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Investigation of Tessellation Patterns in Long-Span Structures

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Abstract

Geometry plays a central role in architecture; contributes to the development of architecture by helping design, construct, analyze and evaluate building forms. Tesselations created with geometric shapes also support design alternatives by using them in the architectural production process. Until today, it is possible to find tessellation patterns on the facades of buildings and load-bearing structure designs in today's architectural environment, as it is used in floor coverings, wall and ceiling decorations, ornaments and landscape designs. From this point of view, in this study, different tessellation patterns in load-bearing system designs are discussed. In particular, it is aimed to analyze the situation through symbolic architectural structures that aim to pass a long span with different tessellation configurations. In the field study, parameters in the understanding of design and application were analyzed depending on the tessellation patterns, construction materials and the span passed in different structures with a long span using the tracking technique and reviewed comparative inferences. At this point, pattern-form, pattern-structure type, structure type-material and material-span properties were examined; the effects of structures on chronological development were observed. As a result, it has been observed that complex tessellation patterns obtained with more than one geometrical form are reflected in mathematical models with the advancement of technology. Anticipating that the usage rates will increase in the future, this study aims to raise awareness about the importance of geometry in load-bearing systems and the use of tessellation configurations.

1. INTRODUCTION

Geometry is part of the architectural design and manufacturing process. This branch of science, which has always been in the main role from primitive times to the present, has brought unlimited geometric freedoms by paving the way for innovations in architecture, especially with the computational design opportunity that has become widespread with the development of technology and provides diversity. There are many methods used in geometric design in architecture, and one of the most basic among them is the tessellation method.

Tessellations with examples in the fields of engineering, mathematics and art have survived from ancient times to the present. This method is seen in the design of architectural structures and objects, geometric figures and mosaic patterns throughout history. The tessellation method is the process of creating complex and detailed patterns from simple geometric forms by making a series of algorithmic calculations by means of translation, rotation and reflection movements without put on top of each other or overlapping regular or freeform geometric shapes [1]. At the design point, the formation mechanisms of tessellation patterns are based on basic design principles. The principle of repetition is the mathematical reproduction of a basic geometric figure. The rhythm principle is a design principle built on repetition. The repetitions that provide the rhythm create a perception of order, balance and orderliness that ensures consistency in the design [2]. The words repetition and pattern are directly related to the concept of rhythm [3]. The basis for tessellation configurations by reconstructing and expanding with rhythm principles (Figure 1).

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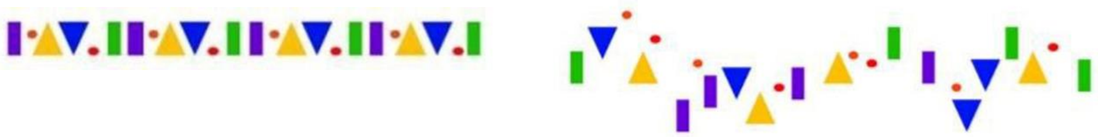


Figure 1. *Example of a pattern on the left, a rhythm on the right [4]*

To understand the concept of tessellation and its patterns, it is necessary to understand geometric shapes and their properties. In this study, it is aimed to make a content analysis specific to the tessellation configurations used in long-span structural system designs.

Tesselation patterns, especially in long span systems and types, were evaluated according to parameters such as structural efficiency, materials and spans, Possible relationships between structures built with different construction techniques are revealed. The usage function (social, cultural, scientific, etc.) of each building is different from each other, and accordingly, the geometrical formations of the designed structures are determined by defining their tessellation patterns. Patterns are reduced to a single module and their mathematical sequences are derived. Interpretations were made by looking at the building construction, materials and spans. According to the parameters, seven structures with similarities and differences were selected and categorized into tables. After giving brief information about each structure, evaluations were made according to criteria such as pattern-form, pattern-structure type, structure type-material, and material-span. As a result, the importance of geometry is emphasized in the long span structural systems discussed by making two parameter comparisons and in this study, the classification of innovative long-span structures, which were role models for the following periods, under different tessellation patterns was revealed.

2. TESSELLATIONS

Angle and edge connections of geometric shapes have an important place in the creation of tessellation configurations. There are certain rules for the creation of the tessellation method. Geometric figures should be placed in a sequential order and without spaces between them, without overlapping. Geometric terms are used in the process of bringing the figures side by side. Tesselation modules created with smooth geometric shapes should complete 360 degrees (Figure 2).

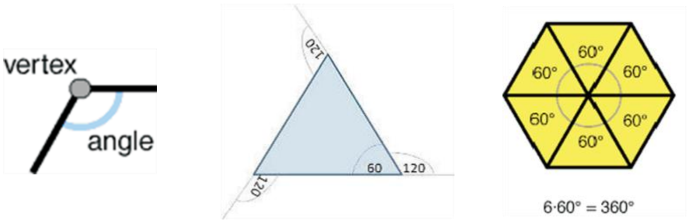


Figure 2. *Triangle geometric form and tesselation module in mathematical context [5]*

As a result, architectural surfaces are obtained by combining tessellation modules with the principle of repetition, which is one of the architectural design principles (Figure 3). A regular tessellation pattern would theoretically cover the entire Euclidean plane with the exact repetition of the same geometric form. If the tessellation arrangement is provided by the successive repetition of a single shape, it is named with the word monohedral. Monohedral is an indication that each piece in the pattern is derived from the same size and shape. Tesselation surfaces arranged with smooth geometric forms can be used in the covering, decoration and processing of any surface.

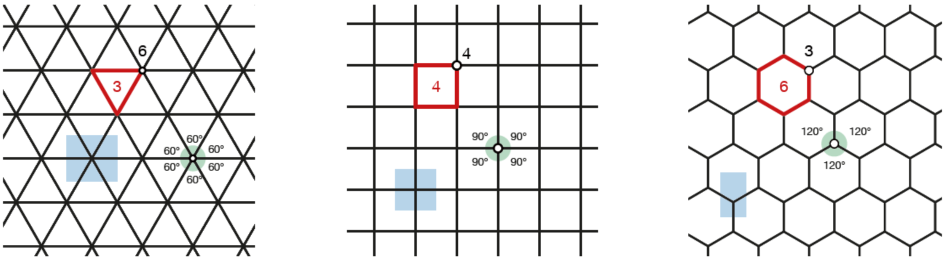


Figure 3. *Geometric tessellation patterns with repetition principle [6]*

There are three different movements of geometric shapes in the creation of tessellation patterns. Various combinations can be obtained with translation, reflection and rotation movements (Figure 4). A tessellation pattern can be created with one of these concepts, or it can be produced with multiple combinations. One of the most important issues to be considered in combinations of the same or different geometric shapes is that the tessellation module should form a full 360-degree angle.

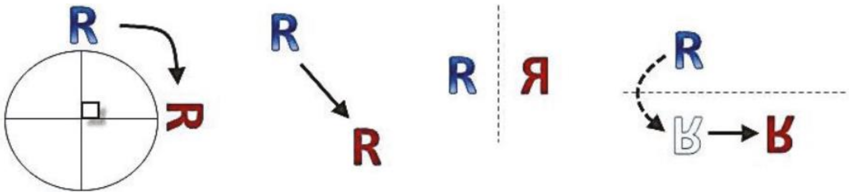


Figure 4. *Translation, reflection and rotational motions [7]*

Multiple geometric shapes can also be used for the application of tessellation method. Even irregular, free-form geometric forms can be edited (Figure 5). At this point, while more stable patterns can be created with regular polygons, more flexible patterns can be revealed with non-regular polygons. The expression of tessellations also changes according to the number of sides of each different geometric shape.

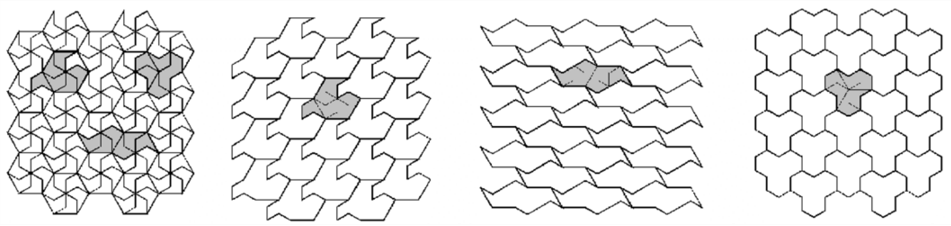


Figure 5. *Tessellation formations of different geometric shapes [8]*

Mathematical formulas have been developed to express tessellations. Patterns are formulated and named with the number of sides of the geometric form. Regular patterns can be designed with polygons combined with each other in a canonical mathematical ratio. Within the same tessellation pattern, different mathematical formulas can be seen as a result of geometric diversity (Figure 6). A tessellation pattern created with regular geometric shapes is called regular tessellation as a geometric expression, and a tessellation pattern obtained with more than one regular geometric shape is called semi-regular tessellation. Expression of tessellations formed by regular polygons in mathematical context; involves writing the number of sides of each polygon around the intersecting midpoint of a module sequentially in clockwise order [1].

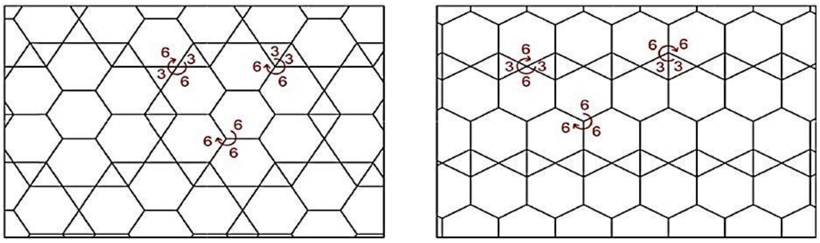


Figure 6. *(3.6.3.6), (3.3.6.6) and (6.6.6) tessellation configurations [9]*

There are also forms in uniform geometric tessellation sequences that cannot form tessellation configuration by repeating a single module. Looking at the pentagon form from regular polygons, it is seen that the pattern does not continue (Figure 7). Since it is a shape with all sides not symmetrical, problems arise in the joints. One of the rules of the tessellation method, the inability to provide 360 degrees, shows the error in configuration. The same is true for regular octagonal figures. Two regular octagons can be joined side by side, but when the 3rd figure or the 4th figure is followed, the pattern is broken due to the angles and side lengths. For this reason, exceptions can be seen in some regular polygons.

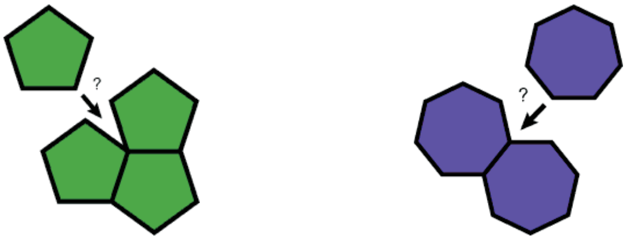


Figure 7. *Cases that do not form a tessellation pattern [5]*

In the study, the geometric sequences of the selected structures were found by tessellation analysis. The formation schemes of the patterns are written with mathematical expressions. The conditions of the structures relative to each other are discussed by looking at these parameters.

3. USE OF TESSELLATION IN LOAD-BEARING SYSTEM DESIGN

Geometry and geometry rules are the most important actors in the basic differentiation of architectural design against the laws of physics that cannot be ignored in load-bearing systems, which are important in the survival of structures. Geometric definition is essential for the implementation and feasibility of both a simple frame system and a uniform shell system design.

Geometry is one of the first features that make up the form of an architectural product and attract attention. In the architectural design process, the general operation and method is initially based on the form, and then the determined construction is reached step by step [10]. Mathematics and geometry have influenced the design of structural and visual forms in architectural phenomena in the historical process. The Parthenon temple in ancient Greek architecture, geometric proportions in Egyptian architecture, symmetries and rhythms based on mathematical operations produced by Vitruvius in Renaissance architecture stand out [11]. Geometric findings seen in classical architectural works continue to develop in modern architecture. For example, significant progress has been made with the ease of applying geometric forms with mathematical expressions in shell structures, which are one of the most basic elements of Modern Architecture.

Shell structures, which have the ability to produce numerous forms, are an effective building system that can be preferred in passing long spans and in terms of material variety. In this type of load-bearing system, where maximum spans can be passed with minimum surface area, economical solutions can be created with minimum material and structural advantages can be provided [12]. Square or rectangular grid, radial geometries or combinations of these can be created with geometric formations in which the tessellation method is used, while triangular grid systems that provide all the features of the shell structure are the most commonly applied geometric attitudes (Figure 8).

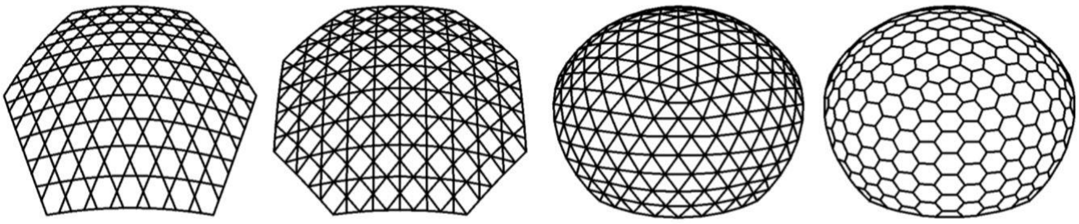


Figure 8. *Examples of semi-regular tesselated grid dome (a, b) and regular tesselated triangle, hexagonal geodesic dome (c, d) [13]*

The basic shaping of the shell and the fact that the shell layer that forms this formation is full body or rod load-bearing is important in the structural performance to be achieved. At this point, one of the types of load-bearing systems in which the geometric shapes on the basis of the study can be read most clearly are the gridshell systems.

Gridshell systems are generally defined as structures that derive their strength and shape from double curvature. Like all shells, these structures are thin-sectioned. Unlike continuous shells, gridshell elements are subdivided into grids or are obtained from lattice meshes (Figure 9). Rectangular grid and combinations are widely used. These structures can be made of materials such as steel, aluminum, wood and composite. Gridshells are produced with large profiles and compound elements in long-span designs, and at the same time, they create structural richness because they can have complex geometries [14].

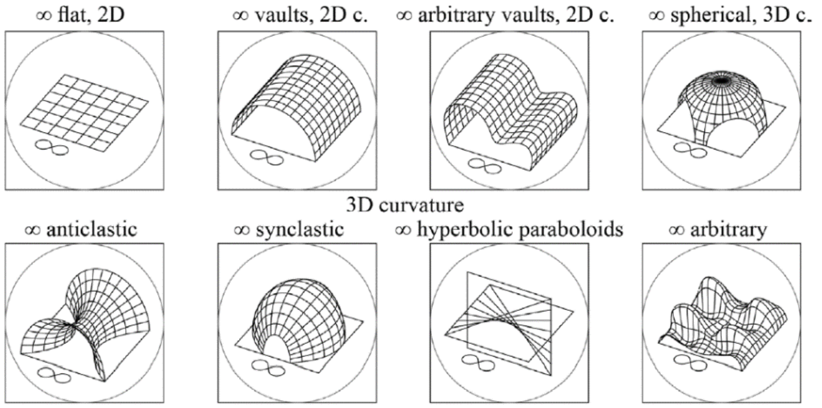


Figure 9. *Types of gridshell structures [15]*

In the study, long-span building types were selected by looking at the tessellation configurations, taking into account the material, structure and span parameters, in order to question the relationship between the systems. Structures consist of shell systems. In the geometric sequence-based structural design, binary parameter analyzes were made in order that the relationship between form, form and material is interconnected, but each of them is a separate design problem.

4. CASE STUDIES

In this research, the efficiency of the geometry-based tessellation concept in the design of the load-bearing system was investigated by using data collection and tracing methods, which are one of the qualitative research methods. In this direction, examples of imaginary structures, which were applied in different chronological periods, opening the door to the next structures, and which are important from the world of architecture, were selected. The tessellation configuration, which is important in design and application, and the understanding of structural use have been evaluated through examples.

4.1. Disney's EPCOT Center – Spaceship Earth

Spaceship Earth opened in Florida in 1982 as a key part of the Walt Disney EPCOT theme park. As a structure, it is 6 steel truss support elements and a complete sphere seated on a hexagonal plane. Its architectural design was by Wallace Floyd Design Group [16].

The diameter of the structure is about 50 meters. The structure, which is approximately 4.5 meters above ground level, is approximately 54 meters high. In the center of the structure, which consists of two dome forms, there is a ring holding the upper dome. The structure is derived from a triangular geometric shape as a construction, similar to Buckminster Fuller's geodesic dome [16]. In the 2-layer construction, there are 11,324 panels called alucobond (a polyethylene plastic layer sandwiched between the aluminum layer) on the surface under the effect of tessellation (Figure 10).

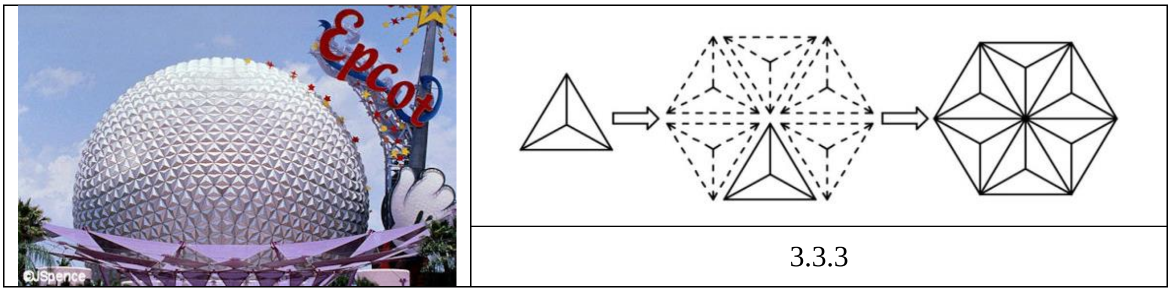


Figure 10. *Spaceship Earth [17], the formation of the tesselation module and the mathematical expression of the tesselation pattern*

4.2. Louvre Pyramid

The renovation project of the museum, which was started in 1983, was carried out by Architect I.M. Pei took charge and proposed a glass pyramid. The reason he chose the pyramid geometry was the idea of creating a small volume concept with a large area as a natural solution. Pei argued that a traditional roof would only provide an advantage in terms of natural light, while a high pointed roof was beneficial in terms of both volume and natural light [18]. The structure, designed from 603 rhombus and 70 triangular geometric glass pieces, reaches a height of 21.6 meters on a 34 m^2 floor (Figure 11) [19].

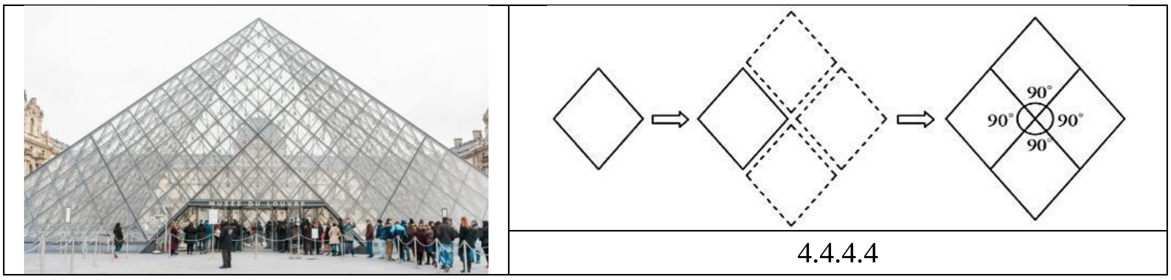


Figure 11. *Louvre Pyramid [20], the formation of tesselation module and mathematical expression of tesselation pattern*

A technique called structural glazing was used in the framework of the pyramid, which was built entirely with glass parts and steel construction. With this technique, glass pieces are fixed on a metal mesh. Thus, a structural structure was obtained that is transparent and receives natural light. The pyramid frame is formed from a mesh of 6,000 structural steel and aluminum bars [21].

4.3. Great Court at the British Museum

The building, which was started to be designed by Norman Foster and Partners architectural office in 1994, was opened in London in 2000. The building in a rectangular plan scheme has a span of 74 meters from east to west and 95 meters from north to south. There is a reading room with a diameter of 44 meters, 3 meters north of the center of the rectangular plan [22]. In line with this situation, the roof of the museum covers an area of 6100 m2 and was designed by making complex mathematical models.

The span of the structure, which is made of 3312 glass triangle panels, has lengths varying between 14 and 40 meters (Figure 12). These different lengths cause the midpoint heights of the roof to vary between 3 and 7 meters. The highest point of the roof from ground level is about 26 meters. Triangular glass panel dimensions vary as 80*150 cm or 220*330 cm. The average area of the glass panels is about 1.85 square meters. The roof lets in natural light, creating controlled light passages to the surrounding galleries and reading room. The structure consists of 10 kilometers of steel material [23].

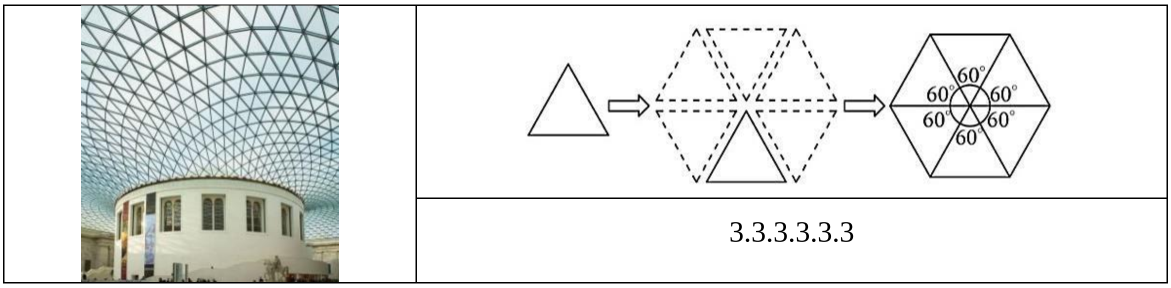


Figure 12. *Great Court at the British Museum [22], the formation of tesselation module and mathematical expression of tesselation pattern*

4.4. Eden Project

The botanical garden structure, which presents plants from different climatic regions of the world with two greenhouse complexes in an open area of 15 hectares, was opened in England in 2001. The domes of the garden, designed by Nicholas Grimshaw Architectural Office, are constructed of double-layered hexagonal geometry (Figure 13). The largest dome of the Eden project is approximately 125 meters in diameter, with a maximum height of 55 meters above ground level. Air-filled ETFE material, which is lighter than glass, was used in this mega structure, which also reduced the weight of steel in the main structure [24]. In addition, more sunlight is taken into the structure with both the selected material and the geometric pattern.

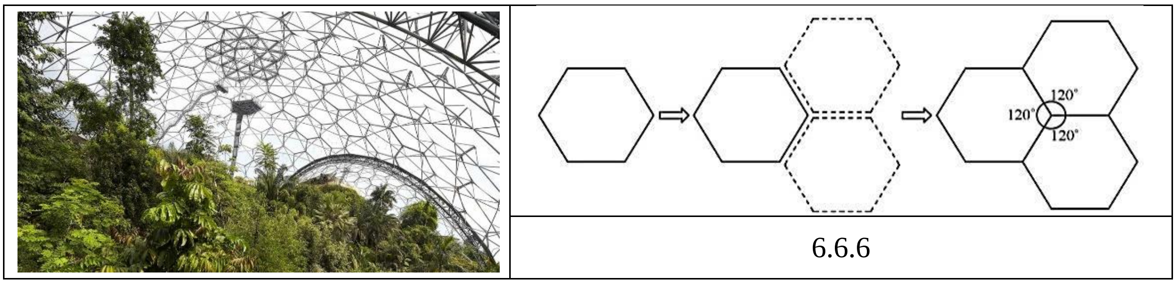


Figure 13. *Eden Project [25], the formation of the tesselation module and the mathematical expression of the tesselation pattern*

The closest distances between geodesic lines and two points on the surface were found during the geometric design. The main steel structure was developed from the MERO space frame system. MERO space frame system is a prefabricated construction technique that allows multi-directional joining of rods around a joint. Complex geometric configurations can be made with this system, with features such as ease of assembly, low tolerances, and fast construction. The total steel weight is 700 tons and the largest hexagonal area is 80 square meters at 11 meters span [24].

4.5. Louvre Abu Dhabi Dome

Louvre Abu Dhabi, designed by Jean Nouvel Architecture Office and completed in Abu Dhabi in 2013, is one of the first examples of universal museums in the Arab world. It has a multi-layered structure that includes Islamic patterns in its structural design in order to support its imaginative meaning (Figure 14). In the dome, which is designed parametrically, it is aimed to obtain an aesthetic appearance called light rain with patterns to filter the desert sun [26], in this direction, two separate tessellation configuration were created.

The dome structure, in which a geometrical order was created with semi-regular tessellation patterns, was formed from a triangular steel frame system. The dome is supported on 4 support elements at a span of 180 meters and reaches a height of 36 meters above ground level [27]. A single piece of geometric pattern reaches up to 5 meters long. Before the building was built, its 1/33 scale prototype was made and its features such as structural efficiency and light optimization were tested. The total weight of the dome is 7,500 tons. It consists of 8 layers, 4 inner and 4 outer. Made of 4 outer layers of steel, 4 inner layers of aluminum [28].

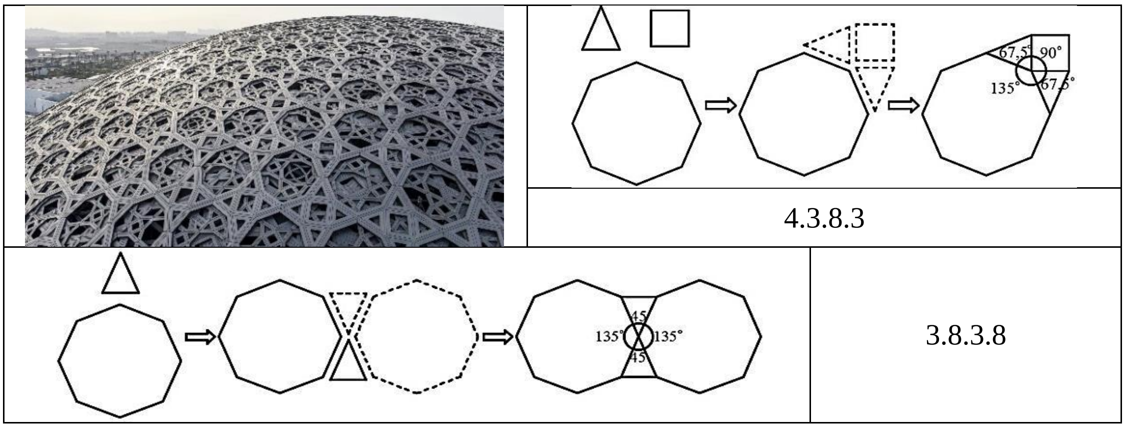


Figure 14. *Louvre Abu Dhabi Dome [27], the formation of tesselation modules and mathematical expression of tesselation pattern*

4.6. La Seine Musicale

Designed by Shigeru Ban and Jean de Gastines in France, the building has been functioning as a music and performing arts center since 2017. The entire main auditorium section was created from wooden elements and carried out with a digital design process [29].

Covering an area of 36,500 m2, the load-bearing system resembles an egg created with a hexagram pattern [30]. This structure is obtained from 15 horizontal ring-shaped beams consisting of 84 diagonals, 42 clockwise and 42 counterclockwise, connected to each other. The entire structure is 70 meters long, 45 meters wide and 27 meters high. The beams have an average cross-sectional area of 32*35 cm and are divided into 1300 individual pieces. All beams are intertwined and sit on the same layer. The façade arrangement on the structure is divided into triangular and regular hexagonal glass panels (Figure 15). Curved timber lumber and flat glass joints are filled with CNC machined lumber pieces. Timber lumber up to 24 meters is used in the most structurally critical parts of the building. 1,700 wooden timber elements were applied in the project [29].

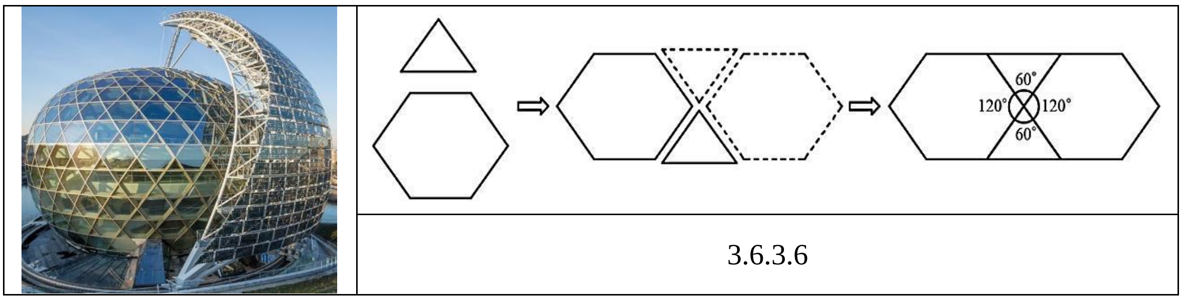


Figure 15. *La Seine Musicale [31], the formation of the tesselation module and the mathematical expression of the tesselation pattern*

4.7. Taiyuan Botanical Garden Domes

The building, which is used as a botanical garden complex in China as of 2020, is the largest example of a wooden gridshell system not built with triangular geometry in the world. Delugan Meissl Associated Architects (DMAA) and StructureCraft companies managed the design, construction and construction process of the complex, which was designed as three separate dome structures [32].

The heights of the three domes are between 12 and 30 meters, and their spans vary from 43 meters to 88 meters. All three parabolic gridshell systems are double curvature structures composed of 2 or 3 cross glulam beams (Figure 16). The domes are 2-layered and covered with glass panels that can be opened in some parts. The smallest structure is built with two layers of glulam wood, while the middle and largest structure is built with three layers of glulam wood. Since the span and height ratio of the largest dome is higher than the others, the use of steel cables is higher than the others [32].

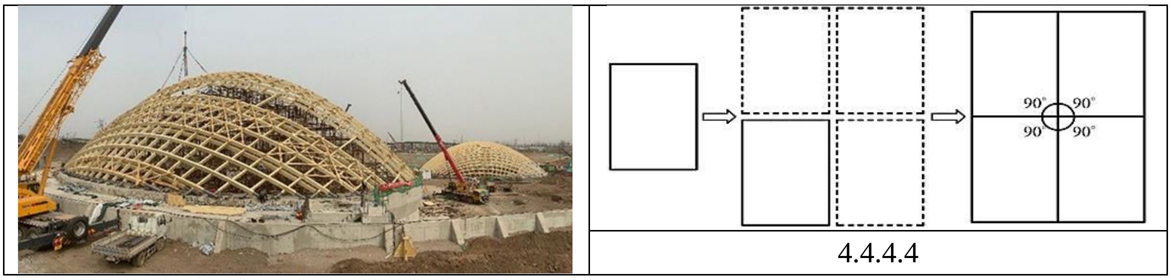


Figure 16. *Taiyuan Botanical Garden Domes [32], the formation of tesselation module and mathematical expression of tesselation pattern*

5. ANALYSIS

Seven structures, which have long spans in geometric sequence-based structural design and have variable parameters according to tessellation patterns, have been tabulated to be evaluated in terms of their similarities and differences according to their tessellation and structure features (Table 1). Although each of the concepts of form, form and material is a separate design problem in structural design, binary parameter analyzes were made in order not to ignore their interrelated relations. At this point, pattern-form, pattern-structure type, structure type-material and material-span features are examined; the effects of the structures on the chronological development were observed.

5.1. Pattern – Form

The structures have different panel sections and non-identical panel geometric shapes. The triangular geometric shape is the most used form in buildings (Figure 17). Triangle geometry is present in La Seine Musicale, Great Court at the British Museum, Louvre Abu Dhabi Dome and EPCOT Center – Spaceship Earth structures. When looking at the structural shapes in buildings, it is seen that the use of curved surfaces for long span systems is more common (Figure 17). Among the circular surface structures, it can be said that there are more long-span structures with geodesic domes in general. Except for the Louvre Pyramid and the Great Court at the British Museum, all buildings are made of circular surfaces. The Great Court at the British Museum construction is not circular, but has a curvilinear formation. Louvre Pyramid, on the other hand, is distinguished from the others as a non-curvilinear long-span structure. When the samples are analyzed, it can be stated that long-span systems with a triangular geometric shape and circular surface are more preferred. EPCOT Center – Spaceship Earth and Eden Project structures, which have the highest height when looking at the structural systems, consist of triangular and hexagonal geometry. Considering that the hexagonal shape is obtained from six triangles, the use of the triangle basic form stands out depending on the height. According to the module formation, Louvre Abu Dhabi Dome, which has the most geometric shapes, stands out because it is the structure with the longest span.

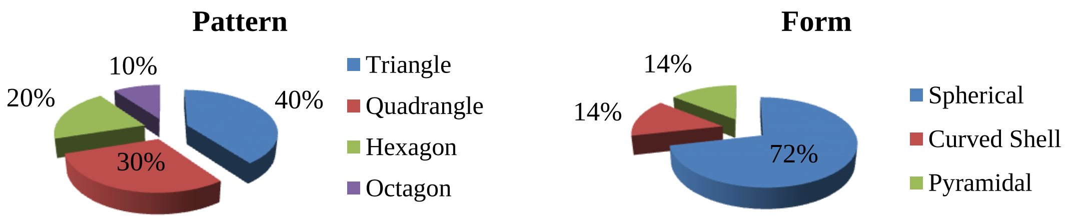


Figure 17. *Percentage ratios of pattern and form properties of long span structural systems*

5.2. Pattern - Structure type

EPCOT Center – Spaceship Earth and Great Court at the British Museum created with triangle geometry, Taiyuan Botanical Garden Domes obtained with rectangular geometry and La Seine Musicale structures formed with triangle – hexagon combination are shell systems. Louvre Pyramid with rectangular geometry, Eden Project with hexagonal geometry and Louvre Abu Dhabi Dome structures produced with triangular and octagonal shapes are lattice systems. It has been observed that there is a tendency from the triangle, which is the basic geometric form, towards more complex geometries (hexagon, octagon, etc.) and from single-layer shell systems to two-layer lattice systems. From here, it can be said that long-span complex patterns made with semi-regular tessellations can be obtained with lattice systems. The Louvre Abu Dhabi Dome structure, which has different semi-regular tessellation patterns, is supported by a lattice structure in the inner layer, allowing different geometric form combinations in the outer layer and provides an advantage in terms of structure span. Most of the structures consist of shell and space lattice system (Figure 18). Eden Project is another architectural work in which the structure span is supported by a geometrical form and structure system, since it is produced with a double-layered hexagonal shape. However, the single-layer structures, in which the structure system also forms the tessellation configuration, are lower in height than the multi-layer systems and the amount of span remains less.

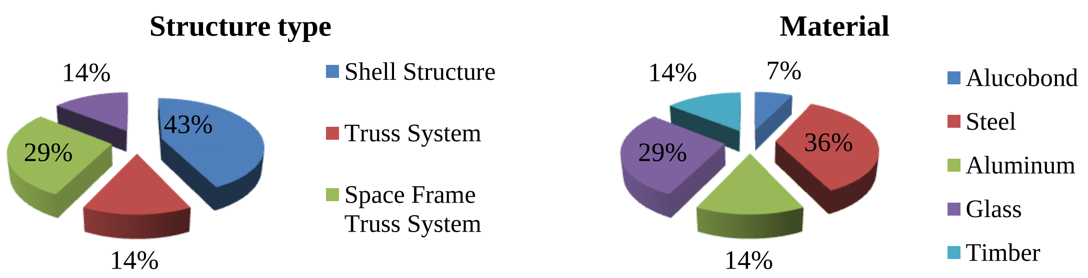
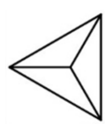
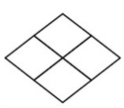
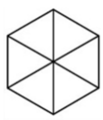
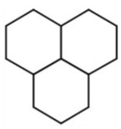
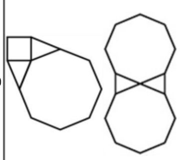
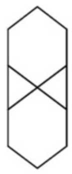
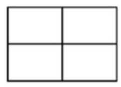


Figure 18. *Percentage ratios of structure type and material properties of long span structural systems*

5.3. Structure type - Material

It shows that architectural designs can be made by supporting a long-span system with building materials such as wood and steel, as well as aluminum and glass building materials in the structures examined (Figure 18). Lattice systems and steel, aluminum and glass materials were used in the Louvre Pyramid, Eden Project and Louvre Abu Dhabi Dome projects. Shell structures and wood, steel, aluminum and glass materials were applied in La Seine Musicale, Great Court at the British Museum, Taiyuan Botanical Garden Domes and EPCOT Center – Spaceship Earth projects. When the samples are analyzed, it has been observed that while steel, aluminum and glass materials can be used together with the structural systems, it has been observed that the wooden material can be applied in shell structures, and its use with lattice systems is weak. It should not be overlooked that the reinforced concrete material is not applied because the investigated long-span systems are gridshells. Reinforced concrete material creates a heavier system in long-span systems compared to other materials, and it is not preferred much due to the necessity of mold application. In order to reduce the weight due to the increase in the amount of span to be passed, there is a tendency from plain web girder systems to open-web girder systems. Also steel, glass, together with the process of modernization and technological developments in building materials such as aluminum, steel-aluminum-composite materials such as glass mixtures as a result of structural efficiency, material gain, lightness, richness and aesthetic structures with benefits such as economic savings are made at rates that cannot be ignored. As a result, the choice of material can determine the construction type of a long span structure.

Table 1. *Properties of structures and tessellation analysis*

	EPCOT Center – Spaceship Earth	**Louvre Pyramid**	**Great Court at the British Museum**	**Eden Project**	**Louvre Abu Dhabi Dome**	**La Seine Musicale**	**Taiyuan Botanical Garden Domes**
Building Function	Social, Entertainment	Social, Cultural, Symbolical	Social, Cultural, Symbolical	Scientific, Environmental, Experimental	Social, Cultural	Social, Entertainment	Scientific, Environmental, Experimental
Panel Shape	Triangular	Diamond	Triangular	Hexagonal	Triangular, Square, Octagonal	Hexagram	Quadrilateral
Tessellation Module							
Module Formation	Rotation, Reflection	Translation	Rotation, Reflection	Translation	Translation, Rotation, Reflection	Translation, Reflection	Translation
Mathematical Sequence	3.3.3	4.4.4.4	3.3.3.3.3.3	6.6.6	4.3.8.3, 3.8.3.8	3.6.3.6	4.4.4.4
Structure Shape	Geodesic Sphere	Pyramidal	Curved Gridshell	Clusters of Spheres	Spherical	Spherical (Glazed geodesic Sphere)	Clusters of Spheres
Structure Type	Double Layer Shell Structure	Grid-Truss Structure with Cable Structure	Shell Structure	Double Layer Space Frame Truss System	Space frame supporting a Cladding System	Egg-Shaped Shell Structure	Parabolic gridshells
Structure Materials	Alucobond and Steel Frame	Structural Steel, Glass Aluminum,	Steel Frame, Glass	ETFE and MERO Steel Frame	Stainless Steel and Aluminum	Latticed Laminated-Timber Frame, Glass	Traditional Chinese Wood, Glass
Structure Height (meter)	54 m	21.6 m	26 m (max.)	55 m (max.)	36 m	27 m	30 m (max.)
Structure Span (meter)	Diameter - 50 m	34*34 m	74*95 m	125 m (max.)	180 m	45*70 m	88 m (max.)

5.4. Material - Span

Looking from the minimum span to the maximum span, the main structure material in the Louvre Pyramid structure, which passes 34 meters, is steel, the supporting materials are aluminum and glass. The main structural materials in the 50-meter span EPCOT Center – Spaceship Earth structure are aluminum and steel. With a span of up to 70 meters, the main structural material of the La Seine Musicale building is wood, with steel and glass as supporting materials. In the 88-meter-span Taiyuan Botanical Garden Domes building, the main structural material is two-layer wood, the supporting materials are steel and glass. In the Great Court at the British Museum, which can reach a span of 95 meters, the main structural material is steel and the support material is glass. Steel is the main structural material in the Eden Project and Louvre Abu Dhabi Dome structures with spans exceeding 100 meters. When analyzed, the main material used for pass the span is mostly steel. Although wood material is also used, wood is not used in structures over 100 meters. It can be seen that very long spans cannot be passed with wooden materials that can be preferred in terms of sustainability. In the Taiyuan Botanical Garden Domes structure, which spans 88 meters, double-layer cross wooden beams are applied instead of single-layer wooden material to provide the span. The structures investigated are mostly architectural works over 40 meters (Figure 19). More complex systems have been developed instead of using a single layer or single material in relation to the increased span. In some structures, double-layer layers are provided to strengthen the system, and in some, cable systems are used to support the main structural structure. While some of them are given strength with composite applications with two or three structure materials, some architectural designs that will survive for many years have been made by using different fasteners.

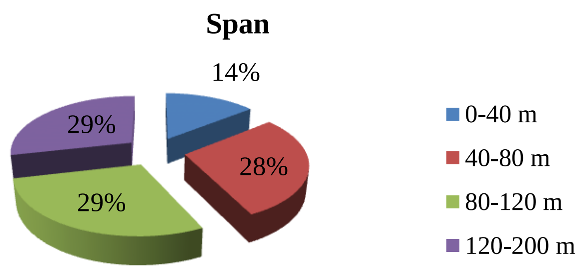


Figure 19. *Percentage ratios of span properties of long span structural systems*

6. DISCUSSION AND CONCLUSION

One of the applications where geometry is included in the architectural design process is the use of the concept of tessellation. The built patterns integrated into architectural design by producing geometric forms in regular patterns in tessellation configurations form the basis of the load-bearing system as well as being used in many applications such as city and regional planning, landscaping, interior decoration works, ornaments and decorations. Especially in long-span structures, different designs are revealed and diversity is provided with this geometric order relationship. As the existing building stock could not meet the sufficient area day by day, long-span building projects were designed and progress was made. Developments in building technology and methodology, together with the support of long-span structural systems with geometric combinations for large organizations, enable the scale of building stock to expand, revealing complex structures within the framework of cooperation in the architectural and engineering sectors.

In the study, the properties of tesselation patterns in long-span building systems were determined, the tessellation cases in different building classes were observed, and the tesselation-structure relationship was evaluated through building examples. Before proceeding to the evaluation part, the tesselation method and its formation stages are introduced. Tessellation configurations in structural design with long spans are generally encountered in gridshell and space lattice systems. Seven different structural buildings meeting these criteria were selected for review. Firstly, the architectural and construction features of the buildings were introduced. Then, the structures were classified and tabulated according to the determined parameters. Based on the table, the tessellation states between the buildings were associated with their structural qualities and the comparative analysis method was used. The conditions of the sample structures that unite on the common denominator in the tessellation method and long-span system parameters are reviewed relative to each other. Construction, material, span and tesselation connections were determined.

As a result of the examinations and analysis, it was determined that while the designs were created with triangular and rectangular shapes, which were the basic geometric forms, in the first systems with long spans when viewed chronologically, geometries such as hexagon and octagon were used by going beyond the ordinary in architectural works close to the present. It has been observed that architectural approaches have increased with the mathematical calculations brought by the tessellation method. The reason for this is the integration of different geometries alone or with more than one geometric figure and it has been determined that the geometric design method tessellation is used in its formation. Louvre Abu Dhabi Dome, which has the longest span among the investigated structures, is an important example of transferring geometric design to a complex structural system with tessellation configurations. It is possible to say that technological developments in the steel, aluminum and glass industries have a significant impact on the production of these configurations in buildings. In addition to regular and semi-regular tessellation patterns, free-form tessellation patterns are also used in long span structures. In the study, the place and properties of tessellation patterns developing in architectural dimension in long-span systems are highlighted. It is aimed to raise awareness by predicting that the complex tessellation patterns obtained with more than one geometrical form will increase in architectural structures by reflecting on mathematical models with the advancement of technology.

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