Investigating Pre-service Mathematics Teachers' Geometric Problem Solving Process in Dynamic Geometry Environment

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Abstract

The aim of this study is to investigate pre-service elementary mathematics teachers' open geometric problem solving process in a Dynamic Geometry Environment. With its qualitative inquiry based research design employed, the participants of the study are three pre-service teachers from 4th graders of the Department of Elementary Mathematics Teaching. In this study, clinical interviews, screencaptures of the problem solving process in the Cabri Geomery Environment, and worksheets included 2 open geometry problems have been used to collect the data. It has been investigated that all the participants passed through similar recursive phases as construction, exploration, conjecture, validate, and justification in the problem solving process. It has been thought that this study provide a new point of view to curriculum developers, teachers and researchers.

Keywords: *Problem solving; teaching geometry; dynamic geometry software; teacher education*

Introduction

Geometry is the set of relationships that affect life with their distinctive qualities, have large impact areas as well as new and flexible structures, and can be visible and discoverable through various tools. Such a structure makes it possible to adopt and apply a variety of experimental strategies. Straesser (2001) suggests that "geometry can be used in different ways if it is taught and learnt in different ways" (p.331). One of the different teaching and learning methods of geometry is obviously the environments provided by dynamic geometry software (DGS).

Software like Cabri II Plus, Geogebra, Sketchpad used in these environments provide new strategies that facilitate students' geometric problem solving via their features.These software create experimental environments for teachers that their students do their experiments easier than other technological environments and traditional paper-pencil environment, observe whether the mathematical relations change or not, construct and verify their own hypothesis (Marrades & Gutiérrez, 2000; Straesser, 2001). These environments transform mathematics classes into scientific laboratories by presenting the opportunity to test the conjectures obtained in mathematics (Leung & Or, 2007). With this transformation students can realize alternative ways to learn mathematical concepts in the environment which they can experiment freely (Marrades & Gutiérrez, 2000) and can research in geometry (Luthuli, 1996 cited in Christou et al., 2005). During this research process, by manipulating figures, students are convinced that any conjecture is valid in all cases (De Villiers, 1993, 2003).

Many researchers (Goldenberg & Cuoco, 1998; Healy & Hoyles, 2001, Güven & Karataş, 2003; Van De Walle, 2004; Bağcivan, 2005; Bintaş, Ceylan & Dönmez, 2006; Karataş & Güven, 2008) have mentioned that dynamic geometry software provide students the opportunity manipulate figures via measuring and dragging tools, to make relationships, to observe the changing features in figures, to conjecture, to explore the theories and their relations, and to test them. Students' skills to discover geometric relationships through this kind of software cannot be improved by any other environment without the aid of computers (Van De Walle, 2004).

In some studies (Or, 2005; Scher, 2002; Köse & Özdaş, 2009), it has been observed that DGS improve students' estimating and reasoning skills and provide them an opportunity of improving their analytical thinking skills. Additionally in some other studies (Christou et. al., 2004, 2005; Healy & Hoyles, 2001) it has been emphasized that the dragging and measuring features of DGS help students to understand the problems clearly and to explore the potential solutions of the given problem, and to construct new concepts.

Accordingly the usage of DGS is a key mediation tool in exploring and researching mathematical concepts. According to the study of Christou et. al. (2005) it has been mentioned that "…DGS as a mediation tool, encourage students to use in problem solving and posing the processes of modeling, conjecturing, experimenting and generalizing…". Besides it has also been mentioned that dynamic geometry environment support the relationship between the construction of new problems and the usage of problem solving strategies and the tendency of high level problem solving.

In such environment the role of in-service and pre-service mathematics teachers is to support and encourage students to discover the mathematical concepts and relations, to construct their own conjectures and justify them with reasoning. In this way, they can choose appropriate problems and develop a variety of strategies and perspectives so that their future students can learn mathematical concepts. Besides, it has been suggested in the reconstructed Turkish Mathematics Instruction Curriculums in 2005 that DGS be used in related objectives of several grades. Considering the fact that teacher training curricula were revised in parallel to the revised primary education curricula, the study sample consisted of pre-service mathematics teachers and the study aimed to explore the processes in which the participating pre-service teachers solved open geometry problems in dynamic geometry environments.

Theoretical Framework

Various phases have been dealt with in the studies examining the processes of problem solving and proving in dynamic geometry environment. Mogetta, Olivero and Jones (1999), in their studies about problem solving, mention that construction, exploration, conjecture, and justification are the main phases of problem solving process. Also Olivero (2001) has mentioned that some researchers (Arzarello, Gallino, Micheletti, Olivero, Paola & Robutti, 1998; Boero, Garuti & Mariotti, 1996) claim that exploration and conjecture are main phases of cognitive process of making proof. As regards proving in dynamic geometry, Edwards (1997) explained the term "conceptual territory before proving" by using conjecturing, verifying, exploring and justifying which are the key elements of formal proof. The researcher also explained that this conceptual territory provides an area for the formation of instinctive ideas that can be tested and verified through formal ways and this situation forms the basis for the proof to be understood better. Another researcher Chazan (1993) emphasized that this approach reflects the quasi-experimental view of mathematics that enable students to transform their understandings from their own conjectures and verifications to formal proof.

Marrades and Gutiérrez (2000) explained the terms explanation, verification, justification, and proof, which are the phases mentioned above, as convincing somebody of the accuracy of a hypothesis. De Villiers (2004) define conjecturing as looking for an inductive pattern, generalization, and verifying as the reality of a statement or conjecture, or obtaining certainty about these. Considering the studies in which these processes are observed in a dynamic geometry environment, it has been observed that open problems or open tasks are noticed to be used in several studies (Arsac, Germain & Mante, 1988; Christou et. al., 2005; Jones, 2000; Olivero, 1998, 2001; Olivero & Sutherland, 2000, Mogetta, Olivero & Jones, 1999; Furinghetti & Paola, 2003). The open-problem expresses a special case that students face with a problem that all the potential solutions are hidden in mathematical research activities. According to Arsac and Mante (1983), for any problem to be any problem considered as an open-problem, it has under-mentioned characteristics:

- The task has to be short and not include a special solution or special problem solving strategy. Additionally, the solution of the problem does not transform the application or the usage of the results presented in the class in any case.
- The problem has to be chosen from the topic that students are less familiar with, has a solution that students can easily find out, and also let students to attempt solving, to conjecture and to find out a solution projects. The questions connected to the problem are like "which relationship can you find between…?" or "What kind of figure can…be transformed into?" types instead of "prove that…" type.

According to Olivero (2001) the phases of open problems are exploring, conjecturing, validating and proving. Students have an environment that they experiment and list the results on their own by the open problems (Olivero, 2001).

Mogetta, Olivero and Jones (1999), in the study carried out by using an open geometry problem about triangles and their properties, mentioned that they observed that problem solving process has four phases as construction, exploration, conjecture, and justification. In addition to this four-phase process given in this research, the role of dragging has been considered and the validation phase has been added. Thus, a five-phase theoretical framework as *construction, exploration, conjecture, validate, and justification* has been taken as a basis in the study.

With all these in mind, the main aim of this research is to investigate pre-service elementary mathematics teachers' open geometric problem solving process in a dynamic geometry environment.

Method

Design of the Study

This study was designed as a case study, a qualitative research method. The case study emerged as a result of the desire "to understand social phenomena while retaining the holistic and meaningful characteristics of everyday events" (Yin, 1994). The case study is a research method that focuses on a specific situation, allows the researcher to focus on a specific subject or case and enables the researcher to explain fine details in terms of cause-effect and mutual relationships among variables by means of the data it provides (Çepni, 2007).

Participants

The participants of the study are three pre-service teachers from fourth graders of the Department of Elementary Mathematics Teaching. When selecting the participants, criterion sampling type of purposeful sampling methods has been utilized. The main aim of considering the criterion sampling is studying with the cases which are related to several sequences of criteria. The criteria taken as basis in this study are as follows;

- The participants of the research have been chosen from the students attending fourth grade of the department of Elementary Mathematics Teaching Training Program because they both have sufficient geometry content knowledge required for the research and have taken the course called as "Technology supported Geometry Teaching" (TSGT).
- The participants have been chosen from three different success levels (low, medium, high) in order to observe whether they have common or different phenomena among the situations varying in the research process. In the selection of students' success levels, the lecturer's opinion about their classroom performances (participation in classroom activities, achieving the given task etc.), 3 research assignments as the requirement of the course, 1 activity presentation assignment and their performances in the mid-term exam have been taken into consideration.

The pre-service teachers take compulsory and elective courses concerning both content knowledge and pedagogical knowledge during their academic life in the faculty of education. In the university where the participants are educated, TSGT course is given among elective courses in the 7th academic year. In this course, basic knowledge about the Cabri Geometry software is primarily given by the lecturer and pre-service teachers are enabled to use this software effectively. The TSGT course the participants took before the study was conducted in the computer lab as 3 hours a week for 14 weeks. They, for this course, prepared an activity with three research assignments, one of which is about quadrilaterals and their properties, and two of which are related to locus. These assignments and activities have been observed by the researchers and three participants with low, medium, and high success levels have been determined in accordance with the researcher and lecturer's opinions. The participants were coded as A, B, and C from high to low level of success.

Data Collection Process

In this study, clinical interviews, screen-captures of the problem solving process in the Cabri Geometry Environment, and worksheets including 2 open geometry problem have been used to collect the data. However, the analysis of only one of the problems has been given a place in the findings of the study because of the page limit.

A clinical interview, which has been used frequently in the qualitative researches, is a technique led by Piaget to explore the construction of the knowledge and reasoning process (Clement, 2000). As a part of these clinical interviews, open problems (as seen in Appendix I) as an interview task and the clinical interview questions (How do you get this result?, Can you explain why do you think this?, etc.) provide researchers to investigate pre-service teachers' problem solving process in detail have been utilized.

After the selection process of the participants, the time schedules of the interviews have been organized for them, and their oral and written permissions have been taken. Later, the interviews have been conducted to each pre-service teacher in the appointed date.

The clinical interviews held have been recorded by the video. The camera has been located to the suitable location which the participant and his/her problem solving process can be observed clearly. In the interviews the activities have been presented to the pre-service teachers in handouts and it has been asked them to cope with the open problems in the paper-pencil environment at first. After the process in the paper-pencil environment observed, it has also been asked again them to complete the same task in DGE.

To observe the construction process of the participants' activities prepared in the Cabri Geomery Environment the screen-captures has been recorded by special capturing software. These records have been used to support the data obtained from video records. Additionally, in clinical interviews, worksheets including the steps when students follow them in the construction phase have been used in order to observe the problem solving process in the paper-pencil environment.

Analyzing Data

After the data obtaining process, the data has been transcribed and checked by the researchers. At first, the phases -as construction, exploration, conjecture, validate and justification- mentioned in the theoretical framework have been related to the pre-service teachers' problem solving process. In the analyzing process data reduction, data display, drawing conclusion and verification steps has been utilized (Miles & Huberman, 1994). The intercoders reliability study has been used for the data coded by two different coders. The disagreements between the two coders have been overcome by discussing and the high percentage common view has been determined (Lincoln & Guba, 1985).

Findings

In this study the problem solving processes of pre-service teachers have been investigated firstly in the paper-pencil environment and successively in the dynamic geometry environment.

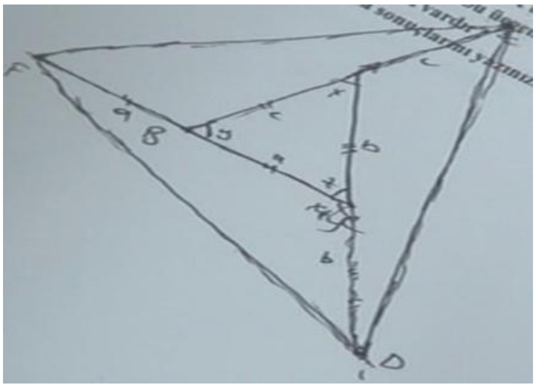


Figure 1. Participant A looks for AAA Similarity in the Process of Problem Solving in Paper-Pencil Environment

It has been observed that the participants have difficulties in solving the given problem (as seen in Appendix I) in paper-pencil environment. Participant A and participant B have tended to look for a similarity (AAA type) between the triangle ABC and the triangle DEF (Figure 1), participant C was looking for any relationship randomly in the paper-pencil environment. However, all three participants moved to the conctruction level in dynamic geometry environment without mathematically verifying their conjectures determined in paper-pencil environment.

In the dynamic geometry environment, it has been investigated that all the participants went through the similar phases in the open problem solving process. As seen on the Figure 2, the recursive processes as construction, exploration, conjecturing, validation and justification of each participant have been schemed and the geometric relationships (relationship between two perimeters, similarity, and relationship between two areas) which participants look for in each phase have been presented with numbers. For example, it has been investigated that participants A and B look for relationships between concepts, make conjectures based on these relationships and validate these conjectures in the exploration phase of their problem solving process. However, participant C has not passed through validation phase for the conjectures made in the process. Additionally, it has been observed that although participant B and C explore the geometric relationships in such conditions, they can not make conjectures about their explorations and tend to look for a new relationship.

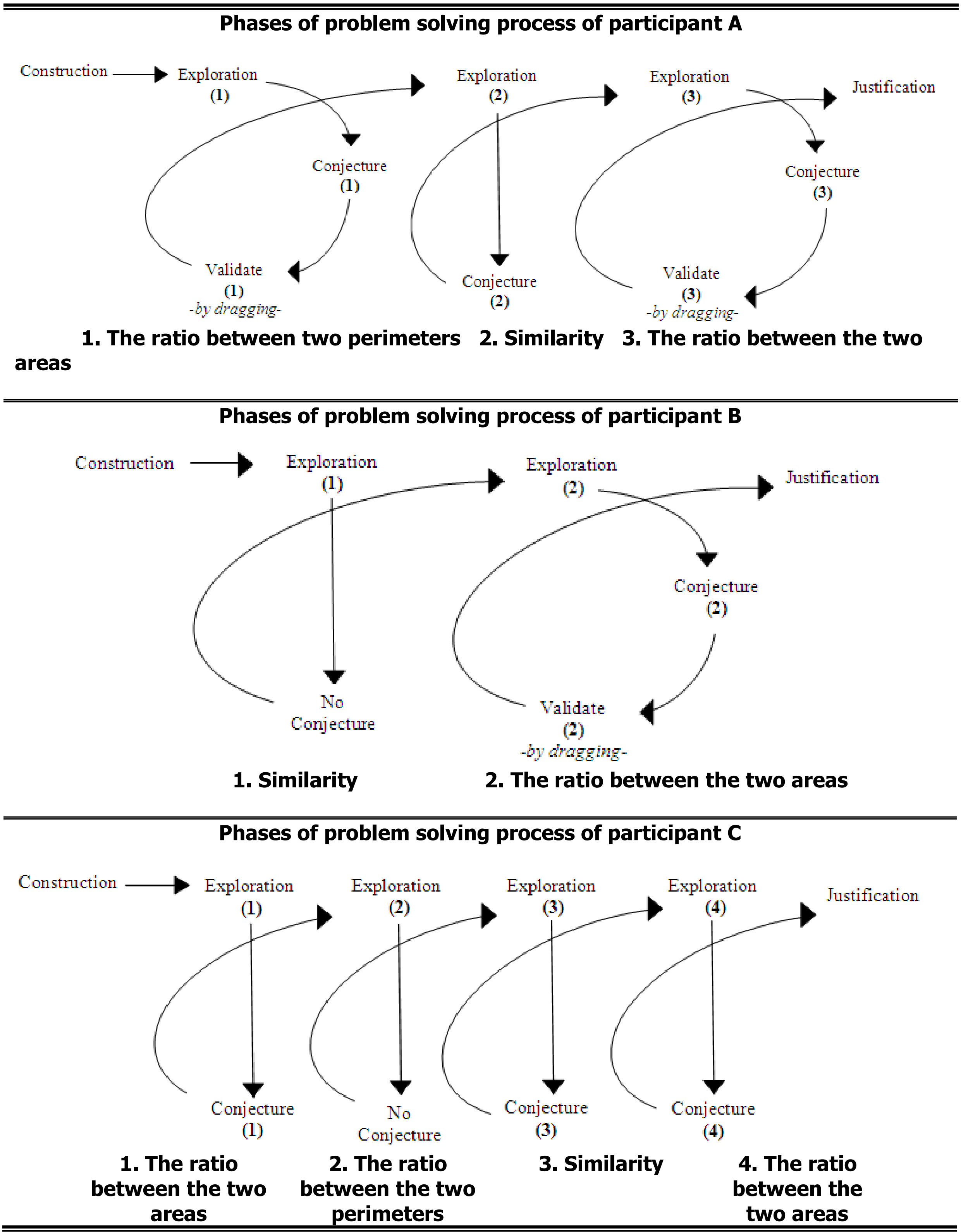
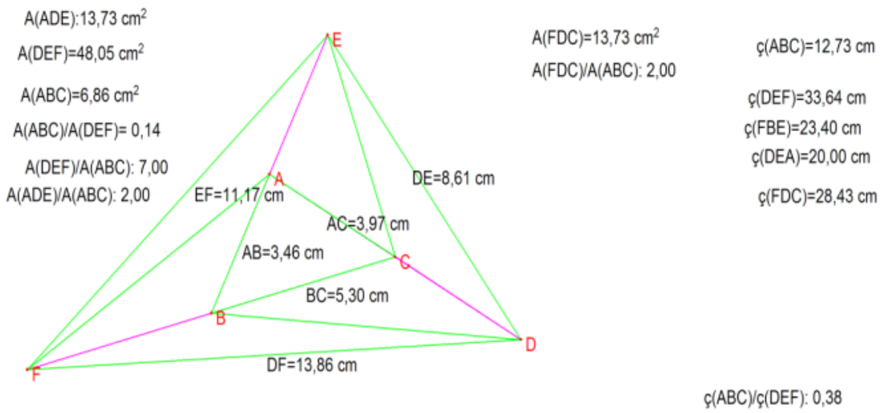


Figure 2. Phases of Problem Solving Processes of Participant A, B, and C.

In the problem solving process, it has been observed that participant A has been focused on the ratio between the perimeters, participant B has been focused on the similarity between two triangles and participant C has been focused on the ratio between the areas of the two triangles in the given problem firstly. Also it has been observed in the whole problem solving process that participant A and participant C have been noticed to research both the ratio between perimeters and areas, and the similarity, but participant B has been noticed to research only the similarity and the ratio between the areas. Nevertheless, it has been determined that at the end of the problem solving process, all the three participants were able to explore the seven times relationship between the areas of the triangle ABC and triangle DEF and to justify that they found.

As seen on the Figure 3, Figure 4 and Figure 5, the geometric relations that students found out in the problem solving process and the mathematical operations related to these relations have been presented in detail.

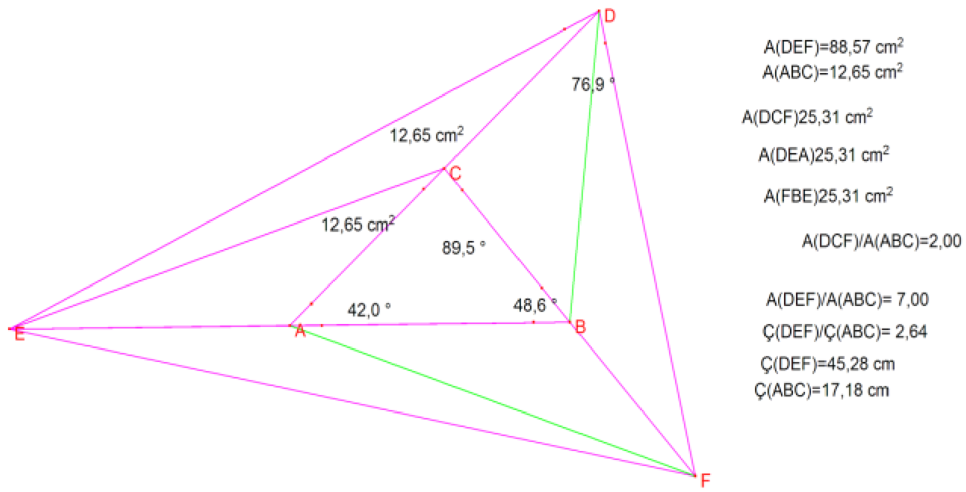
Participant A



1. The ratio between the two perimeters:	P(ABC)/P(DEF)=0.38 and validating the result by using drag test.
2. Similarity:	No AAA or SS similarity found between the triangle DEF and ABC.
3. The ratio between the two areas:	S(ABC)/S(DEF)=0.14 and S(DEF)/S(ABC)=7 Verifying these by using drag test.

Figure 3. Participant A’s problem solving process and mathematical operations they used

Participant C



1. The ratio between the two areas:	S(DEF)/S(ABC)=7
2. The ratio between the two perimeters:	P(DEF)/ P(ABC) = 2.64
3. Similarity:	No AAA similarity between the triangle DEF and the triangle ABC
4. The ratio between the two areas:	S(DCF)/S(ABC)=2

Figure 4. Participant C's Problem Solving Process and Mathematical Operations They Used

Participant A and participant C have justified the seven times relationship that they constructed between the areas through the segment [FA], [DB], and [EC] that they drew into the triangle EFB, CFD and EAD. Therefore, participants constructed 6 more triangles whose bases and heights are at the equal length of those the triangle ABC has and that have the equal areas with the triangle ABC, interior area of the triangle DEF. The below-mentioned part of participant A's clinical interview process has been presented to clarify the justification process.

Participant A (A): [measured the areas of the triangles ADE, DEF, ABC and FDC and the ratios between their areas through a calculator.]… ADE [the area of ADE] was two times of ABC. If one triangle outside comes out two times, then the others will come out so… We can already say this indeed… The big triangle DEF was totally seven times of ABC… When we say ABC is 1S, we say the area of DEF as totally 7S... Similarly, we found when ABC is S then ADE is 2S. Likewise we now find the area of FDC is 2S. There is 2S left for FBE.

Researcher (R): Why do you think these three triangles with the same areas are two times of the area of the triangle ABC? And why the area of the big triangle, the outermost triangle DEF, is seven times of the area of the triangle ABC?

A: …We obtained triangles with two times areas by lengthening equally the sides of the triangle ABC in the same line. …because BF and BC have equal lengths due to symmetry for the point in this line, the outer triangle CDF, triangle with 2S area, here will be separated as S and S, namely the areas of this BDC triangle and BDF triangle will be equal because of this.

R: Can we say, do you think, if their one sides are equal, they have equal areas?

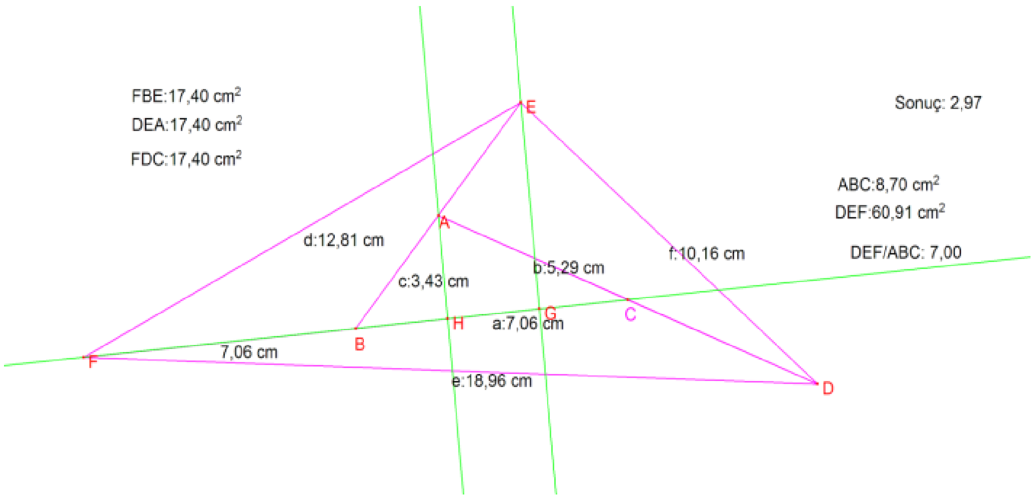
A: Well, now, uhmm, when we divide the outer triangle CDF with 2S area into the opposite side namely to the median from B, we can call them S and S.

R: What is the reason for it?

A: …the reason for it is that the heights are equal. That is the heights we draw to this FC line are equal. …if we think partially, now, we said this FDC will always be two times, namely similarly if we consider ADE, when we draw a segment from E to C... here again we found, uhmmm, ADE is 2S. The 2S area is separated with a median as S and S, as AEC and ECD. ABC was S, uhmm again something comes up, here it comes up as 2S… Namely because of the equality of heights, those drawn from B… Correspondingly we see that ABC and FBA are equal after separating as S and S. The reason for it is the equality of heights again… In this concept the triangle to form at the outermost becomes totally seven times. There is seven times area difference between the areas of triangles DEF and ABC.

Participant B's justifying the relation that he discovered in the given problem differs from that of other participants. This difference results from the two perpendiculars that he drew from the vertice A and vertice E to the line FC. The participant used the heights that were formed and the similarity between the triangle BHA and the triangle BGE formed through these perpendiculars in the comparison of the areas of the triangle ABC and the triangle EBF.

Participant B



1. Similarity:	Try to explore the similarity by using drag test. No similarity found.
2. The ratio between the two areas:	S(DEF)/S(ABC)=7 validating the result by using drag test.

Figure 5. Participant B's Problem Solving Process and Mathematical Operations He Used

As seen in Figure 5, participant B constructed the segment [AH] and segment [EG] by drawing the lines AH and EG perpendicular to the line FC in order to justify the 7 times relationship between the areas of the triangle ABC and the triangle DEF. In the figure he drew, the participant noticed the similarity of the triangle BHA and the triangle BGA from both segment [AB] and segment [AE] being equally long and from the parallelism of perpendiculars that comes down the same line ([AH] // [EG]).

Through this similarity, the student explained the relation between the segments AH and EG by saying *"AB and AE are equal namely here is a one to two ratio. From the similarity between these triangles, EA divided into EB… Because of the ratio that I mentioned [ratio of similarity], [AH] is the half of the [GE]."*

Participant B is, in fact, aware of the fact that the length of the segment [EG], which is the height of the side BF of the triangle EFB, is two times greater than the length of [AH] segment, which is the height of the BC side of the triangle ABC. Thus, the participant, here, was able to justify that the relation that the area of the triangle EFB is two times greater than of the area of the triangle ABC. Based on the two times greater ratio between the area of two triangles, participant B justify the seven times greater ratio between the area of the triangle ABC and the triangle DEF as:

> *Researcher (R): We ask you to find out the relation between the triangles ABC and DEF, can you say anything for that?*
>
> *Participant B (B): That is, when we consider the same process for the all triangles, the same result comes up…so, because we will generalize this to other triangles…. The areas of the triangles formed nearby come up as 2S, 2S and 2S.*
>
> *R: Yes.*
>
> *B: One of them is S, thus the triangle DEF comes up seven times of the triangle ABC… we saw it more clearly with this software.*

In this study, another remarkable finding obtained in the problem solving process is related to the dragging function, which is one of the most distinguishing functions of dynamic geometry software. It has been determined in the process that participant A used the dragging function in the phase of validating the relations that he found out, participant B used this function in looking for relations between the geometric concepts in the exploration phase, and participant C didn't use this function in any phase.

Discussion

The spreading usage of dynamic geometry software in mathematics teaching and learning suggests how teachers of mathematics internalize these technologies. One of the answers of this question lies in the classes given in teacher training programs and the content of these lessons. In line with this thought, we, in this study, examined the problem solving processes of pre- service teachers through a dynamic program like Cabri Geometry, trying to understand how the experiences of these pre-service teachers reflect in their own cognitive processes.

In the solving process of the given open problem, participants firstly have difficulties in the paper-pencil environment and they did not find out any relation between the two triangles mentioned in the problem. In the dynamic geometry environment, participants passed through recursive phases as construction, exploration, conjecture, validate, and justification in the problem solving process. In this study, when the deductions made by participants A, B, and C in the process of problem solving are generally evaluated, participant A and participant B have been observed to make interpretations by making strong geometric relations and to validate the relations that they found out by supporting their geometry knowledge. However, participant C has some difficulties both in mathematically explaining the whole process and especially in justifying the relations that she found out. It can be thought that these difficulties result from the fact that participant C did not pass through the validation phase and she has less and/or unsufficient geometry knowledge.

DGS provide students to construct proper figures and subsequently play a role in helping students to notice the conjectures which cannot been observed easily via its dragging function (Christou et. al., 2004; Laborde, 2000; Or, 2005). Similar to the studies by Christou et. al. (2004), Laborde (2000) and Or (2005), it has been observed in this study that in some phases in the problem solving process participant A and B find the results more easily and observe the conjectures which they cannot observe in the paper-pencil environment, by using DGS's dragging function. Hence, DGS can be said to have a crucial role in problem solving, problem posing and using problem solving strategies (Christou et. al.,2005, Cai & Hwang, 2002). When the studies determined, it has been observed that dragging and measuring features of DGE provide students to understand the problem and to find the potential way of solution.

As seen in the study by Or (2005), the participants have easily constructed the problem in DGE. In the exploration phase, participants have observed the construction that they made and have passed through to the forthcoming conjecture phase based on their observations. The active usage of the dragging feature, which is one of the effective features of DGS, in this phase and the forthcoming validate phase enable participants to justify the conjectures they find out in a strong way. This process is similar to the recursive process consisting of construction, experimentation and conjecturing phase in the study by Or (2005).

In the study, the author expressed that several strategies used by students in the process help them to pass from emprical phase to the theoretical phase and as a consequence to support them to explain sufficiently and/or to prove. The theoretical phase mentioned is defined as the formulation of the proof of the explanation of the validity of students' constructions. In our study, parallel to this, the process of their justification of conjectures enable participants to pass through formal proof by making meaningful arguments about the relations found out.

Results and Suggestions

In this study, it is aimed that the problem solving processes of pre-service primary mathematics teachers in DGE are investigated. To that end, the open problem solving processes of pre-service teachers in DGE have been analyzed and examined in detail.

It has been observed that the problem solving processes of the participants have passes through the recursive phases as construction, exploration, conjecture, validate and justification. In the research process, it has been investigated that almost all participants actively used several features of DGS – like measuring and dragging- and made suitable deductions in the solving process of the open problem given.

In conclusion, it has been thought that this study provide a different point of view to curriculum developers, teachers and researchers. With the help of this study, pre-service teachers were made to be faced with unusal problem solving experience and, via their awareness of this experience they can create richer environments for their students to be thought by them and can support their learning by giving them several ideas. Therefore, it has been thought that this situation can help the pre-service teachers in their proffessional development.

Note: This paper is an extended version of the study presented in 35th Conference of Psychology of Mathematics Education (35th PME) in Ankara, Turkey.

Appendix 1.

OPEN PROBLEM TASK

1. Construct any triangle ABC.

2. Let the symmetry of the point A to the point C –called point D- , let the symmetry of the point B point to the point A -called point E- , and let the symmetry of the point C to the point B –called point F-.

3. Define the triangles DEF, FBE, FDC, and DEA.

4. Compare these triangles that you have constructed, what kind of a relationship is there between the triangles DEF and ABC.

5. Write down your results. How can you justify the results that you have found out?

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