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Interval-valued intuitionistic quadripartitioned neutrosophic soft sets with T, F, C, and U as dependent neutrosophic components and their application in decision-making problem

Somen Debnath[1]

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Soft set, Neutrosophic soft sets, Interval-valued intuitionistic neutrosophic soft sets, Quadripartitioned neutrosophic soft sets, Interval-valued intuitionistic quadripartitioned neutrosophic soft sets

Abstract — Molodtsov introduced a soft set (SS) to model uncertainty parametrically, and Chaterjee et al. proposed the notion of quadripartitioned neutrosophic set (QNS) by dividing indeterminacy into two independent components, namely contradiction (C) and unknown (U). Afterwards, by combining the SS and QNS, a new concept known as quadripartitioned neutrosophic soft set (QNSS) is introduced. In relation to the concept of QNSS, another concept called interval-valued intuitionistic quadripartitioned neutrosophic soft set (in short IVIQNSS) is established to handle more complex indeterminate information parametrically with the restricted conditions. This paper aims to further generalize the existing soft models by introducing an IVIQNSS to explore another kind of imprecise knowledge. The IVIQNSS model can be viewed as a more flexible and powerful framework to encounter indeterminacy parametrically with T, F, C, and U as dependent interval quadripartitioned neutrosophic components where $T, F, C, U \subseteq [0,1]$ such that $\sup T + \sup F \leq 1$, and $\sup C + \sup U \leq 1$. So, by using the IVIQNSS framework we are capable to address the indeterminate, inconsistent, and incomplete information more accurately. Different operations such as complement, AND, OR, union, intersection, etc. are defined on IVIQNSSs. Furthermore, an algorithm is constructed to solve decision-making (DM) problems based on IVIQNSS. Finally, an illustrative example is executed to validate the proposed study.

Subject Classification (2020): 03E72, 03F55.

1. Introduction

Researchers and mathematicians in the world recognized Zadeh's [1] fuzzy set theory as the most appropriate theory to handle uncertainty or vagueness. Over the decades, the fuzzy set theory has progressed rapidly and used successfully by scientists in many practical applications. It plays an important role in the development of computer science, graph theory, image processing, game theory, etc. The fuzzy set is characterized by a membership function $\mu_A(x)$ and it belongs to the closed interval [0,1]. Molodtsov pointed out that the main difficulties in the fuzzy sets and their variations are arises due to the inadequacy of the parameterization tool. In 1999, Molodtsov [2] introduced the soft set (SS)

[1]somen.debnath123@gmail.com (Corresponding Author)
[1]Department of Mathematics, Umakanta Academy, Agartala, India
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theory to eradicate such issues. The absence of any restriction in SS theory makes it flexible, convenient, and easily applicable in practice.

From the above discussion, it is evident that the problem of setting a membership-function and thus has rich potential for application in various directions. It is a general framework to model uncertainty or not clearly defined objects. Using the notion of SS introduced by Molodtsov, Maji et al. [3] defined soft binary operations. Ali et al. [4] gave some new notions of SS. Babitha et al. [5] introduced SS relations. Çağman et al. [6] redefined the operations of SSs. In [7], Maji et al. present an application of SSs in a decision-making problem. Jun et al. [8] gave the relations between soft BCK/BCI-algebras and idealistic soft BCK/BCI algebras. Chen et al. [9] focus on the parameterization reduction of SSs and their applications. Sezgin et al. [10] studied the theoretical aspect of SSs. Aktaş et al. [11] defined SSs and soft groups. Çağman et al. [12] introduced the matrix representation of SS and applied it to the DM problem. Acar et al. [13] presented soft rings. Tahat et al. [14] assessed several soft topological notions etc.

For the unification of the soft set with the fuzzy set and its variations, we discuss the following:

Roy et al. [15] presented an application of fuzzy-soft-sets (FSSs) in DM problems. Majumder et al. [16] generalized FSSs to utilize in medical-diagnosis problems. Xiao et al. [17] proposed an MCDM problem using trapezoidal FSSs. In [18], Yang et al. analyzed a decision problem based on a multi FSS. Çağman et al. [19] showed the practical implication of intuitionistic fuzzy soft sets (IFSSs). In [20], Broumi et al. introduced the notion of the intuitionistic neutrosophic soft set (INSS). Bashir et al. [21] introduced the possibility IFSS. Jiang et al. [22] initiated entropy measures on IFSSs and interval-valued fuzzy soft sets (IVFSSs). Yang et al. [23] introduced an IVFSS. Some applications of IVFSSs were presented in [24-26]. In 2010, Jiang et al. [27] introduced the notion of the interval-valued intuitionistic fuzzy soft sets (IVIFSSs) and studied their properties. Khalid et al. [28] used the distance measure on IVIFSSs. Zhang et al. [29] proposed a novel approach to IVIFSS. Garg et al. [30] introduced a nonlinear programming methodology for the MADM problem using IVIFSS. Some recent works that are based on IVIFSS are given in [31-34].

In 1986, Atanassov [35] introduced IFS as an extension of FS. In IFS, any object in the universe has a membership degree $\mu_A(x)$ and a non-membership degree $\gamma_A(x)$ with $\mu_A(x), \gamma_A(x) \in [0,1]$ such that $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$. But some systems are concerned with indeterminate and inconsistent information that cannot be described by IFS. To handle such problems, Smarandache [36] developed the notion of the neutrosophic set(NS). Every object in the universe under the NS is specified by the truth-membership $\left(T_A(x)\right)$, indeterminacy-membership $\left(I_A(x)\right)$, and the falsity-membership $\left(F_A(x)\right)$ degree and they belong to the real standard or non-standard interval $]-0,\ 1^+[$ such that $-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. But for scientific and engineering applications, we need restricted intervals. To remove such problems, the notion of the single-valued neutrosophic set (SVNS) is initiated by Wang et al. [37]. To make NS more functional and operational, we discuss the following: Maji et al. [38] introduced neutrosophic soft sets in decision-making. Broumi et al. [39] generalized interval neutrosophic soft sets in decision-making problems. Deli [40] defined the interval-valued neutrosophic soft sets. Veerappan et al. [41] presented an application for diagnosing psychiatric disorders by using similarity measures of interval-valued intuitionistic neutrosophic soft sets. Fahmi et al. [42] put forward the geometric operators associated with the linguistic interval-valued intuitionistic neutrosophic fuzzy number. Interval neutrosophic soft sets-based similarity measures are defined in [43]. Deli et al. [44] propounded another similarity measure based on ivnpiv-neutrosophic soft sets. NSM-decision-making approach is found in the literature [45]. Motivating by the Belnap's four-valued logic and Smarandache's neutrosophic logic, where the indeterminacy is split into two parts, namely, 'unknown' viz. neither true nor false and 'contradiction' viz. both true and false, Chatterjee et al. [46] introduced the notion of quadripartitioned neutrosophic set (QNS) and an application of a pattern recognition problem has been shown. The concept of QNS seems to be more functional and operational than that of NS. For example, an expert is being asked to give his/her opinion about a given statement. According to his/her opinion, the given statement is true with degree 0.4, both true and false with degree 0.6, neither true nor false with degree 0.3, and false with degree 0.2. Thus, we give many such instances where the use of QNS is appropriate. It gives a more general framework to model the uncertainty that contains incomplete, indeterminate, and inconsistent information with high precision and accuracy. Some works related to QNS are given in the following: Chaterjee et al. [47] introduced an MCDM algorithm with quadripartitioned neutrosophic weighted aggregation operators using quadripartitioned neutrosophic numbers. Roy et al. [48] defined the similarity measures of quadripartitioned single-valued bipolar neutrosophic sets. Mohansundari et al. [49] initiated the quadripartitioned single-valued neutrosophic Dombi weighted aggregation operators for MADM. Sinha et al. [50] introduced the bipolar quadripartitioned single-valued neutrosophic sets. Mary et al. [51] proposed the quadripartitioned neutrosophic soft set. In [52], Chatterjee et al. propose some uncertainty-based measures that are associated with the interval-valued possibility quadripartitioned single-valued neutrosophic soft sets.

The objectives of the present work are given:

- To introduce the notion of intuitionistic quadripartitioned neutrosophic set (IQNS).
- To introduce the notion of interval-valued intuitionistic quadripartitioned neutrosophic sets (IVIQNSs) and define some set-theoretic operations on them.
- To develop the notion of interval-valued intuitionistic quadripartitioned neutrosophic soft set (IVIQNSS) theory by using IVIQNSs.
- To introduce the operators such as complement, union, intersection, AND, OR on IVIQNSSs.
- To develop some propositions and theorems based on IVIQNSSs.
- To extend the notion proposed in [51].
- To develop a more generalized soft set for accommodating the complex uncertain data present in the belief, expert, and the information system.
- To construct an algorithm-based model for the application of decision-making problems by using IVIQNSSs.

1.1 Motivation

In earlier research works, the concept of fuzzy soft sets (FSSs), intuitionistic fuzzy soft sets (IFSs), interval-valued fuzzy soft sets (IVFSSs), interval-valued intuitionistic fuzzy soft sets (IVIFSSs), neutrosophic soft sets (NSSs), intuitionistic neutrosophic soft sets (INSSs), quadripartitioned neutrosophic soft sets (QNSSs), etc. are used successfully to solve decision-making problems that contain parametric uncertain, incomplete, inconsistent, hesitant or indeterminate data. There is no such work that has been done so far where the indeterminacy can be handled parametrically under the neutrosophic environment by keeping T, F, C, and U as dependent quadripartitioned neutrosophic components. So, the present work is devoted to developing a new methodology to handle indeterminacy parametrically by introducing the interval-valued intuitionistic quadripartitioned neutrosophic soft set (IVIQNSS) theory. This study surely provides a more flexible framework for the decision-makers to explore new decision-making approaches to address the issues under the quadripartitioned neutrosophic soft environment with the inherent restrictions.

To make the proposed model more visible in the real-life scenario, we give a comparative analysis in the following Table 1:

Table 1. Comparative of IVIQNSS model with the existing soft models

Types of soft set	Uncertainty	Falsity	Hesitation	Indeterminacy	Indeterminacy is bifurcated	Indeterminacy is bifurcated and restricted
FSS [15]	✓	×	×	×	×	×
IVFSS [23]	✓	×	×	×	×	×
IFSS [19]	✓	✓	✓	×	×	×
IVIFSS [27]	✓	✓	✓	×	×	×
NSS [38]	✓	✓	×	✓	×	×
INSS [20]	✓	✓	✓	✓	×	×
QNSS [51]	✓	✓	×	✓	✓	×
IVIQNSS (Proposed)	✓	✓	✓	✓	✓	✓

The rest of the paper is arranged in the following way:

Section 2 includes some basic definitions. Section 3 introduces the definition of IVIQNSS and its related sets, operators, propositions, and theorems. Section 4 introduces an algorithm-based model to solve real uncertain decision-making problems by using the IVIQNSS. Section 5 concludes the paper.

2. Preliminaries

In this section, we recall some basic definitions that are essential for the fulfilment of the proposed study. Throughout the section, we denote the set of the universe by Λ, the set of parameters by E, and $A \subseteq E$.

Definition 2.1. [2] Let Λ be a universal set, and the power set of Λ is denoted by $pow(\Lambda)$. Let E be a set of parameters and $A \subseteq E$. Then, a pair (Q, A) is called a SS over Λ is defined as

$$(Q, A) = \{(e, Q(e)) : e \in A \text{ and } Q(e) \in pow(\Lambda)\ \}, \text{ where } Q: A \to pow(\Lambda)$$

Definition 2.2. [53] An IVIFS over Λ is an object of the form $B = \{\langle x, (\mu_B(x), \gamma_B(x))\rangle : x \in \Lambda\}$. Here μ_B and γ_B are respectively called the membership and the non-membership functions in such a manner that μ_B, $\gamma_B: \Lambda \to int([0,1])$ where $int([0,1])$ denotes the set of all closed subintervals of $[0,1]$ satisfying the following condition: $\forall x \in \Lambda$, $\sup(\mu_B(x)) + \sup(\gamma_B(x)) \leq 1$.

Definition 2.3. [27] An IVIFSS over Λ is denoted by a pair (R, A), where R is a mapping given by $R: A \to IVIFS(\Lambda)$, where $IVIFS(\Lambda)$ indicates the set of all IVIFSs of Λ.

Definition 2.4. [36] A NS N over Λ is described by a truth-membership function T_N, an indeterminate membership function I_N, and a falsity-membership function F_N. For a generic element x in Λ, $T_N(x)$, $I_N(x)$, and $F_N(x)$ are real standard or non-standard subintervals of $[0,1]$ and a NS can be written as $N = \{\langle x, (T_N(x), I_N(x), F_N(x))\rangle : x \in \Lambda\}$ such that $T_N, I_N, F_N: \Lambda \to]^{-}0,\ 1^{+}[$ under the condition $^{-}0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3^{+}$. There is no restriction on the supremum of $T_N(x)$,$I_N(x)$, and $F_N(x)$, so for the sake of simplicity, we use the notion proposed in [37], where the above restriction reduces to $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$.

Definition 2.5. [38] Let Λ be an initial universe and $A \subseteq E$. Let N^{Λ} signifies the class of all NSs of Λ. Then, the collection (S, A) is termed to be a NSS over Λ, where $S: A \to N^{\Lambda}$.

Definition 2.6. [54] An INS Hover Λ is represented in the form $H = \langle x, T_H(x), I_H(x), F_H(x)\rangle$, where $\min\{T_H(x), F_H(x)\} \leq 0.5$, $\min\{T_H(x), I_H(x)\} \leq 0.5$, and $\min\{F_H(x), I_H(x)\} \leq 0.5$, for all $x \in \Lambda$ such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 2$.

Definition 2.7. [55] An IVNS M over the space of objects Λ is characterized by a truth-membership function T_M, an indeterminacy-membership function I_M, and a falsity-membership function F_M such that for every $x \in \Lambda$, $T_M(x), I_M(x), F_M(x) \subseteq [0,1]$.

Thus, we represent the IVNS in the form $M = \langle x, T_M(x), I_M(x), F_M(x)\rangle$.

Definition 2.8. [41] An IVINS K in Λ is a set of the form $K = \{\langle x, (T_K(x), I_K(x), F_K(x))\rangle : x \in \Lambda\}$, where the lower and upper bounds of $T_K(x)$, $I_K(x)$, $F_K(x)$ are respectively denoted by $\underline{T}_K(x), \overline{T}_K(x)$; $\underline{I}_K(x), \overline{I}_K(x)$; and $\underline{F}_K(x), \overline{F}_K(x)$, where T_K,I_K,F_K: $\Lambda \to int([0,1])$such that $\overline{T}_K(x) + \overline{F}_K(x) \leq 1$, and $0 \leq \overline{T}_K(x) + \overline{I}_K(x) + \overline{F}_K(x) \leq 2$, $\forall x \in \Lambda$.

Definition 2.9. [41] A pair (L, A) is called an IVINSS over Λ, where $L: A \to IN^{\Lambda}$ and IN^{Λ} denotes the set of all IVINSs of Λ. For any parameter $\varepsilon \in A$, $L(\varepsilon)$ is an IVINSS.

Definition 2.10. [46] A QNS in Λ is a set of the form $D = \{\langle x, (T_D(x), C_D(x), U_D(x), F_D(x))\rangle : x \in \Lambda\}$, where T_D,C_D,U_D, F_D: $\Lambda \to [0,1]$ and $0 \leq T_D(x) + C_D(x) + U_D(x) + F_D(x) \leq 4$, where $T_D(x)$ is the truth-membership degree, $C_D(x)$ is the contradiction-membership degree, $U_D(x)$ is the unknown-membership degree, and $F_D(x)$ is the false-membership degree.

Definition 2.11. [51] Let Λ be an initial universe and $A \subseteq E$. Let QNS^{Λ} denotes the set of all QNSs of Λ. Then, the collection (W, A) is termed to be the QNSS over Λ, where $W: A \to QNS^{\Lambda}$.

Definition 2.12. An intuitionistic quadripartitioned neutrosophic set (IQNS) Z on a universe Λ is an object of the form $Z = \{\langle x, (T_Z(x), C_Z(x), U_Z(x), F_Z(x))\rangle : x \in \Lambda\}$, where T_Z,C_Z,U_Z F_Z: $\Lambda \to [0,1]$ such that $0 \leq T_Z(x) + F_Z(x) \leq 1$ and $0 \leq C_Z(x) + U_Z(x) \leq 1$.

Definition 2.13. An interval-valued intuitionistic quadripartitioned neutrosophic set (IVIQNS) on a universe Λ is an object of the form $Y = \{\langle x, (T_Y(x), C_Y(x), U_Y(x), F_Y(x))\rangle : x \in \Lambda\}$, where T_Y, C_Y, U_Y, F_Y: $\Lambda \to int([0,1])$ where $int([0,1])$ denotes the collection of all closed subintervals of [0,1] satisfying the conditions $\sup(T_Y(x)) + \sup(F_Y(x)) \leq 1$ and $\sup(C_Y(x)) + \sup(U_Y(x)) \leq 1$ such that $\sup(T_Y(x)) + \sup(F_Y(x)) + \sup(C_Y(x)) + \sup(U_Y(x)) \leq 2$. The set of all IVIQNSs over Λ is denoted by $IVIQNS(\Lambda)$.

Let, $S^*, R^* \in IVIQNS(\Lambda)$. Then,

- Their union is denoted by $S^* \overset{\approx}{\cup} R^*$ and defined by

$$S^* \overset{\approx}{\cup} R^* = \left\{ \left\langle x, \begin{pmatrix} \left[\sup\left(\underline{T}_{S^*}(x), \underline{T}_{R^*}(x)\right), \sup\left(\overline{T}_{S^*}(x), \overline{T}_{R^*}(x)\right)\right], \\ \left[\sup\left(\underline{C}_{S^*}(x), \underline{C}_{R^*}(x)\right), \sup\left(\overline{C}_{S^*}(x), \overline{C}_{R^*}(x)\right)\right], \\ \left[\inf\left(\underline{U}_{S^*}(x), \underline{U}_{R^*}(x)\right), \inf\left(\overline{U}_{S^*}(x), \overline{U}_{R^*}(x)\right)\right], \\ \left[\inf\left(\underline{F}_{S^*}(x), \underline{F}_{R^*}(x)\right), \inf\left(\overline{F}_{S^*}(x), \overline{F}_{R^*}(x)\right)\right] \end{pmatrix} \right\rangle : \forall x \in \Lambda \right\}$$

- Their intersection is denoted by $S^* \mathbin{\overset{\approx}{\cap}} R^*$ and defined by

$$S^* \mathbin{\overset{\approx}{\cap}} R^* = \left\{ \left\langle x, \begin{pmatrix} \left[\inf\left(\underline{T}_{S^*}(x), \underline{T}_{R^*}(x)\right), \inf\left(\overline{T}_{S^*}(x), \overline{T}_{R^*}(x)\right)\right], \\ \left[\inf\left(\underline{C}_{S^*}(x), \underline{C}_{R^*}(x)\right), \inf\left(\overline{C}_{S^*}(x), \overline{C}_{R^*}(x)\right)\right], \\ \left[\sup\left(\underline{U}_{S^*}(x), \underline{U}_{R^*}(x)\right), \sup\left(\overline{U}_{S^*}(x), \overline{U}_{R^*}(x)\right)\right], \\ \left[\sup\left(\underline{F}_{S^*}(x), \underline{F}_{R^*}(x)\right), \sup\left(\overline{F}_{S^*}(x), \overline{F}_{R^*}(x)\right)\right] \end{pmatrix} \right\rangle : \forall x \in \Lambda \right\}$$

- The complement of S^* is denoted and defined by

$$(S^*)^c = \{\langle x, (F_{S^*}(x), U_{S^*}(x), C_{S^*}(x), T_{S^*}(x))\rangle : \forall x \in \Lambda\}$$

3. Interval-Valued Intuitionistic Quadripartitioned Neutrosophic Soft Set

In this section, we present the notion of interval-valued intuitionistic quadripartitioned neutrosophic soft set (IVIQNSS) which can be viewed as an extension of a neutrosophic soft set, interval-valued intuitionistic soft set, interval-valued intuitionistic neutrosophic soft set, or quadripartitioned neutrosophic soft set.

Definition 3.1. Let X be an initial universe and E be a set of parameters. Let $IVIQNS(X)$ denotes the set of all IVIQNSs of X, and $A \subseteq E$. Then, a pair (ς, A) is called the IVIQNSS over X, where $\varsigma: A \to IVIQNS(X)$.

In other words, the IVIQNSS is a parameterized family of IVIQN subsets of X. For any parameter $\varepsilon \in A$, $\varsigma(\varepsilon)$ is referred to as the IVIQN value set of ε, and it is written as $\varsigma(\varepsilon) = \{\langle x, T_{\varsigma(\varepsilon)}(x), C_{\varsigma(\varepsilon)}(x), U_{\varsigma(\varepsilon)}(x), F_{\varsigma(\varepsilon)}(x)\rangle : x \in X, \varepsilon \in A\}$, where $T_{\varsigma(\varepsilon)}(x)$, $C_{\varsigma(\varepsilon)}(x)$, $U_{\varsigma(\varepsilon)}(x)$, and $F_{\varsigma(\varepsilon)}(x) \subseteq [0,1]$ such that $\sup(T_{\varsigma(\varepsilon)}(x)) + \sup(F_{\varsigma(\varepsilon)}(x)) \leq 1$, and $\sup(C_{\varsigma(\varepsilon)}(x)) + \sup(U_{\varsigma(\varepsilon)}(x)) \leq 1$, are respectively indicates the truth-membership, contradictory-membership, unknown or ignorance-membership, and falsity-membership degree of an object x that holds on parameter ε, contradict on parameter ε, unknown on parameter ε, and does not hold on parameter ε respectively. The set of all IVIQN value sets is called the IVIQN value class of (ς, A) and it is denoted and defined as $\mathbb{C}_{(\varsigma,A)} = \{\varsigma(\varepsilon) : \varepsilon \in A\}$.

Example 3.2. Let $X = \{c_1, c_2, c_3, c_4, c_5\}$ denotes a set of five cars and $A = \{e_1, e_2, e_3, e_4, e_5\}$ be a parameter set, where e_1=size, e_2=fuel efficiency, e_3=comfort, e_4=colour, and e_5=expensive. Then, the IVIQNSS (ς, A) denotes the "attractiveness of the cars" to the decision-maker.

Suppose

$$\varsigma(e_1) = \begin{Bmatrix} \langle c_1, ([0.25,0.45], [0.35,0.55], [0.15,0.35], [0.2,0.4])\rangle, \\ \langle c_2, ([0.16,0.40], [0.54,0.56], [0.23,0.42], [0.32,0.45])\rangle, \\ \langle c_3, ([0.3,0.4], [0.16,0.18], [0.24,0.54], [0.25,0.28])\rangle, \\ \langle c_4, ([0.34,0.38], [0.14,0.18], [0.25,0.28], [0.3,0.6])\rangle, \\ \langle c_5, ([0.76,0.78], [0.35,0.37], [0.13,0.16], [0.12,0.16])\rangle \end{Bmatrix}$$

$$\varsigma(e_2) = \begin{Bmatrix} \langle c_1, ([0.2,0.4], [0.45,0.56], [0.35,0.37], [0.3,0.5])\rangle, \\ \langle c_2, ([0.26,0.30], [0.34,0.46], [0.3,0.4], [0.42,0.55])\rangle, \\ \langle c_3, ([0.12,0.24], [0.45,0.48], [0.34,0.42], [0.52,0.7])\rangle, \\ \langle c_4, ([0.52,0.58], [0.24,0.38], [0.45,0.48], [0.25,0.34])\rangle, \\ \langle c_5, ([0.56,0.68], [0.25,0.47], [0.33,0.36], [0.22,0.26])\rangle \end{Bmatrix}$$

$$\varsigma(e_3)=\left\{\begin{array}{l}\langle c_1,([0.15,0.25],[0.45,0.65],[0.18,0.25],[0.46,0.67])\rangle,\\ \langle c_2,([0.36,0.42],[0.34,0.46],[0.13,0.32],[0.42,0.48])\rangle,\\ \langle c_3,([0.14,0.26],[0.46,0.58],[0.14,0.34],[0.55,0.57])\rangle,\\ \langle c_4,([0.24,0.28],[0.34,0.45],[0.35,0.38],[0.43,0.64])\rangle,\\ \langle c_5,([0.66,0.68],[0.25,0.36],[0.23,0.36],[0.22,0.26])\rangle\end{array}\right\}$$

$$\varsigma(e_4)=\left\{\begin{array}{l}\langle c_1,([0.45,0.48],[0.38,0.45],[0.26,0.32],[0.36,0.4])\rangle,\\ \langle c_2,([0.36,0.42],[0.48,0.52],[0.24,0.41],[0.31,0.44])\rangle,\\ \langle c_3,([0.21,0.32],[0.26,0.28],[0.35,0.55],[0.35,0.48])\rangle,\\ \langle c_4,([0.14,0.18],[0.34,0.38],[0.55,0.58],[0.42,0.56])\rangle,\\ \langle c_5,([0.16,0.38],[0.34,0.67],[0.23,0.26],[0.52,0.56])\rangle\end{array}\right\}$$

$$\varsigma(e_5)=\left\{\begin{array}{l}\langle c_1,([0.46,0.55],[0.28,0.35],[0.45,0.55],[0.24,0.34])\rangle,\\ \langle c_2,([0.24,0.35],[0.44,0.46],[0.33,0.42],[0.42,0.45])\rangle,\\ \langle c_3,([0.26,0.42],[0.36,0.48],[0.44,0.54],[0.35,0.38])\rangle,\\ \langle c_4,([0.54,0.58],[0.24,0.48],[0.35,0.38],[0.25,0.36])\rangle,\\ \langle c_5,([0.23,0.38],[0.55,0.57],[0.23,0.36],[0.42,0.56])\rangle\end{array}\right\}$$

The IVIQNSS(ς, A) is a parameterized family $\{\varsigma(e_i), i=1,2,3,4,5\}$ of interval-valued intuitionistic quadripartitioned neutrosophic sets (IVIQNSs) on X, and

$$(\varsigma,A)=\left\{\begin{array}{l}
\text{size of the cars}=\left\{\begin{array}{l}\langle c_1,([0.25,0.45],[0.35,0.55],[0.15,0.35],[0.2,0.4])\rangle,\\ \langle c_2,([0.16,0.40],[0.54,0.56],[0.23,0.42],[0.32,0.45])\rangle,\\ \langle c_3,([0.3,0.4],[0.16,0.18],[0.24,0.54],[0.25,0.28])\rangle,\\ \langle c_4,([0.34,0.38],[0.14,0.18],[0.25,0.28],[0.3,0.6])\rangle,\\ \langle c_5,([0.76,0.78],[0.35,0.37],[0.13,0.16],[0.12,0.16])\rangle\end{array}\right\},\\
\text{fuel efficiency of the cars}=\left\{\begin{array}{l}\langle c_1,([0.2,0.4],[0.45,0.56],[0.35,0.37],[0.3,0.5])\rangle,\\ \langle c_2,([0.26,0.30],[0.34,0.46],[0.3,0.4],[0.42,0.55])\rangle,\\ \langle c_3,([0.12,0.24],[0.45,0.48],[0.34,0.42],[0.52,0.7])\rangle,\\ \langle c_4,([0.5,0.6],[0.2,0.3],[0.4,0.5],[0.2,0.3])\rangle,\\ \langle c_5,([0.56,0.68],[0.25,0.47],[0.33,0.36],[0.22,0.26])\rangle\end{array}\right\},\\
\text{comfortable cars}=\left\{\begin{array}{l}\langle c_1,([0.15,0.25],[0.45,0.65],[0.18,0.25],[0.46,0.67])\rangle,\\ \langle c_2,([0.36,0.42],[0.34,0.46],[0.13,0.32],[0.42,0.48])\rangle,\\ \langle c_3,([0.14,0.26],[0.46,0.58],[0.14,0.34],[0.55,0.57])\rangle,\\ \langle c_4,([0.24,0.28],[0.34,0.45],[0.35,0.38],[0.43,0.64])\rangle,\\ \langle c_5,([0.66,0.68],[0.25,0.36],[0.23,0.36],[0.22,0.26])\rangle\end{array}\right\},\\
\text{color of the cars}=\left\{\begin{array}{l}\langle c_1,([0.45,0.48],[0.38,0.45],[0.26,0.32],[0.36,0.4])\rangle,\\ \langle c_2,([0.36,0.42],[0.48,0.52],[0.24,0.41],[0.31,0.44])\rangle,\\ \langle c_3,([0.21,0.32],[0.26,0.28],[0.35,0.55],[0.35,0.48])\rangle,\\ \langle c_4,([0.14,0.18],[0.34,0.38],[0.55,0.58],[0.42,0.56])\rangle,\\ \langle c_5,([0.16,0.38],[0.34,0.67],[0.23,0.26],[0.52,0.56])\rangle\end{array}\right\},\\
\text{expensive cars}=\left\{\begin{array}{l}\langle c_1,([0.46,0.55],[0.28,0.35],[0.45,0.55],[0.24,0.34])\rangle,\\ \langle c_2,([0.24,0.35],[0.44,0.46],[0.33,0.42],[0.42,0.45])\rangle,\\ \langle c_3,([0.26,0.42],[0.36,0.48],[0.44,0.54],[0.35,0.38])\rangle,\\ \langle c_4,([0.54,0.58],[0.24,0.48],[0.35,0.38],[0.25,0.36])\rangle,\\ \langle c_5,([0.23,0.38],[0.55,0.57],[0.23,0.36],[0.42,0.56])\rangle\end{array}\right\}
\end{array}\right\}$$

From the above representation, it has been observed that the precise evaluation for each object on each parameter is unknown, while the lower and upper limits of such evaluations are given. We cannot present the precise truth-membership, contradiction-membership, unknown-membership, and falsity-membership degree of an object.

Definition 3.3. Let (ς_1, A) and (ς_2, B) be two IVIQNSSs over the common initial universe X, and $A, B \subseteq E$. Then, (ς_1, A) is called the IVIQN subset of (ς_2, B) if and only if

(a) $A \subseteq B$

(b) $\forall \varepsilon \in A$, $\varsigma_1(\varepsilon)$ is an IVIQN subset of $\varsigma_2(\varepsilon)$. That is, for all $x \in X$, and $\varepsilon \in A$, $\underline{T}_{\varsigma_1(\varepsilon)}(x) \le \underline{T}_{\varsigma_2(\varepsilon)}(x)$, $\overline{T}_{\varsigma_1(\varepsilon)}(x) \le \overline{T}_{\varsigma_2(\varepsilon)}(x)$; $\underline{C}_{\varsigma_1(\varepsilon)}(x) \le \underline{C}_{\varsigma_2(\varepsilon)}(x)$, $\overline{C}_{\varsigma_1(\varepsilon)}(x) \le \overline{C}_{\varsigma_2(\varepsilon)}(x)$; $\underline{U}_{\varsigma_1(\varepsilon)}(x) \ge \underline{U}_{\varsigma_2(\varepsilon)}(x)$, $\overline{U}_{\varsigma_1(\varepsilon)}(x) \ge \overline{U}_{\varsigma_2(\varepsilon)}(x)$; $\underline{F}_{\varsigma_1(\varepsilon)}(x) \ge \underline{F}_{\varsigma_2(\varepsilon)}(x)$, $\overline{T}_{\varsigma_1(\varepsilon)}(x) \le \overline{T}_{\varsigma_2(\varepsilon)}(x)$.

We denote this relationship by $(\varsigma_1, A) \overset{\cong}{\subseteq} (\varsigma_2, B)$.

Example 3.4. Let (ς_1, A) and (ς_2, B) be two IVIQNSSs over the set of teachers denoted by $X = \{t_1, t_2, t_3, t_4, t_5\}$. We also consider $A = \{e_1 = \text{experience}, e_2 = \text{hardworking}\}$, and $B = \{e_1 = \text{experience}, e_2 = \text{hardworking}, e_3 = \text{creative}\}$.

Suppose

$$\varsigma_1(e_1) = \begin{Bmatrix} \langle t_1, ([0.25,0.45],[0.35,0.55],[0.15,0.35],[0.2,0.4])\rangle, \langle t_2, ([0.16,0.40],[0.54,0.56],[0.23,0.42],[0.32,0.45])\rangle, \\ \langle t_3, ([0.3,0.4],[0.16,0.18],[0.24,0.54],[0.25,0.28])\rangle, \langle t_4, ([0.34,0.38],[0.14,0.18],[0.25,0.28],[0.3,0.6])\rangle, \\ \langle t_5, ([0.76,0.78],[0.35,0.37],[0.13,0.16],[0.12,0.16])\rangle \end{Bmatrix}$$

$$\varsigma_1(e_2) = \begin{Bmatrix} \langle t_1, ([0.2,0.4],[0.45,0.56],[0.35,0.37],[0.3,0.5])\rangle, \langle t_2, ([0.26,0.30],[0.34,0.46],[0.3,0.4],[0.42,0.55])\rangle, \\ \langle t_3, ([0.12,0.24],[0.45,0.48],[0.34,0.42],[0.52,0.7])\rangle, \langle t_4, ([0.52,0.58],[0.24,0.38],[0.45,0.48],[0.25,0.34])\rangle, \\ \langle t_5, ([0.56,0.68],[0.25,0.47],[0.33,0.36],[0.22,0.26])\rangle \end{Bmatrix}$$

$$\varsigma_2(e_1) = \begin{Bmatrix} \langle t_1, ([0.35,0.55],[0.45,0.62],[0.12,0.34],[0.18,0.36])\rangle, \langle t_2, ([0.26,0.42],[0.56,0.66],[0.13,0.32],[0.22,0.35])\rangle, \\ \langle t_3, ([0.35,0.45],[0.26,0.28],[0.14,0.24],[0.15,0.18])\rangle, \langle t_4, ([0.36,0.39],[0.24,0.28],[0.15,0.18],[0.25,0.53])\rangle, \\ \langle t_5, ([0.78,0.8],[0.45,0.47],[0.12,0.15],[0.1,0.12])\rangle \end{Bmatrix}$$

$$\varsigma_2(e_2) = \begin{Bmatrix} \langle t_1, ([0.25,0.45],[0.5,0.6],[0.25,0.27],[0.2,0.3])\rangle, \langle t_2, ([0.3,0.4],[0.4,0.5],[0.2,0.3],[0.4,0.5])\rangle, \\ \langle t_3, ([0.15,0.26],[0.55,0.58],[0.24,0.32],[0.42,0.5])\rangle, \langle t_4, ([0.55,0.6],[0.3,0.4],[0.38,0.4],[0.2,0.3])\rangle, \\ \langle t_5, ([0.6,0.62],[0.3,0.34],[0.23,0.26],[0.12,0.17])\rangle \end{Bmatrix}$$

$$\varsigma_2(e_3) = \begin{Bmatrix} \langle t_1, ([0.15,0.25],[0.45,0.65],[0.18,0.25],[0.46,0.67])\rangle, \langle t_2, ([0.36,0.42],[0.34,0.46],[0.13,0.32],[0.42,0.48])\rangle, \\ \langle t_3, ([0.14,0.26],[0.46,0.58],[0.14,0.34],[0.55,0.57])\rangle, \langle t_4, ([0.24,0.28],[0.34,0.45],[0.35,0.38],[0.43,0.64])\rangle, \\ \langle t_5, ([0.66,0.68],[0.25,0.36],[0.23,0.36],[0.22,0.26])\rangle \end{Bmatrix}$$

By Definition 3.3, $(\varsigma_1, A) \overset{\cong}{\subseteq} (\varsigma_2, B)$.

Definition 3.5. Let (ς_1, A) and (ς_2, B) be two IVIQNSSs over the common initial universe X, and $A, B \subseteq E$. Then, (ς_1, A) and (ς_2, B) are said to be IVIQN soft equal if

(a) $(\varsigma_1, A)$is an IVIQN soft subset of (ς_2, B).

(b) (ς_2, B) is an IVIQN soft subset of (ς_1, A).

That is, for all $x \in X$, and $\varepsilon \in A$, $\underline{T}_{\varsigma_1(\varepsilon)}(x) = \underline{T}_{\varsigma_2(\varepsilon)}(x)$, $\overline{T}_{\varsigma_1(\varepsilon)}(x) = \overline{T}_{\varsigma_2(\varepsilon)}(x)$; $\underline{C}_{\varsigma_1(\varepsilon)}(x) = \underline{C}_{\varsigma_2(\varepsilon)}(x)$, $\overline{C}_{\varsigma_1(\varepsilon)}(x) = \overline{C}_{\varsigma_2(\varepsilon)}(x)$; $\underline{U}_{\varsigma_1(\varepsilon)}(x) = \underline{U}_{\varsigma_2(\varepsilon)}(x)$, $\overline{U}_{\varsigma_1(\varepsilon)}(x) = \overline{U}_{\varsigma_2(\varepsilon)}(x)$; $\underline{F}_{\varsigma_1(\varepsilon)}(x) = \underline{F}_{\varsigma_2(\varepsilon)}(x)$, $\overline{T}_{\varsigma_1(\varepsilon)}(x) = \overline{T}_{\varsigma_2(\varepsilon)}(x)$.

This relationship is denoted by $(\varsigma_1, A) \overset{\cong}{=} (\varsigma_2, B)$.

Now, we define some operations on IVIQNSSs.

Definition 3.6. The complement of an IVIQNSS(ς, A) is denoted by $(\varsigma, A)^c$ and is defined as $(\varsigma, A)^c = (\varsigma^c, \neg A)$, where $\varsigma^c: \neg A \to IVIQNS(X)$ is a mapping given by

$$\varsigma^c(\varepsilon) = \left\langle x, \left(T_{\varsigma(\neg\varepsilon)}(x), C_{\varsigma(\neg\varepsilon)}(x), U_{\varsigma(\neg\varepsilon)}(x), F_{\varsigma(\neg\varepsilon)}(x)\right)\right\rangle$$

Otherwise, $\varsigma^c(\varepsilon) = \left\langle x, \left(F_{\varsigma(\varepsilon)}(x), U_{\varsigma(\varepsilon)}(x), C_{\varsigma(\varepsilon)}(x), T_{\varsigma(\varepsilon)}(x)\right)\right\rangle$.

Definition 3.7. An IVIQNSS (ς, A) over X is said to be a null or void IVIQNSS if $\forall \varepsilon \in A$, and $x \in X$, $T_{\varsigma(\varepsilon)}(x) = [0,0]$, $C_{\varsigma(\varepsilon)}(x) = [0,0]$, $U_{\varsigma(\varepsilon)}(x) = [1,1]$, and $F_{\varsigma(\varepsilon)}(x) = [1,1]$, and it is denoted by $\overset{\cong}{\Phi}$.

Definition 3.8. An IVIQNSS (ς, A) over X is said to be an absolute IVIQNSS if $\forall \varepsilon \in A$, and $x \in X$, $T_{\varsigma(\varepsilon)}(x) = [1,1]$, $C_{\varsigma(\varepsilon)}(x) = [1,1]$, $U_{\varsigma(\varepsilon)}(x) = [0,0]$, and $F_{\varsigma(\varepsilon)}(x) = [0,0]$, and it is denoted by $\overset{\cong}{\Sigma}$.

Definition 3.9. Let (ς_1, A) and (ς_2, B) be two IVIQNSSs over X. Then, "(ς_1, A) **AND** (ς_2, B)" is an IVIQNSS and it is denoted by $(\varsigma_1, A) \wedge (\varsigma_2, B)$ and is defined by $(\varsigma_1, A) \wedge (\varsigma_2, B) = (\varsigma_3, A \times B)$, where $\varsigma_3(a, b) = \varsigma_1(a) \cap \varsigma_2(b)$, $\forall (a, b) \in A \times B$. That is,

$$\varsigma_3(a,b)(x) = \left\langle \begin{array}{l} \left[\inf(\underline{T}_{\varsigma_1(a)}(x), \underline{T}_{\varsigma_2(b)}(x)), \inf\left(\overline{T}_{\varsigma_1(a)}(x), \overline{T}_{\varsigma_2(b)}(x)\right)\right], \\ \left[\inf(\underline{C}_{\varsigma_1(a)}(x), \underline{C}_{\varsigma_2(b)}(x)), \inf\left(\overline{C}_{\varsigma_1(a)}(x), \overline{C}_{\varsigma_2(b)}(x)\right)\right], \\ \left[\sup(\underline{U}_{\varsigma_1(a)}(x), \underline{U}_{\varsigma_2(b)}(x)), \sup\left(\overline{U}_{\varsigma_1(a)}(x), \overline{U}_{\varsigma_2(b)}(x)\right)\right], \\ \left[\sup(\underline{F}_{\varsigma_1(a)}(x), \underline{F}_{\varsigma_2(b)}(x)), \sup\left(\overline{F}_{\varsigma_1(a)}(x), \overline{F}_{\varsigma_2(b)}(x)\right)\right] \end{array} \right\rangle$$

Definition 3.10. Let (ς_1, A) and (ς_2, B) be two IVIQNSSs over X. Then, "(ς_1, A) **OR** (ς_2, B)" is an IVIQNSS and it is denoted and defined by $(\varsigma_1, A) \vee (\varsigma_2, B) = (\varsigma_4, A \times B)$, where $\varsigma_4(a, b) = \varsigma_1(a) \cap \varsigma_2(b)$, $\forall (a, b) \in A \times B$. That is,

$$\varsigma_4(a,b)(x) = \left\langle \begin{array}{l} \left[\sup(\underline{T}_{\varsigma_1(a)}(x), \underline{T}_{\varsigma_2(b)}(x)), \sup\left(\overline{T}_{\varsigma_1(a)}(x), \overline{T}_{\varsigma_2(b)}(x)\right)\right], \\ \left[\sup(\underline{C}_{\varsigma_1(a)}(x), \underline{C}_{\varsigma_2(b)}(x)), \sup\left(\overline{C}_{\varsigma_1(a)}(x), \overline{C}_{\varsigma_2(b)}(x)\right)\right], \\ \left[\inf(\underline{U}_{\varsigma_1(a)}(x), \underline{U}_{\varsigma_2(b)}(x)), \inf\left(\overline{U}_{\varsigma_1(a)}(x), \overline{U}_{\varsigma_2(b)}(x)\right)\right], \\ \left[\inf(\underline{F}_{\varsigma_1(a)}(x), \underline{F}_{\varsigma_2(b)}(x)), \inf\left(\overline{F}_{\varsigma_1(a)}(x), \overline{F}_{\varsigma_2(b)}(x)\right)\right] \end{array} \right\rangle$$

Theorem 3.11. Let (ς_1, A) and (ς_2, B) be two IVIQNSSs over X. Then, we have the following properties:

i. $\left((\varsigma_1, A) \wedge (\varsigma_2, B)\right)^c = (\varsigma_1, A)^c \vee (\varsigma_2, B)^c$

ii. $\left((\varsigma_1, A) \vee (\varsigma_2, B)\right)^c = (\varsigma_1, A)^c \wedge (\varsigma_2, B)^c$

Proof.

i. Let $(\varsigma_1, A) \wedge (\varsigma_2, B) = (\varsigma_3, A \times B)$. Then, $\left((\varsigma_1, A) \wedge (\varsigma_2, B)\right)^c = (\varsigma_3, A \times B)^c = \left({\varsigma_3}^c, \neg(A \times B)\right)$. Also, $(\varsigma_1, A)^c = ({\varsigma_1}^c, \neg A)$ and $(\varsigma_2, B)^c = ({\varsigma_2}^c, \neg B)$. Then, $(\varsigma_1, A)^c \vee (\varsigma_2, B)^c = ({\varsigma_1}^c, \neg A) \vee ({\varsigma_2}^c, \neg B) = (\varsigma_4, \neg A \times \neg B)$.

Now, $\forall (\neg a, \neg b) \in \neg A \times \neg B, x \in X$;

$$T_{\varsigma_4(\neg a,\neg b)}(x) = \left[\sup\left(\underline{T}_{\varsigma_1^c(\neg a)}(x), \underline{T}_{\varsigma_2^c(\neg b)}(x)\right), \sup\left(\bar{T}_{\varsigma_1^c(\neg a)}(x), \bar{T}_{\varsigma_2^c(\neg b)}(x)\right)\right]$$

$$C_{\varsigma_4(\neg a,\neg b)}(x) = \left[\sup\left(\underline{C}_{\varsigma_1^c(\neg a)}(x), \underline{C}_{\varsigma_2^c(\neg b)}(x)\right), \sup\left(\bar{C}_{\varsigma_1^c(\neg a)}(x), \bar{C}_{\varsigma_2^c(\neg b)}(x)\right)\right]$$

$$U_{\varsigma_4(\neg a,\neg b)}(x) = \left[\inf\left(\underline{U}_{\varsigma_1^c(\neg a)}(x), \underline{U}_{\varsigma_2^c(\neg b)}(x)\right), \inf\left(\bar{U}_{\varsigma_1^c(\neg a)}(x), \bar{U}_{\varsigma_2^c(\neg b)}(x)\right)\right]$$

$$F_{\varsigma_4(\neg a,\neg b)}(x) = \left[\inf\left(\underline{F}_{\varsigma_1^c(\neg a)}(x), \underline{F}_{\varsigma_2^c(\neg b)}(x)\right), \inf\left(\bar{F}_{\varsigma_1^c(\neg a)}(x), \bar{F}_{\varsigma_2^c(\neg b)}(x)\right)\right]$$

where

$$\varsigma_1^c(\neg a) = \left\langle x, \left(F_{\varsigma_1(a)}(x), U_{\varsigma_1(a)}(x), C_{\varsigma_1(a)}(x), T_{\varsigma_1(a)}(x)\right)\right\rangle$$

and

$$\varsigma_2^c(\neg b) = \left\langle x, \left(F_{\varsigma_2(b)}(x), U_{\varsigma 2(b)}(x), C_{\varsigma_2(b)}(x), T_{\varsigma_2(b)}(x)\right)\right\rangle$$

Thus, $\underline{T}_{\varsigma_1^c(\neg a)}(x) = \underline{F}_{\varsigma_1(a)}(x)$, $\underline{T}_{\varsigma_2^c(\neg b)}(x) = \underline{F}_{\varsigma_2(b)}(x)$, $\bar{T}_{\varsigma_1^c(\neg a)}(x) = \bar{F}_{\varsigma_1(a)}(x)$, $\bar{T}_{\varsigma_2^c(\neg b)}(x) = \bar{F}_{\varsigma_2(b)}(x)$, $\underline{C}_{\varsigma_1^c(\neg a)}(x) = \underline{U}_{\varsigma_1(a)}(x)$, $\underline{C}_{\varsigma_2^c(\neg b)}(x) = \underline{U}_{\varsigma_2(b)}(x)$, $\bar{C}_{\varsigma_1^c(\neg a)}(x) = \bar{U}_{\varsigma_1(a)}(x)$, and $\bar{C}_{\varsigma_2^c(\neg b)}(x) = \bar{U}_{\varsigma_2(b)}(x)$.

Therefore, we have the following:

$$T_{\varsigma_4(\neg a,\neg b)}(x) = \left[\sup\left(\underline{F}_{\varsigma_1(a)}(x), \underline{F}_{\varsigma_2(b)}(x)\right), \sup\left(\bar{F}_{\varsigma_1(a)}(x), \bar{F}_{\varsigma_2(b)}(x)\right)\right]$$

$$C_{\varsigma_4(\neg a,\neg b)}(x) = \left[\sup\left(\underline{U}_{\varsigma_1(a)}(x), \underline{U}_{\varsigma_2(b)}(x)\right), \sup\left(\bar{U}_{\varsigma_1(a)}(x), \bar{U}_{\varsigma_2(b)}(x)\right)\right]$$

$$U_{\varsigma_4(\neg a,\neg b)}(x) = \left[\inf\left(\underline{C}_{\varsigma_1(a)}(x), \underline{C}_{\varsigma_2(b)}(x)\right), \inf\left(\bar{C}_{\varsigma_1(a)}(x), \bar{C}_{\varsigma_2(b)}(x)\right)\right]$$

$$F_{\varsigma_4(\neg a,\neg b)}(x) = \left[\inf\left(\underline{T}_{\varsigma_1(a)}(x), \underline{T}_{\varsigma_2(b)}(x)\right), \inf\left(\bar{T}_{\varsigma_1(a)}(x), \bar{T}_{\varsigma_2(b)}(x)\right)\right]$$

Again,$\forall(\neg a, \neg b) \in \neg(A \times B)$, we have,

$$\varsigma_3^c(\neg a, \neg b) = \left\langle x, \left(F_{\varsigma_3(a,b)}(x), U_{\varsigma_3(a,b)}(x), C_{\varsigma_3(a,b)}(x), T_{\varsigma_3(a,b)}(x)\right)\right\rangle$$

and

$$(\varsigma_1, A) \wedge (\varsigma_2, B) = (\varsigma_3, A \times B)$$

Therefore,

$$\varsigma_{3(a,b)}(x)=\left\langle\begin{array}{c}\left[\inf\left(\underline{T}_{\varsigma_1(a)}(x),\underline{T}_{\varsigma_2(b)}(x)\right),\inf\left(\bar{T}_{\varsigma_1(a)}(x),\bar{T}_{\varsigma_2(b)}(x)\right)\right],\\ \left[\inf\left(\underline{C}_{\varsigma_1(a)}(x),\underline{C}_{\varsigma_2(b)}(x)\right),\inf\left(\bar{C}_{\varsigma_1(a)}(x),\bar{C}_{\varsigma_2(b)}(x)\right)\right],\\ \left[\sup\left(\underline{U}_{\varsigma_1(a)}(x),\underline{U}_{\varsigma_2(b)}(x)\right),\sup\left(\bar{U}_{\varsigma_1(a)}(x),\bar{U}_{\varsigma_2(b)}(x)\right)\right],\\ \left[\sup\left(\underline{F}_{\varsigma_1(a)}(x),\underline{F}_{\varsigma_2(b)}(x)\right),\sup\left(\bar{F}_{\varsigma_1(a)}(x),\bar{F}_{\varsigma_2(b)}(x)\right)\right]\end{array}\right\rangle$$

$$\varsigma^c_{3(a,b)}(x)=\left\langle\begin{array}{c}\left[sup\left(\underline{F}_{\varsigma_1(a)}(x),\underline{F}_{\varsigma_2(b)}(x)\right),sup\left(\bar{F}_{\varsigma_1(a)}(x),\bar{F}_{\varsigma_2(b)}(x)\right)\right],\\ \left[sup\left(\underline{U}_{\varsigma_1(a)}(x),\underline{U}_{\varsigma_2(b)}(x)\right),\sup\left(\bar{U}_{\varsigma_1(a)}(x),\bar{U}_{\varsigma_2(b)}(x)\right)\right],\\ \left[inf\left(\underline{C}_{\varsigma_1(a)}(x),\underline{C}_{\varsigma_2(b)}(x)\right),\inf\left(\bar{C}_{\varsigma_1(a)}(x),\bar{C}_{\varsigma_2(b)}(x)\right)\right],\\ \left[inf\left(\underline{T}_{\varsigma_1(a)}(x),\underline{T}_{\varsigma_2(b)}(x)\right),\inf\left(\bar{T}_{\varsigma_1(a)}(x),\bar{T}_{\varsigma_2(b)}(x)\right)\right]\end{array}\right\rangle$$

ii. Therefore, ${\varsigma_3}^c$and ς_4 are the same operators and thus $\left((\varsigma_1,A)\wedge(\varsigma_2,B)\right)^c=(\varsigma_1,A)^c\vee(\varsigma_2,B)^c$. By the duality principle for sets, $\left((\varsigma_1,A)\vee(\varsigma_2,B)\right)^c=(\varsigma_1,A)^c\wedge(\varsigma_2,B)^c$.

Theorem 3.12. Let (ς_1,P), (ς_2,Q), and (ς_3,R) be three IVIQNSSs over X. Then, we have the following properties:

i. $(\varsigma_1,P)\wedge\left((\varsigma_2,Q)\wedge(\varsigma_3,R)\right)=\left((\varsigma_1,P)\wedge(\varsigma_2,Q)\right)\wedge(\varsigma_3,R)$
ii. $(\varsigma_1,P)\vee\left((\varsigma_2,Q)\vee(\varsigma_3,R)\right)=\left((\varsigma_1,P)\vee(\varsigma_2,Q)\right)\vee(\varsigma_3,R)$

Proof.

i. Let $(\varsigma_2,Q)\wedge(\varsigma_3,R)=(\varsigma_4,Q\times R)$, where $\varsigma_4(a,b)=\varsigma_2(a)\cap\varsigma_3(b)$, $\forall(a,b)\in Q\times R$. Then, we have

$$\varsigma_4(a,b)(x)=\left\langle\begin{array}{c}\left[\inf\left(\underline{T}_{\varsigma_2(a)}(x),\underline{T}_{\varsigma_3(b)}(x)\right),\inf\left(\bar{T}_{\varsigma_2(a)}(x),\bar{T}_{\varsigma_3(b)}(x)\right)\right],\\ \left[\inf\left(\underline{C}_{\varsigma_2(a)}(x),\underline{C}_{\varsigma_3(b)}(x)\right),\inf\left(\bar{C}_{\varsigma_2(a)}(x),\bar{C}_{\varsigma_3(b)}(x)\right)\right],\\ \left[\sup\left(\underline{U}_{\varsigma_2(a)}(x),\underline{U}_{\varsigma_3(b)}(x)\right),\sup\left(\bar{U}_{\varsigma_2(a)}(x),\bar{U}_{\varsigma_3(b)}(x)\right)\right],\\ \left[\sup\left(\underline{F}_{\varsigma_2(a)}(x),\underline{F}_{\varsigma_3(b)}(x)\right),\sup\left(\bar{F}_{\varsigma_2(a)}(x),\bar{F}_{\varsigma_3(b)}(x)\right)\right]\end{array}\right\rangle$$

Again, we assume that $(\varsigma_1,P)\wedge\left((\varsigma_2,Q)\wedge(\varsigma_3,R)\right)=(\varsigma_1,P)\wedge(\varsigma_4,Q\times R)=\left(\varsigma_5,P\times(Q\times R)\right)$, where $\varsigma_5(c,a,b)=\varsigma_1(c)\cap\varsigma_4(a,b)$, $\forall(c,a,b)\in P\times Q\times R$. Therefore,

$$\varsigma_5(c,a,b)(x)=\left\langle\begin{array}{c}\left[\inf\left(\underline{T}_{\varsigma_1(c)}(x),\underline{T}_{\varsigma_4(a,b)}(x)\right),\inf\left(\bar{T}_{\varsigma_1(c)}(x),\bar{T}_{\varsigma_4(a,b)}(x)\right)\right],\\ \left[\inf\left(\underline{C}_{\varsigma_1(c)}(x),\underline{C}_{\varsigma_4(a,b)}(x)\right),\inf\left(\bar{C}_{\varsigma_1(c)}(x),\bar{C}_{\varsigma_4(a,b)}(x)\right)\right],\\ \left[\sup\left(\underline{U}_{\varsigma_1(c)}(x),\underline{U}_{\varsigma_4(a,b)}(x)\right),\sup\left(\bar{U}_{\varsigma_1(c)}(x),\bar{U}_{\varsigma_4(a,b)}(x)\right)\right],\\ \left[\sup\left(\underline{F}_{\varsigma_1(c)}(x),\underline{F}_{\varsigma_4(a,b)}(x)\right),\sup\left(\bar{F}_{\varsigma_1(c)}(x),\bar{F}_{\varsigma_4(a,b)}(x)\right)\right]\end{array}\right\rangle$$

$$= \left\langle \begin{array}{l} \left[\inf\left(\underline{T}_{\varsigma_1(c)}(x), \inf\left(\underline{T}_{\varsigma_2(a)}(x), \underline{T}_{\varsigma_3(b)}(x)\right)\right), \inf\left(\overline{T}_{\varsigma_1(c)}(x), \inf\left(\overline{T}_{\varsigma_2(a)}(x), \overline{T}_{\varsigma_3(b)}(x)\right)\right)\right], \\ \left[\inf\left(\underline{C}_{\varsigma_1(c)}(x), \inf\left(\underline{C}_{\varsigma_2(a)}(x), \underline{C}_{\varsigma_3(b)}(x)\right)\right), \inf\left(\overline{C}_{\varsigma_1(c)}(x), \inf\left(\overline{C}_{\varsigma_2(a)}(x), \overline{C}_{\varsigma_3(b)}(x)\right)\right)\right], \\ \left[\sup\left(\underline{U}_{\varsigma_1(c)}(x), \sup\left(\underline{U}_{\varsigma_2(a)}(x), \underline{U}_{\varsigma_3(b)}(x)\right)\right), \sup\left(\overline{U}_{\varsigma_1(c)}(x), \sup\left(\overline{U}_{\varsigma_2(a)}(x), \overline{U}_{\varsigma_3(b)}(x)\right)\right)\right], \\ \left[\sup\left(\underline{F}_{\varsigma_1(c)}(x), \sup\left(\underline{F}_{\varsigma_2(a)}(x), \underline{F}_{\varsigma_3(b)}(x)\right)\right), \sup\left(\overline{F}_{\varsigma_1(c)}(x), \sup\left(\overline{F}_{\varsigma_2(a)}(x), \overline{F}_{\varsigma_3(b)}(x)\right)\right)\right] \end{array} \right\rangle$$

We take $(c, a) \in P \times Q$. Let $(\varsigma_1, P) \wedge (\varsigma_2, Q) = (\varsigma_6, P \times Q)$, where $\varsigma_6(c, a) = \varsigma_1(c) \cap \varsigma_2(a)$. Thus,

$$\varsigma_6(c,a)(x) = \left\langle \begin{array}{l} \left[\inf\left(\underline{T}_{\varsigma_1(c)}(x), \underline{T}_{\varsigma_2(a)}(x)\right), \inf\left(\overline{T}_{\varsigma_1(c)}(x), \overline{T}_{\varsigma_2(a)}(x)\right)\right], \\ \left[\inf\left(\underline{C}_{\varsigma_1(c)}(x), \underline{C}_{\varsigma_2(a)}(x)\right), \inf\left(\overline{C}_{\varsigma_1(c)}(x), \overline{C}_{\varsigma_2(a)}(x)\right)\right], \\ \left[\sup\left(\underline{U}_{\varsigma_1(c)}(x), \underline{U}_{\varsigma_2(a)}(x)\right), \sup\left(\overline{U}_{\varsigma_1(c)}(x), \overline{U}_{\varsigma_2(a)}(x)\right)\right], \\ \left[\sup\left(\underline{F}_{\varsigma_1(c)}(x), \underline{F}_{\varsigma_2(a)}(x)\right), \sup\left(\overline{F}_{\varsigma_1(c)}(x), \overline{F}_{\varsigma_2(a)}(x)\right)\right] \end{array} \right\rangle \forall (c,a) \in P \times Q,\ x \in X$$

Since $\left((\varsigma_1, P) \wedge (\varsigma_2, Q)\right) \wedge (\varsigma_3, R) = (\varsigma_6, P \times Q) \wedge (\varsigma_3, R)$, we assume that $(\varsigma_7, (P \times Q) \times R)$, where $\varsigma_7(c, a, b) = \varsigma_6(c, a) \cap \varsigma_3(b)$, $(c, a, b) \in P \times Q \times R$.

Therefore,

$$\varsigma_7(c,a,b)(x) = \left\langle \begin{array}{l} \left[\inf\left(\underline{T}_{\varsigma_6(c,a)}(x), \underline{T}_{\varsigma_3(b)}(x)\right), \inf\left(\overline{T}_{\varsigma_6(c,a)}(x), \overline{T}_{\varsigma_3(b)}(x)\right)\right], \\ \left[\inf\left(\underline{C}_{\varsigma_6(c,a)}(x), \underline{C}_{\varsigma_3(b)}(x)\right), \inf\left(\overline{C}_{\varsigma_6(c,a)}(x), \overline{C}_{\varsigma_3(b)}(x)\right)\right], \\ \left[\sup\left(\underline{U}_{\varsigma_6(c,a)}(x), \underline{U}_{\varsigma_3(b)}(x)\right), \sup\left(\overline{U}_{\varsigma_6(c,a)}(x), \overline{U}_{\varsigma_3(b)}(x)\right)\right], \\ \left[\sup\left(\underline{F}_{\varsigma_6(c,a)}(x), \underline{F}_{\varsigma_3(b)}(x)\right), \sup\left(\overline{F}_{\varsigma_6(c,a)}(x), \overline{F}_{\varsigma_3(b)}(x)\right)\right] \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \left[\inf\left(\inf\left(\underline{T}_{\varsigma_1(c)}(x), \underline{T}_{\varsigma_2(a)}(x)\right), \underline{T}_{\varsigma_3(b)}(x)\right), \inf\left(\inf\left(\overline{T}_{\varsigma_1(c)}(x), \overline{T}_{\varsigma_2(a)}(x)\right), \overline{T}_{\varsigma_3(b)}(x)\right)\right], \\ \left[\inf\left(\inf\left(\underline{C}_{\varsigma_1(c)}(x), \underline{C}_{\varsigma_2(a)}(x)\right), \underline{C}_{\varsigma_3(b)}(x)\right), \inf\left(\inf\left(\overline{C}_{\varsigma_1(c)}(x), \overline{C}_{\varsigma_2(a)}(x)\right), \overline{C}_{\varsigma_3(b)}(x)\right)\right], \\ \left[\sup\left(\sup\left(\underline{U}_{\varsigma_1(c)}(x), \underline{U}_{\varsigma_2(a)}(x)\right), \underline{U}_{\varsigma_3(b)}(x)\right), \sup\left(\sup\left(\overline{U}_{\varsigma_1(c)}(x), \overline{U}_{\varsigma_2(a)}(x)\right), \overline{U}_{\varsigma_3(b)}(x)\right)\right], \\ \left[\sup\left(\sup\left(\underline{F}_{\varsigma_1(c)}(x), \underline{F}_{\varsigma_2(a)}(x)\right), \underline{F}_{\varsigma_3(b)}(x)\right), \sup\left(\sup\left(\overline{F}_{\varsigma_1(c)}(x), \overline{F}_{\varsigma_2(a)}(x)\right), \overline{F}_{\varsigma_3(b)}(x)\right)\right] \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \left[\inf\left(\underline{T}_{\varsigma_1(c)}(x), \inf\left(\underline{T}_{\varsigma_2(a)}(x), \underline{T}_{\varsigma_3(b)}(x)\right)\right), \inf\left(\overline{T}_{\varsigma_1(c)}(x), \inf\left(\overline{T}_{\varsigma_2(a)}(x), \overline{T}_{\varsigma_3(b)}(x)\right)\right)\right], \\ \left[\inf\left(\underline{C}_{\varsigma_1(c)}(x), \inf\left(\underline{C}_{\varsigma_2(a)}(x), \underline{C}_{\varsigma_3(b)}(x)\right)\right), \inf\left(\overline{C}_{\varsigma_1(c)}(x), \inf\left(\overline{C}_{\varsigma_2(a)}(x), \overline{C}_{\varsigma_3(b)}(x)\right)\right)\right], \\ \left[\sup\left(\underline{U}_{\varsigma_1(c)}(x), \sup\left(\underline{U}_{\varsigma_2(a)}(x), \underline{U}_{\varsigma_3(b)}(x)\right)\right), \sup\left(\overline{U}_{\varsigma_1(c)}(x), \sup\left(\overline{U}_{\varsigma_2(a)}(x), \overline{U}_{\varsigma_3(b)}(x)\right)\right)\right], \\ \left[\sup\left(\underline{F}_{\varsigma_1(c)}(x), \sup\left(\underline{F}_{\varsigma_2(a)}(x), \underline{F}_{\varsigma_3(b)}(x)\right)\right), \sup\left(\overline{F}_{\varsigma_1(c)}(x), \sup\left(\overline{F}_{\varsigma_2(a)}(x), \overline{F}_{\varsigma_3(b)}(x)\right)\right)\right] \end{array} \right\rangle$$

$$= \varsigma_5(c,a,b)(x)$$

Thus, ς_5 and ς_7 represent the same operators. Therefore,

$$(\varsigma_1, P) \wedge \big((\varsigma_2, Q) \wedge (\varsigma_3, R)\big) = \big((\varsigma_1, P) \wedge (\varsigma_2, Q)\big) \wedge (\varsigma_3, R)$$

ii. By the duality principle for sets, we have

$$(\varsigma_1, P) \vee \big((\varsigma_2, Q) \vee (\varsigma_3, R)\big) = \big((\varsigma_1, P) \vee (\varsigma_2, Q)\big) \vee (\varsigma_3, R)$$

Definition 3.13. The union of two IVIQNSSs (ς_1, P) and (ς_2, Q) over an initial universe X is an IVIQNSS (ς_3, R), where $R = P \cup Q$ and $\forall \varepsilon \in R$, defined by

$$T_{\varsigma_3(\varepsilon)}(x) = \begin{cases} T_{\varsigma_1(\varepsilon)}(x), & \text{if } \varepsilon \in P - Q,\ x \in X \\ T_{\varsigma_2(\varepsilon)}(x), & \text{if } \varepsilon \in Q - P,\ x \in X \\ \left[\sup\left(\underline{T}_{\varsigma_1(\varepsilon)}(x), \underline{T}_{\varsigma_2(\varepsilon)}(x)\right), \sup\left(\bar{T}_{\varsigma_1(\varepsilon)}(x), \bar{T}_{\varsigma_2(\varepsilon)}(x)\right)\right], & \text{if } \varepsilon \in P \cap Q,\ x \in X \end{cases}$$

$$C_{\varsigma_3(\varepsilon)}(x) = \begin{cases} C_{\varsigma_1(\varepsilon)}(x), & \text{if } \varepsilon \in P - Q,\ x \in X \\ C_{\varsigma_2(\varepsilon)}(x), & \text{if } \varepsilon \in Q - P,\ x \in X \\ \left[\sup\left(\underline{C}_{\varsigma_1(\varepsilon)}(x), \underline{C}_{\varsigma_2(\varepsilon)}(x)\right), \sup\left(\bar{C}_{\varsigma_1(\varepsilon)}(x), \bar{C}_{\varsigma_2(\varepsilon)}(x)\right)\right], & \text{if } \varepsilon \in P \cap Q,\ x \in X \end{cases}$$

$$U_{\varsigma_3(\varepsilon)}(x) = \begin{cases} U_{\varsigma_1(\varepsilon)}(x), & \text{if } \varepsilon \in P - Q,\ x \in X \\ U_{\varsigma_2(\varepsilon)}(x), & \text{if } \varepsilon \in Q - P,\ x \in X \\ \left[\inf\left(\underline{U}_{\varsigma_1(\varepsilon)}(x), \underline{U}_{\varsigma_2(\varepsilon)}(x)\right), \inf\left(\bar{U}_{\varsigma_1(\varepsilon)}(x), \bar{U}_{\varsigma_2(\varepsilon)}(x)\right)\right], & \text{if } \varepsilon \in P \cap Q,\ x \in X \end{cases}$$

$$F_{\varsigma_3(\varepsilon)}(x) = \begin{cases} F_{\varsigma_1(\varepsilon)}(x), & \text{if } \varepsilon \in P - Q,\ x \in X \\ F_{\varsigma_2(\varepsilon)}(x), & \text{if } \varepsilon \in Q - P,\ x \in X \\ \left[\inf\left(\underline{F}_{\varsigma_1(\varepsilon)}(x), \underline{F}_{\varsigma_2(\varepsilon)}(x)\right), \inf\left(\bar{F}_{\varsigma_1(\varepsilon)}(x), \bar{F}_{\varsigma_2(\varepsilon)}(x)\right)\right], & \text{if } \varepsilon \in P \cap Q,\ x \in X \end{cases}$$

And denoted by $(\varsigma_1, P) \overset{\cong}{\cup} (\varsigma_2, Q) = (\varsigma_3, R)$.

Definition 3.14. The intersection of two IVIQNSSs (ς_1, P) and (ς_2, Q) over an initial universe X is an IVIQNSS (ς_3, R), where $R = P \cap Q$ and $\forall \varepsilon \in R$, defined by

$$T_{\varsigma_3(\varepsilon)}(x) = \begin{cases} T_{\varsigma_1(\varepsilon)}(x), & \text{if } \varepsilon \in P - Q,\ x \in X \\ T_{\varsigma_2(\varepsilon)}(x), & \text{if } \varepsilon \in Q - P,\ x \in X \\ \left[\inf\left(\underline{T}_{\varsigma_1(\varepsilon)}(x), \underline{T}_{\varsigma_2(\varepsilon)}(x)\right), \inf\left(\bar{T}_{\varsigma_1(\varepsilon)}(x), \bar{T}_{\varsigma_2(\varepsilon)}(x)\right)\right], & \text{if } \varepsilon \in P \cap Q,\ x \in X \end{cases}$$

$$C_{\varsigma_3(\varepsilon)}(x) = \begin{cases} C_{\varsigma_1(\varepsilon)}(x), & \text{if } \varepsilon \in P - Q,\ x \in X \\ C_{\varsigma_2(\varepsilon)}(x), & \text{if } \varepsilon \in Q - P,\ x \in X \\ \left[\inf\left(\underline{C}_{\varsigma_1(\varepsilon)}(x), \underline{C}_{\varsigma_2(\varepsilon)}(x)\right), \inf\left(\bar{C}_{\varsigma_1(\varepsilon)}(x), \bar{C}_{\varsigma_2(\varepsilon)}(x)\right)\right], & \text{if } \varepsilon \in P \cap Q,\ x \in X \end{cases}$$

$$U_{\varsigma_3(\varepsilon)}(x) = \begin{cases} U_{\varsigma_1(\varepsilon)}(x), & \text{if } \varepsilon \in P - Q,\ x \in X \\ U_{\varsigma_2(\varepsilon)}(x), & \text{if } \varepsilon \in Q - P,\ x \in X \\ \left[\sup\left(\underline{U}_{\varsigma_1(\varepsilon)}(x), \underline{U}_{\varsigma_2(\varepsilon)}(x)\right), \sup\left(\bar{U}_{\varsigma_1(\varepsilon)}(x), \bar{U}_{\varsigma_2(\varepsilon)}(x)\right)\right], & \text{if } \varepsilon \in P \cap Q,\ x \in X \end{cases}$$

$$F_{\varsigma_3(\varepsilon)}(x)=\begin{cases} F_{\varsigma_1(\varepsilon)}(x), & \text{if } \varepsilon\in P-Q,\ x\in X \\ F_{\varsigma_2(\varepsilon)}(x), & \text{if } \varepsilon\in Q-P,\ x\in X \\ \left[\sup\left(\underline{F}_{\varsigma_1(\varepsilon)}(x),\underline{F}_{\varsigma_2(\varepsilon)}(x)\right),\sup\left(\overline{F}_{\varsigma_1(\varepsilon)}(x),\overline{F}_{\varsigma_2(\varepsilon)}(x)\right)\right], & \text{if } \varepsilon\in P\cap Q,\ x\in X \end{cases}$$

And denoted by $(\varsigma_1,P)\ \overset{\cong}{\cap}\ (\varsigma_2,Q)=(\varsigma_3,R)$.

Proposition 3.15. Let X be an initial universe and E be a set of parameters where $P\subseteq E$. If (ς,P) and (ς,E) be two IVIQNSSs over X, then we have the following properties:

i. $(\varsigma,P)\ \overset{\cong}{\cup}\ (\varsigma,P)=(\varsigma,P)$
ii. $(\varsigma,P)\ \overset{\cong}{\cap}\ (\varsigma,P)=(\varsigma,P)$
iii. $(\varsigma,E)\ \overset{\cong}{\cup}\ \overset{\cong}{\Phi}=(\varsigma,E)$
iv. $(\varsigma,E)\ \overset{\cong}{\cap}\ \overset{\cong}{\Phi}=\overset{\cong}{\Phi}$
v. $(\varsigma,E)\ \overset{\cong}{\cup}\ \overset{\cong}{\Sigma}=\overset{\cong}{\Sigma}$
vi. $(\varsigma,E)\ \overset{\cong}{\cap}\ \overset{\cong}{\Sigma}=(\varsigma,E)$

Proof. All proofs are left as an exercise for the readers.

Theorem 3.16. If (ς_1,P) and (ς_2,Q) be two IVIQNSSs over X, then we have the following properties:

i. $$\left((\varsigma_1,P)\ \overset{\cong}{\cup}\ (\varsigma_2,Q)\right)^c=(\varsigma_1,P)^c\ \overset{\cong}{\cap}\ (\varsigma_2,Q)^c$$

ii. $$\left((\varsigma_1,P)\ \overset{\cong}{\cap}\ (\varsigma_2,Q)\right)^c=(\varsigma_1,P)^c\ \overset{\cong}{\cup}\ (\varsigma_2,Q)^c$$

Proof. All proofs can be easily obtained by using Definition 3.6, 3.13, and 3.14.

Theorem 3.17. If (ς_1,P), (ς_2,Q), and (ς_3,R) be three IVIQNSSs, then we have the following properties:

i. $$(\varsigma_1,P)\ \overset{\cong}{\cup}\left((\varsigma_2,Q)\ \overset{\cong}{\cup}\ (\varsigma_3,R)\right)=\left((\varsigma_1,P)\ \overset{\cong}{\cup}\ (\varsigma_2,Q)\right)\overset{\cong}{\cup}\ (\varsigma_3,R)$$

ii. $$(\varsigma_1,P)\ \overset{\cong}{\cap}\left((\varsigma_2,Q)\ \overset{\cong}{\cap}\ (\varsigma_3,R)\right)=\left((\varsigma_1,P)\ \overset{\cong}{\cap}\ (\varsigma_2,Q)\right)\overset{\cong}{\cap}\ (\varsigma_3,R)$$

iii. $$(\varsigma_1,P)\ \overset{\cong}{\cup}\left((\varsigma_2,Q)\ \overset{\cong}{\cap}\ (\varsigma_3,R)\right)=\left((\varsigma_1,P)\ \overset{\cong}{\cup}\ (\varsigma_2,Q)\right)\overset{\cong}{\cap}\left((\varsigma_1,P)\ \overset{\cong}{\cup}\ (\varsigma_3,R)\right)$$

iv. $$(\varsigma_1,P)\ \overset{\cong}{\cap}\left((\varsigma_2,Q)\ \overset{\cong}{\cup}\ (\varsigma_3,R)\right)=\left((\varsigma_1,P)\ \overset{\cong}{\cap}\ (\varsigma_2,Q)\right)\overset{\cong}{\cup}\left((\varsigma_1,P)\ \overset{\cong}{\cap}\ (\varsigma_3,R)\right)$$

Proof. All proofs are left as an exercise for the readers.

4. An Algorithm Based on Interval-Valued Intuitionistic Quadripartitioned Neutrosophic Soft Sets in a Decision-Making Problem

Definition 4.1. Let $X = \{x_1, x_2, \ldots\ldots, x_n\}$ be an initial universe and $E = \{e_1, e_2, \ldots\ldots, e_m\}$ be a set of parameters. Then, for an IVIQNSS(ς, E) over X the degree of truth-membership and the degree of contradiction-membership of an element $x_i$to $\varsigma(e_j)$ denoted by $T_{\varsigma(e_j)}(x_i) = \left[\underline{T}_{\varsigma(e_j)}(x_i), \bar{T}_{\varsigma(e_j)}(x_i)\right]$ and $C_{\varsigma(e_j)}(x_i) = \left[\underline{C}_{\varsigma(e_j)}(x_i), \bar{C}_{\varsigma(e_j)}(x_i)\right]$, respectively. Then, their corresponding score functions are denoted and defined by the following:

$$S_{T_{\varsigma(e_j)}}(x_i) = \sum_{k=1}^{n} \left[\left(\underline{T}_{\varsigma(e_j)}(x_i) + \bar{T}_{\varsigma(e_j)}(x_i)\right) - \left(\underline{T}_{\varsigma(e_j)}(x_k) + \bar{T}_{\varsigma(e_j)}(x_k)\right)\right]$$

$$S_{C_{\varsigma(e_j)}}(x_i) = \sum_{k=1}^{n} \left[\left(\underline{C}_{\varsigma(e_j)}(x_i) + \bar{C}_{\varsigma(e_j)}(x_i)\right) - \left(\underline{C}_{\varsigma(e_j)}(x_k) + \bar{C}_{\varsigma(e_j)}(x_k)\right)\right]$$

Definition 4.2. Let $X = \{x_1, x_2, \ldots\ldots, x_n\}$ be an initial universe and $E = \{e_1, e_2, \ldots\ldots, e_m\}$ be a set of parameters. Then, for an IVIQNSS(ς, E) over X the degree of uncertain membership and the degree of falsity-membership of an element x_i to $\varsigma(e_j)$ denoted by $U_{\varsigma(e_j)}(x_i) = \left[\underline{U}_{\varsigma(e_j)}(x_i), \bar{U}_{\varsigma(e_j)}(x_i)\right]$ and $F_{\varsigma(e_j)}(x_i) = \left[\underline{F}_{\varsigma(e_j)}(x_i), \bar{F}_{\varsigma(e_j)}(x_i)\right]$, respectively. Then, their corresponding score functions are denoted and defined by the following:

$$S_{U_{\varsigma(e_j)}}(x_i) = -\sum_{k=1}^{n} \left[\left(\underline{U}_{\varsigma(e_j)}(x_i) + \bar{U}_{\varsigma(e_j)}(x_i)\right) - \left(\underline{U}_{\varsigma(e_j)}(x_k) + \bar{U}_{\varsigma(e_j)}(x_k)\right)\right]$$

$$S_{F_{\varsigma(e_j)}}(x_i) = -\sum_{k=1}^{n} \left[\left(\underline{F}_{\varsigma(e_j)}(x_i) + \bar{F}_{\varsigma(e_j)}(x_i)\right) - \left(\underline{F}_{\varsigma(e_j)}(x_k) + \bar{F}_{\varsigma(e_j)}(x_k)\right)\right]$$

Definition 4.3. Let $X = \{x_1, x_2, \ldots\ldots, x_n\}$ be an initial universe and $E = \{e_1, e_2, \ldots\ldots, e_m\}$ be a set of parameters. For an IVIQNSS(ς, E) over X, the scores of the truth, contradiction, uncertain, and falsity membership of x_i for each e_j be denoted by $S_{T_{\varsigma(e_j)}}(x_i)$, $S_{C_{\varsigma(e_j)}}(x_i)$, $S_{U_{\varsigma(e_j)}}(x_i)$, and $S_{F_{\varsigma(e_j)}}(x_i)$, respectively. Then, the total score of x_i for each e_j is denoted by $\Omega_{\varsigma(e_j)}(x_i)$ and it is calculated as

$$\Omega_{\varsigma(e_j)}(x_i) = S_{T_{\varsigma(e_j)}}(x_i) + S_{C_{\varsigma(e_j)}}(x_i) + S_{U_{\varsigma(e_j)}}(x_i) + S_{F_{\varsigma(e_j)}}(x_i)$$

Definition 4.4 Let $X = \{x_1, x_2, \ldots\ldots, x_n\}$ be an initial universe and $E = \{e_1, e_2, \ldots\ldots, e_m\}$ be a set of parameters. For an IVIQNSS(ς, E) over X, the scores of the truth, contradict, uncertain, and false membership of x_i for each e_j be denoted by $S_{T_{\varsigma(e_j)}}(x_i)$, $S_{C_{\varsigma(e_j)}}(x_i)$, $S_{U_{\varsigma(e_j)}}(x_i)$, and $S_{F_{\varsigma(e_j)}}(x_i)$, respectively. Then, the accuracy score of x_i for each e_j is denoted by $\Theta_{\varsigma(e_j)}(x_i)$ and it is calculated as

$$\Theta_{\varsigma(e_j)}(x_i) = \frac{S_{T_{\varsigma(e_j)}}(x_i) + S_{C_{\varsigma(e_j)}}(x_i) - S_{U_{\varsigma(e_j)}}(x_i) - S_{F_{\varsigma(e_j)}}(x_i)}{card(E)}$$

where $card(E)$ denotes the number of parameters in E.

Definition 4.5 Let (ς_1, E) and (ς_2, E) be two IVIQNSSs over a common universe X. Then, we have the following properties:

i. If $(\varsigma_1, E) \subseteq (\varsigma_2, E)$, then $S_{T_{\varsigma_1(e_j)}}(x_i) \leq S_{T_{\varsigma_2(e_j)}}(x_i)$, $S_{C_{\varsigma_1(e_j)}}(x_i) \leq S_{C_{\varsigma_2(e_j)}}(x_i)$, $S_{U_{\varsigma_1(e_j)}}(x_i) \geq S_{U_{\varsigma_2(e_j)}}(x_i)$, and $S_{F_{\varsigma_1(e_j)}}(x_i) \geq S_{F_{\varsigma_2(e_j)}}(x_i)$.

ii. If $(\varsigma_1, E) = (\varsigma_2, E)$, then $S_{T_{\varsigma_1(e_j)}}(x_i) = S_{T_{\varsigma_2(e_j)}}(x_i)$, $S_{C_{\varsigma_1(e_j)}}(x_i) = S_{C_{\varsigma_2(e_j)}}(x_i)$, $S_{U_{\varsigma_1(e_j)}}(x_i) = S_{U_{\varsigma_2(e_j)}}(x_i)$, and $S_{F_{\varsigma_1(e_j)}}(x_i) = S_{F_{\varsigma_2(e_j)}}(x_i)$.

Based on the above definitions, we give the steps of the proposed algorithm as follows:

Algorithm:

Step 1. For the universal set $X = \{x_1, x_2, \ldots, x_n\}$ and the parameter set $E = \{e_1, e_2, \ldots, e_m\}$, input the matrix representation of an IVIQNSS(ς, E) in tabular form, according to a decision-maker.

Step 2. Reference to the input matrix obtained in step 1 and using the Definitions 4.1 and 4.2, we compute $S_{T_{\varsigma(e_j)}}(x_i)$, $S_{C_{\varsigma(e_j)}}(x_i)$, $S_{U_{\varsigma(e_j)}}(x_i)$, and $S_{F_{\varsigma(e_j)}}(x_i)$ of x_i for each e_j where $i = 1,2, \ldots, n$; $j = 1,2, \ldots, m$.

Step 3. Taking the results obtained in step 2 and using the Definition 4.3, compute the score $\Omega_{\varsigma(e_j)}(x_i)$ of x_i for each $e_j$where $i = 1,2, \ldots, n$; $j = 1,2, \ldots, m$.

Step 4. Compute the overall score u_i for $x_i$in such a way that

$$u_i = \Omega_{\varsigma(e_1)}(x_i) + \Omega_{\varsigma(e_2)}(x_i) + \ldots\ldots + \Omega_{\varsigma(e_m)}(x_i)$$

Step 5. Find k, for which $u_k = \max_{x_i \in X}\{u_i\}$. Then, $x_k \in X$ is the optimal choice.

Step 6. In case of a tie, either we take both as an optimal choice or we reassess all the values with the expert's advice and repeat all the previous steps.

For the practical application of the above algorithm, we consider the following example.

Illustrative Example 4.6. To implement the proposed algorithm successfully in a real-life context, we consider the following problem:

Suppose Mr. Z wants to purchase a laptop to carry out his official work. But he has limited knowledge to select a good quality laptop. The selection process is complicated due to various conflicting factors involved during decision making. To solve the purpose, Mr. Z consulted with some experts cum decision-makers (DMs) having IT backgrounds. The DMs evaluated the five laptops according to the fixed criteria. To select the best alternative, the evaluation procedure is executed as follows:

Step 1. Consider a set of five laptops be $X = \{x_1, x_2, x_3, x_4, x_5\}$ and a set of parameters be $E = \{e_i : i = 1,2,3,4,5\}$, where e_1 = size, e_2 = color, e_3 = price, e_4 = operating system, and e_5 = RAM. Based on the opinions of the DMs, the decision matrix of the set of five alternatives and five evaluation criteria under the IVIQNSS environment is shown in Table 2.

Table 2. Tabular representation of IVIQNSS to describe the five laptops

X/E	e_1	e_2	e_3	e_4	e_5
x_1	$\langle [0.25,0.45],[0.35,0.55], [0.15,0.35],[0.2,0.4] \rangle$	$\langle [0.2,0.4],[0.45,0.56], [0.35,0.37],[0.3,0.5] \rangle$	$\langle [0.15,0.25],[0.45,0.65], [0.18,0.25],[0.46,0.67] \rangle$	$\langle [0.45,0.48],[0.38,0.45], [0.26,0.32],[0.36,0.4] \rangle$	$\langle [0.46,0.55],[0.28,0.35], [0.45,0.55],[0.24,0.34] \rangle$
x_2	$\langle [0.16,0.40],[0.54,0.56], [0.23,0.42],[0.32,0.45] \rangle$	$\langle [0.26,0.30],[0.34,0.46], [0.3,0.4],[0.42,0.55] \rangle$	$\langle [0.36,0.42],[0.34,0.46], [0.13,0.32],[0.42,0.48] \rangle$	$\langle [0.36,0.42],[0.48,0.52], [0.24,0.41],[0.31,0.44] \rangle$	$\langle [0.24,0.35],[0.44,0.46], [0.33,0.42],[0.42,0.45] \rangle$
x_3	$\langle [0.3,0.4],[0.16,0.18], [0.24,0.54],[0.25,0.28] \rangle$	$\langle [0.12,0.24],[0.45,0.48], [0.34,0.42],[0.52,0.7] \rangle$	$\langle [0.14,0.26],[0.46,0.58], [0.14,0.34],[0.55,0.57] \rangle$	$\langle [0.21,0.32],[0.26,0.28], [0.35,0.55],[0.35,0.48] \rangle$	$\langle [0.26,0.42],[0.36,0.48], [0.44,0.54],[0.35,0.38] \rangle$
x_4	$\langle [0.34,0.38],[0.14,0.18], [0.25,0.28],[0.3,0.6] \rangle$	$\langle [0.5,0.6],[0.2,0.3], [0.4,0.5],[0.2,0.3] \rangle$	$\langle [0.24,0.28],[0.34,0.45], [0.35,0.38],[0.43,0.64] \rangle$	$\langle [0.14,0.18],[0.34,0.38], [0.55,0.58],[0.42,0.56] \rangle$	$\langle [0.54,0.58],[0.24,0.48], [0.35,0.38],[0.25,0.36] \rangle$
x_5	$\langle [0.76,0.78],[0.35,0.37], [0.13,0.16],[0.12,0.16] \rangle$	$\langle [0.56,0.68],[0.25,0.47], [0.33,0.36],[0.22,0.26] \rangle$	$\langle [0.66,0.68],[0.25,0.36], [0.23,0.36],[0.22,0.26] \rangle$	$\langle [0.16,0.38],[0.34,0.67], [0.23,0.26],[0.52,0.56] \rangle$	$\langle [0.23,0.38],[0.55,0.57], [0.23,0.36],[0.42,0.56] \rangle$

Step 2. The score of the truth-membership degrees $S_{T_{\varsigma(e_j)}}(x_i)$ for (ς, E) is shown in Table 3.

Table 3. Tabular representation of the score of truth-membership degree

X/E	e_1	e_2	e_3	e_4	e_5
x_1	−0.72	−0.5	−1.44	1.55	1.04
x_2	−1.42	−1.42	0.46	0.8	−1.06
x_3	−0.72	−2.06	−1.22	-0.45	−0.61
x_4	−0.62	1.64	0.24	−1.5	1.59
x_5	3.48	2.34	3.26	−0.41	−0.96

The score of the contradiction-membership degrees $S_{C_{\varsigma(e_j)}}(x_i)$ for (ς, E) is shown in Table 4.

Table 4. Tabular representation of the score of contradiction-membership degree

X/E	e_1	e_2	e_3	e_4	e_5
x_1	1.09	1.09	1.16	0.05	−1.06
x_2	2.09	0.04	−0.34	0.9	0.29
x_3	−1.71	0.69	0.86	−1.5	−0.01
x_4	−1.81	−1.56	−0.39	−0.5	−0.61
x_5	0.34	−0.36	−1.29	0.95	1.39

The score of the unknown-membership degrees $S_{U_{\varsigma(e_j)}}(x_i)for(\varsigma, E)$ is shown in Table 5.

Table 5. Tabular representation of the score of unknown-membership degree

X/E	e_1	e_2	e_3	e_4	e_5
x_1	0.25	0.17	0.53	0.85	−0.95
x_2	−0.5	0.27	0.43	0.5	0.3
x_3	−1.15	−0.03	0.28	−0.75	−0.85
x_4	0.1	−0.73	−0.97	−1.9	0.4
x_5	1.3	0.42	−0.27	1.3	1.1

The score of the false-membership degrees $S_{F_{\varsigma(e_j)}}(x_i)$ for (ς, E) is shown in Table 6.

Table 6. Tabular representation of the score of false-membership degree

X/E	e_1	e_2	e_3	e_4	e_5
x_1	0.08	0.03	−0.95	0.6	0.87
x_2	−0.77	−0.88	0.2	0.65	−0.58
x_3	0.43	−2.13	−0.9	0.25	0.12
x_4	−1.42	1.47	−0.65	−0.5	0.72
x_5	1.68	1.57	2.3	−1	−1.13

Step 3. By using Table 3 to Table 6, the score $\Omega_{\varsigma(e_j)}(x_i)$ for (ς, E) is exhibited in Table 7.

Table 7. Tabular representation of the score $\Omega_{\varsigma(e_j)}(x_i)$

X/E	e_1	e_2	e_3	e_4	e_5
x_1	0.7	0.79	−0.7	3.05	−0.1
x_2	−0.6	−1.99	0.75	2.85	−1.05
x_3	−3.15	−3.53	−0.98	−2.45	−1.35
x_4	−3.75	0.82	−1.77	−4.4	2.1
x_5	6.8	3.97	4	0.85	0.4

Step 4. Now, we calculate the overall score given as:

$$u_1 = 3.74, u_2 = -0.04, u_3 = -11.46, u_4 = -7, \text{and } u_5 = 16.02$$

Step 5. Thus, $u_k = \max_{x_i \in X}\{u_1, u_2, u_3, u_4, u_5\} = u_5$. Therefore, x_5 is the optimal choice object for the decision maker. If x_5 is not available in the market, then he/she will choose x_1.

Step 6. As there is no tie in the optimal choice so there is no need to reassess data in the given problem.

5. Conclusion and Scope

In this work, firstly, we give the idea of IQNS and IVIQNS. This article aims to develop a new soft model known as IVIQNSSs and investigate some of their fundamental properties and results. The IVIQNSS is a hybrid model, and it is formed by mixing the IVIQNSs (see definition 2.13) and the soft sets [2]. The proposed soft model is developed to address the issues that cannot be tackled by the existing soft models, such as FSSs, IFSSs, NSSs, QNSSs, etc. The present study is useful to provide a particular type of parametric uncertain information to the decision-maker. In IVIQNSS, every object of the universe is described by the truth-membership value $T_A(x)$, falsity-membership value $F_A(x)$ contradiction-membership value $C_A(x)$ and unknown membership value $U_A(x)$ with $T_A(x), F_A(x), C_A(x), U_A(x) \in int([0,1])$ under the condition $0 \leq \sup(T_A(x)) + \sup(F_A(x)) \leq 1$, $0 \leq \sup(C_A(x)) + \sup(U_A(x)) \leq 1$, and $0 \leq \sup(T_A(x)) + \sup(F_A(x)) + \sup(C_A(x)) + \sup(U_A(x)) \leq 2$. Then, we introduce some basic definitions and operations of IVIQNSSs. Some propositions and theorems of IVIQNSSs are developed. We also defined the score functions for the truth-membership, falsity-membership, contradiction-membership, and the unknown-membership. Then, define the total, overall, and accuracy scores for each alternative. Based on these score values, an algorithm is constructed for solving decision-making problems under the IVIQNSS environment. Finally, to validate the proposed algorithm, a real-world-based example is executed to show how it is useful in practical application.

Future works may involve the matrix representation of IVIQNSS for application in various medical diagnosis problems, parameterized reduction of the IVIQNSS for decision-making, necessity, and the possibility operators based on IVIQNSS, similarity measures and entropy on IVIQNSS, weighted operators based on IVIQNSS for decision-making, it can be applied to handle indeterminacy in various fields such as science, engineering, robotics, graph theory, computer science, game theory, and many more.

Author Contributions

The author read and approved the last version of the manuscript.

Conflicts of Interest

The author declares no conflict of interest.

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