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The Modified Exp $\left(-\vartheta(\sigma)\right)$-Expansion Function Method for Exact Solutions of the Simplified MCH Equation and the Getmanou Equation

Şeyma TULUCE DEMIRAY [1,*], Sevgi KASTAL [2]

[1]*Osmaniye Korkut Ata University, Department of Mathematics, 80000, Osmaniye, Türkiye*
seymatuluce@gmail.com, ORCID: 0000-0002-8027-7290
[2]*Osmaniye Korkut Ata University, Department of Mathematics, 80000, Osmaniye, Türkiye*
kastalsevgi@gmail.com, ORCID: 0000-0001-9152-3023

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Abstract

The present research explores, the modified exp $\left(-\vartheta(\sigma)\right)$-expansion function method (MEFM) is tested by applying it to obtain the exact travelling wave solutions for the simplified MCH equation and the Getmanou equation. Dark optical soliton solutions and dark-bright optical soliton solutions of the simplified MCH equation and the Getmanou equation are successfully constructed by using this method. We carry out all the computations and draw the 2D and 3D graph in this paper by Wolfram Mathematica 9. Besides, the graphical representation obviously shows forcefulness of this method.

Keywords: The simplified MCH equation; The Getmanou equation; MEFM; Dark optical soliton solutions; Dark-bright optical soliton solutions; Mathematica.

Basitleştirilmiş MCH Denklemi ve Getmanou Denkleminin Tam Çözümleri için Modifiye Edilmiş Exp $\left(-\vartheta(\sigma)\right)$-Açılım Fonksiyon Metodu

Öz

* Corresponding Author
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Mevcut araştırma, Modifiye edilmiş $\exp(-\vartheta(\sigma))$-açılım fonksiyon metodunu (MEFM), basitleştirilmiş MCH denklemi ve Getmanou denklemi için tam hareketli dalga çözümlerini elde etmek üzere uygulayarak test eder. Basitleştirilmiş MCH denkleminin ve Getmanou denkleminin dark optik soliton ve dark-bright optik soliton çözümleri bu yöntem kullanılarak başarıyla elde edilmiştir. Bu çalışmadaki tüm hesaplamalar Wolfram Mathematica 9 tarafından yapılmış ve 2D ve 3D grafikleri çizilmiştir. Ayrıca, grafiksel gösterim bu yöntemin açıkça gücünü göstermektedir.

Anahtar Kelimeler: Basitleştirilmiş MCH denklemi; Getmanou denklemi; MEFM; Dark optik soliton çözümler; Dark-bright optik soliton çözümler; Mathematica.

1. Introduction

Many phenomena problems in the real world in applied and engineering sciences are structured by nonlinear evolution equations (NLEEs). In recent years, nonlinear evolution equations (NLEEs) have become private species of the branch of partial differential equations (PDEs). Nonlinear evolution equations (NLEEs) are often used to explain a lot of physical events in the areas such as acoustic waves, hydromagnetic waves, chemistry, meteorology, engineering, thermodynamic, biology, physics, fluid mechanic, meteorology, optical fibers, heat transfer, acoustic gravity waves in mathematics. Because of this, most of methods have been developed and applied for these problems. Some of these methods include Extended simple equation method [1, 2], The Paul-Painlevé approach method [3], Multiple Exp-function method [4], ETEM [5], GKM [6]. The goal of this study, MEFM [7, 8] will be used to acquire new exact solutions of the simplified MCH equation and the Getmanou equation.

Firstly, we consider the simplified MCH equation [9, 10],

$$s_t + 2\alpha s_x - s_{xxt} + \beta s^2 s_x = 0 \qquad \alpha \in \Re, \beta > 0 \tag{1}$$

where α and β are constants. Wazwaz [11] investigated a modified form of the Camassa-Holm equation, which was simplified from the MCH equation and the equation expressed in Eqn. (1) is called the simplified MCH equation.

Secondly, we investigate the Getmanou equation [12, 13],

$$s_{xt} + \frac{s_x s_t}{1-s^2} - s\left(1-s^2\right) = 0 \tag{2}$$

Getmanou equation possesses high nonlinearity [13]. Fan studied the single traveling wave solutions based on the complete discrimination system of the fifth-order polynomials [13]. The trial equation method combined with complete discrimination system for polynomial has been used to solve the Getmanou equation [12].

Here, our aim is to find new exact solutions of the simplified MCH equation and the Getmanou equation by way of suggest method. In Section 2, we explain methodology. In Section 3, we apply suggest method to the simplified MCH equation and the Getmanou equation.

2. Materials and Methods

For a known nonlinear partial differential equations are given as follows:

$$K\left(s, s_t, s_x, s_y, s_{tt}, s_{xx}, s_{yy}, \cdots\right) = 0, \tag{3}$$

where $s = s\left(x, y, z, t\right)$ is an obscure function.

Step 1: Getting the transmutation as

$$s\left(x, y, t\right) = S\left(\sigma\right), \quad \sigma = x + y + z - ct, \tag{4}$$

Eqn. (2) is turned into the following nonlinear equation:

$$L\left(S, S', S'', S''', \cdots\right) = 0. \tag{5}$$

Step 2: Taking the following equation for Eqn. (5) as solution:

$$S\left(\sigma\right) = \frac{\sum_{i=0}^{p} A_i \left[\exp\left(-\vartheta\left(\sigma\right)\right)\right]^i}{\sum_{j=0}^{q} B_j \left[\exp\left(-\vartheta\left(\sigma\right)\right)\right]^j} = \frac{A_0 + A_1 \exp\left(-\vartheta\right) + \cdots + A_p \exp\left(p\left(-\vartheta\right)\right)}{B_0 + B_1 \exp\left(-\vartheta\right) + \cdots + B_q \exp\left(q\left(-\vartheta\right)\right)}, \tag{6}$$

where $A_i, B_j, \left(0 \le i \le p, 0 \le j \le q\right)$ are constants, such that $A_p \ne 0, B_q \ne 0,$ and $\vartheta = \vartheta\left(\sigma\right)$ described as;

$$\vartheta'\left(\sigma\right) = \exp\left(-\vartheta\left(\sigma\right)\right) + a \exp\left(\vartheta\left(\sigma\right)\right) + b. \tag{7}$$

Eqn. (7) has the following solution families:

Family 1: When $a \ne 0,\ b^2 - 4a > 0,$

$$\vartheta\left(\sigma\right) = \ln\left(\frac{-\sqrt{b^2 - 4a}}{2a} \tanh\left(\frac{\sqrt{b^2 - 4a}}{2}\left(\sigma + E\right)\right) - \frac{b}{2a}\right). \tag{8}$$

Family 2: When $a \ne 0,\ b^2 - 4a < 0,$

$$\vartheta(\sigma)=\ln\left(\frac{\sqrt{-b^2+4a}}{2a}\tanh\left(\frac{\sqrt{-b^2+4a}}{2}(\sigma+E)\right)-\frac{b}{2a}\right). \tag{9}$$

Family 3: When $a=0,\ b\neq 0,$ and $b^2-4a>0,$

$$\vartheta(\sigma)=-\ln\left(\frac{b}{\exp\left(b(\sigma+E)\right)-1}\right). \tag{10}$$

Family 4: When $a\neq 0,\ b\neq 0,$ and $b^2-4a=0,$

$$\vartheta(\sigma)=\ln\left(-\frac{2b(\sigma+E)+4}{b^2(\sigma+E)}\right). \tag{11}$$

Family 5: When $a=0,\ b=0,$ and $b^2-4a=0,$

$$\vartheta(\sigma)=\ln(\sigma+E). \tag{12}$$

where $A_i, B_j, (0\leq i\leq p, 0\leq j\leq q), E, b, a$ are constants to be obtained later.

Step 3: Setting Eqn. (6) and Eqn. (7) into Eqn. (5), a system of $e^{-\vartheta(\sigma)}$ can be obtained. We solve this system by using Mathematica to identify the coefficients $A_i, B_j, (0\leq i\leq p, 0\leq j\leq q), E, b, a$.

3. Application of MEFM

3.1. Example: The simplified MCH equation

Getting the transformation as

$$s=s(\xi),\quad \xi=x-ct, \tag{13}$$

Eqn. (1) demeans

$$(2\alpha-c)s+cs''+\frac{\beta}{3}s^3=0 \tag{14}$$

By use of balance principle in Eqn. (14), we get

$$p=q+1. \tag{15}$$

If we get $q=1$ so $p=2$, we have

$$S=\frac{A_0+A_1\exp(-\vartheta)+A_2\exp(2(-\vartheta))}{B_0+B_1\exp(-\vartheta)}=\frac{\Upsilon}{\Psi}, \tag{16}$$

and

$$S'=\frac{\Upsilon'\Psi-\Psi'\Upsilon}{\Psi^2}, \tag{17}$$

$$S''=\frac{\Upsilon''\Psi^3-\Psi^2\Upsilon'\Psi'-(\Psi''\Upsilon+\Psi'\Upsilon')\Psi^2+2(\Psi')^2\Upsilon\Psi}{\Psi^4}, \tag{18}$$

$$\vdots$$

Thus, a system of $e^{-\vartheta(\sigma)}$ can be obtained. We solve this system by using Mathematica to identify the coefficients $A_i, B_j, (0\le i\le p, 0\le j\le q), E, b, a$.

Case 1:

$$A_0=\frac{bA_2B_0}{2B_1}, A_1=\frac{1}{2}A_2\left(b+\frac{2B_0}{B_1}\right), c=-\frac{\beta A_2^2}{6B_1^2}, a=\frac{1}{4}\left(2+b^2+\frac{24\alpha B_1^2}{\beta A_2^2}\right). \tag{19}$$

According to Eqn. (19), dark optical soliton solution for Eqn. (1) is gotten

$$s_1(x,t)=\frac{-24\alpha B_1^2+\beta A_2^2\left(-2+b\sqrt{-2-\frac{24\alpha B_1^2}{\beta A_2^2}}\tanh\left[r(x,t)\right]\right)}{2\beta A_2B_1\left(b+\sqrt{-2-\frac{24\alpha B_1^2}{\beta A_2^2}}\tanh\left[r(x,t)\right]\right)}, \tag{20}$$

where

$$r(x,t)=\frac{1}{2}\left(E+x+\frac{t\beta A_2^2}{6B_1^2}\right)\sqrt{-2-\frac{24\alpha B_1^2}{\beta A_2^2}}.$$

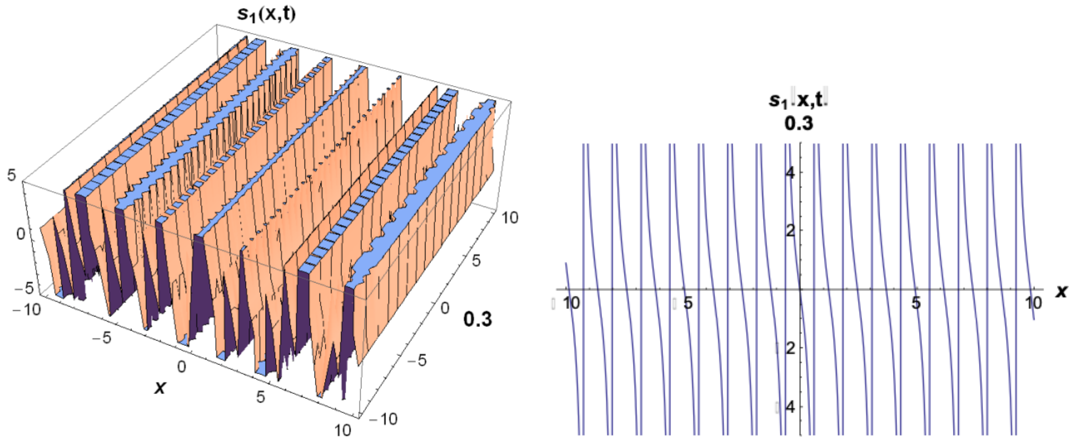

Figure 1: Three-dimensional and two-dimensional plots of imaginary values of Eqn. (20) for $A_2 = 4, B_1 = 4,\ E = 0.5, b = 0.3, \alpha = 3, \beta = 3, -10 < x < 10, -10 < t < 10$ and $t = 0.3$ for 2D plot

Case 2:

$$A_0 = -\frac{6\alpha B_0 B_1}{\beta A_1} + A_1\left(\frac{B_0}{2B_1} - \frac{B_1}{8B_0}\right), A_2 = \frac{A_1 B_1}{2B_0}$$

$$b = \frac{2B_0}{B_1}, a = -\frac{1}{4} + B_0^2\left(-\frac{12\alpha}{\beta A_1^2} + \frac{1}{B_1^2}\right), c = -\frac{\beta A_1^2}{24B_0^2}. \tag{21}$$

According to Eqn. (21), dark-bright optical soliton solution for Eqn. (1) is found

$$s_2(x,t) = -\frac{\operatorname{sech}\left[z(x,t)\right]^2\left(\beta A_1^2 + 48\alpha B_0^2\right)\left(-P + \beta A_1^2\left(B_0^2 - B_1^2\right)\right)}{K\left(2B_0 + MB_1 \tanh\left[k(x,t)\right]\right)\left(P + L\left(B_1 + 2B_0 M \tanh\left[k(x,t)\right]\right)\right)}, \tag{22}$$

where $M = \sqrt{1 + \frac{48\alpha B_0^2}{\beta A_1^2}},\ P = 48\alpha B_0^2 B_1,\ z(x,t) = \frac{\left(t\beta A_1^2 + 24(E+x)B_0^2\right)M}{48B_0^2}$ and

$$k(x,t) = \frac{1}{2}\left(E + x + \frac{t\beta A_1^2}{24B_0^2}\right)M, \quad K = 4\beta A_1 B_0, \ L = \beta A_1^2.$$

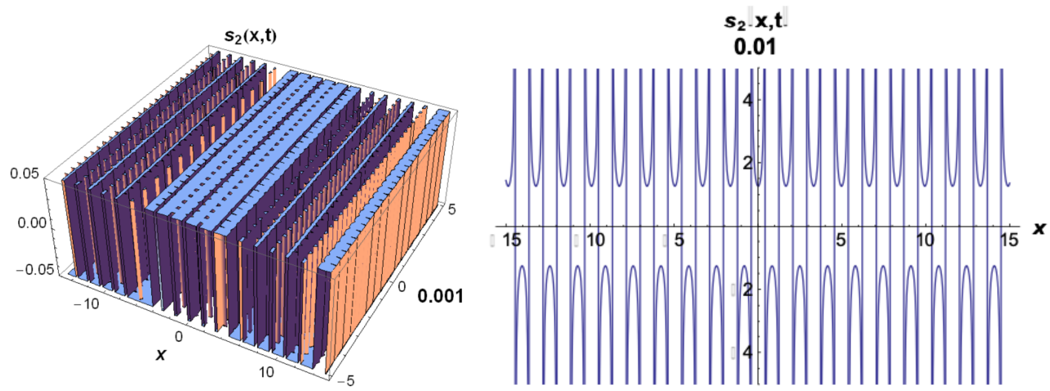

Figure 2: Three-dimensional and two-dimensional plots of imaginary values of Eqn. (22) for $A_2 = 0.2$, $B_0 = 0.3, B_1 = 0.1,\ E = 0.1,\ b = 0.3,\ \alpha = 1, \beta = -7, -15 < x < 15, -5 < t < 5$ and $t = 0.01$ for 2D plot

Case 3:

$$A_0 = \frac{1}{4}\left(-1+b^2\right)A_2 - \frac{12\alpha B_0^2}{\beta b^2 A_2}, A_1 = bA_2, B_1 = \frac{2B_0}{b},$$
$$c = -\frac{\beta b^2 A_2^2}{24B_0^2}, a = \frac{1}{4}\left(-1+b^2-\frac{48\alpha B_0^2}{\beta b^2 A_2^2}\right). \tag{23}$$

According to Eqn. (23), dark-bright optical soliton solution for Eqn. (1) is procured

$$s_3\left(x,t\right) = -\frac{\operatorname{sech}\left[h\left(x,t\right)\right]^2\left(\beta b^2\left(-1+b^2\right)A_2^2 - 48\alpha B_0^2\right)\left(\beta b^2 A_2^2 + 48\alpha B_0^2\right)}{L\left(b+P\tanh\left[h\left(x,t\right)\right]\right)\left(M+\beta b^2 A_2^2\left(1+bP\tanh\left[h\left(x,t\right)\right]\right)\right)}, \tag{24}$$

where $P = \sqrt{1+\frac{48\alpha B_0^2}{\beta b^2 A_2^2}}$, $h\left(x,t\right) = \frac{1}{2}\left(E + x + \frac{t\beta b^2 A_2^2}{24B_0^2}\right)P$, $L = 4\beta b A_2 B_0$, $M = 48\alpha B_0^2$.

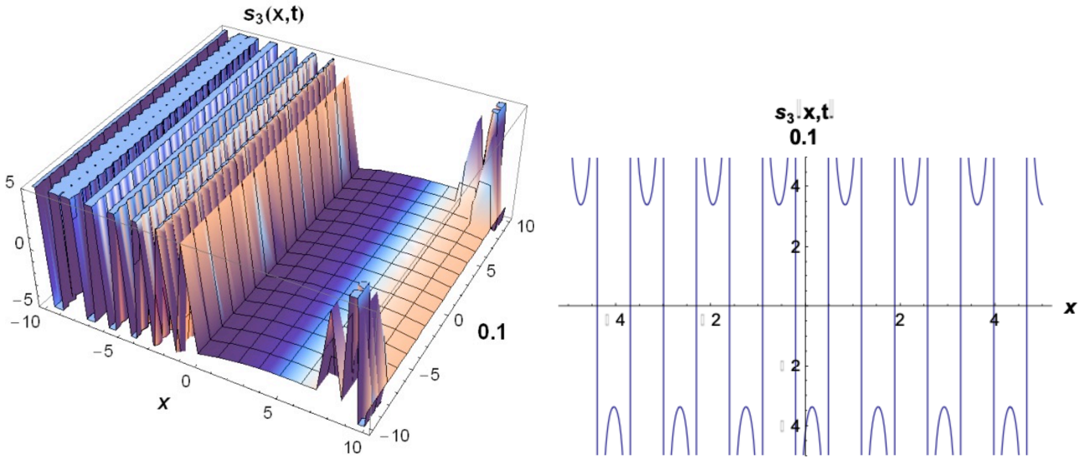

Figure 3: Three-dimensional and two-dimensional plots of real values of Eqn. (24) for $A_2 = 3, B_0 = 2, B_1 = 3, E = 0.1, b = -1, \alpha = -1, \beta = 1, -10 < x < 10, -10 < t < 10$ and $t = 0.1$ for 2D plot

Case 4:

$$A_0 = \frac{B_0\left(A_1 - \frac{i\sqrt{6c}B_0}{\sqrt{\beta}}\right)}{B_1}, A_2 = \frac{i\sqrt{6c}B_1}{\sqrt{\beta}}, b = -\frac{\frac{i\sqrt{6c}B_0}{\sqrt{\beta}} + 6B_0}{3B_1},$$

$$a = \frac{-\beta A_1^2 + 2i\sqrt{6c\beta}A_1B_0 + 6cB_0^2 + 3(c-2\alpha)B_1^2}{6cB_1^2}. \tag{25}$$

According to Eqn. (25), dark optical soliton solution for Eqn. (1) is obtained

$$s_4(x,t) = \frac{3\sqrt{6}(c-2\alpha)B_1 + 3\sqrt{c}\sqrt{-1+\frac{2\alpha}{c}}\left(i\sqrt{2\beta}A_1 + 2\sqrt{3c}B_0\right)\tanh\left[f(x,t)\right]}{\sqrt{6}\beta A_1 - 3i\sqrt{c}\sqrt{\beta}\left(2B_0 - \sqrt{-2+\frac{4\alpha}{c}}B_1\tanh\left[f(x,t)\right]\right)}, \tag{26}$$

where $f(x,t) = (E - ct + x)\sqrt{-\frac{1}{2} + \frac{\alpha}{c}}$.

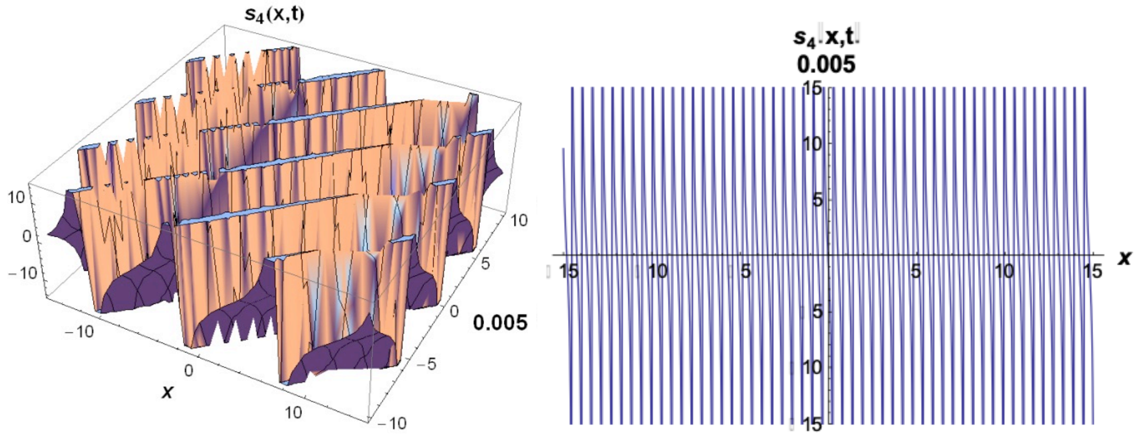

Figure 4: Three-dimensional and two-dimensional plots of real values of Eqn. (26) for $c = -0.1, \alpha = 3$, $\beta = -0.5, -15 < x < 15, -10 < t < 10, E = 0.3$ and $t = 0.005$ for 2D plot

Case 5:

$$A_0 = A_2\left(-\frac{1}{4}+\frac{B_0^2}{B_1^2}\right)-\frac{3\alpha B_1^2}{\beta A_2}, A_1 = \frac{2A_2B_0}{B_1},$$

$$c = -\frac{\beta A_2^2}{6B_1^2}, b = \frac{2B_0}{B_1}, a = -\frac{1}{4}+\frac{B_0^2}{B_1^2}-\frac{3\alpha B_1^2}{\beta A_2^2}. \tag{27}$$

According to Eqn. (27), dark-bright optical solution for Eqn. (1) is attained

$$s_5(x,t) = \frac{\operatorname{sech}\left[f(x,t)\right]^2\left(\beta A_2^2+12\alpha B_1^2\right)\left(12\alpha B_1^4+\beta A_2^2\left(-4B_0^2+B_1^2\right)\right)}{P\left(2B_0+MB_1\tanh\left[k(x,t)\right]\right)\left(K+\beta A_2^2\left(B_1+2B_0M\tanh\left[k(x,t)\right]\right)\right)}, \tag{28}$$

where $M = \sqrt{1+\frac{12\alpha B_1^2}{\beta A_2^2}}$, $P = 2\beta A_2 B_1$, $f(x,t) = \frac{\left(t\beta A_2^2+6(E+x)B_1^2\right)M}{12B_1^2}$ and

$$k(x,t) = \frac{1}{2}\left(E+x+\frac{t\beta A_2^2}{6B_1^2}\right)M \;, \quad K = 12\alpha B_1^3.$$

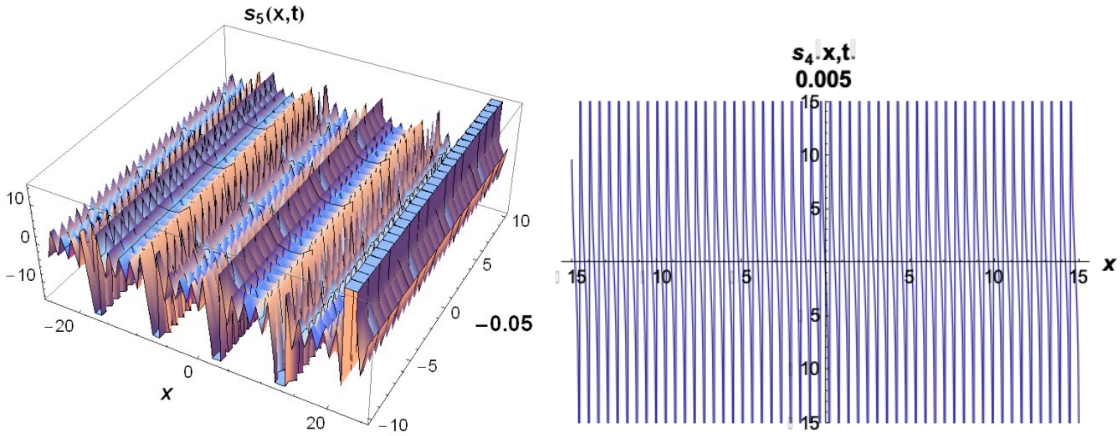

Figure 5: Three-dimensional and two-dimensional plots of real values of Eqn. (28) for $A_2 = 4, B_0 = -1,$ $B_1 = -5, E = 0.3, \alpha = 3,\ \beta = -5,\ -25 < x < 25$ $-10 < t < 10$ and $t = -0.05$ for 2D plot

In Figs. 1-2, we plot two and three dimensional graphics of imaginary values of Eqn. (20) and Eqn. (22), which demonstrate the vitality of solutions with suitable parametric choices. Then, in Figs. 3-5, we draw two and three dimensional graphics of real values of Eqn. (24), Eqn. (26) and Eqn. (28), which indicate the dynamic of solutions with suitable parametric choices.

3.2. Example: The Getmanou equation

Getting the transformation as

$$s = s(\xi), \quad \xi = x - ct, \tag{29}$$

Eqn. (2) demeans

$$c\left(s^2 - 1\right)s'' - c\left(s'\right)^2 - s^5 + 2s^3 - s = 0. \tag{30}$$

By use of balance principle in Eqn. (30), we get

$$p = q + 1\,. \tag{31}$$

If we get $q = 1$ so $p = 2$, we have

$$S = \frac{A_0 + A_1 \exp(-\vartheta) + A_2 \exp\left(2(-\vartheta)\right)}{B_0 + B_1 \exp(-\vartheta)} = \frac{\Upsilon}{\Psi}, \tag{32}$$

and

$$S' = \frac{\Upsilon'\Psi - \Psi'\Upsilon}{\Psi^2}, \tag{33}$$

$$S'' = \frac{\Upsilon''\Psi^3 - \Psi^2\Upsilon'\Psi' - \left(\Psi''\Upsilon + \Psi'\Upsilon'\right)\Psi^2 + 2\left(\Psi'\right)^2 \Upsilon\Psi}{\Psi^4}, \tag{34}$$

$$\vdots$$

Thus, a system of $e^{-\vartheta(\sigma)}$ can be obtained. We solve this system by using Mathematica to identify the coefficients $A_i, B_j, \left(0 \le i \le p, 0 \le j \le q\right), E, b, a$.

Case 1:

$$A_0 = \frac{2B_0}{3} + \frac{A_2 B_0^{\,2}}{B_1^{\,2}} + \frac{B_1^{\,2}}{36A_2}, A_1 = \frac{2A_2B_0}{B_1} + \frac{2B_1}{3}, c = \frac{A_2^{\,2}}{2B_1^{\,2}}$$

$$b = \frac{2B_0}{B_1} + \frac{5B_1}{3A_2}, a = \frac{5B_0}{3A_2} + \frac{B_0^{\,2}}{B_1^{\,2}} + \frac{B_1^{\,2}}{36A_2} \tag{35}$$

According to Eqn. (35), dark-bright optical soliton solution for Eqn. (2) is procured

$$s_1\left(x,t\right) = \left(\begin{array}{c} \dfrac{12\left(-1+8\,\mathrm{sech}\left[f\left(x,t\right)\right]^2\right)A_2B_0B_1^2}{\left(\left(B_1^2 + K\left(5-2\sqrt{6}\tanh\left[f\left(x,t\right)\right]\right)\right)\left(K + B_1^2\left(5+2\sqrt{6}\tanh\left[f\left(x,t\right)\right]\right)\right)\right)} \\ + \dfrac{B_1^4\left(-5+4\,\mathrm{sech}\left[f\left(x,t\right)\right]^2 - 2\sqrt{6}\tanh\left[f\left(x,t\right)\right]\right)}{\left(\left(B_1^2 + K\left(5-2\sqrt{6}\tanh\left[f\left(x,t\right)\right]\right)\right)\left(K + B_1^2\left(5+2\sqrt{6}\tanh\left[f\left(x,t\right)\right]\right)\right)\right)} \\ + \dfrac{36A_2^2B_0^2\left(-5+4\,\mathrm{sech}\left[f\left(x,t\right)\right]^2 + 2\sqrt{6}\tanh\left[f\left(x,t\right)\right]\right)}{\left(\left(B_1^2 + K\left(5-2\sqrt{6}\tanh\left[f\left(x,t\right)\right]\right)\right)\left(K + B_1^2\left(5+2\sqrt{6}\tanh\left[f\left(x,t\right)\right]\right)\right)\right)} \end{array} \right), \tag{36}$$

where $f\left(x,t\right) = \sqrt{\dfrac{2}{3}}\left(EE + x - \dfrac{tA_2^2}{2B_1^2}\right)\dfrac{B_1}{A_2}, \quad K = 6A_2B_0.$

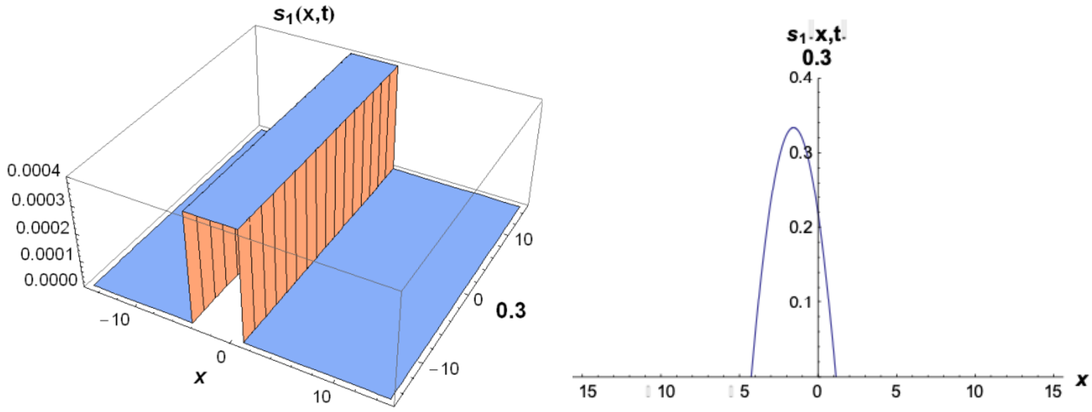

Figure 6: Three-dimensional and two-dimensional plots of solution Eqn. (36) for the values $A_2 = 5, B_0 = 0.2, B_1 = 2, E = 3, -15 < x < 15, -15 < t < 15$ and $t = 0.3$ for 2D plot

Case 2:

$$A_0 = \frac{2B_0}{3} + \frac{\sqrt{2c}B_0{}^2}{B_1} + \frac{B_1}{36\sqrt{2c}}, A_1 = \frac{2}{3}\left(3\sqrt{2c}B_0 + B_1\right), A_2 = \sqrt{2c}B_1$$

$$b = \frac{5}{3\sqrt{2c}} + \frac{2B_0}{B_1}, a = \frac{1}{72}\left(\frac{1}{c} + \frac{12B_0\sqrt{c} + 5\sqrt{2}B_1}{\sqrt{c}B_1^2}\right) \tag{37}$$

According to Eqn. (37), dark-bright optical soliton solution for Eqn. (2) is found

$$s_2(x,t) = \frac{8\operatorname{sech}\left[g(x,t)\right]^2 P - 2K + 8\sqrt{3}\left(72cB_0^2 - B_1^2\right)\tanh\left[g(x,t)\right]}{\left(2B_1 + M\left(L - 4\sqrt{3}\tanh\left[g(x,t)\right]\right)\right)\left(M + B_1\left(L + 4\sqrt{3}\tanh\left[g(x,t)\right]\right)\right)} \tag{38}$$

where $g(x,t) = \frac{EE - ct + x}{\sqrt{3c}}$, $K = 360\sqrt{2}cB_0^2 + 24\sqrt{c}B_0B_1 + 5\sqrt{2}B_1^2$, $L = 5\sqrt{2}$ and

$$P = 72\sqrt{2}cB_0^2 + 48\sqrt{c}B_0B_1 + \sqrt{2}B_1^2, M = 12\sqrt{c}B_0.$$

In Figs. 6-7, we plot two and three dimensional graphics of Eqn. (36) and Eqn. (38), which show the vitality of solutions with suitable parametric choices.

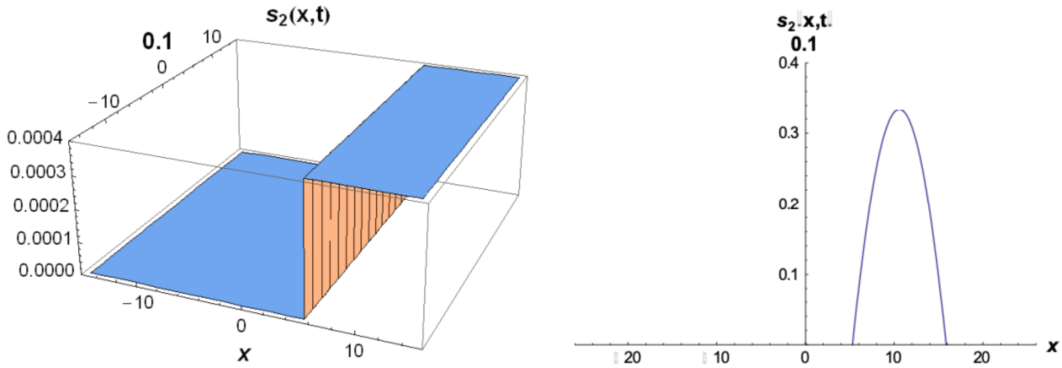

Figure 7: Three-dimensional and two-dimensional plots of solution Eqn. (38) for the values $A_2 = 0.4, B_0 = -8, B_1 = 0.015, E = -2.5, -15 < x < 15, -15 < t < 15$ and $t = 0.1$ for 2D plot

4. Conclusion

In this study, the simplified MCH equation and the Getmanou equation are researched by via a MEFM. After, we draw 2D and 3D graphs of dark optical soliton solutions, dark-bright optical soliton solutions of this equation by use of Mathematica. Soliton solutions are of two types as dark soliton and bright soliton. If there is a solution of type sech hyperbolic function, it is called a bright soliton solution and if there is a solution of type tanh hyperbolic function, it is called a dark soliton solution [14]. Solutions are called dark-bright solitons if they contain the sech and tanh functions at the same time.

From the obtained results, it has been deduced that MEFM is highly credible and strong in the sense that finding exact solutions. The solutions we obtained from equations are new solutions brought to the literature. The research shows that the MEFM algorithm is productive and can be used for many other NLEEs in mathematical physics.

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