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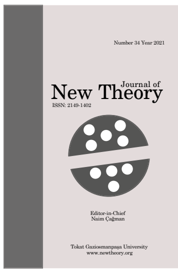

Operability-Oriented Configurations of the Soft Decision-Making Methods Proposed between 2013 and 2016 and Their Comparisons

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Research Article

Abstract − The concept of fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) is a mathematical tool coming into prominence with its ability to model decision-making problems. Therefore, in the present study, we configure soft decision-making (SDM) methods having been constructed with soft sets, soft matrices, and their fuzzy hybrid versions and introduced between 2013 and 2016 to operate them in *fpfs*-matrices space faithfully to the original. We then analyse the decision-making performances of the configured methods herein by using five test cases containing totally ordered alternatives. Thus, we determine the methods producing a valid ranking order according to all the test cases and apply the determined methods to a performance-based value assignment (PVA) problem in which the filters are to be ranked in terms of their image denoising performances. Therefore, we compare the performance ranking of the filters by using the methods. Finally, we discuss the need for further research.

Keywords *– Fuzzy sets, soft sets, soft matrices, fpfs-matrices, soft decision-making, PVA problem*

Mathematics Subject Classification (2020) − 03E72, 15B15

1. Introduction

The soft decision-making (SDM) methods, constructed with the concepts of soft sets [1], fuzzy soft sets [2,3], fuzzy parameterized soft sets [4], fuzzy parameterized fuzzy soft sets (*fpfs*-sets) [5], soft matrices [6], fuzzy soft matrices [7], and fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) [8], are widely used to model uncertainties mathematically. The relationship between these concepts is provided as ordered from the general to the specific in Fig. 1. Moreover, many researchers have focused on these concepts in various areas, such as algebra [9-12], topology [13-17], analysis and function theory [18,19], decision-making [3,20], and data classification [21,22]. In literature, what studies related to SDM primarily lack is usually its application to a hypothetical problem instead of a real-life problem. A limited number of studies, including methods applied to a real problem, can be summarised as follows: In [23], the authors have used soft sets to attain shoreline resources evaluation rules. [24] has attracted attention to this theory using soft set theory in the computerised classification of malignant and normal micro-calcifications on mammograms. In [25], the scholars have proposed a method via fuzzy soft sets to classify numerical data. [26] has introduced a classification method to classify medical data using fuzzy soft sets. In [21], the researchers have applied an SDM method constructed by *fpfs*-matrices to monolithic columns classification. [22] has applied a data classification problem in machine learning by using *fpfs*-matrices.

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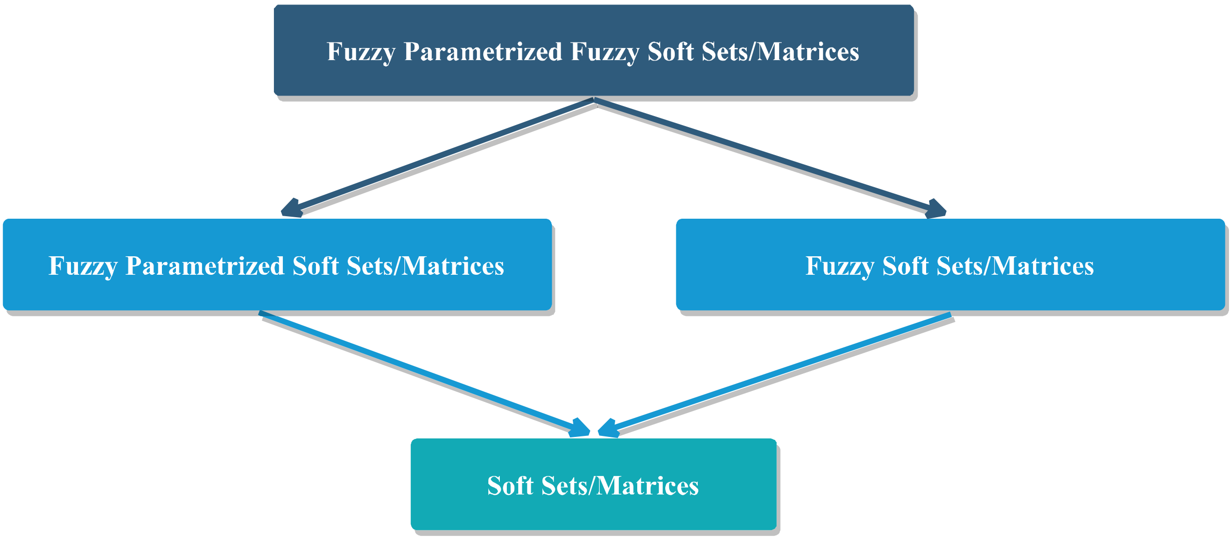

Fig. 1. Relationship between *fpfs*-sets/matrices and their substructures

Recently, the concept of *fpfs*-matrices [8] has stood out among the others due to its modelling success, the uncertainties in the decision-making problems where alternatives or parameters are fuzzy. Thus, the configurations of SDM methods constructed with the aforesaid concepts to operate them in *fpfs*-matrices space have become a popular study subject. To this end, over 50 SDM methods constructed with the aforesaid concepts have been configured [27-30] in *fpfs*-matrices space, faithfully to the original. Thereby, the configurations of the methods having been constructed with the abovementioned concepts and which were proposed between 1999 and 2012 have been completed. Furthermore, in [31-40], the authors have improved some of the configured methods to make them run faster and to simplify them mathematically. In [27,29], although some of the SDM methods proposed after 2012 have been configured, their configurations have not been completed yet. The present study aims to complete the configurations of the SDM methods having been constructed with soft sets, soft matrices, and their fuzzy hybrid versions and introduced between 2013 and 2016. To this end, we consider the SDM methods provided in [41-72].

The following tables provide some information about the preconfigured SDM methods. Table 1 explains the abbreviations used in Table 2-5. Table 2, 3, 4, and 5 show the unabbreviated forms of the previously configured SDM methods employing single, double, triple, and multiple matrices and their spaces in which they have been first put forward, respectively. Moreover, Table 6 lists the SDM methods constructed in the *fpfs*-matrices space. Lastly, Table 7 presents the SDM methods with the same configurations.

Table 1. Abbreviations of the considered spaces

SS	Soft Sets
SM	Soft Matrices
FSS	Fuzzy Soft Sets
FSM	Fuzzy Soft Matrices
FPSS	Fuzzy Parameterized Soft Sets
FPFSS	Fuzzy Parameterized Fuzzy Soft Sets
FPFSM	Fuzzy Parameterized Fuzzy Soft Matrices

Table 2. SDM methods employing single *fpfs*-matrix

Configured SDM Methods	**Original Spaces of the Configured SDM Methods**						**Descriptions**
FPFSM	FPFSS	FPSS	FSM	FSS	SM	SS	
CCE10 [27]	✓						Çağman, Çıtak, Enginoğlu 2010
CCE11 [27]		✓					Çağman, Çıtak, Enginoğlu 2011
CEC11 [29]				✓			Çağman, Enginoğlu, Çıtak 2011
F10(z) [28]				✓			Feng 2010
FJLL10 [29]				✓			Feng, Jun, Liu, Li 2010
FJLL10/2 [29]				✓			Feng, Jun, Liu, Li 2010
FJLL10/3 [29]				✓			Feng, Jun, Liu, Li 2010
FJLL10/4 [29]				✓			Feng, Jun, Liu, Li 2010
KKT13 [27]			✓				Khan, Khan, Thakur 2013
KM11 [29]				✓			Kalaichelvi, Malini 2011
KS10 [28]				✓			Kalayathankal, Singh 2010
KSM10 [28]				✓			Kuang, Shu, Mou 2010
KWW11(w, z) [28]				✓			Kong, Wang, Wu 2011
M11 [29]				✓			Mou 2011
MBR01 [27]				✓			Maji, Biswas, Roy 2001
MRB02 [27]						✓	Maji, Roy, Biswas 2002
MS10 [29]						✓	Majumdar, Samantha 2010
SM11 [28]				✓			Sun, Ma 2011
WW11 [29]						✓	Wu, Wang 2011
YE12 [37]		✓					Yılmaz, Eraslan 2012

Table 3. SDM methods employing double *fpfs*-matrices

Configured SDM Methods	**Original Spaces of the Configured SDM Methods**						**Descriptions**
FPFSM	FPFS	FPSS	FSM	FSS	SM	SS	
BMM12 [30]			✓				Basu, Mahapatra, Mondal 2012
BMM12/3 [30]					✓		Basu, Mahapatra, Mondal 2012
CD12/3 [30]		✓					Çağman, Deli 2012
CD12/4 [30]		✓					Çağman, Deli 2012
CE10 [27]						✓	Çağman, Enginoğlu 2010
CE10-2 [27]					✓		Çağman, Enginoğlu 2010
CE12 [27]			✓				Çağman, Enginoğlu 2012
CE10an [39]						✓	Çağman, Enginoğlu 2010
CE10on [39]						✓	Çağman, Enginoğlu 2010
FLC12 [30]						✓	Feng, Li, Çağman 2012
ICJ17 [29]					✓		Inthumathi, Chitra, Jayasree
KWW11/2(w, z) [30]						✓	Kong, Wang, Wu 2011
NS11 [30]			✓				Neog, Sut 2011
VR13 [27]					✓		Vijayabalaji, Ramesh 2013
Z14 [29]					✓		Zhang 2014
ZZ16 [27]	✓						Zhu, Zhan 2016
ZZ16/2 [27]	✓						Zhu, Zhan 2016

Table 4. SDM methods employing triple *fpfs*-matrices

Configured SDM Methods	**Original Spaces of the Configured SDM Methods**						**Descriptions**
FPFSM	FPFSS	FPSS	FSM	FSS	SM	SS	
BMM12/2 [30]					✓		Basu, Mahapatra, Mondal 2012
KGW09 [30]				✓			Kong, Gao, Wang 2009
QYZ12 [30]				✓			Qin, Yang, Zhang 2012
RM07a [30]				✓			Roy, Maji 2007
RM07o [30]				✓			Roy, Maji 2007
RM11 [27]						✓	Razak, Mohamad 2011
RM13 [27]				✓			Razak, Mohamad 2013

Table 5. SDM methods employing multiple *fpfs*-matrices

Configured SDM Methods	**Original Spaces of the Configured SDM Methods**						**Descriptions**
FPFSM	FPFSS	FPSS	FSM	FSS	SM	SS	
BNS12 [29]			✓				Borah, Neog, Sut 2012
CD12 [27]		✓					Çağman, Deli 2012
CD12-2 [27]		✓					Çağman, Deli 2012
DB12 [27]				✓			Das, Borgohain 2012
E15 [27]						✓	Eraslan 2015
EK15 [27]				✓			Eraslan, Karaaslan 2015
MR13 [29]			✓				Mondal, Roy 2013
MR13/2 [29]			✓				Mondal, Roy 2013
MR13/3 [29]			✓				Mondal, Roy 2013
NB14 [29]			✓				Nagarajan, Balamurugan 2014
NKY17 [29]				✓			Nagarani, Kalyani, Yookesh
S12 [29]				✓			Sut 2012
YJ11 [29]			✓				Yang, Ji 2011
YJ11/2 [29]			✓				Yang, Ji 2011

It can be seen from Table 2, 3, 4, and 5 that the fuzzy soft sets space, one of the substructures of *fpfs*-sets, is widely used in decision-making problems.

Table 6. SDM methods constructed in *fpfs*-matrices space

Proposed SDM Methods	**Number of Employed Matrices**				**Descriptions**
	Single	Double	Triple	Multiple	
EM20o [36]			✓		Enginoğlu, Memiş 2020
EMA18on [32]		✓			Enginoğlu, Memiş, Arslan 2018
EMC19o [34]		✓			Enginoğlu, Memiş, Çağman 2019
EMK19 [35]				✓	Enginoğlu, Memiş, Karaaslan 2019
EMO18o [40]		✓			Enginoğlu, Memiş, Öngel 2018
EC20 (PEM) [8]	✓				Enginoğlu, Çağman 2020 (Prevalence Effect Method)
Simplified SDM Methods					
EM20a [36]			✓		Enginoğlu, Memiş 2020
EMA18an [39]		✓			Enginoğlu, Memiş, Arslan 2018
EMC19a [34]		✓			Enginoğlu, Memiş, Çağman 2019
EMO18a [33]		✓			Enginoğlu, Memiş, Öngel 2018
sDB12 [38]				✓	Simplified DB12
sMBR01 [31]	✓				Simplified MBR01

Table 7. SDM methods with the same configurations

CE10-2	CE12
RM11	RM13
MR13	NB14

In Section 2 of the present study, we present some of the basic definitions of *fpfs*-matrices to be needed in the following sections of the paper. In Section 3, we configure the SDM methods provided in [41-72]. In Section 4, we propound five test cases to examine the consistency of the SDM methods employing fpfs-matrices. We then determine the considered SDM methods producing a valid ranking order in all the test cases. In Section 5, we apply the determined methods to a performance-based value assignment (PVA) problem in which the filters are ranked with regard to their salt-and-pepper noise (SPN) removal performances. Therefore, we compare the ranking order performances of the methods in the PVA problem. Finally, we discuss the need for further research.

2. Preliminaries

In this section, firstly, we present the concept of *fpfs*-matrices [8]. Throughout this paper, let E be a parameter set, $F(E)$ be the set of all the fuzzy sets over E, and $\mu \in F(E)$. Here, a fuzzy set is denoted by $\left\{ {}^{\mu(x)}x \mid x \in E \right\}$.

Definition 2.1. [5] Let U be a universal set, $\mu \in F(E)$, and α be a function from μ to $F(U)$. Then, the set $\left\{ \left({}^{\mu(x)}x, \alpha\left({}^{\mu(x)}x \right) \right) \mid x \in E \right\}$, being the graphic of α, is called a fuzzy parameterized fuzzy soft set (*fpfs*-set) parameterized via E over U (or briefly over U).

In the present paper, the set of all the *fpfs*-sets over U is denoted by $FPFS_E(U)$. In $FPFS_E(U)$, since the $\text{graph}(\alpha)$ and α generate each other uniquely, the notations are interchangeable. Therefore, as long as it causes no confusion, we denote an *fpfs*-set $\text{graph}(\alpha)$ by α.

Example 2.2. Let $E = \{x_1, x_2, x_3\}$ and $U = \{u_1, u_2, u_3, u_4\}$. Then,

$$\alpha = \{({}^{0.4}x_1, \{{}^{0.2}u_2, {}^{0.4}u_3, {}^{0.7}u_4\}), ({}^{0.9}x_2, \{{}^{0.5}u_1, {}^{0.3}u_2, {}^{0.6}u_3, {}^{0.4}u_4\}), ({}^{0.7}x_3, \{{}^{0.2}u_1, {}^{0.9}u_3, {}^{1}u_4\})\}$$

is an *fpfs*-set over U.

Definition 2.3. [8] Let $\alpha \in FPFS_E(U)$. Then, $[a_{ij}]$ is called *fpfs*-matrix of α and is defined by

$$[a_{ij}] = \begin{bmatrix} a_{01} & a_{02} & a_{03} & \cdots & a_{0n} & \cdots \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \end{bmatrix}$$

such that for $i \in \{0,1,2,\cdots\}$ and $j \in \{1,2,\cdots\}$,

$$a_{ij} := \begin{cases} \mu(x_j), & i = 0 \\ \alpha\left({}^{\mu(x_j)}x_j \right)(u_i), & i \neq 0 \end{cases}$$

Here, if $|U| = m - 1$ and $|E| = n$, then $[a_{ij}]$ has order $m \times n$.

From now on, the set of all the *fpfs*-matrices parameterized via E over U is denoted by $FPFS_E[U]$.

Example 2.4. The *fpfs*-matrix of α provided in Example 2.2 is as follows:

$$[a_{ij}] = \begin{bmatrix} 0.4 & 0.9 & 0.7 \\ 0 & 0.5 & 0.2 \\ 0.2 & 0.3 & 0 \\ 0.4 & 0.6 & 0.9 \\ 0.7 & 0.4 & 0.1 \end{bmatrix}$$

Definition 2.5. [8] Let $[a_{ij}]_{m \times n_1} \in FPFS_{E_1}[U]$, $[b_{ik}]_{m \times n_2} \in FPFS_{E_2}[U]$, and $[c_{ip}]_{m \times n_1 n_2} \in FPFS_{E_1 \times E_2}[U]$ such that $p = n_2(j-1) + k$. For all i and p, if $c_{ip} := \min\{a_{ij}, b_{ik}\}$, then $[c_{ip}]$ is called and-product of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}] \wedge [b_{ik}]$.

Definition 2.6. Let $[a_{ij}]_{m\times n_1} \in FPFS_{E_1}[U]$, $[b_{ik}]_{m\times n_2} \in FPFS_{E_2}[U]$, and $[c_{ip}]_{m\times n_1 n_2} \in FPFS_{E_1\times E_2}[U]$ such that $p = n_2(j-1)+k$. For all i and p, if $c_{ip} := \frac{a_{ij}+b_{ik}}{2}$, then $[c_{ip}]$ is called mean-product of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}] \times_m [b_{ik}]$.

Definition 2.7. Let $[s_{i1}] \in M_{(m-1)\times 1}(\mathbb{R})$ such that $m \geq 2$. Then, normalisation $[\hat{s}_{i1}]$ of $[s_{i1}]$ is defined by

$$\hat{s}_{i1} := \begin{cases} \dfrac{s_{i1} - \min\limits_k s_{k1}}{\max\limits_k s_{k1} - \min\limits_k s_{k1}}, & \max\limits_k s_{k1} \neq \min\limits_k s_{k1} \\ 1, & \max\limits_k s_{k1} = \min\limits_k s_{k1} \end{cases}$$

To obtain an increasing sequence consisting of all the elements of an index set, being a subset of $\mathbb{N}^n$, we present a linear ordering relation over $\mathbb{N}^n$ as follows:

Definition 2.8. [30] Let $(j_1, j_2, \dots, j_n), (k_1, k_2, \dots, k_n) \in \mathbb{N}^n$. Then, the relation "$\leq$" is called a linear ordering relation and is defined by

$$(j_1, j_2, \dots, j_n) \leq (k_1, k_2, \dots, k_n) \Leftrightarrow [j_1 < k_1 \vee (j_1 = k_1 \wedge j_2 < k_2) \vee \dots \vee (j_1 = k_1 \wedge j_2 = k_2 \wedge \dots \wedge j_{n-1} = k_{n-1} \wedge j_n \leq k_n)]$$

3. Configurations of Soft Decision-Making Methods

In this section, we configure the SDM methods constructed by soft sets [41-47], fuzzy soft sets [41,46,48-63], fuzzy parameterized soft sets [64,65], *fpfs*-sets [66,67], soft matrices [47,68], and fuzzy soft matrices [43,69-72]. From now on, $I_n = \{1,2,\cdots,n\}$ and $I_n^* = \{0,1,2,\cdots,n\}$.

[69] has employed fuzzy soft matrices to determine an eligible candidate in the recruitment process just as [70] has utilised them to select an environment with healthy living conditions. We configure the proposed methods therein as follows:

Algorithm 3.1. BSD13 and SR15

BSD13, SR15, and NS11 [30] are the same. Therefore, we prefer the notation NS11.

In [41], the authors have suggested a new method based on spatial distance and fuzzy soft sets. Moreover, they have presented the method for two soft sets. We configure the proposed methods therein as follows:

Algorithm 3.2. CXL13($\boldsymbol{\lambda}$)

Step 1. Construct an *fpfs*-matrix $[a_{ij}]_{m\times n}$

Step 2. Construct the parameters' optimum solution matrix $\lambda := [\lambda_{1j}]_{1\times n}$ such that $0 \leq \lambda_{1j} \leq 1$, for all $j \in I_n$

Step 3. Obtain $[b_{i1}]_{(m-1)\times 1}$ defined by

$$b_{i1} := \sqrt{\sum_{j=1}^{n} \left(\lambda_{1j} - a_{0j}a_{ij}\right)^2}, \quad i \in I_{m-1}$$

Step 4. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by $s_{i1} := \max\limits_k b_{k1} - b_{i1}$ such that $i \in I_{m-1}$

Step 5. Obtain the decision set $\{^{\hat{s}_{k1}}u_k | u_k \in U\}$

Algorithm 3.3. CXL13/2($\boldsymbol{\lambda}$)

Step 1. Construct two *fpfs*-matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$

Step 2. Obtain $[c_{ij}]_{m\times n}$ defined by $c_{ij} := \frac{a_{ij}+b_{ij}}{2}$ such that $i \in I_{m-1}^*$ and $j \in I_n$

Step 3. Apply CXL13 to $[c_{ij}]$

[48] has studied the selection of a suitable house with fuzzy soft sets. We configure the proposed method therein as follows:

Algorithm 3.4. GLF13($\boldsymbol{R}$)

Step 1. Construct *fpfs*-matrices $[a_{ij}^1]_{m\times n}, [a_{ij}^2]_{m\times n}, \cdots, [a_{ij}^t]_{m\times n}$

Step 2. Determine a set R of indices such that $R \subseteq I_n$. Moreover, let (r_k) denote the increasing sequence of the elements of R.

Step 3. Obtain $K_r = \{v \in I_t : \exists i \ni a_{0r}^v a_{ir}^v \neq 0\}$, for all $r \in R$. For $r \in R$, if $K_r = \emptyset$, then K_r is chosen as $\{0\}$. Furthermore, let (u_k^r) stand for the increasing sequence of the elements of K_r, for all $r \in R$.

Step 4. Obtain $[b_{ik}^z]_{m\times|K_r|}$ defined by

$$b_{ik}^z := \begin{cases} a_{ir_z}^{u_k^{r_z}}, & \forall z \in I_{|R|}, u_k^{r_z} \neq 0 \\ 1, & otherwise \end{cases}$$

such that $i \in I_{m-1}^*$, $z \in I_{|R|}$, and $k \in I_{|K_r|}$

Here, $|R|$ and $|K_r|$ denote the cardinality of R and K_r, respectively.

Step 5. Obtain $[c_{ij}]_{m\times|R|}$ defined by

$$c_{ij} := \min_k \{b_{ik}^j\}$$

such that $i \in I_{m-1}^*$ and $j \in I_{|R|}$

Step 6. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by

$$s_{i1} := \min_{j\in I_{|R|}} \{c_{0j} c_{ij}\}, \quad i \in I_{m-1}$$

Step 7. Obtain the decision set $\{{}^{\hat{s}_{k1}}u_k | u_k \in U\}$

In [42], the authors have developed a pruning method using soft sets. We configure it as follows:

Algorithm 3.5. HG13

Step 1. Construct two *fpfs*-matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$

Step 2. Obtain $[c_{i1}]_{(m-1)\times 1}$, $[d_{i1}]_{(m-1)\times 1}$, $[e_{i1}]_{(m-1)\times 1}$, and $V = \left\{u_i : e_{i1} = \max_{k\in I_{m-1}} e_{k1}\right\}$ such that

$$c_{i1} := \sum_{j=1}^{n} a_{0j} a_{ij}, \quad d_{i1} := \sum_{j=1}^{n} b_{0j} b_{ij}, \quad \text{and} \quad e_{i1} := c_{i1} + d_{i1}, \quad i \in I_{m-1}$$

Step 3. For all $u_i \in V$, obtain $\bar{u}_i := \{u_j \in V : (c_{i1}, d_{i1}) = (c_{j1}, d_{j1}) \vee (c_{i1}, d_{i1}) = (d_{j1}, c_{j1})\}$

Step 4. Obtain $W = \left\{ u_i \in V : |\bar{u}_i| = \min_{u_k\in V} |\bar{u}_k| \right\}$

Here, $|\bar{u}_i|$ denotes the cardinality of $\bar{u}_i$.

Step 5. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by $s_{i1} := \begin{cases} \frac{1+e_{k1}}{1+\sum_j (a_{0j}+b_{0j})}, & u_k \in W \\ \frac{e_{k1}}{\sum_j (a_{0j}+b_{0j})}, & u_k \in U - W \end{cases}, \quad i \in I_{m-1}$

Step 6. Obtain the decision set $\{{}^{\hat{s}_{k1}}u_k | u_k \in U\}$

[49] has proposed a method based on group decision-making and applied it to a company's staff selection problem. We configure the proposed method therein as follows:

Algorithm 3.6. SM13($\boldsymbol{w},\boldsymbol{\alpha}$)

Step 1. Construct *fpfs*-matrices $[a_{ij}^1]_{m\times n}, [a_{ij}^2]_{m\times n}, \cdots, [a_{ij}^t]_{m\times n}$ such that $a_{0j}^1 = a_{0j}^2 = \cdots = a_{0j}^t = a_{0j}$

Step 2. Construct $w \coloneqq [w_{1k}]_{1\times t}$ such that $0 \le w_{1k} \le 1$ and $\sum_{k=1}^{t} w_{1k} = 1$, for $k \in I_t$

Step 3. Obtain $[b_{1j}]_{1\times n}$ defined by

$$b_{1j} \coloneqq \begin{cases} \dfrac{a_{0j}}{\sum_{k=1}^{n} a_{0k}}, & \displaystyle\sum_{k=1}^{n} a_{0k} \neq 0 \\ \dfrac{1}{n}, & otherwise \end{cases}, \quad j \in I_n$$

Step 4. Obtain $[c_{kr}]_{t\times t}$ defined by

$$c_{kr} \coloneqq \sum_{j=1}^{n} b_{1j}\, z_{kr}^{j}, \quad k,r \in I_t$$

such that

$$z_{kr}^{j} \coloneqq \begin{cases} \dfrac{\sum_{i=1}^{m-1} \min\{a_{ij}^k, a_{ij}^r\}}{\sum_{i=1}^{m-1} \max\{a_{ij}^k, a_{ij}^r\}}, & \displaystyle\sum_{i=1}^{m-1} \max\{a_{ij}^k, a_{ij}^r\} \neq 0 \\ 1, & otherwise \end{cases}$$

Step 5. Obtain $[d_{1k}]_{1\times t}$ defined by

$$d_{1k} \coloneqq \frac{1}{t-1} \sum_{r=1, r\neq k}^{t} c_{kr}, \quad k \in I_t$$

Step 6. Obtain $[e_{1k}]_{1\times t}$ defined by

$$e_{1k} \coloneqq \begin{cases} \dfrac{d_{1k}}{\sum_{l=1}^{t} d_{1l}}, & \displaystyle\sum_{l=1}^{t} d_{1l} \neq 0 \\ \dfrac{1}{t}, & otherwise \end{cases}, \quad k \in I_t$$

Step 7. For $\alpha \in [0,1]$, obtain $[\lambda_{1k}]_{1\times t}$ defined by

$$\lambda_{1k} \coloneqq \alpha w_{1k} + (1-\alpha) e_{1k}, \quad k \in I_t$$

Step 8. Obtain $[f_{ij}]_{(m-1)\times n}$ defined by

$$f_{ij} \coloneqq \sum_{k=1}^{t} \lambda_{1k} a_{ij}^k$$

such that $i \in I_{m-1}$, $j \in I_n$, and $k \in I_t$

Step 9. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by

$$s_{i1} \coloneqq 1 - \sqrt{\sum_{j=1}^{n} b_{1j} \left(f_{ij} - \max_{k\in I_{m-1}} f_{kj}\right)^2}, \quad i \in I_{m-1}$$

Step 10. Obtain the decision set $\{^{\hat{s}_{k1}}u_k | u_k \in U\}$

[43] has proposed two SDM methods via soft sets and fuzzy soft matrices. Moreover, [67] has suggested an SDM method constructed with *fpfs*-sets. We configure the proposed methods therein as follows:

Algorithm 3.7. GDC14 and RH16/2

GDC14, RH16/2, and MRB02 [27] are the same. Therefore, we prefer the notation MRB02.

Algorithm 3.8. GDC14/2(λ)

GDC14/2 and NKY17(λ) [29] are the same. Therefore, we prefer the notation NKY17(λ).

In [44], the author has utilised soft sets to determine an optimal alternative. We configure the proposed method therein as follows:

Algorithm 3.9. K14

Step 1. Construct two *fpfs*-matrices $\left[a_{ij}\right]_{m\times n}$ and $\left[b_{ij}\right]_{m\times n}$

Step 2. Obtain $\left[c_{ij}\right]_{(m-1)\times n}$ defined by $c_{ij} := \min\left\{a_{0j}a_{ij}, b_{0j}b_{ij}\right\}$ such that $i \in I_m$ and $j \in I_n$

Step 3. Obtain $V = \left\{u_i \in U : \sum_{j=1}^{n} c_{ij} \neq 0\right\}$

Step 4. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by

$$s_{i1} := \begin{cases} \left(\sum_{j=1}^{n} a_{0j}a_{ij}\right)\left(\sum_{j=1}^{n} b_{0j}b_{ij}\right) - \sum_{j=1}^{n} a_{0j}a_{ij} - \sum_{j=1}^{n} b_{0j}b_{ij}\,, & u_i \in V \\ 0, & u_i \in U - V \end{cases}$$

such that $i \in I_{m-1}$

Step 5. Obtain the decision set $\left\{{}^{\hat{s}_{k1}}u_k | u_k \in U\right\}$

[64] has studied financial decision-making problems using fuzzy parameterized soft sets, although it is stated that fuzzy soft sets are used. We configure the proposed method as follows:

Algorithm 3.10. MM14

MM14 and CCE10 [27] are the same. Therefore, we prefer the notation CCE10.

In [50], the authors have proposed an algorithm using fuzzy soft sets to determine the optimal decision program. We configure the proposed method therein as follows:

Algorithm 3.11. WQ14($\boldsymbol{\kappa}$)

Step 1. Construct an *fpfs*-matrix $\left[a_{ij}\right]_{m\times n}$

Step 2. Obtain $[b_{ik}]_{(m-1)\times(m-1)}$ defined by

$$b_{ik} := \frac{1}{n}\sum_{j=1}^{n}\left(1 - a_{0j}\left|a_{ij} - a_{kj}\right|\right), \quad i,k \in I_{m-1}$$

Step 3. Obtain $[c_{ik}]_{(m-1)\times(m-1)}$ defined by

$$c_{ik} := \max_{j}\left\{\min\left\{b_{ij}, b_{jk}\right\}\right\}, \quad i,j,k \in I_{m-1}$$

Step 4. Obtain the set D of all the entries of $[c_{ik}]$

Step 5. Obtain the descending-sorted matrix $\left[e_{1j}\right]_{1\times|D|}$ of the D's elements such that $j \in I_{|D|}$

Step 6. Obtain $[f_{ik}^{j}]_{(m-1)\times(m-1)}$ defined by

$$f_{ik}^{j} := \begin{cases} 1, & c_{ik} \geq e_{1j} \\ 0, & c_{ik} < e_{1j} \end{cases}$$

such that $i,k \in I_{m-1}$ and $j \in I_{|D|}$

Step 7. Obtain $[g_{1j}]_{1\times|D|}$ defined by

$$g_{1j} := \sum_{k=1}^{m-1} \chi(j,k), \quad j \in I_{|D|}$$

such that

$$\chi(j,k) := \begin{cases} 1, & 1 < \sum_{i=1}^{m-1} f_{ik}^{j} < m-1 \\ 0, & otherwise \end{cases}$$

Step 8. Obtain $[h_{1j}]_{1\times(|D|-1)}$ defined by

$$h_{1j} := \begin{cases} \dfrac{e_{1j} - e_{1(j+1)}}{g_{1(j+1)} - g_{1j}}, & g_{1(j+1)} > g_{1j} \\ 0, & g_{1(j+1)} \leq g_{1j} \end{cases}, \quad j \in I_{(|D|-1)}$$

Step 9. Obtain $\lambda := e_{1p}$ such that $p := 1 + \underset{j}{\operatorname{argmax}}\, h_{1j}$

Here, $\underset{j}{\operatorname{argmax}}\, h_{1j}$ is an index of the h_{1j} being maximum for all $j \in I_{(|D|-1)}$.

Step 10. For all $u_i \in U$, obtain $\bar{u}_i := \{u_s \in U : \forall k \in I_n, j \in I_{|D|}(f_{ik}^{j} = f_{sk}^{j})\}$

Step 11. Obtain the clustering set $C = \{\bar{u}_i : u_i \in U\}$

Step 12. For all $r \in I_n$, obtain $[a_{ij}^{r}]_{m\times(n-1)}$ deleting r^{th} column of $[a_{ij}]$

Step 13. For all $r \in I_n$, obtain $[b_{ik}^{r}]_{(m-1)\times(m-1)}$ applying Step 3 to $[a_{ij}^{r}]$

Step 14. For all $r \in I_n$, obtain $[c_{ik}^{r}]_{(m-1)\times(m-1)}$ applying Step 4 to $[b_{ik}^{r}]$

Step 15. Obtain $[\tilde{f}_{ik}^{r}]_{(m-1)\times(m-1)}$ defined by

$$\tilde{f}_{ik}^{r} := \begin{cases} 1, & c_{ik}^{r} \geq \lambda \\ 0, & c_{ik}^{r} < \lambda \end{cases}$$

such that $i,k \in I_{m-1}$ and $r \in I_n$

Step 16. For all $u_i \in U$, obtain $\bar{u}_i^{r} := \{u_s \in U : \forall k \in I_n, (\tilde{f}_{ik}^{r} = \tilde{f}_{sk}^{r})\}$

Step 17. For all $r \in I_n$, obtain the clustering set $C^r = \{\bar{u}_i^{r} : u_i \in U\}$

Step 18. Obtain $[\sigma_{1j}]_{1\times n}$ defined by

$$\sigma_{1j} := 1 - \frac{|C \cap C^j|}{m-1}, \quad j \in I_n$$

Step 19. Obtain $[\beta_{1j}]_{1\times n}$ defined by

$$\beta_{1j} := \begin{cases} \dfrac{\sigma_{1j}}{\sum_{k=1}^{n} \sigma_{1k}}, & \displaystyle\sum_{k=1}^{n} \sigma_{1k} \neq 0 \\ \dfrac{1}{n}, & otherwise \end{cases}, \quad j \in I_n$$

Step 20. Obtain $[w_{1j}]_{1\times n}$ defined by

$$w_{1j} := \kappa a_{0j} + (1-\kappa)\beta_{1j}, \quad j \in I_n$$

Here, κ is Bias coefficient chosen by decision-maker and $\kappa \in [0,1]$

Step 21. Obtain $[\tilde{a}_{ij}]_{m\times n}$ defined by $\tilde{a}_{0j} := w_{1j}$ and $\tilde{a}_{ij} := a_{ij}$, for all $i \in I_{m-1}$ and $j \in I_n$

Step 22. Apply MRB02 [27] to $[\tilde{a}_{ij}]$

[51,55,56] have introduced the same methods for fuzzy soft sets by combining grey relational analysis with the Dempster-Shafer theory of evidence and applied them to medical diagnosis. We configure the proposed methods therein as follows:

Algorithm 3.12. XWL14(α, q), LWX15(α, q), and T15(α, q)

Step 1. Construct an *fpfs*-matrix $[a_{ij}]_{m\times n}$

Step 2. Obtain $[b_{i1}]_{(m-1)\times 1}$ defined by

$$b_{i1} := \frac{1}{n}\sum_{j=1}^{n} a_{0j}a_{ij}, \quad i \in I_{m-1}$$

Step 3. Obtain $[c_{ij}]_{(m-1)\times n}$ defined by

$$c_{ij} := |a_{0j}a_{ij} - b_{i1}|$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 4. For $\alpha \in [0,1]$, obtain $[d_{ij}]_{(m-1)\times n}$ defined by

$$d_{ij} := \begin{cases} \dfrac{\min\limits_{k\in I_{m-1}} c_{kj} + \alpha \max\limits_{k\in I_{m-1}} c_{kj}}{c_{ij} + \alpha \max\limits_{k\in I_{m-1}} c_{kj}}, & \max\limits_{k\in I_{m-1}} c_{kj} \neq 0 \\ 1, & otherwise \end{cases}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 5. For $q \in \mathbb{N}^+$, obtain $[e_{1j}]_{1\times n}$ defined by

$$e_{1j} := \frac{1}{m-1}\left(\sum_{i=1}^{m-1} (d_{ij})^q\right)^{\frac{1}{q}}, \quad j \in I_n$$

Step 6. Obtain $[f_{ij}]_{(m-1)\times n}$ defined by

$$f_{ij} := \begin{cases} \dfrac{a_{0j}a_{ij}}{\sum_{k=1}^{m-1} a_{0j}a_{kj}}, & \displaystyle\sum_{k=1}^{m-1} a_{0j}a_{kj} \neq 0 \\ \dfrac{1}{m-1}, & otherwise \end{cases}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 7. Obtain $[g_{ij}]_{(m-1)\times n}$ and $[h_{1j}]_{1\times n}$ defined by

$$g_{ij} \coloneqq (1 - e_{1j})f_{ij}$$

and

$$h_{1j} \coloneqq 1 - \sum_{i=1}^{m-1} g_{ij}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 8. Obtain $[Bel_{i1}^{n-1}]_{m\times 1}$ defined by

$$Bel_{i1}^{j} \coloneqq \begin{cases} \dfrac{g_{i1}g_{i2} + g_{i1}h_{12} + h_{11}g_{i2}}{\left|1 - \sum_{k=1}^{m-1}\sum_{l=1,k\neq l}^{m-1} g_{k1}g_{l2}\right|}, & i \in I_{m-1} \text{ and } j = 1 \\ \dfrac{h_{11}h_{12}}{\left|1 - \sum_{k=1}^{m-1}\sum_{l=1,k\neq l}^{m-1} g_{k1}g_{l2}\right|}, & i = m \text{ and } j = 1 \\ \dfrac{Bel_{i1}^{j-1}g_{i(j+1)} + Bel_{i1}^{j-1}h_{1(j+1)} + Bel_{m1}^{j-1}g_{i(j+1)}}{\left|1 - \sum_{k=1}^{m-1}\sum_{l=1,k\neq l}^{m-1} Bel_{k1}^{j-1}g_{l(j+1)}\right|}, & i \in I_{m-1}, j \in I_{n-1}, \text{and } j \neq 1 \\ \dfrac{Bel_{m1}^{j-1}h_{1(j+1)}}{\left|1 - \sum_{k=1}^{m-1}\sum_{l=1,k\neq l}^{m-1} Bel_{k1}^{j-1}g_{l(j+1)}\right|}, & i = m, j \in I_{n-1}, \text{and } j \neq 1 \end{cases}$$

Step 9. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by $s_{i1} \coloneqq Bel_{i1}^{n-1}$ such that $i \in I_{m-1}$

Step 10. Obtain the decision set $\{{}^{\hat{s}_{k1}}u_k | u_k \in U\}$

XWL14(α, q), LWX15(α, q), and T15(α, q) are the same. Therefore, we prefer the notation XWL14(α, q).

Algorithm 3.13. XWL14/2$(\boldsymbol{\alpha}, \boldsymbol{q})$, LWX15/2$(\boldsymbol{\alpha}, \boldsymbol{q})$, and T15/2$(\boldsymbol{\alpha}, \boldsymbol{q})$

Step 1. Construct two *fpfs*-matrices $[a_{ij}]_{m\times n_1}$ and $[b_{ik}]_{m\times n_2}$

Step 2. Find and-product *fpfs*-matrix $[c_{ip}]_{m\times n_1 n_2}$ of $[a_{ij}]$ and $[b_{ik}]$

Step 3. Apply XWL14 to $[c_{ip}]$

XWL14/2(α, q), LWX15/2(α, q), and T15/2(α, q) are the same. Therefore, we prefer the notation XWL14/2(α, q).

In [52], the scholars have suggested a new SDM method based on grey relational analysis and fuzzy soft sets. We configure the proposed method therein as follows:

Algorithm 3.14. YHX14$(\boldsymbol{\alpha}, \boldsymbol{\beta})$

Step 1. Construct an *fpfs*-matrix $[a_{ij}]_{m\times n}$

Step 2. Obtain $[\lambda_{1j}]_{1\times n}$ defined by

$$\lambda_{1j} \coloneqq \frac{1}{m-1}\sum_{i=1}^{m-1} a_{ij}, \quad i \in I_n$$

Step 3. Obtain $[b_{ij}]_{(m-1)\times n}$ defined by

$$b_{ij} \coloneqq \begin{cases} a_{0j}, & a_{ij} \geq \lambda_{1j} \\ 0, & a_{ij} < \lambda_{1j} \end{cases}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 4. Obtain $[c_{i1}]_{(m-1)\times 1}$ defined by

$$c_{i1} \coloneqq \sum_{j=1}^{n} b_{ij}\,, \quad i \in I_{m-1}$$

Step 5. Obtain $[d_{1j}]_{1\times n}$ defined by

$$d_{1j} \coloneqq \max_{i\in I_{m-1}} \{a_{0j}a_{ij}\}, \quad j \in I_n$$

Step 6. Obtain $[e_{i1}]_{(m-1)\times 1}$ defined by

$$e_{i1} \coloneqq -\frac{1}{n\ln(2)}\sum_{j=1}^{n}\left(a_{0j}a_{ij}\ln\left(\frac{\varepsilon + a_{0j}a_{ij}}{\varepsilon + \frac{1}{2}(a_{0j}a_{ij} + d_{1j})}\right) + (1 - a_{0j}a_{ij})\ln\left(\frac{1+\varepsilon - a_{0j}a_{ij}}{1+\varepsilon - \frac{1}{2}(a_{0j}a_{ij} + d_{1j})}\right)\right.$$
$$\left. + d_{1j}\ln\left(\frac{\varepsilon + d_{1j}}{\varepsilon + \frac{1}{2}(d_{1j} + a_{0j}a_{ij})}\right) + (1 - d_{1j})\ln\left(\frac{1+\varepsilon - d_{1j}}{1+\varepsilon - \frac{1}{2}(d_{1j} + a_{0j}a_{ij})}\right)\right), \quad i \in I_{m-1}$$

Here, if $a_{0j}a_{ij} = 0$, $d_{1j} = 0$, $a_{0j}a_{ij} = 1$, or $d_{1j} = 1$, then $\ln\left(\frac{a_{0j}a_{ij}}{\frac{1}{2}(a_{0j}a_{ij}+d_{1j})}\right)$, $\ln\left(\frac{d_{1j}}{\frac{1}{2}(d_{1j}+a_{0j}a_{ij})}\right)$, $\ln\left(\frac{1-a_{0j}a_{ij}}{1-\frac{1}{2}(a_{0j}a_{ij}+d_{1j})}\right)$, or $\ln\left(\frac{1-d_{1j}}{1-\frac{1}{2}(d_{1j}+a_{0j}a_{ij})}\right)$ are undefined, respectively. To cope with these drawbacks, we modify them as $\ln\left(\frac{\varepsilon+a_{0j}a_{ij}}{\varepsilon+\frac{1}{2}(a_{0j}a_{ij}+d_{1j})}\right)$, $\ln\left(\frac{\varepsilon+d_{1j}}{\varepsilon+\frac{1}{2}(d_{1j}+a_{0j}a_{ij})}\right)$, $\ln\left(\frac{1+\varepsilon-a_{0j}a_{ij}}{1+\varepsilon-\frac{1}{2}(a_{0j}a_{ij}+d_{1j})}\right)$, and $\ln\left(\frac{1+\varepsilon-d_{1j}}{1+\varepsilon-\frac{1}{2}(d_{1j}+a_{0j}a_{ij})}\right)$, respectively, such that $\varepsilon \ll 1$ is a positive constant, e.g., $\varepsilon = 0.0001$.

Step 7. For $\alpha \in [0,1]$, obtain $[f_{i1}]_{(m-1)\times 1}$ and $[g_{i1}]_{(m-1)\times 1}$ defined by

$$f_{i1} \coloneqq \begin{cases} \dfrac{\min\limits_{k\in I_{m-1}}\{n - c_{k1}\} + \alpha \max\limits_{k\in I_{m-1}}\{n - c_{k1}\}}{n - c_{i1} + \alpha \max\limits_{k\in I_{m-1}}\{n - c_{k1}\}}, & \max\limits_{k\in I_{m-1}}\{n - c_{k1}\} \neq 0 \\ 1, & otherwise \end{cases}, \quad i \in I_{m-1}$$

and

$$g_{i1} \coloneqq \begin{cases} \dfrac{\min\limits_{k\in I_{m-1}} e_{k1} + \alpha \max\limits_{k\in I_{m-1}} e_{k1}}{e_{i1} + \alpha \max\limits_{k\in I_{m-1}} e_{k1}}, & \max\limits_{k\in I_{m-1}} e_{k1} \neq 0 \\ 1, & otherwise \end{cases}, \quad i \in I_{m-1}$$

Step 8. For $\beta \in [0,1]$, obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by

$$s_{i1} \coloneqq \beta f_{i1} + (1-\beta) g_{i1}, \quad i \in I_{m-1}$$

Step 9. Obtain the decision set $\{^{\hat{s}_{k1}}u_k | u_k \in U\}$

[45] has applied an SDM method constructed via soft sets to a problem related to a company's recruitment scenario. Moreover, the following method is a version constructed with t *fpfs*-matrices of Z14 provided in [29]. We configure the proposed method therein as follows:

Algorithm 3.15. Z14/2

Step 1. Construct *fpfs*-matrices $[a_{ij}^1]_{m\times n}, [a_{ij}^2]_{m\times n}, \cdots, [a_{ij}^t]_{m\times n}$

Step 2. Obtain $[b_{ij}]_{m\times n}$ defined by $b_{ij} \coloneqq \max\limits_{k\in I_t} a_{ij}^k$ such that $i \in I_{m-1}^*$ and $j \in I_n$

Step 3. Apply MRB02 [27] to $[b_{ij}]$

Z14 [29] is a special version of Z14/2.

In [53], the researcher has availed of the concept of fuzzy soft sets. We configure the proposed method therein as follows:

Algorithm 3.16. A15

Step 1. Construct an *fpfs*-matrix $\left[a_{ij}\right]_{m\times n}$

Step 2. Obtain $[b_{ik}]_{(m-1)\times(m-1)}$ defined by

$$b_{ik} := \begin{cases} \displaystyle\sum_{j=1}^{n} a_{0j}\chi\left(a_{ij}, a_{kj}\right), & i \neq k \\ 0, & i = k \end{cases}$$

such that

$$\chi\left(a_{ij}, a_{kj}\right) := \begin{cases} 1, & a_{ij} > a_{kj} \\ 0, & a_{ij} \leq a_{kj} \end{cases}, \quad i,k \in I_{m-1}$$

Step 3. Obtain $[c_{ik}]_{(m-1)\times(m-1)}$ defined by

$$c_{ik} := \begin{cases} b_{ik}, & i \neq k \\ n(m-2) - \displaystyle\sum_{l=1,}^{m-1} b_{lk}, & i = k \end{cases}, \quad i,k \in I_{m-1}$$

Step 4. Obtain sum of the eigenvectors $[s_{i1}]_{(m-1)\times 1}$ associated with the dominant eigenvalues $\lambda := n(m-2)$

Step 5. Obtain the decision set $\left\{{}^{\hat{s}_{k1}}u_k | u_k \in U\right\}$

[65] has modelled a car purchasing problem through fuzzy parameterized soft sets. We configure the proposed method therein as follows:

Algorithm 3.17. DC15($\boldsymbol{\alpha}$)

Step 1. Construct an *fpfs*-matrix $\left[a_{ij}\right]_{m\times n}$

Step 2. Obtain increasing sequence (r_t) consisting of all the elements of $R := \left\{j : a_{0j} \geq \alpha\right\}$ such that $\alpha \in [0,1]$. If $R = \emptyset$, then $\alpha := \frac{1}{n}\sum_{j=1}^{n} a_{0j}$.

Step 3. Obtain $\left[b_{ip}\right]_{m\times|R|^2}$ defined by $b_{ip} = \min\left\{a_{ir_j}, a_{ir_k}\right\}$ such that $p = n(j-1) + k$, $i \in I_{m-1}^*$, and $j,k \in I_{|R|}$. Here, $|R|$ denote the cardinality of R.

Step 4. Obtain the score matrix $[s_{i1}]_{m\times 1}$ defined by

$$s_{i1} := \frac{1}{|R|^2}\sum_{p=1}^{|R|^2} b_{0p}b_{ip}, \quad i \in I_{m-1}$$

Step 5. Obtain the decision set $\left\{{}^{\hat{s}_{k1}}u_k | u_k \in U\right\}$

[54,58] have proposed an SDM method based on fuzzy soft sets. We configure the proposed methods therein as follows:

Algorithm 3.18. HJ15($\boldsymbol{\lambda}$) and H16($\boldsymbol{\lambda}$)

Step 1. Construct two *fpfs*-matrices $\left[a_{ij}\right]_{m\times n}$ and $\left[b_{ij}\right]_{m\times n}$

Step 2. For $\lambda \in [0,1]$, obtain $\left[c_{ij}\right]_{m\times n}$ and $\left[d_{ij}\right]_{m\times n}$ defined by

$$c_{ij} \coloneqq \begin{cases} a_{ij}, & a_{ij} \geq \lambda \\ 0, & a_{ij} < \lambda \end{cases} \text{ and } d_{ij} \coloneqq \begin{cases} b_{ij}, & b_{ij} \geq \lambda \\ 0, & b_{ij} < \lambda \end{cases}$$

such that $i \in I_{m-1}^*$ and $j \in I_n$

Step 3. Apply CE10a [27] to $[c_{ij}]$ and $[d_{ij}]$

In [71], the scholars have suggested two SDM methods based on grey relational analysis and fuzzy soft matrices. We configure the proposed methods therein as follows:

Algorithm 3.19. XHL15$(\boldsymbol{\alpha}, \boldsymbol{q})$

Step 1. Construct an *fpfs*-matrix $[a_{ij}]_{m\times n}$

Step 2. Obtain $[b_{ij}]_{(m-1)\times n}$ defined by

$$b_{ij} \coloneqq (2a_{0j} - 1)(2a_{ij} - 1)$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 3. Obtain $[c_{i1}]_{(m-1)\times 1}$ defined by

$$c_{i1} \coloneqq \frac{1}{n}\sum_{j=1}^{n} b_{ij}, \quad i \in I_{m-1}$$

Step 4. Obtain $[d_{ij}]_{(m-1)\times n}$ defined by

$$d_{ij} \coloneqq |b_{ij} - c_{i1}|$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 5. For $\alpha \in [0,1]$, obtain $[e_{ij}]_{(m-1)\times n}$ defined by

$$e_{ij} \coloneqq \begin{cases} \dfrac{\min\limits_{k\in I_{m-1}} d_{kj} + \alpha \max\limits_{k\in I_{m-1}} d_{kj}}{d_{ij} + \alpha \max\limits_{k\in I_{m-1}} d_{kj}}, & \max\limits_{k\in I_{m-1}} d_{kj} \neq 0 \\ 1, & otherwise \end{cases}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 6. For $q \in \mathbb{N}^+$, obtain $[f_{1j}]_{1\times n}$ defined by

$$f_{1j} \coloneqq \frac{1}{m-1}\left(\sum_{i=1}^{m-1} (e_{ij})^q\right)^{\frac{1}{q}}, \quad j \in I_n$$

Step 7. Obtain $[g_{1j}]_{1\times n}$ defined by

$$g_{1j} \coloneqq 1 - f_{1j}, \quad j \in I_n$$

Step 8. Obtain $[h_{ij}]_{(m-1)\times n}$ defined by

$$h_{ij} \coloneqq b_{ij} g_{1j}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 9. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$

$$s_{i1} \coloneqq \delta_i(1,2,\dots,n), \quad i \in I_{m-1}$$

such that

$$\delta_i(1,2,\dots,n) \coloneqq \frac{\delta_i(1,2,\dots,n-1)+h_{in}}{1+\delta_i(1,2,\dots,n-1)h_{in}}, \quad i \in I_{m-1} \text{ and } n \geq 2$$

and

$$\delta_i(1,2) \coloneqq \frac{h_{i1}+h_{i2}}{1+h_{i1}h_{i2}}, \quad i \in I_{m-1}$$

Step 10. Obtain the decision set $\{^{\hat{s}_{k1}}u_k | u_k \in U\}$

Algorithm 3.20. XHL15/2($\boldsymbol{\alpha}, \boldsymbol{q}$)

Step 1. Construct two *fpfs*-matrices $[a_{ij}]_{m\times n_1}$ and $[b_{ik}]_{m\times n_2}$

Step 2. Find and-product *fpfs*-matrix $[c_{ip}]_{m\times n_1 n_2}$ of $[a_{ij}]$ and $[b_{ik}]$

Step 3. Apply XHL15(α, q) to $[c_{ip}]$

[66] has benefited *fpfs*-sets to fill an announced position in a company. We configure the proposed methods therein as follows:

Algorithm 3.21. ZXZ15($\boldsymbol{\alpha}$)

Step 1. Construct two *fpfs*-matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$

Step 2. Obtain $[c_{ij}]_{m\times n}$ defined by

$$c_{0j} \coloneqq \sqrt{\frac{(a_{0j})^2+(b_{0j})^2}{2}} \text{ and } c_{ij} \coloneqq \max\{a_{ij}, b_{ij}\}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 3. Obtain $[d_{ij}]_{m\times n}$ defined by

$$d_{0j} \coloneqq c_{0j} \text{ and } d_{ij} \coloneqq \begin{cases} c_{ij}, & c_{ij} \geq \alpha \\ 0, & c_{ij} < \alpha \end{cases}$$

such that $\alpha \in [0,1]$, $i \in I_{m-1}$, and $j \in I_n$

Step 4. Apply CCE10 [27] to $[d_{ij}]$

Algorithm 3.22. ZXZ15/2($\boldsymbol{\alpha}$)

Step 1. Construct two *fpfs*-matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$

Step 2. Obtain $[c_{ij}]_{m\times n}$ defined by

$$c_{0j} \coloneqq \sqrt{\frac{(a_{0j})^2+(b_{0j})^2}{2}} \text{ and } c_{ij} \coloneqq \min\{a_{ij}, b_{ij}\}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 3. Obtain $[d_{ij}]_{m\times n}$ defined by

$$d_{0j} \coloneqq c_{0j} \text{ and } d_{ij} \coloneqq \begin{cases} c_{ij}, & c_{ij} \geq \alpha \\ 0, & c_{ij} < \alpha \end{cases}$$

such that $\alpha \in [0,1]$, $i \in I_{m-1}$, and $j \in I_n$

Step 4. Apply CCE10 [27] to $[d_{ij}]$

In [46], the researchers have revised two SDM methods based on soft sets and fuzzy soft sets. We configure the proposed methods therein as follows:

Algorithm 3.23. ZZ15

Step 1. Construct an *fpfs*-matrix $[a_{ij}]_{m\times n}$

Step 2. Obtain $[b_{ij}]_{m\times n}$defined by $b_{0j} := \begin{cases} \frac{a_{0j}}{\sum_{k=1}^{n} a_{0k}}, & \sum_{k=1}^{n} a_{0k} \neq 0 \\ \frac{1}{n}, & otherwise \end{cases}$ and $b_{ij} := a_{ij}$ such that $i \in I_{m-1}$ and $j \in I_n$

Step 3. Apply MRB02 [27] to $[b_{ij}]$

Algorithm 3.24. ZZ15/2(λ)

Step 1. Construct an *fpfs*-matrix $[a_{ij}]_{m\times n}$

Step 2. Construct $\lambda := [\lambda_{1j}]_{1\times n}$ such that $0 \leq \lambda_{1j} \leq 1$ for $j \in I_n$

Step 3. Obtain $[b_{ij}]_{m\times n}$ defined by

$$b_{ij} := \begin{cases} a_{0j}, & i = 0 \\ 1, & i \neq 0 \text{ and } a_{ij} \geq \lambda_{1j} \\ 0, & i \neq 0 \text{ and } a_{ij} < \lambda_{1j} \end{cases}$$

such that $i \in I_{m-1}^*$ and $j \in I_n$

Step 4. Apply ZZ15 to $[b_{ij}]$

[57] has propounded a novel approach associated with fuzzy soft sets. We configure the proposed method therein as follows:

Algorithm 3.25. A16

Step 1. Construct *fpfs*-matrices $[a^1_{ij_1}]_{m\times n_1}, [a^2_{ij_2}]_{m\times n_2}, \cdots, [a^t_{ij_t}]_{m\times n_t}$

Step 2. Find and-product *fpfs*-matrix $[b_{ij}]_{m\times n}$ of $[a^1_{ij_1}], [a^2_{ij_2}], \cdots, [a^t_{ij_t}]$ such that $n = n_1 n_2 \ldots n_t$

Step 3. Obtain $[c_{ik}]_{(m-1)\times(m-1)}$ defined by

$$c_{ik} := \sum_{j=1}^{n} b_{0j}\chi(b_{ij}, b_{kj}), \quad i,k \in I_{m-1}$$

such that

$$\chi(b_{ij}, b_{kj}) := \begin{cases} \frac{b_{ij} - b_{kj}}{\max\limits_{t\in I_{m-1}} \{b_{tj}\}}, & b_{ij} > b_{kj} \\ 0, & b_{ij} \leq b_{kj} \end{cases}$$

Step 4. Apply Step 3-6 of MBR01 [27] to $[c_{ik}]$

[47] has assessed the eligibility of a group of students for a scholarship using soft sets and soft matrices. Moreover, [67] has revised the SDM method in [3] via *fpfs*-sets. We configure the proposed method therein as follows:

Algorithm 3.26. AC16, AC16/2, RH16

AC16, AC16/2, RH16, and CEC11 [29] are the same. Therefore, we prefer the notation CEC11.

In [72], the researchers have studied the fuzzy soft matrices by using different t-norms. We configure the proposed methods therein as follows:

Algorithm 3.27. AM16

AM16 is the same as MR13 [29]. Therefore, we prefer notation MR13.

Algorithm 3.28. AM16/2

AM16/2 is the same as MR13/2 [29]. Therefore, we prefer notation MR13/2.

Algorithm 3.29. AM16/3

AM16/3 is the same as MR13/3 [29]. Therefore, we prefer notation MR13/3.

In [59], the authors have applied the concept of fuzzy soft sets to a problem concerning selecting an investing area. We configure the proposed method therein as follows:

Algorithm 3.30. NRM16($\boldsymbol{R}$)

Step 1. Construct an *fpfs*-matrix $[a_{ij}]_{m\times n}$

Step 2. Determine a set R of indices such that $R \subseteq I_n$.

Step 3. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by

$$s_{i1} := \prod_{j\in R} a_{0j}a_{ij}\,, \quad i \in I_{m-1}$$

Step 4. Obtain the decision set $\{{}^{\hat{s}_{k1}}u_k | u_k \in U\}$

[67] has used *fpfs*-sets to decide on a car purchase problem. We configure the proposed method therein as follows:

Algorithm 3.31. RH16/3($\boldsymbol{R}$)

Step 1. Construct two *fpfs*-matrices $[a_{ij}]_{m\times n_1}$ and $[b_{ik}]_{m\times n_2}$

Step 2. Determine a set R of indices such that $R \subseteq I_{n_1} \times I_{n_2}$

Step 3. Obtain increasing sequence (r_t) consisting of all the elements of R such that $r_t := (u_t, v_t)$

Step 4. Obtain $[d_{it}]_{m\times|R|}$ defined by

$$d_{it} := \min_{r_t\in R}\{a_{iu_t}, b_{iv_t}\}, \quad i \in I^*_{m-1}$$

Here, $|R|$ denotes the cardinality of R.

Step 5. Obtain $[e_{it}]_{m\times|R|}$ defined by

$$e_{0t} := d_{0t} \text{ and } e_{it} := d_{0t}d_{it}$$

such that $i \in I_{m-1}$ and $t \in I_{|R|}$

Step 6. Apply MBR01 [27] to $[e_{it}]$

In [60], the researchers have employed fuzzy soft sets to choose practical and reliable social network sites. We configure the proposed method therein as follows:

Algorithm 3.32. RK16

RK16 is the same as MBR01 [27]. Therefore, we prefer the notation MBR01.

[61] has constructed an SDM method being a generalisation of MRB02 by using multiple fuzzy soft sets. We configure the proposed method therein as follows:

Algorithm 3.33. RS16

Step 1. Construct *fpfs*-matrices $\left[a_{ij}^1\right]_{m\times n}, \left[a_{ij}^2\right]_{m\times n}, \cdots, \left[a_{ij}^t\right]_{m\times n}$

Step 2. Obtain $\left[b_{i1}^1\right]_{(m-1)\times 1}, \left[b_{i1}^2\right]_{(m-1)\times 1}, \dots, \left[b_{i1}^t\right]_{(m-1)\times 1}$ defined by

$$b_{i1}^k \coloneqq \sum_{j=1}^{n} a_{0j}^k a_{ij}^k$$

such that $i \in I_{m-1}$ and $k \in I_t$

Step 3. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by

$$s_{i1} := \sum_{k=1}^{t} b_{i1}^k\,, \quad i \in I_{m-1}$$

Step 4. Obtain the decision set $\left\{{}^{\hat{s}_{k1}}u_k | u_k \in U\right\}$

In [62], the authors have drawn on fuzzy soft sets to solve a group decision-making problem. We configure the proposed method therein as follows:

Algorithm 3.34. SMT16

Step 1. Construct *fpfs*-matrices $\left[a_{ij}^1\right]_{m\times n}, \left[a_{ij}^2\right]_{m\times n}, \dots, \left[a_{ij}^t\right]_{m\times n}$

Step 2. Obtain $\left[b_{ik}^1\right]_{(m-1)\times(m-1)}, \left[b_{ik}^2\right]_{(m-1)\times(m-1)}, \dots, \left[b_{ik}^t\right]_{(m-1)\times(m-1)}$ defined by

$$b_{ik}^r \coloneqq \sum_{j=1}^{n} a_{0j}^r \chi\left(a_{ij}^r, a_{kj}^r\right), \quad i,k \in I_{m-1}, r \in I_t$$

such that

$$\chi\left(a_{ij}^r, a_{kj}^r\right) \coloneqq \begin{cases} 1, & a_{ij}^r \geq a_{kj}^r \\ 0, & a_{ij}^r < a_{kj}^r \end{cases}$$

Step 3. Obtain $\left[c_{i1}^1\right]_{(m-1)\times 1}, \left[c_{i1}^2\right]_{(m-1)\times 1}, \dots, \left[c_{i1}^t\right]_{(m-1)\times 1}$ defined by

$$c_{i1}^r \coloneqq \sum_{k=1}^{m-1} b_{ik}^r, \quad i \in I_{m-1}$$

Step 4. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by

$$s_{i1} \coloneqq \sum_{r=1}^{t} c_{i1}^r\,, \quad i \in I_{m-1}$$

Step 5. Obtain the decision set $\left\{{}^{\hat{s}_{k1}}u_k | u_k \in U\right\}$

[68] has developed an SDM method based on mean-product and max-max decision-making via soft matrices. We configure the proposed method therein as follows:

Algorithm 3.35. VMH16

Step 1. Construct two *fpfs*-matrices $\left[a_{ij}\right]_{m\times n_1}$ and $[b_{ik}]_{m\times n_2}$

Step 2. Find the mean-product matrix $\left[c_{ip}\right]_{m\times n_1 n_2}$ of $\left[a_{ij}\right]$ and $[b_{ik}]$

Step 3. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by

$$s_{i1} \coloneqq \max_{k\in I_{n_1}} \begin{cases} \max\limits_{p\in I_k}\left(c_{0p}c_{ip}\right), & I_k \neq \emptyset \\ 0, & I_k = \emptyset \end{cases}$$

such that $i \in I_{m-1}$ and $I_k \coloneqq \left\{p \mid \exists i \ni c_{0p}c_{ip} \neq 0, (k-1)n_2 < p \leq kn_2\right\}$

Step 4. Obtain the decision set $\left\{{}^{\hat{s}_{k1}}u_k | u_k \in U\right\}$

[63] has propounded two novel methods based upon ambiguity measure and Dempster-Shafer theory of evidence in the fuzzy soft sets space. We configure the proposed methods therein as follows:

Algorithm 3.36. WHXDD16

Step 1. Construct an *fpfs*-matrix $\left[a_{ij}\right]_{m\times n}$

Step 2. Obtain $\left[b_{ij}\right]_{(m-1)\times n}$ defined by

$$b_{ij} \coloneqq \begin{cases} \dfrac{a_{0j}a_{ij}}{\sum_{k=1}^{m-1} a_{0j}a_{kj}}, & \displaystyle\sum_{k=1}^{m-1} a_{0j}a_{kj} \neq 0 \\ \dfrac{1}{m-1}, & otherwise \end{cases}$$

such that $i \in I_{m-1}$ and $j \in I_n$.

Step 3. Obtain $\left[c_{1j}\right]_{1\times n}$ defined by

$$c_{1j} \coloneqq -\sum_{i=1}^{m-1} b_{ij} \log_2\left(\varepsilon + b_{ij}\right), \quad j \in I_n$$

Here, if $b_{ij} = 0$, then $c_{1j} \coloneqq -\sum_{i=1}^{m-1} b_{ij} \log_2 b_{ij}$ is undefined. To cope with this drawback, we modify it as $c_{1j} \coloneqq -\sum_{i=1}^{m-1} b_{ij} \log_2\left(\varepsilon + b_{ij}\right)$ such that $\varepsilon \ll 1$ is a positive constant, e.g., $\varepsilon = 0.0001$.

Step 4. Obtain $\left[d_{1j}\right]_{1\times n}$ defined by

$$d_{1j} \coloneqq \begin{cases} \dfrac{c_{1j}}{\sum_{k=1}^{n} c_{1k}}, & \displaystyle\sum_{k=1}^{n} c_{1k} \neq 0 \\ \dfrac{1}{n}, & otherwise \end{cases}, \quad j \in I_n$$

Step 5. Obtain $\left[e_{ij}\right]_{(m-1)\times n}$ defined by

$$e_{ij} \coloneqq b_{ij}\left(1 - d_{1j}\right)$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 6. Obtain $\left[f_{1j}\right]_{1\times n}$ defined by

$$f_{1j} \coloneqq 1 - \sum_{i=1}^{m-1} e_{ij}, \quad j \in I_n$$

Step 7. Apply Step 7-10 of XWL14 to $\left[f_{1j}\right]$

Algorithm 3.37. WHXDD16/2

Step 1. Construct two *fpfs*-matrices $[a_{ij}]_{m\times n_1}$ and $[b_{ik}]_{m\times n_2}$

Step 2. Find and-product *fpfs*-matrix $[c_{ip}]_{m\times n_1 n_2}$ of $[a_{ij}]$ and $[b_{ik}]$

Step 3. Apply WHXDD16 to $[c_{ip}]$

4. Test Cases for the Comparison of the SDM Methods

This section proposes five test cases to compare decision-making performances of SDM methods. SDM methods employ single, double, or multiple *fpfs*-matrices. Therefore, each test case consists of t *fpfs*-matrices $[a_{ij}^1], [a_{ij}^2], \dots, [a_{ij}^t]$, which has order $m \times n$ and manifest the same ranking order of alternatives without employing SDM methods. If an SDM method employs a single *fpfs*-matrix, we only use $[a_{ij}^1]$. Similarly, if double, we use $[a_{ij}^1]$ and $[a_{ij}^2]$. If an SDM method produces the ranking order provided in a test case, then it is said to accomplish the test case. In this section, let $t = 3$, $m = 5$, $n = 4$, $U = \{u_1, u_2, u_3, u_4\}$ be the set of alternatives, and $E = \{x_1, x_2, x_3, x_4\}$ be the set of parameters.

4.1. Test Case 1

Test Case 1 constructs three *fpfs*-matrices $[a_{ij}^1]_{5\times4}$, $[a_{ij}^2]_{5\times4}$, and $[a_{ij}^3]_{5\times4}$ such that for all $j \in I_4$ and $k \in I_3$, $a_{01}^k = a_{02}^k = a_{03}^k = a_{04}^k$ and $a_{1j}^k < a_{2j}^k < a_{3j}^k < a_{4j}^k$. Therefore, $a_{0j}^k a_{1j}^k < a_{0j}^k a_{2j}^k < a_{0j}^k a_{3j}^k < a_{0j}^k a_{4j}^k$, for all $j \in I_4$ and $k \in I_3$. For each *fpfs*-matrix herein, the ranking order of alternatives is $u_1 \prec u_2 \prec u_3 \prec u_4$. For example,

$$[a_{ij}^1] \coloneqq \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 \\ 0.7 & 0.6 & 0.5 & 0.4 \\ 0.8 & 0.7 & 0.6 & 0.5 \\ 0.9 & 0.8 & 0.7 & 0.6 \\ 1 & 0.9 & 0.8 & 0.7 \end{bmatrix}, \quad [a_{ij}^2] \coloneqq \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 \\ 0.3 & 0.2 & 0.1 & 0 \\ 0.4 & 0.3 & 0.2 & 0.1 \\ 0.5 & 0.4 & 0.3 & 0.2 \\ 0.6 & 0.5 & 0.4 & 0.3 \end{bmatrix}, \quad \text{and} \quad [a_{ij}^3] \coloneqq \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 \\ 0.5 & 0.4 & 0.3 & 0.2 \\ 0.6 & 0.5 & 0.4 & 0.3 \\ 0.7 & 0.6 & 0.5 & 0.4 \\ 0.8 & 0.7 & 0.6 & 0.5 \end{bmatrix}$$

4.2. Test Case 2

Test Case 2 constructs three *fpfs*-matrices $[b_{ij}^1]_{5\times4}$, $[b_{ij}^2]_{5\times4}$, and $[b_{ij}^3]_{5\times4}$ such that for all $j \in I_4$ and $k \in I_3$, $b_{01}^k = b_{02}^k = b_{03}^k = b_{04}^k$ and $b_{4j}^k < b_{3j}^k < b_{2j}^k < b_{1j}^k$. Therefore, $b_{0j}^k b_{4j}^k < b_{0j}^k b_{3j}^k < b_{0j}^k b_{2j}^k < b_{0j}^k b_{1j}^k$, for all $j \in I_4$ and $k \in I_3$. For each *fpfs*-matrix herein, the ranking order of alternatives is $u_4 \prec u_3 \prec u_2 \prec u_1$. For example,

$$[b_{ij}^1] \coloneqq \begin{bmatrix} 0.7 & 0.7 & 0.7 & 0.7 \\ 1 & 0.9 & 0.8 & 0.7 \\ 0.9 & 0.8 & 0.7 & 0.6 \\ 0.8 & 0.7 & 0.6 & 0.5 \\ 0.7 & 0.6 & 0.5 & 0.4 \end{bmatrix}, \quad [b_{ij}^2] \coloneqq \begin{bmatrix} 0.7 & 0.7 & 0.7 & 0.7 \\ 0.6 & 0.5 & 0.4 & 0.3 \\ 0.5 & 0.4 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.2 & 0.1 \\ 0.3 & 0.2 & 0.1 & 0 \end{bmatrix}, \quad \text{and} \quad [b_{ij}^3] \coloneqq \begin{bmatrix} 0.7 & 0.7 & 0.7 & 0.7 \\ 0.8 & 0.7 & 0.6 & 0.5 \\ 0.7 & 0.6 & 0.5 & 0.4 \\ 0.6 & 0.5 & 0.4 & 0.3 \\ 0.5 & 0.4 & 0.3 & 0.2 \end{bmatrix}$$

4.3. Test Case 3

Test Case 3 constructs three *fpfs*-matrices $[c_{ij}^1]_{5\times4}$, $[c_{ij}^2]_{5\times4}$, and $[c_{ij}^3]_{5\times4}$ such that for all $i, j \in I_4$ and $k \in I_3$, $c_{01}^k < c_{02}^k < c_{03}^k < c_{04}^k$, $c_{ii}^k = \lambda \in [0,1]$, and if $i \neq j$, then $c_{ij}^k = 0$. Therefore, $c_{01}^k c_{11}^k < c_{02}^k c_{22}^k < c_{03}^k c_{33}^k < c_{04}^k c_{44}^k$ and if $i \neq j$, then $c_{0j}^k c_{ij}^k = 0$, for all $j \in I_4$ and $k \in I_3$. For each *fpfs*-matrix herein, the ranking order of alternatives is $u_1 \prec u_2 \prec u_3 \prec u_4$. For example,

$$[c_{ij}^1] \coloneqq \begin{bmatrix} 0.6 & 0.7 & 0.8 & 0.9 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad [c_{ij}^2] \coloneqq \begin{bmatrix} 0.4 & 0.5 & 0.6 & 0.7 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad [c_{ij}^3] \coloneqq \begin{bmatrix} 0.2 & 0.3 & 0.4 & 0.5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.4. Test Case 4

Test Case 4 constructs three *fpfs*-matrices $[d_{ij}^1]_{5\times4}$, $[d_{ij}^2]_{5\times4}$, and $[d_{ij}^3]_{5\times4}$ such that for all $i,j \in I_4$ and $k \in I_3$, $d_{04}^k < d_{03}^k < d_{02}^k < d_{01}^k$, $d_{ii}^k = \lambda \in [0,1]$, and if $i \neq j$, then $d_{ij}^k = 0$. Therefore, $d_{04}^k d_{44}^k < d_{03}^k d_{33}^k < d_{03}^k d_{33}^k < d_{01}^k d_{11}^k$ and if $i \neq j$, then $d_{0j}^k d_{ij}^k = 0$, for all $j \in I_4$ and $k \in I_3$. For each *fpfs*-matrix herein, the ranking order of alternatives is $u_4 \prec u_3 \prec u_2 \prec u_1$. For example,

$$[d_{ij}^1] \coloneqq \begin{bmatrix} 0.9 & 0.8 & 0.7 & 0.6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad [d_{ij}^2] \coloneqq \begin{bmatrix} 0.7 & 0.6 & 0.5 & 0.4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad [d_{ij}^3] \coloneqq \begin{bmatrix} 0.5 & 0.4 & 0.3 & 0.2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.5. Test Case 5

Test Case 4 constructs three *fpfs*-matrices $[e_{ij}^1]_{5\times4}$, $[e_{ij}^2]_{5\times4}$, and $[e_{ij}^3]_{5\times4}$ such that for all $i,j \in I_4$ and $k \in I_3$, $e_{ij}^k = \lambda \in [0,1]$. For each *fpfs*-matrix herein, the ranking order of alternatives is $u_1 \approx u_2 \approx u_3 \approx u_4$. Here, $\approx$ denotes the same ranking order. For example,

$$[e_{ij}^1] \coloneqq \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}, \quad [e_{ij}^2] \coloneqq \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}, \quad \text{and} \quad [e_{ij}^3] \coloneqq \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$$

4.6. Results of Test Cases

In this subsection, we test the configured SDM methods using the aforesaid five test cases. Here, the methods working with a single matrix employ the first *fpfs*-matrices in each test case. Similarly, the methods working with double matrices utilise the first two *fpfs*-matrices. Moreover, the other methods use all the *fpfs*-matrices. For example, in Test Case 1, the methods employing single matrix, double matrices, and multiple matrices use the first *fpfs*-matrix $[a_{ij}^1]$, the first two *fpfs*-matrices $[a_{ij}^1]$ and $[a_{ij}^2]$, and all the *fpfs*-matrices $[a_{ij}^1]$, $[a_{ij}^2]$, and $[a_{ij}^3]$, respectively.

Table 8 indicates in which test cases the methods are successful. It can be seen from Table 8 that 20 of 37 methods, namely MBR01, MRB02, CCE10, CEC11, CXL13(λ_1), WQ14(κ), YHX14(α,β), DC15(α), ZZ15, CXL13/2(λ_1), HG13, ZXZ15(α), VMH16, MR13, MR13/2, SM13(w,α), Z14/2, RS16, SMT16, and NKY17(λ_2), pass all the tests. Moreover, the numbers of the passed tests are provided in the last column of Table 8. Here, $\alpha = 0.5$, $\beta = 0.5$, $\kappa = 0.4$, $\lambda = 0.5$, $\lambda_1 = [1\ 1\ 1\ 1]$, $\lambda_2 = [0.25\ 0.25\ 0.25\ 0.25]$, $\lambda_3 = [0.5\ 0.5\ 0.5\ 0.5]$, $q = 2$, $R = \{1, 2, 3, 4\}$, and $w = [0.34\ 0.34\ 0.34]$.

Table 8. Success of the methods in the test cases

	Algorithms\Test Cases	Test Case 1	Test Case 2	Test Case 3	Test Case 4	Test Case 5	Passed Test's Numbers
1	NS11 [BSD13, SR15]		✓	✓	✓	✓	4
2	CXL13(λ_1)	✓	✓	✓	✓	✓	5
3	CXL13/2(λ_1)	✓	✓	✓	✓	✓	5
4	GLF13(R)	✓	✓			✓	3
5	HG13	✓	✓	✓	✓	✓	5
6	SM13(w, α)	✓	✓	✓	✓	✓	5
7	MRB02 [GDC14, RH16/2]	✓	✓	✓	✓	✓	5
8	NKY17(λ_2) [GDC14/2]	✓	✓	✓	✓	✓	5
9	K14		✓			✓	2
10	CCE10 [MM14]	✓	✓	✓	✓	✓	5
11	WQ14(κ)	✓	✓	✓	✓	✓	5
12	XWL14(α, q) [LWX15(α, q), T15(α, q)]	✓	✓			✓	3
13	XWL14/2(α, q) [LWX15/2(α, q), T15/2(α, q)]	✓	✓			✓	3
14	YHX14(α, β)	✓	✓	✓	✓	✓	5
15	Z14/2	✓	✓	✓	✓	✓	5
16	A15			✓	✓	✓	3
17	DC15(α)	✓	✓	✓	✓	✓	5
18	HJ15(λ) [H16(λ)]		✓			✓	2
19	XHL15(α, q)		✓	✓	✓	✓	4
20	XHL15/2(α, q)		✓	✓	✓	✓	4
21	ZXZ15(α)	✓	✓	✓	✓	✓	5
22	ZXZ15/2(α)			✓	✓	✓	3
23	ZZ15	✓	✓	✓	✓	✓	5
24	ZZ15/2(λ_3)			✓	✓	✓	3
25	A16		✓		✓	✓	3
26	CEC11 [AC16, AC16/2, RH16]	✓	✓	✓	✓	✓	5
27	MR13 [AM16]	✓	✓	✓	✓	✓	5
28	MR13/2 [AM16/2]	✓	✓	✓	✓	✓	5
29	MR13/3 [AM16/3]					✓	1
30	NRM16(R)	✓	✓			✓	3
31	RH16/3(R)	✓	✓			✓	3
32	MBR01 [RK16]	✓	✓	✓	✓	✓	5
33	RS16	✓	✓	✓	✓	✓	5
34	SMT16	✓	✓	✓	✓	✓	5
35	VMH16	✓	✓	✓	✓	✓	5
36	WHXDD16					✓	1
37	WHXDD16/2	✓				✓	2
	Total	26	31	26	27	37	

5. An Application of Some of the Configured Methods to a PVA Problem

This section applies the configured methods herein to a PVA problem concerning the salt-and-pepper noise (SPN) removal performance of the filters provided in [73]. Therefore, firstly, we present the results of the filters in [73] produced by the quality metrics Peak Signal-to-Noise Ratio (PSNR), Structural Similarity (SSIM) [74], and Visual Information Fidelity (VIF) [75] for 20 traditional images at noise density occurring between 10% and 90% in Table 9, 10, and 11, respectively. Moreover, the bold values in the tables signify the filters with the best performance.

Table 9. Mean-PSNR results for the 20 traditional images with different noise densities

Filters/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
DBAIN	37.52	34.29	31.96	29.83	27.86	25.89	23.90	21.55	18.55
MDBUTMF	36.80	32.18	29.02	28.48	28.81	28.34	26.95	23.42	15.29
BPDF	36.98	33.54	31.03	28.88	26.82	24.60	21.98	17.74	10.51
NAFSMF	36.08	33.27	31.49	30.15	29.02	27.96	26.82	25.47	22.34
AWMF	36.34	35.00	33.83	32.69	31.47	30.14	28.68	26.99	24.70
DAMF	39.58	36.33	34.14	32.45	30.99	29.64	28.28	26.69	24.35
ARmF	**40.04**	**37.12**	**35.14**	**33.53**	**31.99**	**30.45**	**28.86**	**27.08**	**24.74**

Table 10. Mean-SSIM results for the 20 traditional images with different noise densities

Filters/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
DBAIN	0.9796	0.9584	0.9315	0.8968	0.8520	0.7949	0.7213	0.6265	0.4966
MDBUTMF	0.9774	0.9197	0.8117	0.7973	0.8399	0.8410	0.8025	0.7023	0.3566
BPDF	0.9783	0.9536	0.9229	0.8838	0.8323	0.7634	0.6680	0.5096	0.2585
NAFSMF	0.9748	0.9504	0.9248	0.8973	0.8666	0.8320	0.7910	0.7357	0.6190
AWMF	0.9728	0.9622	0.9484	0.9315	0.9098	0.8816	0.8437	0.7904	0.7028
DAMF	0.9854	0.9699	0.9516	0.9303	0.9051	0.8748	0.8368	0.7846	0.6964
ARmF	**0.9868**	**0.9735**	**0.9581**	**0.9400**	**0.9173**	**0.8880**	**0.8491**	**0.7947**	**0.7056**

Table 11. Mean-VIF results for the 20 traditional images with different noise densities

Filters/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
DBAIN	0.8548	0.7319	0.6179	0.5119	0.4095	0.3128	0.2229	0.1365	0.0635
MDBUTMF	0.8272	0.6713	0.5044	0.4420	0.4310	0.3978	0.3302	0.2212	0.0730
BPDF	0.8188	0.6858	0.5659	0.4564	0.3529	0.2541	0.1614	0.0783	0.0334
NAFSMF	0.7902	0.6751	0.5828	0.5030	0.4307	0.3604	0.2897	0.2129	0.1226
AWMF	0.7896	0.7366	0.6789	0.6181	0.5533	0.4833	0.4066	0.3129	0.1928
DAMF	0.8787	0.7816	0.6943	0.6162	0.5437	0.4731	0.3998	0.3096	0.1913
ARmF	**0.8832**	**0.7975**	**0.7210**	**0.6474**	**0.5741**	**0.4974**	**0.4158**	**0.3182**	**0.1955**

In this PVA problem, the alternatives are indicated as $u_1 :=$ "DBAIN", $u_2 :=$ "MDBUTMF", $u_3 :=$ "BPDF", $u_4 :=$ "NAFSMF", $u_5 :=$ "AWMF", $u_6 :=$ "DAMF", and $u_7 :=$ "ARmF" such that $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$. Moreover, the parameters are denoted by $x_1 :=$ "SPN ratio 10%", $x_2 :=$ "SPN ratio 20%", $x_3 :=$ "SPN ratio 30%", $x_4 :=$ "SPN ratio 40%", $x_5 :=$ "SPN ratio 50%", $x_6 :=$ "SPN ratio 60%", $x_7 :=$ "SPN ratio 70%", $x_8 :=$ "SPN ratio 80%", and $x_9 :=$ "SPN ratio 90%" such that $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$.

Suppose that the noise removal performances of the filters at high noise densities are more significant than at the other densities. In such a case, it is anticipated that the membership degrees at high noise densities are greater than at the other noise densities. In other words, the first rows of the *fpfs*-matrices are considered to be [0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9] herein. Furthermore, while the SSIM and VIF values are in the interval [0,1], the PSNR values are not. Hence, the PSNR values are normalised via the maximum value provided in Table 9 to construct the *fpfs*-matrix $[a_{ij}]$. Thus, Table 9, 10, and 11 can be represented with *fpfs*-matrices $[a_{ij}]_{8\times 9}$, $[b_{ij}]_{8\times 9}$, and $[c_{ij}]_{8\times 9}$ as follows:

$$
[a_{ij}] \coloneqq \begin{bmatrix}
0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
0.9371 & 0.8564 & 0.7982 & 0.7450 & 0.6958 & 0.6466 & 0.5969 & 0.5382 & 0.4633 \\
0.9191 & 0.8037 & 0.7248 & 0.7113 & 0.7195 & 0.7078 & 0.6731 & 0.5849 & 0.3819 \\
0.9236 & 0.8377 & 0.7750 & 0.7213 & 0.6698 & 0.6144 & 0.5490 & 0.4431 & 0.2625 \\
0.9011 & 0.8309 & 0.7865 & 0.7530 & 0.7248 & 0.6983 & 0.6698 & 0.6361 & 0.5579 \\
0.9076 & 0.8741 & 0.8449 & 0.8164 & 0.7860 & 0.7527 & 0.7163 & 0.6741 & 0.6169 \\
0.9885 & 0.9073 & 0.8526 & 0.8104 & 0.7740 & 0.7403 & 0.7063 & 0.6666 & 0.6081 \\
1.0000 & 0.9271 & 0.8776 & 0.8374 & 0.7990 & 0.7605 & 0.7208 & 0.6763 & 0.6179
\end{bmatrix}
$$

$$
[b_{ij}] \coloneqq \begin{bmatrix}
0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
0.9796 & 0.9584 & 0.9315 & 0.8968 & 0.8520 & 0.7949 & 0.7213 & 0.6265 & 0.4966 \\
0.9774 & 0.9197 & 0.8117 & 0.7973 & 0.8399 & 0.8410 & 0.8025 & 0.7023 & 0.3566 \\
0.9783 & 0.9536 & 0.9229 & 0.8838 & 0.8323 & 0.7634 & 0.6680 & 0.5096 & 0.2585 \\
0.9748 & 0.9504 & 0.9248 & 0.8973 & 0.8666 & 0.8320 & 0.7910 & 0.7357 & 0.6190 \\
0.9728 & 0.9622 & 0.9484 & 0.9315 & 0.9098 & 0.8816 & 0.8437 & 0.7904 & 0.7028 \\
0.9854 & 0.9699 & 0.9516 & 0.9303 & 0.9051 & 0.8748 & 0.8368 & 0.7846 & 0.6964 \\
0.9868 & 0.9735 & 0.9581 & 0.9400 & 0.9173 & 0.8880 & 0.8491 & 0.7947 & 0.7056
\end{bmatrix}
$$

and

$$
[c_{ij}] \coloneqq \begin{bmatrix}
0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
0.8548 & 0.7319 & 0.6179 & 0.5119 & 0.4095 & 0.3128 & 0.2229 & 0.1365 & 0.0635 \\
0.8272 & 0.6713 & 0.5044 & 0.4420 & 0.4310 & 0.3978 & 0.3302 & 0.2212 & 0.0730 \\
0.8188 & 0.6858 & 0.5659 & 0.4564 & 0.3529 & 0.2541 & 0.1614 & 0.0783 & 0.0334 \\
0.7902 & 0.6751 & 0.5828 & 0.5030 & 0.4307 & 0.3604 & 0.2897 & 0.2129 & 0.1226 \\
0.7896 & 0.7366 & 0.6789 & 0.6181 & 0.5533 & 0.4833 & 0.4066 & 0.3129 & 0.1928 \\
0.8787 & 0.7816 & 0.6943 & 0.6162 & 0.5437 & 0.4731 & 0.3998 & 0.3096 & 0.1913 \\
0.8832 & 0.7975 & 0.7210 & 0.6474 & 0.5741 & 0.4974 & 0.4158 & 0.3182 & 0.1955
\end{bmatrix}
$$

Nine of the SDM methods having passed all the test cases, namely MBR01, MRB02, CCE10, CEC11, CXL13(λ_1), WQ14(κ), YHX14(α,β), DC15(α), and ZZ15, employ only an *fpfs*-matrix. Similarly, CXL13/2(λ_1), HG13, ZXZ15(α), and VMH16 utilise two *fpfs*-matrices, and the others work with multiple *fpfs*-matrices. Moreover, we consider the variables $\alpha = 0.5$, $\beta = 0.5$, $\kappa = 0.4$, $\lambda_1 = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$, $\lambda_2 = \left[\frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\right]^T$, and $w = [1\ 1\ 1]$.

Secondly, we apply the SDM methods to the aforesaid *fpfs*-matrices $[a_{ij}]_{8\times 9}$, $[b_{ij}]_{8\times 9}$, and $[c_{ij}]_{8\times 9}$. The decision sets and ranking orders produced by these SDM methods are manifested in Table 12 and 13, respectively. The last column in Table 13 demonstrates the number of the methods producing the same ranking order.

Table 12. Decision sets produced by SDM methods (in the event of more-importance-attached noise removal performance at high noise densities)

Algorithms	**Matrices**	**Decision Sets**
MBR01	$[a_{ij}]$	$\{{}^{0.2227}\text{DBAIN}, {}^{0.2422}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.3828}\text{NAFSMF}, {}^{0.7891}\text{AWMF}, {}^{0.6719}\text{DAMF}, {}^{1}\text{ARmF}\}$
MRB02	$[a_{ij}]$	$\{{}^{0.8391}\text{DBAIN}, {}^{0.8446}\text{MDBUTMF}, {}^{0.7361}\text{BPDF}, {}^{0.9143}\text{NAFSMF}, {}^{0.9835}\text{AWMF}, {}^{0.9776}\text{DAMF}, {}^{1}\text{ARmF}\}$
CCE10	$[a_{ij}]$	$\{{}^{0.8391}\text{DBAIN}, {}^{0.8446}\text{MDBUTMF}, {}^{0.7361}\text{BPDF}, {}^{0.9143}\text{NAFSMF}, {}^{0.9835}\text{AWMF}, {}^{0.9776}\text{DAMF}, {}^{1}\text{ARmF}\}$
CEC11	$[a_{ij}]$	$\{{}^{0.8450}\text{DBAIN}, {}^{0.8523}\text{MDBUTMF}, {}^{0.7523}\text{BPDF}, {}^{0.9137}\text{NAFSMF}, {}^{0.9818}\text{AWMF}, {}^{0.9772}\text{DAMF}, {}^{1}\text{ARmF}\}$
CXL13(λ_1)	$[a_{ij}]$	$\{{}^{0.4077}\text{DBAIN}, {}^{0.4027}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.6642}\text{NAFSMF}, {}^{0.9219}\text{AWMF}, {}^{0.9125}\text{DAMF}, {}^{1}\text{ARmF}\}$
WQ14(κ)	$[a_{ij}]$	$\{{}^{0.8483}\text{DBAIN}, {}^{0.8480}\text{MDBUTMF}, {}^{0.7550}\text{BPDF}, {}^{0.9124}\text{NAFSMF}, {}^{0.9788}\text{AWMF}, {}^{0.9776}\text{DAMF}, {}^{1}\text{ARmF}\}$
YHX14(α,β)	$[a_{ij}]$	$\{{}^{0.2630}\text{DBAIN}, {}^{0.2779}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.6484}\text{NAFSMF}, {}^{0.9819}\text{AWMF}, {}^{0.9750}\text{DAMF}, {}^{1}\text{ARmF}\}$
DC15(α)	$[a_{ij}]$	$\{{}^{0.4628}\text{DBAIN}, {}^{0.4727}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.7927}\text{NAFSMF}, {}^{0.9880}\text{AWMF}, {}^{0.9526}\text{DAMF}, {}^{1}\text{ARmF}\}$
ZZ15	$[a_{ij}]$	$\{{}^{0.8391}\text{DBAIN}, {}^{0.8446}\text{MDBUTMF}, {}^{0.7361}\text{BPDF}, {}^{0.9143}\text{NAFSMF}, {}^{0.9835}\text{AWMF}, {}^{0.9776}\text{DAMF}, {}^{1}\text{ARmF}\}$
CXL13/2(λ_1)	$[a_{ij}]$, $[b_{ij}]$	$\{{}^{0.4358}\text{DBAIN}, {}^{0.3630}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.6997}\text{NAFSMF}, {}^{0.9437}\text{AWMF}, {}^{0.9327}\text{DAMF}, {}^{1}\text{ARmF}\}$
HG13	$[a_{ij}]$, $[b_{ij}]$	$\{{}^{0.3674}\text{DBAIN}, {}^{0.3409}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.6317}\text{NAFSMF}, {}^{0.8651}\text{AWMF}, {}^{0.8459}\text{DAMF}, {}^{1}\text{ARmF}\}$
ZXZ15(α)	$[a_{ij}]$, $[b_{ij}]$	$\{{}^{0.7437}\text{DBAIN}, {}^{0.7582}\text{MDBUTMF}, {}^{0.6995}\text{BPDF}, {}^{0.9321}\text{NAFSMF}, {}^{0.9925}\text{AWMF}, {}^{0.9878}\text{DAMF}, {}^{1}\text{ARmF}\}$
VMH16	$[a_{ij}]$, $[b_{ij}]$	$\{{}^{0.2722}\text{DBAIN}, {}^{0.5393}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.7090}\text{NAFSMF}, {}^{0.9870}\text{AWMF}, {}^{0.9513}\text{DAMF}, {}^{1}\text{ARmF}\}$
MR13	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	$\{{}^{0.6585}\text{DBAIN}, {}^{0.7256}\text{MDBUTMF}, {}^{0.5448}\text{BPDF}, {}^{0.7414}\text{NAFSMF}, {}^{0.9616}\text{AWMF}, {}^{0.9627}\text{DAMF}, {}^{1}\text{ARmF}\}$
MR13/2	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	$\{{}^{0.4119}\text{DBAIN}, {}^{0.5444}\text{MDBUTMF}, {}^{0.2777}\text{BPDF}, {}^{0.6026}\text{NAFSMF}, {}^{0.9569}\text{AWMF}, {}^{0.9290}\text{DAMF}, {}^{1}\text{ARmF}\}$
SM13(w,α)	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	$\{{}^{0.3799}\text{DBAIN}, {}^{0.3521}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.6412}\text{NAFSMF}, {}^{0.9208}\text{AWMF}, {}^{0.9287}\text{DAMF}, {}^{1}\text{ARmF}\}$
Z14/2	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	$\{{}^{0.8608}\text{DBAIN}, {}^{0.8483}\text{MDBUTMF}, {}^{0.7614}\text{BPDF}, {}^{0.9321}\text{NAFSMF}, {}^{0.9925}\text{AWMF}, {}^{0.9878}\text{DAMF}, {}^{1}\text{ARmF}\}$
RS16	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	$\{{}^{0.3523}\text{DBAIN}, {}^{0.3832}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.6067}\text{NAFSMF}, {}^{0.9411}\text{AWMF}, {}^{0.9278}\text{DAMF}, {}^{1}\text{ARmF}\}$
SMT16	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	$\{{}^{0.2237}\text{DBAIN}, {}^{0.2666}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.3641}\text{NAFSMF}, {}^{0.7867}\text{AWMF}, {}^{0.6723}\text{DAMF}, {}^{1}\text{ARmF}\}$
NKY17(λ_2)	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	$\{{}^{0.2412}\text{DBAIN}, {}^{0.2490}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.3696}\text{NAFSMF}, {}^{0.7899}\text{AWMF}, {}^{0.6732}\text{DAMF}, {}^{1}\text{ARmF}\}$

The ranking orders in Table 13 manifest that MBR01, MRB02, CCE10, CEC11, YHX14(α,β), DC15(α), ZZ15, ZXZ15(α), VMH16, MR13/2, RS16, SMT16, and NKY17(λ_2) produce the same ranking order just as CXL13(λ_1), WQ14(κ), CXL13/2(λ_1), HG13, and Z14/2 do. Moreover, the ranking orders produced by MR13 and SM13(w,α) except for those of MDBUTMF and DBAIN tend to generate the same pattern. The results show that the decision-making abilities of SDM methods herein agree that ARmF outperforms the other filters and BPDF exhibits the minimum performance compared to the others.

Table 13. Ranking orders produced by SDM methods (in the event of more-importance-attached noise removal performance at high noise densities)

Algorithms	Matrices	Ranking Orders	Frequency
MBR01	$[a_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
MRB02	$[a_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
CCE10	$[a_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
CEC11	$[a_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
CXL13(λ_1)	$[a_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺DAMF≺AWMF≺ARmF	5
WQ14(κ)	$[a_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺DAMF≺AWMF≺ARmF	5
YHX14(α,β)	$[a_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
DC15(α)	$[a_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
ZZ15	$[a_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
CXL13/2(λ_1)	$[a_{ij}]$, $[b_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺DAMF≺AWMF≺ARmF	5
HG13	$[a_{ij}]$, $[b_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺DAMF≺AWMF≺ARmF	5
ZXZ15(α)	$[a_{ij}]$, $[b_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
VMH16	$[a_{ij}]$, $[b_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
MR13	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺AWMF≺DAMF≺ARmF	1
MR13/2	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
SM13(w,α)	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺AWMF≺DAMF≺ARmF	1
Z14/2	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺DAMF≺AWMF≺ARmF	5
RS16	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
SMT16	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13
NKY17(λ_2)	$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$	BPDF≺ DBAIN ≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	13

On the other hand, assume that the noise removal performances of the filters at low noise densities are more significant than at the higher densities. In such a case, it is anticipated that the membership degrees at low noise densities are greater than at the higher noise densities. In other words, the first rows of the *fpfs*-matrices are considered to be $[0.9\ 0.8\ 0.7\ 0.6\ 0.5\ 0.4\ 0.3\ 0.2\ 0.1]$ herein. Therefore, Table 9, 10, and 11 can be represented with *fpfs*-matrices $[d_{ij}]_{8\times 9}$, $[e_{ij}]_{8\times 9}$, and $[f_{ij}]_{8\times 9}$ as follows:

$$[d_{ij}] \coloneqq \begin{bmatrix} 0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.4 & 0.3 & 0.2 & 0.1 \\ 0.9371 & 0.8564 & 0.7982 & 0.7450 & 0.6958 & 0.6466 & 0.5969 & 0.5382 & 0.4633 \\ 0.9191 & 0.8037 & 0.7248 & 0.7113 & 0.7195 & 0.7078 & 0.6731 & 0.5849 & 0.3819 \\ 0.9236 & 0.8377 & 0.7750 & 0.7213 & 0.6698 & 0.6144 & 0.5490 & 0.4431 & 0.2625 \\ 0.9011 & 0.8309 & 0.7865 & 0.7530 & 0.7248 & 0.6983 & 0.6698 & 0.6361 & 0.5579 \\ 0.9076 & 0.8741 & 0.8449 & 0.8164 & 0.7860 & 0.7527 & 0.7163 & 0.6741 & 0.6169 \\ 0.9885 & 0.9073 & 0.8526 & 0.8104 & 0.7740 & 0.7403 & 0.7063 & 0.6666 & 0.6081 \\ 1.0000 & 0.9271 & 0.8776 & 0.8374 & 0.7990 & 0.7605 & 0.7208 & 0.6763 & 0.6179 \end{bmatrix}$$

$$[e_{ij}] \coloneqq \begin{bmatrix} 0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.4 & 0.3 & 0.2 & 0.1 \\ 0.9796 & 0.9584 & 0.9315 & 0.8968 & 0.8520 & 0.7949 & 0.7213 & 0.6265 & 0.4966 \\ 0.9774 & 0.9197 & 0.8117 & 0.7973 & 0.8399 & 0.8410 & 0.8025 & 0.7023 & 0.3566 \\ 0.9783 & 0.9536 & 0.9229 & 0.8838 & 0.8323 & 0.7634 & 0.6680 & 0.5096 & 0.2585 \\ 0.9748 & 0.9504 & 0.9248 & 0.8973 & 0.8666 & 0.8320 & 0.7910 & 0.7357 & 0.6190 \\ 0.9728 & 0.9622 & 0.9484 & 0.9315 & 0.9098 & 0.8816 & 0.8437 & 0.7904 & 0.7028 \\ 0.9854 & 0.9699 & 0.9516 & 0.9303 & 0.9051 & 0.8748 & 0.8368 & 0.7846 & 0.6964 \\ 0.9868 & 0.9735 & 0.9581 & 0.9400 & 0.9173 & 0.8880 & 0.8491 & 0.7947 & 0.7056 \end{bmatrix}$$

and

$$[f_{ij}] \coloneqq \begin{bmatrix} 0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.4 & 0.3 & 0.2 & 0.1 \\ 0.8548 & 0.7319 & 0.6179 & 0.5119 & 0.4095 & 0.3128 & 0.2229 & 0.1365 & 0.0635 \\ 0.8272 & 0.6713 & 0.5044 & 0.4420 & 0.4310 & 0.3978 & 0.3302 & 0.2212 & 0.0730 \\ 0.8188 & 0.6858 & 0.5659 & 0.4564 & 0.3529 & 0.2541 & 0.1614 & 0.0783 & 0.0334 \\ 0.7902 & 0.6751 & 0.5828 & 0.5030 & 0.4307 & 0.3604 & 0.2897 & 0.2129 & 0.1226 \\ 0.7896 & 0.7366 & 0.6789 & 0.6181 & 0.5533 & 0.4833 & 0.4066 & 0.3129 & 0.1928 \\ 0.8787 & 0.7816 & 0.6943 & 0.6162 & 0.5437 & 0.4731 & 0.3998 & 0.3096 & 0.1913 \\ 0.8832 & 0.7975 & 0.7210 & 0.6474 & 0.5741 & 0.4974 & 0.4158 & 0.3182 & 0.1955 \end{bmatrix}$$

Thirdly, we apply the SDM methods to the *fpfs*-matrices $[d_{ij}]_{8\times 9}$, $[e_{ij}]_{8\times 9}$, and $[f_{ij}]_{8\times 9}$. The decision sets and ranking orders generated by the SDM methods are provided in Table 14 and 15, respectively. The last column in Table 15 demonstrates the number of the methods producing the same ranking order.

Table 14. Decision sets produced by SDM methods (in the event of more-importance-attached noise removal performance at low noise densities)

Algorithms	**Matrices**	**Decision Sets**
MBR01	$[d_{ij}]$	$\{^{0.2546}\text{DBAIN}, {}^{0}\text{MDBUTMF}, {}^{0.0093}\text{BPDF}, {}^{0.1111}\text{NAFSMF}, {}^{0.5556}\text{AWMF}, {}^{0.6944}\text{DAMF}, {}^{1}\text{ARmF}\}$
MRB02	$[d_{ij}]$	$\{^{0.8964}\text{DBAIN}, {}^{0.8784}\text{MDBUTMF}, {}^{0.8610}\text{BPDF}, {}^{0.9041}\text{NAFSMF}, {}^{0.9555}\text{AWMF}, {}^{0.9774}\text{DAMF}, {}^{1}\text{ARmF}\}$
CCE10	$[d_{ij}]$	$\{^{0.8964}\text{DBAIN}, {}^{0.8784}\text{MDBUTMF}, {}^{0.8610}\text{BPDF}, {}^{0.9041}\text{NAFSMF}, {}^{0.9555}\text{AWMF}, {}^{0.9774}\text{DAMF}, {}^{1}\text{ARmF}\}$
CEC11	$[d_{ij}]$	$\{^{0.9023}\text{DBAIN}, {}^{0.9805}\text{MDBUTMF}, {}^{0.8717}\text{BPDF}, {}^{0.9030}\text{NAFSMF}, {}^{0.9512}\text{AWMF}, {}^{0.9779}\text{DAMF}, {}^{1}\text{ARmF}\}$
CXL13(λ_1)	$[d_{ij}]$	$\{^{0.2896}\text{DBAIN}, {}^{0.2641}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.4731}\text{NAFSMF}, {}^{0.8380}\text{AWMF}, {}^{0.8623}\text{DAMF}, {}^{1}\text{ARmF}\}$
WQ14(κ)	$[d_{ij}]$	$\{^{0.8894}\text{DBAIN}, {}^{0.8760}\text{MDBUTMF}, {}^{0.8450}\text{BPDF}, {}^{0.9057}\text{NAFSMF}, {}^{0.9576}\text{AWMF}, {}^{0.9780}\text{DAMF}, {}^{1}\text{ARmF}\}$
YHX14(α,β)	$[d_{ij}]$	$\{^{0.1511}\text{DBAIN}, {}^{0.0652}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.1412}\text{NAFSMF}, {}^{0.5084}\text{AWMF}, {}^{0.9463}\text{DAMF}, {}^{1}\text{ARmF}\}$
DC15(α)	$[d_{ij}]$	$\{^{0.2460}\text{DBAIN}, {}^{0}\text{MDBUTMF}, {}^{0.0489}\text{BPDF}, {}^{0.2624}\text{NAFSMF}, {}^{0.7380}\text{AWMF}, {}^{0.7937}\text{DAMF}, {}^{1}\text{ARmF}\}$
ZZ15	$[d_{ij}]$	$\{^{0.8964}\text{DBAIN}, {}^{0.8784}\text{MDBUTMF}, {}^{0.8610}\text{BPDF}, {}^{0.9041}\text{NAFSMF}, {}^{0.9555}\text{AWMF}, {}^{0.9774}\text{DAMF}, {}^{1}\text{ARmF}\}$
CXL13/2(λ_1)	$[d_{ij}]$, $[e_{ij}]$	$\{^{0.3143}\text{DBAIN}, {}^{0.2127}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.5349}\text{NAFSMF}, {}^{0.8828}\text{AWMF}, {}^{0.9884}\text{DAMF}, {}^{1}\text{ARmF}\}$
HG13	$[d_{ij}]$, $[e_{ij}]$	$\{^{0.2511}\text{DBAIN}, {}^{0.0346}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.3467}\text{NAFSMF}, {}^{0.6800}\text{AWMF}, {}^{0.7787}\text{DAMF}, {}^{1}\text{ARmF}\}$
ZXZ15(α)	$[d_{ij}]$, $[e_{ij}]$	$\{^{0.9318}\text{DBAIN}, {}^{0.9022}\text{MDBUTMF}, {}^{0.9126}\text{BPDF}, {}^{0.9583}\text{NAFSMF}, {}^{0.9871}\text{AWMF}, {}^{0.9901}\text{DAMF}, {}^{1}\text{ARmF}\}$
VMH16	$[d_{ij}]$, $[e_{ij}]$	$\{^{0.3679}\text{DBAIN}, {}^{0.1858}\text{MDBUTMF}, {}^{0.2334}\text{BPDF}, {}^{0}\text{NAFSMF}, {}^{0.0406}\text{AWMF}, {}^{0.8737}\text{DAMF}, {}^{1}\text{ARmF}\}$
MR13	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	$\{^{0.8361}\text{DBAIN}, {}^{0.8030}\text{MDBUTMF}, {}^{0.7619}\text{BPDF}, {}^{0.8153}\text{NAFSMF}, {}^{0.9337}\text{AWMF}, {}^{0.9715}\text{DAMF}, {}^{1}\text{ARmF}\}$
MR13/2	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	$\{^{0.8222}\text{DBAIN}, {}^{0.7156}\text{MDBUTMF}, {}^{0.7516}\text{BPDF}, {}^{0.7477}\text{NAFSMF}, {}^{0.8485}\text{AWMF}, {}^{0.9575}\text{DAMF}, {}^{1}\text{ARmF}\}$
SM13(w,α)	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	$\{^{0.3023}\text{DBAIN}, {}^{0.1238}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.3911}\text{NAFSMF}, {}^{0.7184}\text{AWMF}, {}^{0.8759}\text{DAMF}, {}^{1}\text{ARmF}\}$
Z14/2	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	$\{^{0.9435}\text{DBAIN}, {}^{0.9112}\text{MDBUTMF}, {}^{0.9188}\text{BPDF}, {}^{0.9583}\text{NAFSMF}, {}^{0.9871}\text{AWMF}, {}^{0.9901}\text{DAMF}, {}^{1}\text{ARmF}\}$
RS16	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	$\{^{0.2930}\text{DBAIN}, {}^{0.0990}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.3115}\text{NAFSMF}, {}^{0.7384}\text{AWMF}, {}^{0.8701}\text{DAMF}, {}^{1}\text{ARmF}\}$
SMT16	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	$\{^{0.2734}\text{DBAIN}, {}^{0.0230}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.1229}\text{NAFSMF}, {}^{0.5300}\text{AWMF}, {}^{0.6959}\text{DAMF}, {}^{1}\text{ARmF}\}$
NKY17(λ_2)	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	$\{^{0.3049}\text{DBAIN}, {}^{0.0717}\text{MDBUTMF}, {}^{0}\text{BPDF}, {}^{0.1121}\text{NAFSMF}, {}^{0.5995}\text{AWMF}, {}^{0.7040}\text{DAMF}, {}^{1}\text{ARmF}\}$

The ranking orders in Table 15 show that MRB02, CCE10, CXL13(λ_1), WQ14(κ), ZZ15, CXL13/2(λ_1), HG13, SM13(w, α), and RS16 produce the same ranking order. The ranking orders produced by MBR01 and MR13/2 except for those of NAFMSF and BPDF tend to exhibit the same pattern. Moreover, YHX14(α, β), MR13, SMT16, and NKY17(λ_2) have the same ranking order just as DC15(α), ZXZ15, and Z14/2 do even though CEC11 and VMH16 have more incoherent ranking order than the others. Despite the different decision-making skills of all the SDM methods herein, all the methods validate that ARmF performs better than the other filters, and all the SDM methods except for MBR01, DC15(α), ZXZ15, VMH16, MR13/2, and Z14/2 indicate that BPDF displays the minimum performance.

Table 15. Ranking orders produced by SDM methods (in the event of more-importance-attached noise removal performance at low noise densities)

Algorithms	**Matrices**	**Ranking Orders**	**Frequency**
MBR01	$[d_{ij}]$	MDBUTMF≺BPDF≺NAFMSF≺ DBAIN ≺AWMF≺DAMF≺ARmF	1
MRB02	$[d_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺AWMF≺DAMF≺ARmF	9
CCE10	$[d_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺AWMF≺DAMF≺ARmF	9
CEC11	$[d_{ij}]$	BPDF≺ DBAIN ≺NAFMSF≺AWMF≺DAMF≺MDBUTMF≺ARmF	1
CXL13(λ_1)	$[d_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺AWMF≺DAMF≺ARmF	9
WQ14(κ)	$[d_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺AWMF≺DAMF≺ARmF	9
YHX14(α, β)	$[d_{ij}]$	BPDF≺MDBUTMF≺NAFMSF≺ DBAIN ≺AWMF≺DAMF≺ARmF	4
DC15(α)	$[d_{ij}]$	MDBUTMF≺BPDF≺ DBAIN ≺NAFMSF≺AWMF≺DAMF≺ARmF	3
ZZ15	$[d_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺AWMF≺DAMF≺ARmF	9
CXL13/2(λ_1)	$[d_{ij}]$, $[e_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺AWMF≺DAMF≺ARmF	9
HG13	$[d_{ij}]$, $[e_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺AWMF≺DAMF≺ARmF	9
ZXZ15(α)	$[d_{ij}]$, $[e_{ij}]$	MDBUTMF≺BPDF≺ DBAIN ≺NAFMSF≺AWMF≺DAMF≺ARmF	3
VMH16	$[d_{ij}]$, $[e_{ij}]$	NAFSMF≺AWMF≺MDBUTMF≺BPDF≺ DBAIN ≺DAMF≺ARmF	1
MR13	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	BPDF≺MDBUTMF≺NAFMSF≺ DBAIN ≺AWMF≺DAMF≺ARmF	4
MR13/2	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	MDBUTMF≺NAFMSF≺BPDF≺ DBAIN ≺AWMF≺DAMF≺ARmF	1
SM13(w, α)	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺AWMF≺DAMF≺ARmF	9
Z14/2	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	MDBUTMF≺BPDF≺ DBAIN ≺NAFMSF≺AWMF≺DAMF≺ARmF	3
RS16	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	BPDF≺MDBUTMF≺ DBAIN ≺NAFSMF≺AWMF≺DAMF≺ARmF	9
SMT16	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	BPDF≺MDBUTMF≺NAFMSF≺ DBAIN ≺AWMF≺DAMF≺ARmF	4
NKY17(λ_2)	$[d_{ij}]$, $[e_{ij}]$, $[f_{ij}]$	BPDF≺MDBUTMF≺NAFMSF≺ DBAIN ≺AWMF≺DAMF≺ARmF	4

6. Conclusion

In this study, we configured SDM methods constructed with the concepts of soft sets, fuzzy soft sets, fuzzy parameterized soft sets, *fpfs*-sets, soft matrices, fuzzy soft matrices, and fuzzy parameterized soft matrices, faithfully to the original. Hereby, in 2013 and 2016, we completed the configurations of the methods proposed via these concepts to the *fpfs*-matrices space. Afterwards, we implemented the configured methods to five test cases. By doing so, we determined the methods passing all the test cases. We then applied them to a PVA problem to order the state-of-the-art filters with respect to their noise removal performance.

SDM methods constructed by the superstructures of *fpfs*-sets were not included in this study. In the future, researchers can also configure methods constructed via these superstructures to convenient spaces, such as intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices space [76] and interval-valued intuitionistic fuzzy parameterized interval-valued intuitionistic fuzzy soft sets/matrices space [77]. Furthermore, the configured methods can be compared by applying them to decision-making problems in different fields, such as medical diagnosis. Besides, it will be possible to compare all the SDM methods put forward via the aforesaid concepts in the literature and apply them to different decision-making problems once these methods have been configured.

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