \sqrt{1-\mu_0^2}\sin(\varphi_0), \mu_0\right)$ that is scattered by angles Θ_b and Φ_b with respect to the initial direction. The new direction is given by [22,23]:

$$\mu = \mu_0\mu_b - \sqrt{1-\mu_0^2}\sqrt{1-\mu_b^2}\,cos(\Phi_b) \tag{21}$$

$$cos(\varphi) = \frac{1}{\sqrt{1-\mu^2}}\left[\sqrt{1-\mu_0^2} \times \mu_b\, cos(\varphi_0) + \sqrt{1-\mu_b^2} \times [\mu_0\, cos(\Phi_b)\, cos(\varphi_0) - sin(\Phi_b)\, sin(\varphi_0)]\right] \tag{22}$$

$$sin(\varphi) = \frac{1}{\sqrt{1-\mu^2}}\left[\sqrt{1-\mu_0^2} \times \mu_b\, sin(\varphi_0) + \sqrt{1-\mu_b^2} \times [\mu_0\, cos(\Phi_b)\, sin(\varphi_0) + sin(\Phi_b)\, cos(\varphi_0)]\right] \tag{23}$$

In this case, first of all, the μ value is obtained from the Eq. (21), then by using this value the best azimuthal angle that satisfies both Eqs. (22) and (23) are calculated.

To track the transport of the scattered neutron, z_0 , w_0, μ_0 and φ_0 values are updated to z $(\Sigma_s/\Sigma_t)w_{int}$, μ and φ respectively. The transport of each neutron is monitored until it is killed by Russian-Roulette method.

In the Russian-Roulette method, a threshold weight and a survival weight which are designated by w_{rr} and w_{sur}, respectively, are selected. For the cases that the scattered neutron weight is less than the threshold weight ($w_0 < w_{rr}$) a random number is generated; if this random number be less than the $P_{sur}(= w_0/w_{sur})$ the particle with the new weight of $w_0(= w_{sur})$ is survived; otherwise, the neutron is killed and transport of the other neutrons is simulated. In this manuscript, the threshold and survival weight are set to

0.25 $w_{av_{inc}}$ and 0.5$w_{av_{inc}}$, where the $w_{av_{inc}}$ denotes the average weight of the incident neutron and is equal to unity.

Albedo is defined as the reflection probability of the incident neutron and denoted by α. The transmission factor is defined as the transmission probability of the incident neutrons that leak out the scattering region at $z = a$ and denoted by κ. These quantities are tallied as follows:

$$\alpha = \frac{\sum_{i=1}^{N_{L_l}} w_{0_i}\, e^{\Sigma_t\, d_{s_i}}}{N_n \times 1} \tag{24}$$

$$\kappa = \frac{\sum_{j=1}^{N_{L_r}} w_{0_j}\, e^{\Sigma_t\, d_{s_j}}}{N_n \times 1} \tag{25}$$

where i and j count the number of leakage events that occur at the left and right boundaries of the slab media, respectively.

The absorption probability of the incident neutrons is also scored as follows:

$$P_a = \frac{\sum_{k=1}^{N_{int}} w_{0_k} \left(\frac{\Sigma_a}{\Sigma_t}\right)}{N_n \times 1} \tag{26}$$

where k specifies the number of interactions.

3. RESULTS AND DİSCUSSİONS

In order to test the validity of the proposed Monte Carlo method, the uncollided and once-collided partial currents at the boundaries are obtained from both Monte Carlo and successive approximation methods and their ratio to the total inlet current are compared with each other. The obtained results for the uncollided and once-collided angular fluxes from the successive approximation method are as follows.

$$\psi^{(0)}(z, \mu > 0) = \mu^{\beta} e^{\frac{-\Sigma_t}{\mu}(z+a)}$$

$$\psi^{(0)}(z, \mu < 0) = 0.0$$

$$\psi^{(1)}(z, \mu > 0) = e^{\frac{-\Sigma_t}{\mu}z} \left[\frac{\Sigma_{s0}}{2} \int_{-a}^{z} A(z') e^{\frac{\Sigma_t}{\mu}z'}\, dz'\right] + e^{\frac{-\Sigma_t}{\mu}z} \left[\frac{3\Sigma_{s1}}{2} \int_{-a}^{z} B(z')\, e^{\frac{\Sigma_t}{\mu}z'}\, dz'\right]$$

$$\psi^{(1)}(z, \mu < 0) = e^{\frac{-\Sigma_t}{\mu}z} \left[\frac{\Sigma_{s0}}{2} \int_{a}^{z} A(z') e^{\frac{\Sigma_t}{\mu}z'}\, dz'\right] + e^{\frac{-\Sigma_t}{\mu}z} \left[\frac{3\Sigma_{s1}}{2} \int_{a}^{z} B(z')\, e^{\frac{\Sigma_t}{\mu}z'}\, dz'\right]$$

where

$$A(z) = \int_{-1}^{0} \psi^{(0)}(z, \mu')\, d\mu' + \int_{0}^{1} \psi^{(0)}(z, \mu')\, d\mu'$$

$$B(z) = \int_{-1}^{0} \mu' \psi^{(0)}(z, \mu')\, d\mu' + \int_{0}^{1} \mu' \psi^{(0)}(z, \mu')\, d\mu'$$

In this manuscript, in order to compare our results with the results presented in the literature the total neutron cross section is taken equal to $1.0\ cm^{-1}$[10]. To implement the provided Monte Carlo simulation the scattering and absorption cross sections are required. Using the mean number of secondary neutrons per collision(c) which is used as an input parameter in the reference article [10], these cross sections are obtained as $\Sigma_{s_0} = c\Sigma_t$ and $\Sigma_a = \Sigma_t - \Sigma_{s_0}$.

The number of incident neutrons in the Monte Carlo simulation is set to $2.0E + 6$. In order to compare the Monte Carlo and successive approximation methods, for different selected parameters, the ratios of the partial currents at boundaries to inlet current are presented in Tables 1 and 2. It is seen that the results are in good agreement with each other.

Table 1. Ratio of the partial currents to inlet current for $\Sigma_t = 1.0\ cm^{-1}, \beta = 0.0, f_1 = 0.1, \tau = 1.0\ cm$.

	Monte Carlo Method	Successive approximation method
$\frac{J^{-(0)}(-a)}{J^{+(0)}(-a)}$	0.0	0.0
$\frac{J^{+(0)}(a)}{J^{+(0)}(-a)}$	0.2194642	0.2193840
$\frac{J^{-(1)}(-a)}{J^{+(0)}(-a)}$	0.0174553	0.0174908
$\frac{J^{+(1)}(a)}{J^{+(0)}(-a)}$	0.0131680	0.0131473

Table 2. Ratio of the partial currents to inlet current for $\Sigma_t = 1.0\ cm^{-1}, \beta = 1.0, f_1 = 0.1 , \tau = 2.0\ cm$.

	Monte Carlo Method	Successive approximation method
$\frac{J^{-(0)}(-a)}{J^{+(0)}(-a)}$	0.0	0.0
$\frac{J^{+(0)}(a)}{J^{+(0)}(-a)}$	0.0750605	0.0750684
$\frac{J^{-(1)}(-a)}{J^{+(0)}(-a)}$	0.0161776	0.0161959
$\frac{J^{+(1)}(a)}{J^{+(0)}(-a)}$	0.0063186	0.0063312

For different input parameters, the mean value of all scattering cosines that are sampled during the Monte Carlo simulation is shown in Table 3. It is observed that, the mean value of the sampled scattering cosines becomes almost equal to mean scattering cosine (f_1) value. Also it is seen that by increasing the slab thickness the results become more accurate.

Table 3. Average value for the all scattering cosines sampled during the Monte Carlo simulation.

	τ	$0.1\ cm$	$0.5\ cm$	$1.0\ cm$	$2.0\ cm$
$\beta = 0.0, c = 0.8 , f_1 = 0.00$	$< \mu_b >$	-0.000696	-0.000651	-0.000578	-0.000164
$\beta = 1.0, c = 0.1, f_1 = 0.10$	$< \mu_b >$	0.097965	0.100966	0.099243	0.099892
$\beta = 3.0, c = 0.8 , f_1 = 0.50$	$< \mu_b >$	0.49941	0.500270	0.499993	0.499996

For different types of inlet current, different f_1 values, and different thicknesses the albedo and transmission factor are obtained from the Monte Carlo and compared with the results calculated from the H_N deterministic method presented by S. Bulut and M. Ç. Güleçyüz [10]. The H_N solution method depends on the use of the angular distributions of the method of elementary solutions; where the orthogonality relations of the singular eigenfunctions together with the values of the angular distributions at the boundaries of the given medium lead to the solution of the problem.

3.1. Case 1: Direction-independent inlet

In this test case the β value is taken zero, that is, the incident neutron does not have any directional preference. Tables 4 and 5 represent the comparison between the deterministic and Monte Carlo methods. For the constant f_1 values, it is seen that by increasing the thickness the albedo increases and transmission factor decreases, where the results of both methods show the same performance.

On the other hand, it is known that by increasing the mean scattering cosine value, the forward scattering contribution increases; leading to a reduction in albedo and an increase in the transmission factor. As seen in both Tables, the Monte Carlo results satisfy this condition. But the H_N deterministic method cannot satisfy this physical property generally, where as shown in Table 4, for the case $\tau = 1.0\ cm$, by increasing the f_1 value the transmission factor goes up and then decreases.

In addition, it is observed that by increasing the c value, both the albedo and transmission factor experience an increase.

Table 4. Comparison of Monte Carlo and deterministic methods for $\beta = 0.0$ and. $c = 0.1$.

f_1	Method	$\tau = 0.1\ cm$		$\tau = 0.5\ cm$		$\tau = 1.0\ cm$		$\tau = 2.0\ cm$	
		α	κ	α	κ	α	κ	α	κ
0.0	Monte Carlo	0.007184	0.839513	0.017830	0.457757	0.020850	0.231683	0.021716	0.065773
	$H_{N=7}$ method	0.007182	0.839537	0.017770	0.437934	0.020747	0.231808	0.021624	0.065840
0.1	Monte Carlo	0.006552	0.840201	0.015878	0.459721	0.018514	0.233548	0.019272	0.0666778
	$H_{N=7}$ method	0.006556	0.840028	0.015973	0.459480	0.018056	**0.268279**	0.019297	0.066656
0.5	Monte Carlo	0.003807	0.843025	0.008787	0.466894	0.010064	0.240746	0.010387	0.070729
	$H_{N=7}$ method	0.004041	0.842978	0.008691	0.466414	0.009595	0.240028	0.009769	0.070218

Table 5. Comparison of Monte Carlo and deterministic methods for $\beta = 0.0$ and $c = 0.8$.

f_1	Method	$\tau = 0.1\ cm$		$\tau = 0.5\ cm$		$\tau = 1.0\ cm$		$\tau = 2.0\ cm$	
		α	κ	α	κ	α	κ	α	κ
0.0	Monte Carlo	0.064994	0.896493	0.205770	0.621731	0.280209	0.416248	0.328250	0.197271
	$H_{N=7}$ method	0.064925	0.896574	0.205616	0.621975	0.280152	0.416245	0.327951	0.197270
0.1	Monte Carlo	0.059861	0.901529	0.190666	0.636773	0.261523	0.434061	0.308712	0.212082
	$H_{N=7}$ method	0.059874	0.901624	0.190654	0.636790	0.261711	0.433792	0.309080	0.212005
0.5	Monte Carlo	0.036724	0.924655	0.121410	0.706122	0.172801	0.521132	0.212524	0.294540

	$H_{N=7}$ method	0.039096	0.922396	0.123621	0.703233	0.173106	0.518755	0.211456	0.293647

3.2. Case 2: Direction-dependent inlet I

In this test case, β value is taken equal to one, that is, the angular flux boundary condition due to incoming neutrons becomes equal to $\psi(-a,\mu > 0) = \mu$. The albedo and transmission factor resulted from the Monte Carlo method for $c = 0.1$ and $c = 0.8$ are given in Tables 6 and 7, respectively, and compared with the results obtained from the $H_{N=7}$ deterministic method.

It is seen that the results are in good agreement. But for the case $\tau = 1.0\ cm$ and $c = 0.1$, as shown in Table 6, by increasing the f_1 value the transmission factor obtained from $H_{N=7}$ method goes up and then decreases whereas we observe a logical increase in the Monte Carlo results.

Table 6. Comparison of Monte Carlo and deterministic methods for $\beta = 1.0$ and $c = 0.1$.

f_1	Method	$\tau = 0.1\ cm$		$\tau = 0.5\ cm$		$\tau = 1.0\ cm$		$\tau = 2.0\ cm$	
		α	κ	α	κ	α	κ	α	κ
0.0	Monte Carlo	0.005843	0.868926	0.015785	0.509205	0.018950	0.270416	0.019927	0.080970
	$H_{N=7}$ method	0.005848	0.868836	0.015727	0.509319	0.018855	0.270514	0.019849	0.081037
0.1	Monte Carlo	0.005181	0.869655	0.013734	0.511156	0.016468	0.272425	0.017333	0.082007
	$H_{N=7}$ method	0.005211	0.869280	0.013833	0.510934	0.016179	**0.284335**	0.017342	0.081973
0.5	Monte Carlo	0.0027459	0.872340	0.006972	0.518952	0.008219	0.280663	0.008562	0.086964
	$H_{N=7}$ method	0.002650	0.872475	0.006158	0.518446	0.006924	0.279635	0.007079	0.086095

Table 7. Comparison of Monte Carlo and deterministic methods for $\beta = 1.0$ and $c = 0.8$.

f_1	Method	$\tau = 0.1\ cm$		$\tau = 0.5\ cm$		$\tau = 1.0\ cm$		$\tau = 2.0\ cm$	
		α	κ	α	κ	α	κ	α	κ
0.0	Monte Carlo	0.052922	0.915698	0.183236	0.6602346	0.258764	0.451589	0.309459	0.218208
	$H_{N=7}$ method	0.080289	0.902352	0.183145	0.660511	0.258691	0.451620	0.309551	0.218042
0.1	Monte Carlo	0.047691	0.920650	0.167552	0.676176	0.238683	0.470466	0.289054	0.234098
	$H_{N=7}$ method	0.047750	0.920789	0.167461	0.676060	0.239103	0.470341	0.289368	0.234003
0.5	Monte Carlo	0.026928	0.941748	0.099528	0.746841	0.148704	0.563601	0.190718	0.324647
	$H_{N=7}$ method	0.026608	0.941926	0.097190	0.745789	0.144947	0.560960	0.185259	0.319629

3.3. Case 3: Direction-dependent inlet II

In this test case, β value is taken equal to three. The results are presented in Tables 8 and 9. It is observed that the results of both methods are close to each other. But as shown in bold in Table 9, in the case of isotropic scattering and $\tau = 0.1\ cm$, the calculated transmission factor from the H_7 method is equal to 0.993709 where by increasing the f_1 value it experience a decrease and then goes up. In contrast the corresponding result in the Monte Carlo method is obtained equal to 0.927789 where by increasing the f_1 value the transmission factor increases.

Table 8. Comparison of Monte Carlo and deterministic methods for $\beta = 3.0$ and c= 0.1

f_1	Method	$\tau = 0.1\ cm$		$\tau = 0.5\ cm$		$\tau = 1.0\ cm$		$\tau = 2.0\ cm$	
		α	κ	α	κ	α	κ	α	κ
0.0	Monte Carlo	0.004994	0.887883	0.014155	0.553539	0.017367	0.309422	0.018466	0.098940
	$H_{N=7}$ method	0.004988	0.887634	0.014115	0.553506	0.017296	0.309458	0.018390	0.098890
0.1	Monte Carlo	0.004335	0.888510	0.012068	0.555518	0.014815	0.311538	0.015706	0.100110
	$H_{N=7}$ method	0.004353	0.887972	0.012147	0.555132	0.014563	0.311705	0.015728	0.100041
0.5	Monte Carlo	0.0021180	0.891006	0.005578	0.563558	0.006727	0.320598	0.007067	0.106049
	$H_{N=7}$ method	0.0017653	0.891570	0.004178	0.563240	0.004727	0.319439	0.004835	0.104768

Table 9. Comparison of Monte Carlo and deterministic methods for $\beta = 3.0$ and $c = 0.8$

f_1	Method	$\tau = 0.1\ cm$		$\tau = 0.5\ cm$		$\tau = 1.0\ cm$		$\tau = 2.0\ cm$	
		α	κ	α	κ	α	κ	α	κ
0.0	Monte Carlo	0.045200	0.927789	0.165102	0.692578	0.240004	0.485261	0.293050	0.240200
	$H_{N=7}$ method	0.042604	**0.993709**	0.164980	0.692655	0.239889	0.485157	0.293087	0.240092
0.1	Monte Carlo	0.039965	0.932963	0.148645	0.708985	0.219498	0.504614	0.271900	0.257293
	$H_{N=7}$ method	0.040014	0.933062	0.148720	0.708792	0.219289	0.504927	0.271676	0.257259
0.5	Monte Carlo	0.020807	0.952535	0.082267	0.779291	0.128642	0.602215	0.170715	0.356147
	$H_{N=7}$ method	0.018643	0.954429	0.075862	0.781155	0.120234	0.600592	0.161206	0.348426

4. CONCLUSIONS

A rigorous non-analog Monte Carlo method is proposed to simulate the albedo and transmission factor of an infinite non-multiplying slab medium. Using the corresponding expression for the inlet current, a new probability density function is generated to sample the direction of the incident neutrons. To minimize the imposed variances due to either complete leak or complete absorption of a neutron, forced collision and implicit capture variance reduction techniques are employed. Furthermore, the Russian-Roulette technique is used to reduce simulation time. For neutron scattering, sampling is performed on the scattering cosine to determine the new direction of the scattered neutron. The validity of the proposed method is confirmed through comparison with solutions for the uncollided and once-collided angular fluxes resulted from the successive approximation method. The Monte Carlo results are also compared with the results of the $H_{N=7}$ deterministic method, and it is observed that the results of the Monte Carlo are physically meaningful.

5. ACKNOWLEDGEMENT

I would like to express my gratitude to my supervisor Prof. Mehmet Tombakoğlu for all his guidance and helpful advice throughout this study.

REFERENCES

[1] Hoffman, A. J. (2013),” A Time-Dependent Method of Characteristics Formulation with Time Derivative Propagation”, PhD thesis, University of Michigan.

[2] Duderstadt, J. J. (1976), Nuclear reactor analysis. Wiley.

[3] Hanuš, M. (2014), Mathematical Modeling of Neutron Transport, PhD thesis, University of West Bohemia.

[4] Wu, Yican. (2019), Neutronics of advanced nuclear systems. Springer.

[5] Kalos, M. H., & Whitlock, P. A. (2009), Monte carlo methods. John Wiley & Sons.

[6] Seco, J., & Verhaegen, F. (2013), Monte Carlo techniques in radiation therapy, CRC press.

[7] Dupree, S. A., & Fraley, S. K. (2012), A Monte Carlo primer: A Practical approach to radiation transport. Springer Science & Business Media.

[8] Haghighat, A. (2016), Monte Carlo Methods for Particle Transport, Crc Press.

[9] Saidi, P., Sadeghi, M., and Tenreiro, C. (2013), “Variance reduction of Monte Carlo simulation in nuclear engineering”. In Theory and Applications of Monte Carlo Simulations”. Intech Open.

[10] Bulut, S., and M. Ç. Güleçyüz. (2005),”The HN method for slab albedo problem for linearly anisotropic scattering”, Kerntechnik 70.5-6, 301-308.

[11] Karataşlı, Muhammet, Tahsin Özer, and Ahmet Varinlioğlu. (2015),” Second Type Chebyshev Polynomial Approximation to Linearly Anisotropic Neutron Transport Equation in Slab Geometry”, Suleyman Demirel University Journal of Science 10.2.

[12] Güleçyüz, M. Ç., A. Kaşkaş, and C. Tezcan. (1999),” Slab albedo problem for anisotropic scattering using singular eigenfunction solution of the CN equations”, Journal of Quantitative Spectroscopy and Radiative Transfer, 61.3, 329-338.

[13] Grandjean, P., and C. E. Siewert. (1979), “The FN method in neutron-transport theory. Part II: applications and numerical results”, Nuclear Science and Engineering 69.2, 161-168.

[14] Stacey, Weston M. (2007), Nuclear reactor physics. Vol. 2. Weinheim: wiley-vch.

[15] Prinja A.K., Larsen E.W. (2010), General Principles of Neutron Transport. In: Cacuci D.G. (eds) Handbook of Nuclear Engineering. Springer, Boston, MA.

[16] Schwenk-Ferrero, (1986), Aleksandra.”GANTRAS: A system of codes for the solution of the multigroup transport equation with a rigorous treatment of anisotropic neutron scattering. Plane and spherical geometry, gscs.

[17] INÖNÜ, Erdal. (1970),”Orthogonality of a set of polynomials encountered in neutron-transport and radiative-transfer theories”. Journal of Mathematical Physics, 11.2, 568-577.

[18] MIKA, Janusz R. (1961),”Neutron transport with anisotropic scattering”, Nuclear science and engineering, 11.4: 415-427.

[19] Coveyou, R. R. (1965), "A monte carlo technique for selecting neutron scattering angles from anisotropic distributions", Nuclear science and engineering, 260-262.

[20] Lux, Iván. (1982), "Semicontinuous selection of scattering angles from low-order Pn scattering densities", Nuclear science and Engineering 82.3,332-337.

[21] Lux, I. (1991), and K. Koblinger. Monte Carlo Particle Transport Methods: Neutron and Photon Calculations, CRC Boca Raton.

[22] Bielajew, Alex F. (2001), "Fundamentals of the Monte Carlo method for neutral and charged particle transport", The University of Michigan.

[23] Singkarat, Somsorn. (1990), "Study of the multiple scattering effect in TEBENE using the Monte Carlo method". No. CTH-RF--69. Chalmers Univ. of Tech.