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Araştırma Makalesi / Research Article

MEFM For Exact Solutions Of The (3+1) Dimensional KZK Equation and (3+1) Dimensional JM Equation

Seyma TULUCE DEMİRAY[1*], Sevgi KASTAL[2]

[1] *Department of Mathematics, Faculty of Science and Letters, Osmaniye Korkut Ata University, Osmaniye,* Turkey.

[2] *Department of Mathematics, Faculty of Science and Letters, Osmaniye Korkut Ata University, Osmaniye,* Turkey.

Corresponding author[*] e-posta[1]: seymatuluce@gmail.com ORCID ID: http://orcid.org/0000-0002-8027-7290

*e-posta[2]: kastalsevgi@gmail.com ORCID ID: http://orcid.org/*0000-0001-9152-3023

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The (3+1) dimensional KZK Equation; (3+1)-Dimensional Jimbo-Miwa Equation; MEFM; Dark Soliton Solution; Trigonometric Function Solutions; Hyperbolic Solutions.

Abstract

In this study, we have applied the modified exp $\left(-\Omega(\xi)\right)$-expansion function method (MEFM) to obtain the exact travelling wave solutions for the (3+1) dimensional Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation and (3+1)-dimensional Jimbo-Miwa (JM) equation. Dark soliton solution, dark-bright soliton solution, hyperbolic function solution and trigonometric function solution of the (3+1) dimensional KZK equation and (3+1)-dimensional JM equation have been found by using this method. After that, we have scratched the 2D and 3D graphs for all exact solutions obtained in this study by using Wolfram Mathematica 9. Thus, the graphical simulations openly show force of this method.

(3+1) Boyutlu KZK Denklemi ve (3+1) Boyutlu JM Denkleminin Tam Çözümleri için MEFM

Anahtar kelimeler
(3 + 1) boyutlu KZK Denklemi; (3 + 1) - Boyutlu Jimbo-Miwa Denklemi; MEFM; Dark Soliton Çözümü; Trigonometrik Fonksiyon Çözümleri; Hiperbolik Çözümler.

Öz

Bu çalışmada, (3+1) boyutlu Khokhlov – Zabolotskaya – Kuznetsov (KZK) denklemi ve (3+1) boyutlu Jimbo-Miwa (JM) denklemlerinin yürüyen tam dalga çözümlerini elde etmek için, modifiye edilmiş exp $\left(-\Omega(\xi)\right)$-açılım fonksiyon metodunu (MEFM) uyguladık. Bu yöntemle (3+1) boyutlu KZK denkleminin ve (3+1) boyutlu JM denkleminin dark soliton çözümü, dark-bright soliton çözümü, hiperbolik fonksiyon çözümü ve trigonometrik fonksiyon çözümü bulunmuştur. Daha sonra, bu çalışmada elde edilen tüm kesin çözümler için Wolfram Mathematica 9'u kullanarak 2 boyutlu ve 3 boyutlu grafikleri çizdik. Böylece, grafiksel gösterimler bu yöntemin gücünü açıkça göstermektedir.

1.Introduction

In recent years nonlinear evolution equations (NLEEs) have become exclusive type of the branch of partial differential equations (PDEs). NLEEs are frequently used to expound a lot of physical events in the fields such as chemical kinematics, thermodynamic, fluid mechanic, optical fibers, heat transfer. Therefore, most of methods are established and applied for these problems. Some of these methods are Sine-Gordon expansion method (Ali *et al.* 2020), Hirota bilinear method (Ismael *et al.* 2020), Bifurcation method (Zheng *et al.* 2021), The modified auxiliary equation method (Akbar *et al.* 2020), Generalized Kudryashov method (Tuluce Demiray 2020), Simple Hirota's method (Ismael and Bulut 2020), (m + 1/G')-expansion method (Durur *et al.* 2020), finite difference method (Yokus and Kaya 2020), Sinh–Gordon function method (Yokus *et al.* 2020), (1/G')-expansion method (Yokus *et al.* 2020). The aim of this article, MEFM (Tuluce Demiray 2020, Tuluce Demiray and Bulut 2018) will be used to obtain new exact solutions (3+1) dimensional KZK equation and (3+1)-dimensional JM equation.

We consider the (3+1) dimensional KZK equation (Akcagil and Aydemir 2016)

$$s_{xt}+s_x^2+ss_{xx}+rs_{xxx}+ns_{yy}+ms_{zz}=0, \tag{1}$$

where m,n,r is real constants and $r\neq 0$.

Then, we investigate the (3+1)-dimensional JM equation (Zhang *et al.* 2009, Zayed *et al.* 2013)

$$s_{xxxy}+6s_xs_y+3sv_{xx}+3s_{xx}v+3s_{yt}-3s_{zz}=0, \tag{2}$$
$$s_y=v_x.$$

Eq. (1) was natively created as a device for the defination of nonlinear acoustic beams (Rozanova-Pierrat 2006).

Our target is to get new exact solutions of (3+1) dimensional KZK equation and (3+1)-dimensional JM equation by using suggested method. In Sec. 2, we clarify the technique. In Sec. 3, we apply suggested method to (3+1) dimensional KZK equation and (3+1)-dimensional JM equation.

2. Explanation of the Method

For a known nonlinear partial differential equation is given as follows:

$$K\left(s,s_t,s_x,s_y,s_{tt},s_{xx},s_{yy},\cdots\right)=0, \tag{3}$$

where $s=s\left(x,y,z,t\right)$ is an obscure function.

Step 1: Getting the transformation as

$$s\left(x,y,t\right)=S\left(\sigma\right),\ \sigma=x+y+z-ct, \tag{4}$$

Eq. (2) is turned into the following nonlinear equation:

$$L\left(S,S',S'',S''',\cdots\right)=0. \tag{5}$$

Step 2: Taking the following equation for Eq. (5) as solution:

$$S(\sigma)=\frac{\sum_{i=0}^{p}A_i\left[\exp\left(-\vartheta(\sigma)\right)\right]^i}{\sum_{j=0}^{q}B_j\left[\exp\left(-\vartheta(\sigma)\right)\right]^j}=\frac{A_0+A_1\exp(-\vartheta)+\cdots+A_p\exp\left(p(-\vartheta)\right)}{B_0+B_1\exp(-\vartheta)+\cdots+B_q\exp\left(q(-\vartheta)\right)}, \tag{6}$$

where $A_i, B_j, \left(0\le i\le p, 0\le j\le q\right)$ are constants, such that $A_p\neq 0, B_q\neq 0$. Also, $\vartheta=\vartheta(\sigma)$ is described as;

$$\vartheta'(\sigma)=\exp\left(-\vartheta(\sigma)\right)+a\exp\left(\vartheta(\sigma)\right)+b. \tag{7}$$

Eq. (7) has the following solution families:

Family1: For $a\neq 0$, $b^2-4a>0$,

$$\vartheta(\sigma)=\ln\left(\frac{-\sqrt{b^2-4a}}{2a}\tanh\left(\frac{\sqrt{b^2-4a}}{2}(\sigma+E)\right)-\frac{b}{2a}\right). \tag{8}$$

Family2: When $a\neq 0$, $b^2-4a<0$,

$$\vartheta(\sigma)=\ln\left(\frac{\sqrt{-b^2+4a}}{2a}\tanh\left(\frac{\sqrt{-b^2+4a}}{2}(\sigma+E)\right)-\frac{b}{2a}\right). \tag{9}$$

Family3: When $a=0$, $b\neq 0$ and $b^2-4a>0$,

$$\vartheta(\sigma)=-\ln\left(\frac{b}{\exp\left(b(\sigma+E)\right)-1}\right). \tag{10}$$

Family4: When $a\neq 0$, $b\neq 0$ and $b^2-4a=0$,

$$\vartheta(\sigma)=\ln\left(-\frac{2b(\sigma+E)+4}{b^2(\sigma+E)}\right). \tag{11}$$

Family5: When $a=0$, $b=0$ and $b^2-4a=0$,

$$\vartheta(\sigma)=\ln(\sigma+E). \tag{12}$$

where $A_i, B_j, (0 \le i \le p, 0 \le j \le q), E, b, a$ are constants to be obtained later.

Step 3: Setting Eq. (6) and Eq. (7) into Eq. (5), a system of $e^{-\vartheta(\sigma)}$ can be obtained. We solve this system by using Wolfram Mathematica 9 to identify the coefficients $A_i, B_j, (0 \le i \le p, 0 \le j \le q)$, E, b, a.

3. Application to (3+1) dimensional KZK equation

Getting the transformation as

$$s(x,y,t)=S(\sigma), \quad \sigma=x+y+z-ct, \tag{13}$$

Eq. (1) demeans

$$(m+n-c)s+\frac{1}{2}s^2+rs'=0. \tag{14}$$

By use of balance principle in Eq. (14), we get

$$p=q+1. \tag{15}$$

If we get $q=1$ so $p=2$, we have

$$S=\frac{A_0+A_1\exp(-\vartheta)+A_2\exp(2(-\vartheta))}{B_0+B_1\exp(-\vartheta)}=\frac{\Upsilon}{\Psi}, \tag{16}$$

(16)

and

$$S'=\frac{\Upsilon'\Psi-\Psi'\Upsilon}{\Psi^2}, \tag{17}$$

$$S''=\frac{\Upsilon''\Psi^3-\Psi^2\Upsilon'\Psi'-(\Psi''\Upsilon+\Psi'\Upsilon')\Psi^2+2(\Psi')^2\Upsilon\Psi}{\Psi^4}, \tag{18}$$

$$\vdots$$

Thus, a system of $e^{-\vartheta(\sigma)}$ can be obtained. We solve this system by using Wolfram Mathematica 9 to identify the coefficients $A_i, B_j, (0 \le i \le p, 0 \le j \le q), E, b, a$.

Case 1:

$$A_0=\frac{i\sqrt{a}\sqrt{A_1^3}}{2\sqrt{A_1-8rB_0}}, \quad A_2=-\frac{i\sqrt{A_1}\sqrt{A_1-8rB_0}}{2\sqrt{a}},$$

$$B_1=-\frac{i\sqrt{A_1}\sqrt{A_1-8rB_0}}{4r\sqrt{a}}, \tag{19}$$

$$\lambda=\frac{8ir\sqrt{a}B_0}{\sqrt{A_1}\sqrt{A_1-8rB_0}}, \quad c=m+n+\frac{4ir\sqrt{a}(A_1-4rB_0)}{\sqrt{A_1}\sqrt{A_1-8rB_0}}.$$

According to Eq. (19), we find trigonometric function solution for Eq. (1) as follows:

$$s_1(x,y,z,t)=\frac{\left(2r\sqrt{a}A_1^{3/2}K\left(-i+\tan\left[g(x,y,z,t)\right]\right)^2\right)}{\left(P\left(D-K\tan\left[g(x,y,z,t)\right]\right)\left(A_1+D\left(i+\tan\left[g(x,y,z,t)\right]\right)\right)\right)}, \tag{20}$$

where

$$g(x,y,z,t)=\frac{\sqrt{a}(A_1-4rB_0)\left(E+x+y+z-t\left(m+n+\frac{4ir\sqrt{a}(A_1-4rB_0)}{\sqrt{A_1}\sqrt{A_1-8rB_0}}\right)\right)}{\sqrt{A_1}\sqrt{A_1-8rB_0}}$$

$K=(A_1-4rB_0)$, $P=\sqrt{A_1-8rB_0}$ and $D=4irB_0$.

Case 2:

$$A_0=\frac{4ir\sqrt{a}B_0(A_1-2rB_0)}{\sqrt{A_1}\sqrt{A_1-8rB_0}}, \quad A_2=-\frac{i\sqrt{A_1}\sqrt{A_1-8rB_0}}{2\sqrt{a}},$$

$$B_1=-\frac{i\sqrt{A_1}\sqrt{A_1-8rB_0}}{4r\sqrt{a}},$$

$$b=\frac{i\sqrt{a}(3A_1^2-8rA_1B_0+16r^2B_0^2)}{2\sqrt{A_1}\sqrt{A_1-8rB_0}(A_1-2rB_0)},$$

$$c=m+n+\frac{ir\sqrt{a}(A_1-4rB_0)(5A_1-4rB_0)}{\sqrt{A_1}\sqrt{A_1-8rB_0}(A_1-2rB_0)}. \tag{21}$$

According to Eq. (21), we find trigonometric function solution for Eq. (1) as follows:

$$s_2(x,y,z,t)=\frac{4ir\sqrt{a}\left(A_1-2rB_0+\dfrac{2A_1(A_1-8rB_0)(A_1-2rB_0)}{P+iM\tan\left[\tau(x,y,z,t)\right]}\right)}{\sqrt{A_1}\sqrt{A_1-8rB_0}},\quad (22)$$

where $M=(A_1-4rB_0)(5A_1-4rB_0)$,

$P=3A_1^2-8rA_1B_0+16r^2B_0^2$ and

$$\tau(x,y,z,t)=\frac{\sqrt{a}M\left(E+x+y+z-t\left(m+n+\dfrac{ir\sqrt{a}(A_1-4rB_0)(5A_1-4rB_0)}{2\sqrt{A_1}\sqrt{A_1-8rB_0}(A_1-2rB_0)}\right)\right)}{\sqrt{A_1}\sqrt{A_1-8rB_0}}.$$

Case 3:

$$A_0=\frac{B_0(A_1-2rB_0)}{B_1},\ A_2=2rB_1,$$

$$c=m+n-rb+\frac{A_1-2rB_0}{B_1},$$

$$a=-\frac{(A_1-2rB_0)(A_1-2r(B_0+bB_1))}{4r^2B_1^2}.\qquad (23)$$

According to Eq. (23), we get dark optical soliton solution for Eq. (1) as follows:

$$s_3(x,y,z,t)=\frac{(A_1-2rB_0)\left(B_1+\dfrac{A_1-2r(B_0+bB_1)}{A_1-r(2B_0+bB_1)\tanh\left[k(x,y,x,t)\right]}\right)}{B_1^2},\quad (24)$$

where

$$k(x,y,z,t)=\frac{\left(E+x+y+z-t\left(m+n-rb+\dfrac{A_1-2rB_0}{B_1}\right)\right)\left(A_1-r(2B_0+bB_1)\right)}{2rB_1}.$$

4. Application to (3+1) dimensional JM Equation

Getting the transformation as

$$s=s(\xi),\ \xi=kx+my+nz+wt,\qquad (25)$$

Eq. (2) demeans

$$k^3ms''+3kms^2+\left(3wm-3n^2\right)s=0.\qquad (26)$$

By use of balance principle in Eq. (26), one can procured

$$p=q+2.\qquad (27)$$

If we get $q=1$ so $p=3$, we have

$$S=\frac{A_0+A_1\exp(-\vartheta)+A_2\exp(2(-\vartheta))+A_3\exp(3(-\vartheta))}{B_0+B_1\exp(-\vartheta)+B_2\exp(-\vartheta)}=\frac{\Upsilon}{\Psi},\quad (28)$$

and

$$S'=\frac{\Upsilon'\Psi-\Psi'\Upsilon}{\Psi^2},\qquad (29)$$

$$S''=\frac{\Upsilon''\Psi^3-\Psi^2\Upsilon'\Psi'-(\Psi''\Upsilon+\Psi'\Upsilon')\Psi^2+2(\Psi')^2\Upsilon\Psi}{\Psi^4}.\quad (30)$$

$$\vdots$$

Thus, a system of $e^{-\vartheta(\sigma)}$ can be obtained. We solve this system by using Wolfram Mathematica 9 to identify the coefficients $A_i, B_j, (0\le i\le p, 0\le j\le q), E, b, a$.

Case 1:

$$A_0=-2k^2aB_0,\ A_1=\frac{A_2B_0+2k^2\left(B_0^2-aB_1^2\right)}{B_1},$$

$$A_3=-2k^2B_1,\ \lambda=-\frac{A_2+2k^2B_0}{2k^2B_1},$$

$$w=\frac{n^2}{m}+\frac{4k^3a}{3}-\frac{\left(A_2+2k^2B_0\right)^2}{12k^2B_1^2}.\qquad (31)$$

According to Eq. (31), we obtain dark-bright optical soliton solution for Eq. (2) as follows:

$$s_1(x,y,z,t)=\frac{2k^2a\,\mathrm{sech}\left[r(x,y,z,t)\right]^2\left(\left(A_2+2k^2B_0\right)^2-16k^4aB_1^2\right)}{\left(A_2+k^2\left(2B_0-2\sqrt{-4a+\frac{\left(A_2+2k^2B_0\right)^2}{4k^4B_1^2}}B_1\tanh\left[r(x,y,z,t)\right]\right)\right)^2},\quad(32)$$

where

$$r(x,y,z,t)=\frac{1}{2}\left(E+kx+my+nz+t\left(\frac{m^2}{n}+\frac{4k^3a}{3}-\frac{\left(A_2+2k^2B_0\right)^2}{12kB_1^2}\right)\right)M$$

and

$$M=\sqrt{-4a+\frac{\left(A_2+2k^2B_0\right)^2}{4k^4B_1^2}}.$$

Case 2:

$$A_0=-2k^2aB_0,\ A_1=aA_3-2k^2bB_0,$$

$$A_2=bA_3-2k^2B_0,$$

$$B_1=-\frac{A_3}{2k^2},\ w=\frac{n^2}{m}-\frac{1}{3}k^3\left(b^2-4a\right).\quad(33)$$

According to Eq. (33), we find hyperbolic function solution for Eq. (2) as follows:

$$s_2(x,y,z,t)=\frac{2k^2\left(b^2-4a\right)a}{\left(b\cosh\left[h(x,y,z,t)\right]+\sqrt{b^2-4a}\sinh\left[h(x,y,z,t)\right]\right)^2},\quad(34)$$

where

$$h(x,y,z,t)=\frac{1}{2}\left(E+\frac{n^2t}{m}+kx+my+nz-\frac{1}{3}k^3t\left(b^2-4a\right)\right)\sqrt{b^2-4a}\,.$$

Case 3:

$$A_0=\frac{aA_3B_0}{B_1},\ A_1=A_3\left(a+\frac{bB_0}{B_1}\right),$$

$$A_2=A_3\left(b+\frac{B_0}{B_1}\right),\ k=-\frac{i\sqrt{A_3}}{\sqrt{2B_1}},$$

$$w=\frac{-i\sqrt{2}m\left(b^2-4a\right)\sqrt{A_3^3}+12n^2\sqrt{B_1^3}}{12m\sqrt{B_1^3}}.\quad(35)$$

According to Eq. (35), we obtain hyperbolic function solution for Eq. (2) as follows:

$$s_3(x,y,z,t)=\frac{a\left(-b^2+4a\right)A_3}{\left(b\cosh\left[f(x,y,z,t)\right]+\sqrt{b^2-4a}\sinh\left[f(x,y,z,t)\right]\right)^2B_1},\quad(36)$$

where

$$f(x,y,z,t)=\frac{\sqrt{P}\left(-i\sqrt{2}mtP\sqrt{A_3^3}-6i\sqrt{2A_3}mxB_1+12\left(n^2t+m\left(E+my+nz\right)\right)\sqrt{B_1^3}\right)}{24m\sqrt{B_1^3}}$$

and $P=\left(b^2-4a\right)$.

Case 4:

$$A_0=\frac{B_0\left(-8k^4a-\frac{\left(A_2+2k^2B_0\right)^2}{B_1^2}\right)}{12k^2},$$

$$A_1=-\frac{A_2^2-8k^2A_2B_0+4k^4\left(-5B_0^2+2aB_1^2\right)}{12k^2B_1},$$

$$A_3=-2k^2B_1,\ b=-\frac{A_2+2k^2B_0}{2k^2B_1},$$

$$w=\frac{n^2}{m}-\frac{4k^3a}{3}+\frac{\left(A_2+2k^2B_0\right)^2}{kB_1^2}.\quad(37)$$

According to Eq. (37), we find dark optical soliton solution for Eq. (2) as follows:

$$s_4(x,y,z,t)=\left(\frac{\left(4k^2a^2\left(-24k^4B_1^2-\frac{M\left(L+K\tanh\left[h(x,y,z,t)\right]\right)^2}{4a^2}\right)\right)}{\left(3\left(A_2+k^2\left(2B_0-2K\tanh\left[h(x,y,z,t)\right]\right)\right)^2\right)}+\frac{\left(4k^2a^2\left(\frac{N\left(A_2+k^2\left(2B_0-2KB_1\tanh\left[h(x,y,z,t)\right]\right)\right)}{a}\right)\right)}{\left(3\left(A_2+k^2\left(2B_0-2K\tanh\left[h(x,y,z,t)\right]\right)\right)^2\right)}\right),\quad(38)$$

where

$$h(x,y,z,t)=\frac{1}{2}K\left(E+\frac{q^2t}{p}+kx+my+nz-\frac{4}{3}k^3ta+\frac{t\left(A_2+2k^2B_0\right)^2}{12kB_1^2}\right)$$

and

$$K=\sqrt{-4a+\frac{\left(A_2+2k^2B_0\right)^2}{4k^4B_1^2}},$$

$$M=\left(\left(A_2+2k^2B_0\right)^2+8k^4aB_1^2\right),$$

$$N=3\left(A_2+2k^2B_0\right), L=-\frac{A_2+2k^2B_0}{2k^2B_1}.$$

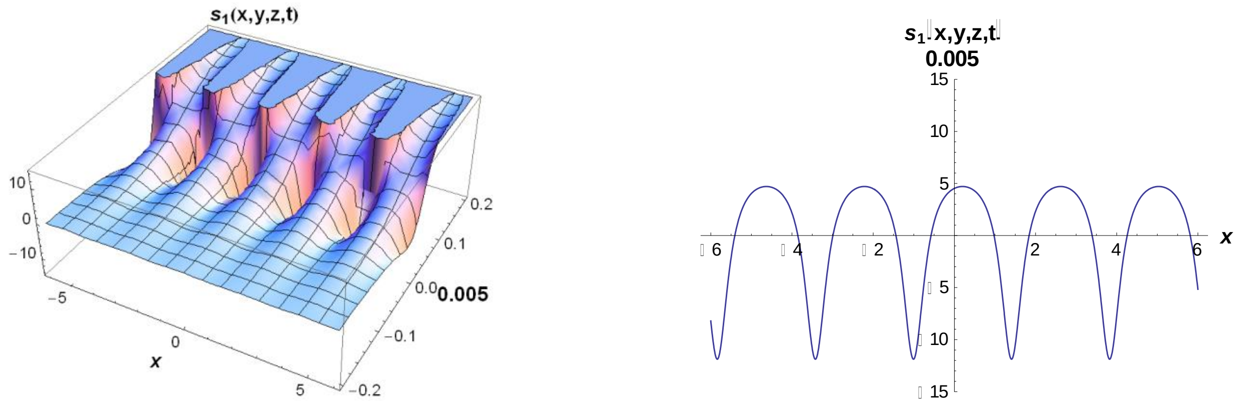

Fig.1. Three-dimensional and two-dimensional plots of imaginary values of Eq. (20) for $y=3$, $z=-5$, $m=1$, $n=1$, $r=3$, $E=0.6$, $A_1=4$, $B_0=-0.1$, $a=0.4$, $-6<x<6$, $-0.2<t<0.2$ and $t=0.005$ for 2D plot.

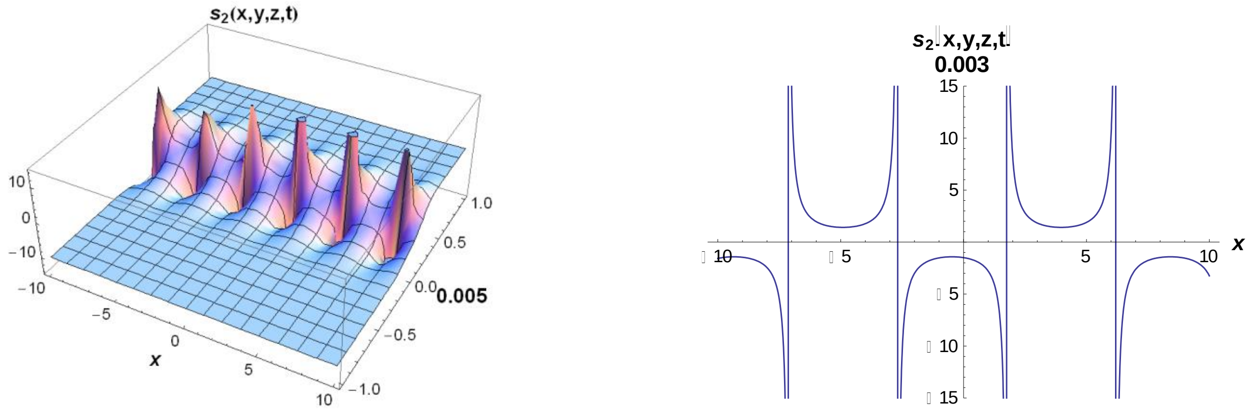

Fig.2. Three-dimensional and two-dimensional plots of imaginary values of Eq. (22) for $a=0.4$, $y=5$, $z=-6$, $r=-3$, $A_1=20$, $E=0.05$, $-10<x<10$, $-1<t<1$ and $t=0.005$ for 2D plot.

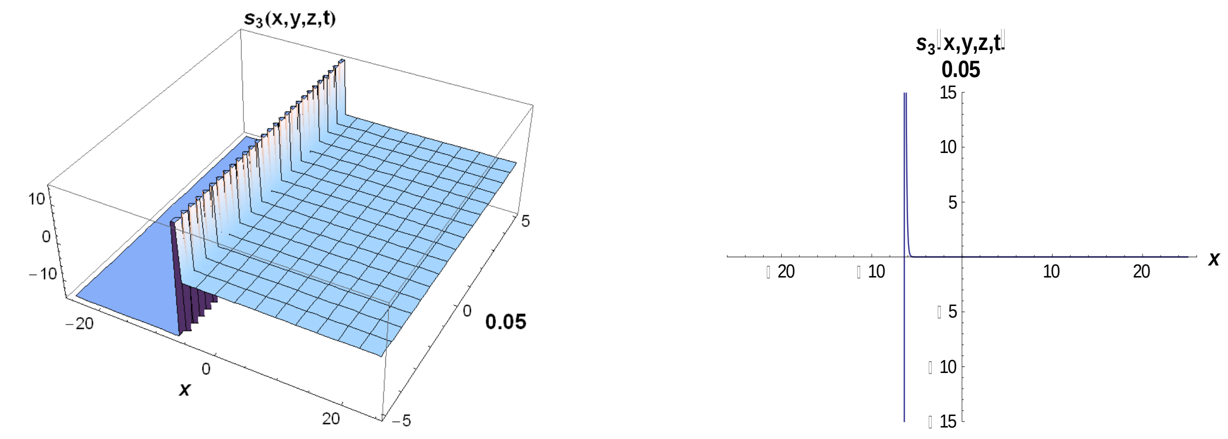

Fig.3. Three-dimensional and two-dimensional plots of real values of Eq. (24) for $b=5$, $y=3$, $z=1.2$, $m=3$, $n=2$, $r=5$, $A_1=4$, $B_0=-0.3$, $B_1=-0.3$, $E=-0.1$, $-25<x<25$, $-5<t<5$ and $t=0.005$ for 2D plot.

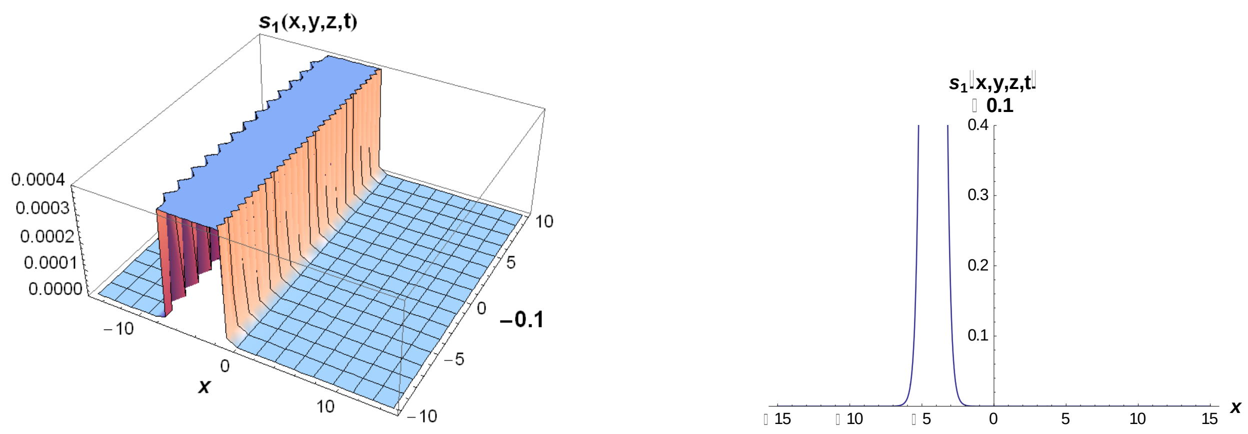

Fig.4. Three-dimensional and two-dimensional plots of Eq. (32) for $A_2=-2$, $B_0=-3$, $B_1=1.2$, $E=-0.2$, $k=0.2$, $y=-1$, $z=1$, $m=0.5$, $a=5$, $n=3$, $-25<x<25$, $-10<t<10$ and $t=-0.1$ for 2D plot.

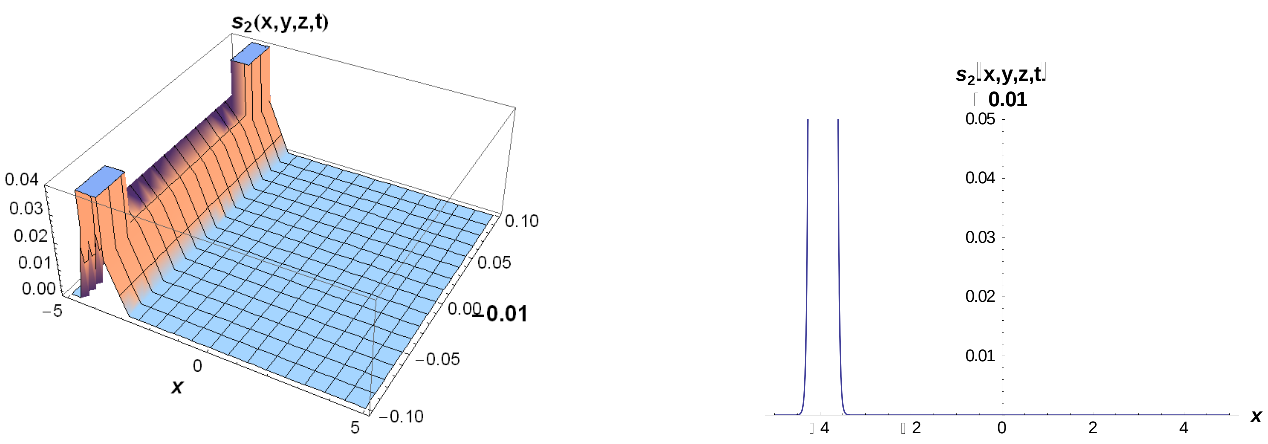

Fig.5. Three-dimensional and two-dimensional plots of Eq. (34) for $y=-2$, $z=0.5$, $m=-4$, $n=-1.2$, $E=-5.3$, $k=5$, $a=2$, $b=7$, $-5<x<5$, $-0.1<t<0.1$ and $t=-0.01$ for 2D plot.

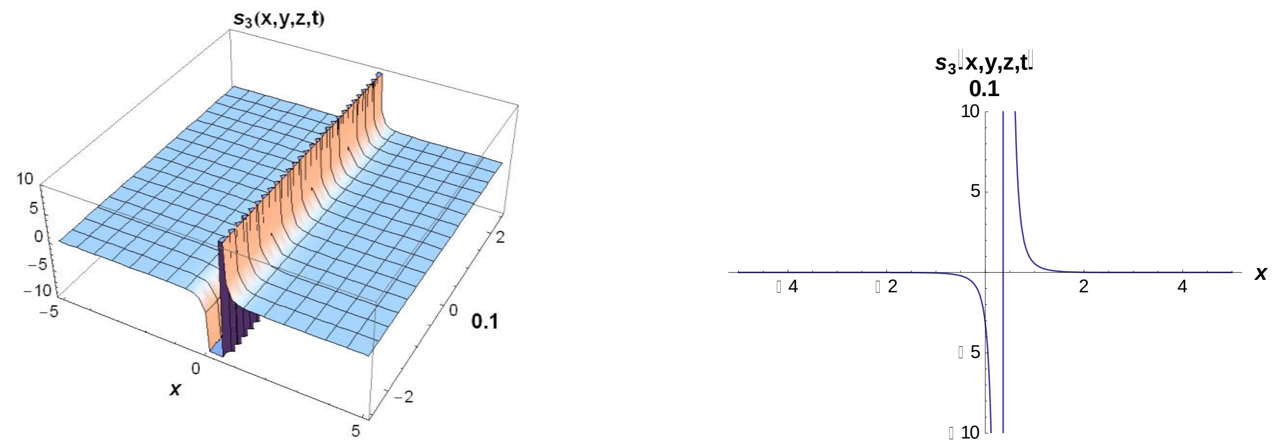

Fig.6. Three-dimensional and two-dimensional plots of Eq. (36) for $y = 0.3$, $z = -0.002$, $m = -3.5$, $n = 20.2$, $E = -0.1$, $a = 3$, $b = 2$, $-5 < x < 5$, $-0.1 < t < 0.1$ and $t = -0.01$ for 2D plot.

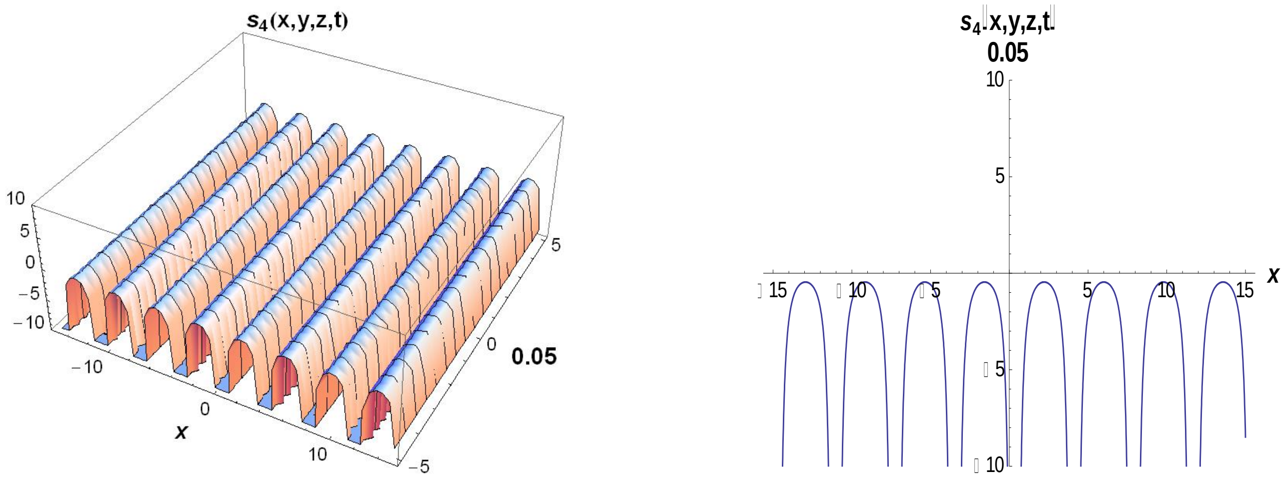

Fig.7. Three-dimensional and two-dimensional plots of Eq. (38) for $y = 3.2$, $z = 2$, $m = 0.5$, $n = 0.2$, $E = 3.1$, $a = 5$, $b = 2$, $-15 < x < 15$, $-5 < t < 5$ and $t = 0.05$ for 2D graph.

5. Results and Discussion

We obtain some exact solutions of (3+1) dimensional KZK equation and (3+1)-dimensional JM equation by applying MEFM. These solutions were controlled in Wolfram Mathematica 9. We prove their accuracy by graphically representing these obtained results by aid of Wolfram Mathematica 9. MEFM, which is easier to apply than other methods, is a very effective and reliable method for finding solutions to NLEEs.

6. Conclusion

In this paper, we make use of MEFM to seek solutions of (3+1) dimensional KZK equation and (3+1)-dimensional JM equation. Then, we draw two and three dimensional graphs of dark optical soliton solutions, trigonometric function solution, dark-bright optical soliton solution and hyperbolic function solutions of these equations via Wolfram Mathematica 9.

As a result of these datas, it has been deduced that MEFM is extremely credible and strong in the sense that finding exact solutions. The paper shows that the MEFM algorithm is efficient and can be used for many other NLEEs in mathematical physics.

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