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Benchmarking Urban Energy Efficiency with Deterministic and Stochastic Methods

Kentsel Enerji Verimliliğinin Deterministik ve Stokastik Yöntemlerle Kıyaslanması

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Abstract

In urban sustainability researches, benchmarking methods have become the most needed ways to measure urban energy efficiency. Benchmarking the efficiency of urban energy with parametric and non-parametric methods are important cases within the energy field. Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) are ideal approaches to measure performance of various industries with multiple indicators. Stochastic method considers the noise in data and evaluates the critical success parameters of energy efficiency by separating noise from efficiency scores. This study evaluatesurban energy efficiency by deterministic and stochastic ways with deploying DEA and SFA methodologies. The aim of the study is to show the effects and results of deterministic and stochastic approaches in urban energy efficiency measurement and to evaluate how Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) can be used to derive measures of efficiency and productivity change over time in complex multi-input contexts in the production and consumption of energy services. Using data gathered from Turkish Statistical Institute (TURKSTAT) and Energy Market Regulatory Authority (EMRA) Development Reports. In the study, 30 cities, which are accepted as metropolitans of Turkey by government, are selected as Decision Making Units (DMUs) of both methods. As a result, different efficiency estimates are presented and evaluated within the scope of statistical noise, multiple inputs and outputs by DEA and SFA methods.
Keywords: Urban Energy Efficiency; Urban Sustainability; Optimization for Energy Efficiency; Stochastic Frontier Analysis; Data Envelopment Analysis

Öz

Kentsel sürdürülebilirlik araştırmalarında, kentsel enerji verimliliğini ölçmek için en çok ihtiyaç duyulan yöntemler kıyaslama yöntemleridir. Kentsel enerji verimliliğinin parametrik ve parametrik olmayan yöntemlerle kıyaslanması, enerji alanındaki önemli ihtiyaçlardandır. Veri Zarflama Analizi (VZA) ve Stokastik Sınır Analizi (SSA), çeşitli endüstrilerin performansını çoklu göstergelerle ölçmek için ideal yaklaşımlardır. Stokastik yöntem, verilerdeki gürültüyü dikkate alır ve gürültüyü verimlilik puanlarından ayırarak, enerji verimliliğinin kritik başarı parametrelerini ortaya koyar. Bu çalışma, kentsel enerji verimliliğini VZA ve SSA metodolojilerini kullanarak deterministik ve stokastik yollarla değerlendirmektedir. Çalışmanın amacı, kentsel enerji verimliliği ölçümünde deterministik ve stokastik yaklaşımların sonuçlarını sunmak, VZA ve SSA etkinlik ölçümlerinin, zaman içinde çok çıktılı ve çok girdili enerji hizmetlerinin üretim ve tüketiminde, nasıl değişebileceğini göstermektir. Veri analizlerinde, Türkiye İstatistik Kurumu (TÜİK) ve Enerji Piyasası Düzenleme Kurumu (EPDK) Kalkınma Raporlarından faydalanılmıştır. Türkiye'nin 30 metropolitanı her iki yöntemde de Karar Verme Birimleri (KVB) olarak seçilmiştir. Sonuç olarak, VZA ve SSA yöntemleriyle, istatistiksel gürültü, çoklu girdiler ve çıktılar kapsamında farklı verimlilik tahminleri sunulmuş ve değerlendirilmiştir.
Anahtar Kelimeler: Kentsel Enerji Verimliliği; Kentsel Sürdürülebilirlik; Enerji Verimliliğinde Optimizasyon; Stokastik Sınır Analizi; Veri Zarflama Analizi

I. INTRODUCTION

Metropolitans are centers of energy demand, investment and consumption centers in the World [1]. Since there is a very rapid urbanization, urban energy efficiency becomes an important concept of urban sustainability. Measuring urban energy sustainability is a case for metropolitans to realize more sustainable urban energy development and to take consideration important achievement metrics of energy efficiency. Urban energy efficiency requires using less energy to produce expected factors. High-energy efficiency will reduce high investments and increase the social urban sustainability, necessary investments and environmental protection of world cities.

Realizing the needed determination of energy efficiency is critical to implement different approaches that maximize outputs and decrease investments [2]. Patterson who was the first researcher about energy efficiency, mentioned that there is no quantitative evaluation of efficiency which should be measured by a series of inputs and outputs [3].

Previous researches have indicated deterministic and classical methods, which present limited insight on the randomness and complexity of urban energy efficiency. Hence, these studies failed to gain stochastic techniques to measure the noise and uncertainty in data.

In recent years, more researches have been introduced different measuring methodologies to find solution for the complex questions of urban energy efficiency. Some of them studied economic situations depending on mathematical assumptions, the others implemented various kinds of models to see the high energy-consuming effect through critical and various parameters.

Between the econometric models and mathematical programming, two critical differences are the formulation of a production frontier function and the measurement of efficiency. The stochastic econometric model recognizes the effects of statistical noise in imprecise data, but since it is parametric, it requires determining a functional structure according to the type of process to be measured. It also provides for a statistical testing of hypotheses and the results of confidence intervals [4]. The mathematical programming approach is deterministic and non-parametric as DEA, and does not take error of data into calculations. It is a detailed model of multiple input output evaluation [5].

In this study, data analysis and benchmarking were made between 2018 and 2019, for the urban energy efficiency performances of 30 metropolitans in Turkey. There is a comparison of different deterministic and stochastic results of Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA) for measurement of urban energy efficiency. This paper shows the strengths and weaknesses in estimating urban energy efficiency of these methods through noisy in energy data. Technical efficiencies of cities differ between two methods. Because in deterministic DEA approach there is no consideration of the uncertainty in data on the frontier. On the other hand, the SFA method allows calculating error in data. These approaches can be classified as parametric and non-parametric methods. In the parametric SFA method a cost or production frontier formulation is estimated statistically, but in the nonparametric DEA method mathematical programming techniques are used. This study is the first study used to show the effects and results of deterministic and stochastic approaches in urban energy efficiency measurement within BCC, CCR and Cobb-Douglas implementations.

The structure of the paper is organized as follows: The relevant literature was reviewed in Section 2 as 'Literature review'. The proposed approaches are presented in Section 3 as 'Methodology'. The evaluation for urban energy efficiency of DMUs is implemented in Section 4 as 'Case study', and the paper is highlighted in Section 5 as 'Conclusion' and further research areas.

II. LITERATURE REVIEW

The cities, which are economic and cultural activity centers, provide important life factors to both developed and developing countries [6], they are the consumer of energy and the determinants of urban energy efficiency. Urban energy consumption is also related with urban energy investments, climate, population density and economy. Hence local energy efficiency inspections, investment decisions and plans can increase global energy efficiency.

Doherty et al. [9] has presented the energy consumption of cities into three categories as; operational, embodied and transport energy.

World Energy Council [10], stated urban energy consumption through "economic improvement and the distribution of income, urban form and density profiles, urban culture and climate, demographic development, transition and age situation."

According to these definitions and perspectives, urban energy efficiency components in cities must be an integration of energy usage within lifecycle needs.

Forsström, et al. [11] introduced energy efficiency as the ratio of energy output and the energy input of the related factors. In this study, complex energy measurement output indicators were defined for service, process, goods and consumption. Energy efficiency is an important component of environmental sustainability and urban sustainable development. Natural energy resources are the results of ecological efficiency that can be defined as a part of sustainability. Hence, they said that; energy efficiency is a subset of ecological efficiency and sustainable development encompasses both of them [11].

There are different input and output parameters as energy efficiency measurement indicators. Patterson [3] has stated general energy efficiency factors as "thermodynamic, physical-thermodynamic, economic-thermodynamic and economic" groups. However, each group has strengths and weaknesses and in each of them allows choosing alternative indicators. For instance, energy intensity as energy consumption per unit of Gross Domestic Product (GDP) indicator shows total industry sectors' energy need to produce one unit of expected economic output.

There are many researches in literature on energy efficiency measurements. Keirstead [8] implemented three different approaches to investigate urban energy efficiency in United Kingdom. Keirstead [8] used ratio calculation, regression residuals and DEA methods.

The input indicators of DEA model were total energy consumption, land area, climate and population, while the output indicators were life expectancy, carbon dioxide emissions as undesirable output and access time to services.

Dizdarevic et al. [12] measured energy efficiency in the EU countries between 2000 and 2010. They implemented input-oriented CCR DEA model within capital, labour, and energy use as input parameters and gross domestic product as the output. Here, GDP was the most important factor because of the energy economy policies.

Li et al. [2] made energy efficiency calculation of high energy using industry by discussing the models BCC and CCR methods of DEA and SFA. This study presented an overview of energy efficiency measurement in high energy using industries. Economy, environmental factors, energy consumption, GDP, energy price and employees can be considered as indicators of linear frontier models. On the other hand, as technological indicators energy demand, resources can be considered as variables of constraint models. According to this study different implementations of energy efficiency measurement, DEA models are best approaches for multiple factors. In addition, energy-economic models enableauthorities to make decisions and plans for future energy policies.

Yang et al. [13] represented a method of determining the energy intensive urban built environment for improving energy efficiency and understanding of how urban buildings consume energy. Building energyuse may include imprecise, random error term shows unpredictable conditions like energy supply and demand imbalance, system faults, service quality, climate, weather uncertainty and data input failures. This randomness may have either positive or negative effect on measurement and can be interpreted as stochastic differences of energy efficiency. Since the random factors, uncertain situations and error in data have important impacts on building energy performance, this study utilized SFA to find out the efficiency frontier and to remove influences of random error [13].

Baycan and İlhan [14] aimed to measure urban energy efficiency by nonparametric DEA method and they calculated efficiency scores for 81 provinces of Turkey. They used population, area, energy

consumption, heating and cooling degrees as input factors, annual income, CO2 emissions, average life expectancy as output factors. According to study; to make detailed and reliable studies in urban energy efficiency, there should be correct and high quality energy data, planned economical decisions and registered outputs, regular data evaluations of CO2 emissions and energy efficient buildings in each city. Kuosmanen et al. [15] tried to show the best benchmarking method of energy efficiency with comparison of DEA, SFA and stochastic nonparametric envelopment of data (StoNED). They emphasized the importance of stochastic studiesbeside deterministic ones. Moutinho et al. [16]analyzed effect of urban air pollution in ecologicalefficiency through DEA and SFA in Germany. They found out that randomness and noisy in data like inclimate change effects efficiency results.

In summary, SFA methodology contains calculation errors and provides the random instability of variables [2]. It leads to succession of measurement errors. On the other hand, it is hard to understand the determination of the error structure [18].

The deterministic approach DEA is a nonparametric linear programming methodology, which obtains an efficiency frontier using convex linear combination of factors. Besides, SFA requires a parametric expression of efficiency frontier function and supposes a compound error term, which stands for deviations from the frontier function. The compound error term means the sum of the stochastic inefficiencies and stochastic noise that is data error. StoNED is similar toSFA and DEA methods, with a compound stochastic error term and with a nonparametric, piecewise linear frontier function [22]. Lopes and Mesquita [23] showed that these models are preferred among the energy efficiency calculations for benchmarking.

In the study there are different measurement scores of urban energy efficiency with DEA and SFA models for 30 metropolitans in Turkey. These cities accepted as metropolitans by government according to their number of districts, service limits, total population, physical settlement status and economic development levels [17].

As a summary of the literature review, researches on energy efficiency benchmarking with DEA and SFA methods is presented below with Table1.

Table 1. Literature review summary of the study.

Author(s)	Category	Study Descriptions
Moutinho et al. (2021)	Effect of urban air pollution in ecological efficiency through DEA and SFA in Germany.	Study found out that randomness and noisy in data like in climate change effects efficiency results.
Yang et al. (2018)	Energy efficiency and understanding of how urban buildings consume energy.	Building energy use may include imprecise, random error terms and this randomness may have either positive or negative effect on measurement and can be interpreted as stochastic differences of energy efficiency. Since the random factors, uncertain situations and error in data have important impacts on building energy performance, this study utilized SFA to find out the efficiency frontier and to remove influences of random error.
Gil et al. (2017)	Brazilian energy distribution benchmarking.	DEA efficiency scores of electricity distribution companies are calculated with higher scores than the original scores.
Li et al. (2017)	Energy efficiency calculation of high energy using industry by discussing the models BCC and CCR methods of DEA and SFA.	According to this study different implementations of energy efficiency measurement, DEA models are best approaches for multiple factors. In addition, energy-economic models enable authorities to make decisions and plans for future energy policies.
Baycan and İlhan (2015)	Urban energy efficiency measurement by nonparametric DEA method.	According to study; to make detailed and reliable studies in urban energy efficiency, there should be correct and high quality energy data, planned economical decisions and registered outputs, regular data evaluations of CO_2 emissions and energy efficient buildings in each city.
Kuosmanen et al. (2013)	Study on the best benchmarking methods of energy efficiency with comparison of DEA, SFA and stochastic nonparametric envelopment of data (StoNED).	They emphasized the importance of stochastic studies beside deterministic ones.
Lopes and Mesquita (2015)	Study on benchmarking methods of energy efficiency with comparison of DEA, SFA and stochastic nonparametric envelopment of data (StoNED).	Showed that DEA, SFA and stochastic nonparametric envelopment of data (StoNED) models are preferred among the energy efficiency calculations for benchmarking.
Doherty et al. (2013)	Energy consumption in urban environments.	According to study; energy consumption of cities can be measured into three categories as; operational, embodied and transport energy
Keirstead (2013)	Benchmarking Urban Energy Efficiency	Study used ratio calculation, regression residuals and DEA methods. The input indicators of DEA model were total energy consumption, land area, climate and population, while the output indicators were life expectancy, carbon dioxide emissions as undesirable output and access time to services.

Dizdarevic et al. (2012)	Energy efficiency measurement in the EU countries.	They implemented input-oriented CCR DEA model within capital, labour, and energy use as input parameters and gross domestic product as the output. Here, GDP was the most important factor because of the energy economy policies.
Forsström, et al. (2011)	Energy efficiency study through, complex energy measurement output indicators defined for service, process, goods and consumption.	Energy efficiency is an important component of environmental sustainability and urban sustainable development. Natural energy resources are the results of ecological efficiency that can be defined as a part of sustainability. Hence, they said that; energy efficiency is a subset of ecological efficiency and sustainable development encompasses both of them
Proposed Model	In the paper, there are different benchmarking scores of urban energy efficiency measurement with deterministic and stochastic methods.	Study compared different deterministic and stochastic results of Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA) for measurement of urban energy efficiency. This paper shows the strengths and weaknesses in estimating urban energy efficiency of these methods through noisy on the frontier. Technical efficiencies of cities differ between two methods.

III. METHODOLOGY

As mentioned and overviewed in the literature part, there are two fundamental approaches, which are parametric analysis and nonparametric methods, applied to calculation of frontiers. The flowchart of the models are presented in Figure 1. Here SFA is the parametric analysis, DEA is the nonparametric method. The aim of using DEA and SFA methodologies in urban energy efficiency problem is measuring inefficiency of a DMU as the distance between an efficient DMU frontier and actual performance of the DMU and showing the divergent efficiency results of stochastic and deterministic methods. However, the two methods had different advantages and disadvantages. DEA needs no assumptions about the probability density of inputs and outputs on frontier. It joins noise as part of the efficiency score and assumes no errors and deviations from the efficient frontier. SFA allows deviations from the efficient frontier into a random error term that is statistical noise and a one-sided error term representing inefficiency. SFA needs the determination of a functional form for the frontier and assumptions about the distributions of the random error and inefficiency error terms. SFA can separate random noise from frontier.

3.1. Data Envelopment Analysis

DEA is a mathematical analysis method that is used to evaluate the effectiveness of decision making units, that use multiple inputs to obtain multiple outputs.DEA was first presented by Farrell [19], then, the approach was introduced by Charnes et al. [20] who led the basis of a literature through operational research and economics. Charnes, Cooper and Rhodes [20] developed the first DEA application that was named the CCR model. Banker, Charnes and Cooper [21] studied the DEA to get a variable returns to scale

formulation of the CCR model that was named BCC model.

In this context, in our study a performance comparison was made by input oriented BCC and CCR DEA approaches. CCR model was used to analyze the set of DMUs that were using the production function with constant returns to scale. The reason for using CCR was to provide the possibility of separately calculating technical efficiency globally.

Technical efficiency calculates the DMU's overall success with related inputs. CCR model calculates the sector efficiency of a decision unit which contains technical and scale efficiency. In this approach the assumption is that outputs increase with an increase in inputs [2].

Mathematical equations of the input oriented and output oriented CCR models are given below. It is supposed that there are n homogenous DMUs (DMU_j , $j = 1,....n$) such that all of them use m inputs x_{ij} ($i =1,2,.....m$) to obtain s outputs y_{rj} ($r =1,2,.....s$), $x_j = (x_{1j}, ..., x_{mj})$ and $y_j = (x_{1j}, ..., y_{sj})$ which are nonnegative and nonzero vectors. The CCR model's production possibility set suggested by Charnes, Cooper, Rhodes in 1978 [20] is as follows:

$$T_{CCR} = \{(X,Y)/ \sum_{j=1}^{n} (X_j \lambda_j) \leq X , \ \sum_{j=1}^{n} (Y_j \lambda_j) \geq Y , \ \lambda_j \geq 0, j = 1,....n \ \}$$

CCR efficiency scores can be obtained by using the envelopment input-oriented and input-oriented equation (1), respectively where x_{i0} and y_{r0} represent the ith input and the rth output indicator vector of DMU_0 under calculation in models.
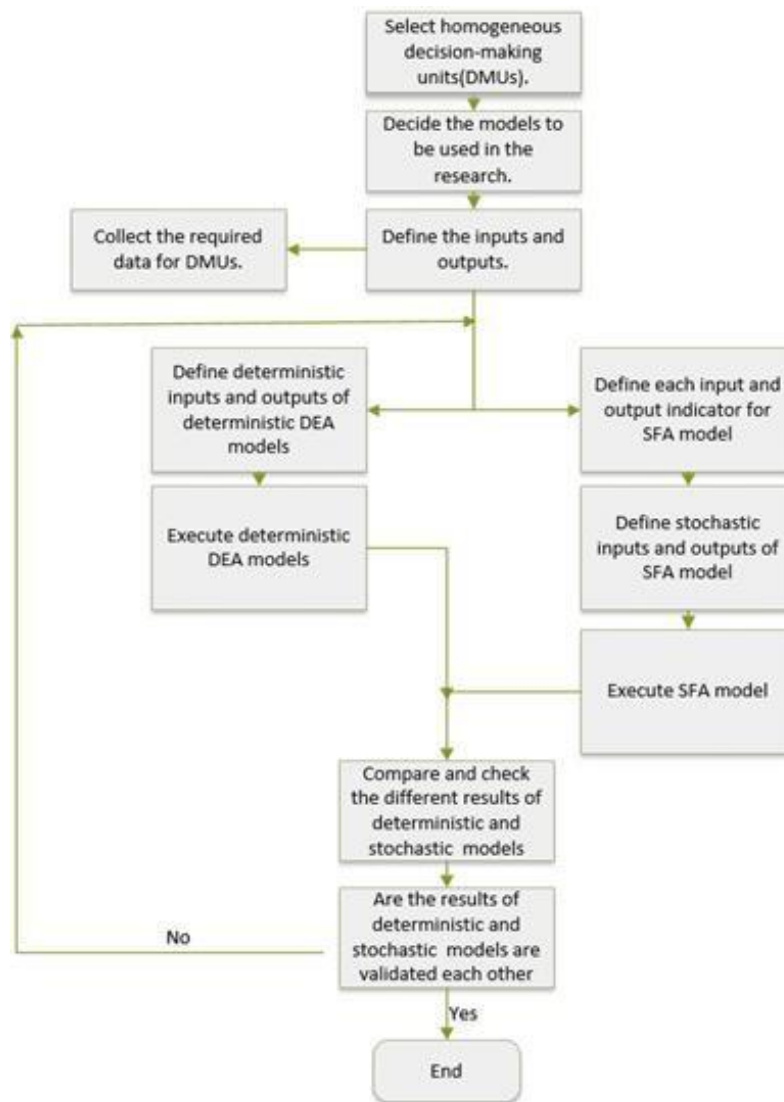
Figure 1. The flowchart of the models for performance benchmarking of DMUs.

A *DMU* is named input oriented CCR efficient if its expected value in equation (1) is equal to unity.

$$min \; \theta_0 + \varepsilon \left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right)$$

$s.t.$

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta_0 \; x_{i0}$$
$$\sum_{j=1}^{n} y_{rj}\lambda_j - y_{r0} - s_r^+ = 0$$

(1)

$$\lambda_j \geq 0 \; , \; s_i^- \geq 0 \; , \; s_r^+ \geq 0, \quad i=1,2,....,m \; ,$$
$$r= 1,2,.......,s \; , \; j=1,2,......,n$$

The BCC model was based on variable returns to scale and derived by including CCR model convexity constraint. The effective BCC frontier, expressed in a tighter envelope, is closer to the decision units than the CCR frontier [21]. The BCC model obtains the pure technical efficiency of decision making unit locally. In the model assumption of variable returns to

scale means that the output indicators of model will not increase proportionally with an increase in input indicators [2].

The BCC production possibility set introduced by Banker, Charnes and Cooper in 1984 [21] is as follows:

$$T_{BCC}=\{(X,Y) / \sum_{j=1}^{n}(X_j\lambda_j) \leq X \; , \; \sum_{j=1}^{n}(Y_j\lambda_j) \geq Y \; ,$$
$$\sum_{j=1}^{n} \lambda_j = 1, \; \lambda_j \geq 0, j = 1,....n \; \}$$

A unit is called input oriented BCC efficient if its expected efficiency value in equation (2) is equal to 1.

$$min \; \theta_0 - \varepsilon \left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right)$$

$s.t.$

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta_0 x_{i0}$$
$$\sum_{j=1}^{n} y_{rj}\lambda_j - y_{r0} - s_r^+ = 0 \qquad (2)$$
$$\sum_{j=1}^{n} \lambda_j = 1$$
$$, \quad \geq 0, s_r^+ \geq 0,$$

$\lambda_j \geq 0 \quad s_i^- \qquad\qquad i=1,2,....,m$,
$r= 1,2,........,s$, $j=1,2,.......,n$

Here the λ_j represent structural variables, the s_i^-, s_r^+ represent slacks and $\varepsilon > 0$ is a ''non-Archimedean infinitesimal'' determined to be smaller than any positive real number. This means that ε is not a real number.

3.2. Stochastic Frontier Analysis

This section aims to introduce Stochastic Frontier Analysis, which is a parametric approach is used in efficiency and productivity measurement in the framework of mathematical and econometric assumptions. According to literature search, an overview of SFA models has been introduced in this part.

Researchers tried improving urban energy efficiency by considering investment and production. Farrel [24] presented SFA model to fill the gap between theoretical and empirical searches. This approach showed us that there was a parametric relation between model's input and output indicators. In 1970s, SFA was first implemented in the evaluation of production function frontier by Aigner et al. [25].

The main feature of SFA is the estimation of a conventional function and the determination of efficiency or inefficiency by calculating the distance of each decision unit to the curve created by this function.

As emphasized before, the DEA method has deterministic structure so it ignores measurement errors. In SFA, frontier emphasizes the limit of production and stochastic term implies calculated error. Literature searches showed that the random indicators may affect production. Hence, the statistical error, which has normal and one sided distribution, included into the model. The SFA method fixes the disadvantages of measurement errors.

Aigner et al. [25] used cross-sectional data to estimate a production frontier, that was named a Cobb-Douglas production frontier. The Cobb-Douglas form of SFA model is a generellay preferred functional form in stochastic frontier analysis studies. In SFA studies Cobb-Douglas production function is preferredbecause its various advantages such as its

understandable structure. Lau [33] has presented that, "this model makes computations easy and has the features of explicit representability, uniformity, parsimony and flexibility."

Cobb-Douglas production frontier function's form is given as in equation (3):

$$\ln(y_i) = \beta_0 + \sum_{n=1}^{N} \beta_n lnx_{ni} - u_i \, , \; i = 1,2,..., N \quad (3)$$

where y_i is the vector size of output that is produced by ith DMU, x_{ni} is the vector size of nth input that is used by ith DMU, β is unknown parameter and u_i is positive random variable that indicates technical inefficiency. But the equation (3) is a deterministic frontier as $exp(\beta_0 + \sum_{n=1}^{N} \beta_n lnx_{ni})$.

This deterministic frontier counts out the mathematical possibility of measurement error and statistical noises, and remarks all deviations from the frontier as only technical inefficiency [25]. Hence, stochastic production frontier function was presented by specifying random indicators that represented statistical noise.

In 1977, Aigner et al. [26] defined the model's production function in deterministic and stochastic ways. Broeck et al. [27] introduced a new stochastic production function. They included symmetric random error for calculation of statistical noise. The formula is as follows.

$$y_i = f(x_i, \beta_i) + v_i - u_i = f(x_i, \beta_i) + \varepsilon_i, i = 1,2,...,N \qquad (4)$$

where v_i independent random variable showing the $N(0, \sigma_i^2)$ distribution and $\varepsilon_i \leq 0$. In the equation (4) it is assumed that ε_i is a composite error parameter consisting of two independent parameters, v_i and u_i. v_i includes noisy, errors that occur in determining the production function and also omissions caused by the independent parameter x.

If a DMU provides expected production with full efficiency, the technical efficiency is "1", but if it produces expected outputs under the optimal capacity its efficiency measure is less than 1, that is this DMU is inefficient. Another point to be considered is how the efficiency is calculated. If the problem is output maximization (production maximization), then the composite error term calculation is valid and it is $\varepsilon_i = v_i - u_i$. If the efficiency problem is input minimization (cost function), then the equation $\varepsilon_i = v_i + u_i$ is valid [26].

In the paper FRONTIER Version 4.1 software was used for stochastic frontier analysis that transforms equation (4) to a logarithmic function as below equation (5) as in Broeck et al. [27].

$$\ln(y_i) = \beta_0 + \sum_{n=1}^{N} \beta_n lnx_{ni} + v_i - u_i, i = 1,2,\dots,N \quad (5)$$

Equation (5) represents the logarithm of inputs and outputs.

According to Coelli et.al. [28], Cobb-Douglas stochastic frontier function is as below:

$$\ln(y_i) = \beta_0 + \beta_1 lnx_i + v_i - u_i \ , \ i = 1,2,\dots,N \quad (6)$$
$$y_i = \exp(\beta_0 + \beta_1 lnx_i + v_i - u_i) \ , \ i = 1,2,\dots,N \quad (7)$$

Since the study wants to use a production function and maximize our production, the usage of Cobb-Douglas production frontier function including cross-sectional data and supposing a half normal distribution is provided. In the urban energy efficiency study, the usage of the Cobb-Douglas production frontier is as below:

$$\ln(y_i) = \ln(x_i, \beta) + v_i - u_i \ , \ i = 1,2,\dots,N \quad (8)$$

The SFA form of Cobb-Douglas production function is implemented for the years 2018-2019 and is modeled as follows.

$$\ln(y_i) = \beta_0 + \beta_1(lnx_1) + \beta_2(lnx_2) + \beta_3(lnx_3)$$
$$+ \beta_4(lnx_4) + v_i - u_i \quad (9)$$

In equation (8), y_i is the ith metropolitan's outputs' log, is the ith metropolitan's inputs' logs, u_i is the ith metropolitan's inefficiency. In equation (9) x_1, x_2, x_3 implies respectively; invoiced consumption (MWh), total installed power (MW), line length (Km) and population.

IV. CASE STUDY

Performance evaluation of urban energy efficiency includes the indicators of energy supply and demand balance, energy consumption, energy generation, energy investment components as installed power, line length and economy. With the optimal results of energy efficiency, GDP and investment are two important components. In energy efficiency studies, the importance of GDP had been realized by Hu and Wang [31].

Methodologies need to consider indicators to obtain the optimal outputs, Incomplete and uncertain data of energy industry is an important issue for efficiency measurement. Such that, according to deterministic models, the theory of probability is needed. As discussed widely in the literature, the major sources of uncertainty in the data used in the performance measurement of urban energy efficiency are costs, economy, data faults, energy demand and supply, regulatory issues such as unpredictability in the orders and laws. Deterministic solutions miss versatile decision-making and analysis processes. Because of these, in many industrial problems, managers deal with imprecise situations and random data. The noise

and randomness in data usually causes errors in frontier function and efficiency results. In these cases, data analysts can consider imprecise data as random indicators. By working with random inputs and outputs and realizing the possibility of uncertain situations, different perspectives of the available information can be detected in energy efficiency studies to make right energy policy. The main utility of random data in SFA models is the prediction of efficiencies in future optimization problems.

This study compares stochastic Cobb-Douglas production frontier including cross sectional data to maximize output, and input oriented deterministic CCR and BCC models. DEA and SFA are techniques that can be best measurement of urban energy efficiency of cities with multiple inputs and outputs.

4.1. Indicators, Data Set and Data Statistics
In this part of the study, there is implementation of dataset from 2018 to 2019 to evaluate urban energy efficiency of Turkey's 30 metropolitans within stochastic and deterministic framework. Inputs are invoiced consumption (MWh), total installed power (MW), line length (Km) and population, while outputs are total generation (MWh), number of consumers and GDP per capita.

Descriptive statistics of the input and output indicators are presented in Table 2.

Table 3 and Table 4 show the logarithm of data that were added to Cobb-Douglas SFA model.

In addition to data set and basic descriptive statistics, correlation coefficients between variables used in the models were also calculated with SPSS in Table 5. This shows us an idea about the direction and size of the relationship between inputs and outputs.

Correlation coefficients of the data were obtained by using the averages of data indicators. According to Table 5, inputs and outputs have directly proportional relationship. It is also seen that all variables have a positive correlation with each other.

4.2. Estimation of Results
In this section, efficiency results of models are discussed. The results of deterministic CCR model of expression (1) and BCC model of expression (2) were implemented on General Algebraic Modeling System (GAMS) mathematical programming and optimization system and SFA model of (9) were obtained from FRONTIER 4.1. Deterministic and stochastic results are included in Table 6 and Table 7 as follows.

Table 2. Descriptive statistics of the indicators

Inputs	Mean	Median	Std. Deviation	Minimum	Maximum
x1-Invoiced Consumption (MWh)	6.290.560,04	3.893.568,91	75.065.449,78	879.856	40.452.119
x2-Total Installed Power (MW)	1.965,17	2.002,95	1.328,02	100	4.527
x3-Line Length (Km)	24.853,73	20.782,00	14.186,71	10.593	71.709
x4-Population	2.116.510,47	1.362.698,00	2.665.698,78	767.848	15.067.724
Outputs	**Mean**	**Median**	**Std. Deviation**	**Minimum**	**Maximum**
y1-Total Generation (MWh)	6.388.647,71	5.690.800,06	5.434.535,62	16.639	18.584.686
y2-Number of Consumers	1.114.852,00	705.365,00	1.411.251,58	282.108	7.883.441
y3-GDP per capita	8.261,07	7.666,00	3.294,46	3.382	16.707

Table 3. Estimated input and output parameters of 2018

	x1	x2	x3	x4	y1	y2	y3
ADANA	15,71863	8,259919	10,23225	14,61307	16,72174	13,89078	8,838697
ANKARA	16,47614	7,947527	11,00471	15,52098	16,2819	14,89265	9,450144
ANTALYA	15,92878	7,550624	10,72322	14,7019	15,37748	14,34382	9,204825
AYDIN	14,79935	7,213547	9,926813	13,90877	15,54773	13,45426	8,826294
BALIKESİR	15,01754	7,949247	9,807362	14,01974	16,21013	13,76917	8,97221
BURSA	16,28544	7,876471	9,857025	14,91229	15,98165	14,30661	9,305741
DENİZLİ	15,0751	7,478481	9,965147	13,84291	15,57676	13,27341	9,116579
DİYARBAKIR	14,82812	7,734121	9,807307	14,36502	15,36927	13,19688	8,360539
ERZURUM	13,68751	6,202596	9,774233	13,55135	14,11538	12,81392	8,645762
ESKİŞEHİR	14,90925	6,560125	9,411565	13,67761	13,85659	13,29199	9,221379
GAZİANTEP	15,82764	6,528689	9,609787	14,52284	13,76599	13,5022	8,873608
HATAY	15,33529	7,918425	9,806536	14,29166	16,52292	13,53636	8,780941
İSTANBUL	17,51563	8,014197	11,18037	16,52807	15,94592	15,88028	9,696586
İZMİR	16,61626	8,416362	10,44599	15,27889	16,73785	14,72486	9,34688
KAHRAMANMARAŞ	15,211	8,417715	9,74373	13,95079	16,01713	13,08671	8,649974
KAYSERİ	15,11067	6,818793	10,10135	14,14458	14,40703	13,47854	8,983565
KOCAELİ	16,14421	7,651577	9,380421	14,46072	15,61601	13,73824	9,723583
KONYA	15,66414	6,725058	10,74231	14,60651	13,96152	13,9752	8,932609
MALATYA	14,31964	5,123369	10,07234	13,58866	12,56287	12,98026	8,583917
MANİSA	15,34944	7,758163	9,980078	14,17294	16,05141	13,62902	9,149634
MARDİN	14,37754	5,096018	9,338558	13,62821	9,719519	12,55005	8,485496
MERSİN	15,30502	6,958306	9,93953	14,4113	15,09836	13,82598	8,956351
MUĞLA	15,10634	7,74119	10,16439	13,78246	16,26748	13,34397	9,139703
ORDU	14,0491	6,172869	10,10545	13,55665	13,96561	13,17894	8,596374
SAKARYA	15,13731	7,838182	9,417111	13,82615	16,47421	13,18572	9,120525
SAMSUN	14,97997	8,276003	10,23218	14,10498	15,76155	13,61079	8,781555
ŞANLIURFA	15,3735	8,211953	10,29675	14,5264	15,56095	13,33635	8,129175
TEKİRDAĞ	15,71038	7,374942	9,267949	13,845	14,8575	13,31559	9,393162
TRABZON	14,22131	6,355013	9,94415	13,6022	14,21329	13,25662	8,929568
VAN	13,77971	4,602567	9,776903	13,93221	12,05714	12,81033	8,126223

Table 4. Estimated input and output parameters of 2019

	x1	x2	x3	x4	y1	y2	y3
ADANA	15,72576	8,26380	10,25171	14,62107	16,68923	13,91799	8,77550
ANKARA	16,46120	7,91317	11,02580	15,54523	16,07575	14,91065	9,34991
ANTALYA	15,96831	7,61494	10,75081	14,73647	15,42432	14,37155	9,12706
AYDIN	14,80828	7,18267	9,95802	13,92075	15,67355	13,48459	8,76695
BALIKESİR	15,03267	7,98557	9,81891	14,02140	16,06117	13,79342	8,92656
BURSA	16,28476	7,89266	9,88466	14,93266	15,81965	14,33795	9,24257
DENİZLİ	15,05028	7,52474	9,98806	13,85204	15,53683	13,31178	9,06347
DİYARBAKIR	14,76965	7,74527	9,84776	14,37875	16,01321	13,24440	8,28864
ERZURUM	13,69901	6,78294	9,77968	13,54378	14,05736	12,84174	8,60290
ESKİŞEHİR	14,91706	6,50031	9,48189	13,69614	13,96560	13,32437	9,15011
GAZİANTEP	15,88966	6,52258	9,63763	14,54275	14,02400	13,54389	8,83407
HATAY	15,29485	7,91739	9,83162	14,30341	16,42892	13,56610	8,74715
İSTANBUL	17,49228	7,94069	11,18238	16,55759	15,79041	15,90287	9,62791
İZMİR	16,55707	8,58435	10,47813	15,28964	16,42709	14,75890	9,29779
KAHRAMANMARAŞ	15,17722	8,42674	9,75063	13,95883	16,15144	13,12572	8,59841
KAYSERİ	15,10096	6,81161	10,13539	14,15726	14,61633	13,51981	8,88784
KOCAELİ	16,11513	7,62393	9,40179	14,48490	15,20160	13,77604	9,65778
KONYA	15,63730	6,80549	10,78230	14,61858	14,33773	14,00989	8,84645
MALATYA	14,30469	5,21602	10,08134	13,59257	13,02776	12,99926	8,51945
MANİSA	15,28788	8,01597	10,01118	14,18058	16,32692	13,65599	9,13580
MARDİN	14,36473	5,20872	9,36152	13,63970	11,34265	12,60175	8,42247
MERSİN	15,34276	7,01728	9,97562	14,42551	15,33807	13,85243	8,88854
MUĞLA	15,05618	7,75022	10,19062	13,79851	16,33687	13,37487	9,06843
ORDU	14,02365	6,19644	10,11286	13,53341	14,11983	13,21284	8,58093
SAKARYA	15,10651	7,84569	9,43052	13,84361	14,55491	13,22862	9,04674
SAMSUN	14,92495	8,14014	10,24409	14,11453	15,64022	13,65184	8,72328
ŞANLIURFA	15,36226	8,23454	10,32302	14,54480	16,28768	13,38375	8,07689
TEKİRDAĞ	15,74325	7,36822	9,44668	13,86944	14,46854	13,35311	9,33061
TRABZON	14,21124	6,39145	9,94228	13,60352	14,11226	13,27810	8,88672
VAN	13,81124	4,86738	9,80797	13,94369	12,44851	12,85042	8,07372

Table 5. Correlation coefficients between variables

		x1	x2	x3	x4	y1	y2	y3
x1	Pearson Correlation	1	,379*	,731**	,971**	,321	,971**	,708**
	Sig. (2-tailed)		,039	,000	,000	,083	,000	,000
	N	30	30	30	30	30	30	30
x2	Pearson Correlation	,379*	1	,274	,316	,819**	,316	,238
	Sig. (2-tailed)	,039		,142	,089	,000	,089	,204
	N	30	30	30	30	30	30	30
x3	Pearson Correlation	,731**	,274	1	,792**	,209	,811**	,383*
	Sig. (2-tailed)	,000	,142		,000	,267	,000	,037
	N	30	30	30	30	30	30	30
x4	Pearson Correlation	,971**	,316	,792**	1	,251	,991**	,576**
	Sig. (2-tailed)	,000	,089	,000		,181	,000	,001
	N	30	30	30	30	30	30	30
y1	Pearson Correlation	,321	,819**	,209	,251	1	,272	,300
	Sig. (2-tailed)	,083	,000	,267	,181		,146	,107
	N	30	30	30	30	30	30	30
y2	Pearson Correlation	,971**	,316	,811**	,991**	,272	1	,623**
	Sig. (2-tailed)	,000	,089	,000	,000	,146		,000
	N	30	30	30	30	30	30	30
y3	Pearson Correlation	,708**	,238	,383*	,576**	,300	,623**	1
	Sig. (2-tailed)	,000	,204	,037	,001	,107	,000	
	N	30	30	30	30	30	30	30

Table 6. Estimated results of 2018

	CCR	BCC	SFA
ADANA	0.8994	1.0000	0.7225
ANKARA	0.9802	1.0000	0.7806
ANTALYA	1.0000	1.0000	0.8963
AYDIN	0.9824	0.9859	0.7243
BALIKESİR	1.0000	1.0000	0.6466
BURSA	1.0000	1.0000	0.7985
DENİZLİ	0.8548	0.8752	0.8538
DİYARBAKIR	0.6529	0.8149	0.3987
ERZURUM	1.0000	1.0000	0.4585
ESKİŞEHİR	1.0000	1.0000	0.8429
GAZİANTEP	0.8059	0.9255	0.8599
HATAY	1.0000	1.0000	0.6701
İSTANBUL	1.0000	1.0000	0.7444
İZMİR	0.9929	1.0000	0.8140
KAHRAMANMARAŞ	0.6627	0.8072	0.6908
KAYSERİ	0.7822	0.7848	0.8049
KOCAELİ	1.0000	1.0000	0.8803
KONYA	0.8941	0.9042	0.8992
MALATYA	1.0000	1.0000	0.8870
MANİSA	0.8958	0.9117	0.7936
MARDİN	1.0000	1.0000	0.6797
MERSİN	0.9772	0.9941	0.7210
MUĞLA	1.0000	1.0000	0.8948
ORDU	1.0000	1.0000	0.6904
SAKARYA	1.0000	1.0000	0.7734
SAMSUN	0.8111	0.8267	0.5942
ŞANLIURFA	0.4435	0.5172	0.5698
TEKİRDAĞ	1.0000	1.0000	0.9281
TRABZON	1.0000	1.0000	0.6991
VAN	1.0000	1.0000	0.4699

Table 7. Estimated results of 2019

	CCR	BCC	SFA
ADANA	0.9508	1.0000	0.7135
ANKARA	0.9204	1.0000	0.7387
ANTALYA	0.9994	1.0000	0.8849
AYDIN	1.0000	1.0000	0.7480
BALIKESİR	1.0000	1.0000	0.5371
BURSA	1.0000	1.0000	0.7641
DENİZLİ	0.8602	0.8785	0.7684
DİYARBAKIR	1.0000	1.0000	0.3407
ERZURUM	1.0000	1.0000	0.2405
ESKİŞEHİR	1.0000	1.0000	0.7561
GAZİANTEP	0.8436	0.9461	0.9203
HATAY	1.0000	1.0000	0.6447

İSTANBUL	1.0000	1.0000	0.7011
İZMİR	0.9694	1.0000	0.6517
KAHRAMANMARAŞ	0.9614	1.0000	0.4940
KAYSERİ	0.8142	0.8142	0.7132
KOCAELİ	1.0000	1.0000	0.8444
KONYA	0.8784	0.9023	0.8290
MALATYA	1.0000	1.0000	0.9035
MANİSA	1.0000	1.0000	0.6817
MARDİN	1.0000	1.0000	0.3925
MERSİN	0.9994	0.9994	0.7631
MUĞLA	1.0000	1.0000	0.9997
ORDU	1.0000	1.0000	0.5616
SAKARYA	0.9212	1.0000	0.3825
SAMSUN	0.8551	0.9016	0.3762
ŞANLIURFA	0.7385	0.7396	0.4742
TEKİRDAĞ	1.0000	1.0000	0.9520
TRABZON	1.0000	1.0000	0.5284
VAN	1.0000	1.0000	0.3729

Table 8. Analysis results of Cobb-Douglas production function model

Parameter	Coefficient	Standard Error	T Ratio
β_0	2.283	1.930	1.182
β_1	0.320	0.205	1.563
β_2	-0.399	0.138	-2.884
β_3	1.126	0,148	7.610
β_4	-0.072	0.131	-0.548
σ^2	0.202	0.103	1.954
γ	0.850	0.243	3.497
Log-likelihood	-6.064		
LR test of the one-sided error	3.768		

The average results of Table 6 and Table 7 are presented and discussed under Table 9. In Table 8 the Cobb-Douglas production function analysis results can be observed. According to Table 8 the parameter γ=0.850 is represents statistical significance at the 1% level. That means most sources of inefficiency, in combined error term (ε) caused with 85% of technical inefficiency and 15% of random errors. In this context, although the technical inefficiency has a high rate within the combined error term, the existence of random errors cannot be ignored. The likelihood ratio test shows that inefficiency scores are statistically significant according to urban energy efficiency among metropolitans were identified. In Table 8 parameter coefficients of independent variables are also significant. The one-sided error LR test to evaluate the technical efficiency of DMUs can be taken into consideration. LR ratio was found to be approximately 3.768 and this value should be

compared with the table value of 2.706 in the Kodde-Palm with a restriction of 1 at 0.05 significance level [32].

$$H_0 : \gamma = 0$$
$$H_1 : \gamma \neq 0$$

Since 3.768 score is bigger than the table value of 2.706 in the Kodde-Palm, H_0 hypothesis is rejected. This situation implies that, there is a statistically significant technical inefficiency in the model. The analyzed coefficients are also consistent with results from literature studies; for example, economically and socially underdeveloped cities inefficient according to developed ones.

Table 9 and Figure 1 show us average estimated 2018 and 2019 results and ranks of the CCR, BCC and SFA models.

Table 9. Average efficiency estimated results of models

|	CCR	BCC	SFA	Rank
ADANA	0.9251	1.0000	0.7180	22
ANKARA	0.9503	1.0000	0.7596	18
ANTALYA	0.9997	1.0000	0.8906	5
AYDIN	0.9912	0.9929	0.7361	10
BALIKESİR	1.0000	1.0000	0.5918	24
BURSA	1.0000	1.0000	0.7813	7
DENİZLİ	0.8575	0.8768	0.8111	19
DİYARBAKIR	0.8265	0.9074	0.3697	29
ERZURUM	1.0000	1.0000	0.3495	26
ESKİŞEHİR	1.0000	1.0000	0.7995	6
GAZİANTEP	0.8248	0.9358	0.8901	14
HATAY	1.0000	1.0000	0.6574	12
İSTANBUL	1.0000	1.0000	0.7227	9
İZMİR	0.9812	1.0000	0.7328	11
KAHRAMANMARAŞ	0.8121	0.9036	0.5924	27
KAYSERİ	0.7982	0.7995	0.7590	25
KOCAELİ	1.0000	1.0000	0.8623	4
KONYA	0.8863	0.9032	0.8641	13
MALATYA	1.0000	1.0000	0.8952	3
MANİSA	0.9479	0.9558	0.7376	15
MARDİN	1.0000	1.0000	0.5361	21
MERSİN	0.9883	0.9967	0.7420	8
MUĞLA	1.0000	1.0000	0.9472	1
ORDU	1.0000	1.0000	0.6260	16
SAKARYA	0.9606	1.0000	0.5779	20
SAMSUN	0.8331	0.8641	0.4852	28
ŞANLIURFA	0.5910	0.6284	0.5220	30
TEKİRDAĞ	1.0000	1.0000	0.9400	2
TRABZON	1.0000	1.0000	0.6137	17
VAN	1.0000	1.0000	0.4214	23
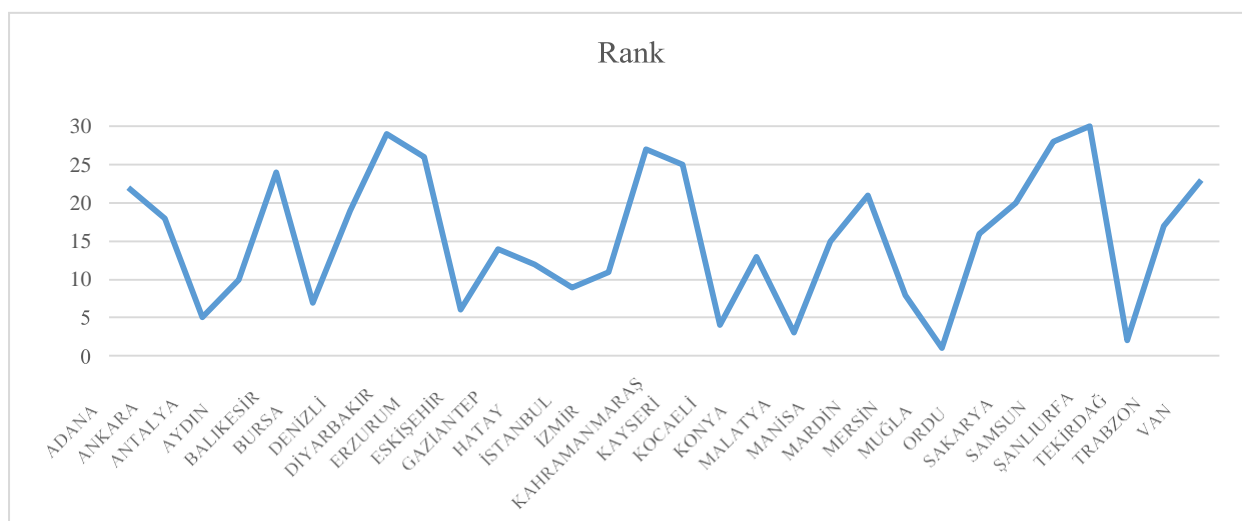
Figure 2. Average efficiency estimated ranks of models for each metropolitan.

As seen in Table 9 in which average results and ranks of DMUs are included, Balıkesir, Bursa, Erzurum, Eskişehir, Hatay, İstanbul, Kocaeli Malatya, Mardin, Muğla, Ordu, Tekirdağ, Trabzon, Van are permanently efficient metropolitans in both the deterministic input-oriented CCR and BCC models. That is, they are technical efficient under constant returns to scale conditions when data errors are ignored. On the other hand, in deterministic framework Adana, Ankara, Antalya, İzmir, Sakarya are BCC efficient but, CCR inefficient DMUs. CCR model evaluates global and technical efficiencies of units according to the efficiency frontier. BCC model evaluates local and pure technical efficiencies. In summary, while the CCR model calculates both technical and scale efficiency scores, the BCC model estimates pure technical efficiency of units against the efficiency production function. Therefore, technical efficiency results of the BCC results are either higher than or equal to the CCR results. As can be seen from the deterministic results, it is easier for a decisionmaking unit to be BCC efficient than CCR efficient. Therefore, CCR efficient DMUs are also efficient in BCC model. The opposite is not always true for input oriented models. According to the results of the study, the efficiency values in the BCC model were equal to or greater than the CCR model. Since there is no significant difference between deterministic CCR and BCC technical efficiency results, Turkey's metropolitans have urban energy efficiency in scale except Aydın, Denizli, Diyarbakır, Gaziantep, Kahramanmaraş, Kayseri, Konya, Manisa, Mersin, Samsun, Şanlıurfa. These cities are inefficient in both models.

According to results of Cobb-Douglas output maximization stochastic frontier Tekirdağ and Muğla have the best performance. Diyarbakır, Erzurum, Samsun, Van have the lowest scores, and the other energy efficiency performances are higher than fifty percentages in stochastic framework. In addition, Erzurum and Van are locally and globally efficient in deterministic CCR and BCC models but their performance is under fifty percentages in stochastic framework. This confirms the situation of our γ parameter in which most sources of stochastic inefficiency, caused with technical operations and some sources caused with random errors. Consequently, study can inference that reliable urban energy efficiency evaluation highly relates with stochastic models.

Considering the inputs and outputs of models, the number of customers, population, total generation, invoiced consumption that impact unregistered energy usage because of loss-leakage ratio and energy supply-demand imbalance, GDP that impacts the correct investment decisions, line length that impacts the low amount of energy distribution issues seem to be the cause of inefficiency, but it can be said that the

inefficiency will be eliminated by increasing the values of these parameters.

V. CONCLUSION

Evaluation of urban energy efficiency performance has different forms, aims and implementations. It can be seen from literature, there is no definite methodology to evaluate energy efficiency on urban researches. To the best of our knowledge, this is the first study estimating the urban energy efficiency of Turkey's metropolitans with both parametric and nonparametric, deterministic and stochastic models.

The CCR model compares companies that have operations in homogenous or non-homogeneous way and creates an overall/global efficiency scores, while the BCC model compares companies with operations in homogeneous way and creates local efficiency scores. By the study, it has been shown that the uncertainty of the data causes stochastic inefficiency. These results suggest that more attention should be paid to error levels to evaluate stochastic efficiency.

According to the results, permanently inefficient cities need agricultural irrigation in large areas due to the regions where they are located. This situation causes excessive waste due to unregistered electricity use in agricultural irrigation. The wasteful consumption leads excessive load on power lines and transformers. There is also loss-leakage in energy usage in these cities. Loss-leakage ratio, causes unregistered electricity use and this means high load on power lines also. The other inefficiency reasons are; insufficient line lengths in large areas, investment that is not well planned and not on time, supply-demand imbalance due to the excess number of subscribers and loss-leakage ratio.

The issues that causes inefficiency can be overcame with; increasing investments in networks through inspection, directing the audits in a way that minimizes the inputs and maximizes the outputs of the cities, making legal and deterrent regulations in combating unregistered electricity use, increasing service quality by renewing network infrastructures and commissioning new technology projects and facilities, taking into account the parameters as area, line length, population and number of subscribers while making the investment decision.

As a result; examining the effect of uncertainty with a high tendency in societies such as our country, avoiding uncertainty is important in terms of predicting reactive or proactive approaches of organizations in uncertain environments. This study shows us that, stochastic models, which take into account uncertainties and random variables, can be used to measure the urban energy efficiency for reliable decision making in energy policies. Study can offer for policy implications; using remote, smart electricity

meters, e-invoice applications, taking into account the loss ratios differences of electricity consumption, gaining available and beneficial data because of registering the accurate consumption. In future studies, the application of random variables to other stochastic models for the urban energy efficiency of decision making units and the effect of changes in input and output parameters on modeling can be examined.

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