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A NEW VIEW ON FIXED POINT

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Abstract. In this paper, we examine a view on fixed point with near soft mapping. First, we study the relationship between. soft mapping and almost smooth mapping. Also, the notion of near soft point, near soft mappings, a different approach to the study of near soft topological spaces. Shows how a near soft fixed point is derived from near soft topological spaces. Finally, many cases such as conservation of near soft compact topological spaces under near soft continuous mapping have been obtained.

1. Introduction

Near sets is a concept given by Peters [4] who deals with the proximity of objects. Here it causes the sample objects to be divided by the feature selection. The nearness of sets foundation on object definitions can be seen by introducing the near approximation space and finding nearby sets there.

The soft set concept, another concept proposed by Molodtsov [5], has been studied by many scientists [2, 6, 7, 8, 9, 10]. The soft sets and soft topological spaces and some of their related concepts have studied by Shabir and Hussain in [1, 2]. Wardowski [14], studied on a fixed points of soft mapping. The notion of near soft set emerges by considering the soft sets approximation and the near set theory as a common concept. Tasbozan [3] introduce the soft topology and sets based on a nearness approximation space. And many studies have been conducted on this subject [13, 11, 12]. The aim of this article is to create different concepts on nearness approximation space. In this study, we create the near soft point notion of near soft set and near soft mapping. These new concepts are explained with examples. The notions of near soft point, near soft topological space are described and their basic properties are explored with the help of examples. New definitions and theorems about near soft continuous mapping and near soft compactness have been obtained. Also, discuss the contrasting image and properties of an image in near soft mapping, based on the presented near soft element concept. In the last part

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of this study, a near soft compact Hausdorff topological space, near soft mapping, and a new fixed point result were created.

2. Near Soft Sets and Near Soft Topology

Definition 2.1. Let $(\mathcal{O}, \mathcal{F}, \sim_{Br}, N_r, \upsilon_{N_r})$ be a nearness approximation space(NAS) and $\sigma = (F, B)$ be a soft set(SS) over $\mathcal{O}$.

$N_r*((F,B)) = (N_r*(F(\phi) = \cup\{x \in \mathcal{O} : [x]_{Br} \subseteq F(\phi)\}, B))$ and $N_r^*((F,B)) = (N_r^*(F(\phi) = \cup\{x \in \mathcal{O} : [x]_{Br} \cap F(\phi) \neq \emptyset\}, B))$ are lower and upper near approximation operators. The SS $N_r((F,B))$ with $Bnd_{N_r(B)}((F,B)) \geqslant 0$ called a near soft set(NSS) [3].

Definition 2.2. Let $\mathcal{O}$ be an initial universe set, E be the universe set of parameters and $A, B \subseteq E$

(1) (F, A) is called a relative null NSS if $F(\phi) = \emptyset$, $\forall c \in A$.
(2) (G, B) is called a relative whole NSS if $G(\phi) = \mathcal{O}$, $\forall \phi \in B$. [3]

Definition 2.3. $(F, A)^c = (F^c, A)$ NSS is a complement of (F, A) if $F^c(\phi) = \mathcal{O} - F(\phi)$ $\forall \phi \in A$ [3].

Definition 2.4. Let (F, B) be a NSS over $\mathcal{O}$ and τ be the collection of near soft subsets NSs of $\mathcal{O}$, if if the following are provided

i): $(\emptyset, B)$, $(\mathcal{O}, B) \in \tau$
ii): $(F_1, B), (F_2, B) \in \tau$ then $(F_1, B) \cap (F_2, B) \in \tau$
iii): $(F_i, B), \forall \phi \in B$ then $\underset{i}{\cup}(F_i, B) \in \tau$

Then $(\mathcal{O}, \tau, B)$ is a near soft topological space($NSTS$) [3].

Definition 2.5. Let $(\mathcal{O}, \tau, B)$ be a $NSTS$ over $\mathcal{O}$, then the members of τ are said to be near soft open sets ($NSOS$) in $\mathcal{O}$. If its complement is open and a member of τ then a NSs of $(\mathcal{O}, \tau, B)$ is called near soft closed(NSC).

Definition 2.6. Let (F, B) be a NSS over $\mathcal{O}$. If for the element $\phi \in B$, $F(\phi) = \{x\}$ and $F(\phi^{'}) = \emptyset$, $\forall \phi^{'} \in B - \{\phi\}$ then NSS (F, B) is a near soft point (NSP), denoted by (x, ϕ).

Proposition 1. Let $(\mathcal{O}, \tau, B)$ be a $NSTS$ over $\mathcal{O}$, then the collection $\tau_\phi = \{F(\phi) : (F, B) \in \tau\}$ for each $\phi \in B$, defines a topology on $\mathcal{O}$.

Definition 2.7. Let $(\mathcal{O}, \tau, B)$ be a $NSTS$ over $\mathcal{O}$ and (F, B) be a NSS over $\mathcal{O}$. Then the near soft closure $(F, B)^c$ is the intersection of all NSC super sets of (F, B).

Definition 2.8. Let $(\mathcal{O}, \tau, B)$ be a $NSTS$ over $\mathcal{O}$ and (F, B) be a NSS over $\mathcal{O}$. Then the near soft interior $(F, B)^\circ$ is the collection of all $NSOS$ of (F, B).

Example 2.9. $\mathcal{O} = \{x_1, x_2, x_3\}, B = \{\phi_1, \phi_2\} \subseteq \mathcal{F}$ be denote a set of objects and a set of parameters respectively. Let (F, B) be a SS defined by $(F, B) = \{(\phi_2, x_2)\}$. Then $\sigma = (F, B)$ is a NSS with $r = 1$.

$$
\begin{aligned}
[x_1]_{\phi_1} &= \{x_1, x_2\}, [x_2]_{\phi_2} = \{x_2\} \\
[x_3]_{\phi_1} &= \{x_3\}, [x_1]_{\phi_2} = \{x_1, x_3\}
\end{aligned}
$$

Then $N_*(\sigma) = N_*(F(\phi), B) = (F_*(\phi), B) = \{(\phi_2, \{x_2\})\}$,$N^*(\sigma) = N^*(F(\phi), B) = (F, B)$ and $Bnd_N(\sigma) \geq 0$.Thus (F, B) is a NSS.

Then $\sigma = (F, B)$ is a NSS with $r = 2$.

$$[x_1]_{\phi_1,\phi_2} = \{x_1\}, [x_2]_{\phi_1,\phi_2} = \{x_2\}, [x_3]_{\phi_1,\phi_2} = \{x_3\}$$

$N^*(\sigma) = N_*(\sigma) = (F, B)$.Thus (F, B) is a NSS. Also $\phi_2 \in B, F(\phi_2) = \{x_2\}$ and $\phi_2' \in B - \{\phi_2\}, F(\phi_2') = \emptyset$. Thus (F, B) is a NSP and denote (x_2, ϕ_2) or $(x_2)_{\phi_2}$.

Definition 2.10. Let $(\mathcal{O}, \tau, B)$ be a $NSTS$ over $\mathcal{O}$. If there exists a $NSOS$ (G, B) such that $(x_\phi, B) \in (G, B) \subset (F, B)$ then a NSS (F, B) in $(\mathcal{O}, \tau, B)$ is a near soft neighbourhood of the NSP $(x_\phi, B) \in (F, B)$.

3. Near Soft Compactness, Near Soft Mapping and Its Fixed Points

In this section, we will give some definitions using NSP.

Definition 3.1. Let $(\mathcal{O}, \tau, B)$ be a $NSTS$ and $x, y \in \mathcal{O}$ such that $x \neq y$. $(\mathcal{O}, \tau, B)$ is a near soft Hausdorff space($NSHS$) if for each $NSOS$ $(F, D), (G, C) \in (\mathcal{O}, B)$ such that $x \in (F, D)$, $y \in (G, C)$ and $(F, D) \cap (G, C) = \emptyset$. Similarly for each NSP $(x_\phi, B), (y_{\phi'}, B) \in (\mathcal{O}, B)$ such that $(x_\phi, B) \neq (y_{\phi'}, B)$ there are $NSOS$ $(F_1, B), (F_2, B) \subset (\mathcal{O}, B)$ so that $(x_\phi, B) \in (F_1, B), (y_{\phi'}, B) \in (F_2, B)$ and $(F_1, B) \cap (F_2, B) = (\emptyset, B)$.

Definition 3.2. Two NSS (F, B) and (G, B) in $(\mathcal{O}, B)$ are near soft disjoint denoted by $(F, B) \cap (G, B) = (\emptyset, B)$, if $F(\phi) \cap G(\phi) = \emptyset$, $\forall \phi \in B$.

Definition 3.3. Two NSP (x_ϕ, B) and $(y_{\phi'}, B)$ over a common universe $\mathcal{O}$ are distinct, written $(x, \phi) \neq (y, \phi^{'})$ if there corresponding NSS (F, B) and (G, B) are disjoint.

Definition 3.4. Let $(\mathcal{O}, \tau, B)$ be a $NSTS$ and a NSS $(V, B) \subseteq (\mathcal{O}, B)$ is near soft open $\Leftrightarrow$ for each a NSS $(W, B) \in \tau$ which $\alpha \in (W, B) \subseteq (V, B)$.

Definition 3.5. Let $(\mathcal{O}, \tau, B)$ be a $NSTS$ and $G \subseteq \mathcal{O}$. The near soft topology (NST) on (G, B) incited by the NST τ is the family τ_G of the NSs of G of the shape $\tau_G = \{V \cap G : V \in \tau\}$. Thus (G, τ_G, B) is a near soft topological subspace of $(\mathcal{O}, \tau, B)$.

Definition 3.6. Let $(\mathcal{O}, \tau, B)$ be a $NSTS$ and $C \subseteq \mathcal{O}$. If $(C, B) \subseteq \cup_{i\in I}(V_i, B)$ then $\{V_i\}_{i\in I} \subseteq \tau$ is a NSO cover of (C, B).

Definition 3.7. If for each NSO cover $\{V_i\}_{i\in I}$ of (C, B) there exists $i_1, i_2, ..., i_k \in I$, $k \in N$ such that $(C, B) \subseteq \cup_{n=1}^{k}(V_{in}, B)$ then (C, τ, B) is a near soft compact space ($NSCoS$).

Definition 3.8. Let $(\mathcal{O}, \tau, B)$ be a $NSTS$ and $C \subseteq \mathcal{O}$. If the (G, τ_G, B) is $NSCoS$ then the (G, B) NSS is compact in $(\mathcal{O}, \tau, B)$.

Definition 3.9. Let $(\mathcal{O}, \tau, B)$ be a $NSHS$. Then every $NSCo$ set in $(\mathcal{O}, \tau, B)$ is NSC in $(\mathcal{O}, \tau, B)$.

Proof. Let (C, B) be a $NSCo$ set in $(\mathcal{O}, \tau, B)$ and $x \in C^{'}$. For every $y \in C$ let $x, y \in \mathcal{O}$ and $x \in (F, D), y \in (G, P), (F, D), (G, P) \in (\mathcal{O}, B)$ such that $(F, D) \cap (G, P) = \emptyset$. From the near soft compactness of (C, B) there exists $y_1, y_2, ..., y_k \in (C, B)$ such that $(C, B) \subseteq (G_1, P) \cup ... \cup (G_k, C)$. Denote $(F, D) = (F_1, D) \cup ... \cup (F_k, D)$ and $(G_1, P) \cup ... \cup (G_k, C) = (G, P)$ then $F \cap G = \emptyset$ and thus $F \cap C = \emptyset$, which $x \in F \subseteq C^{'}$ thus (C, B) is NSC. □

Definition 3.10. $NS(\mathcal{O},B)$ denotes the family of all NSS over $(\mathcal{O},B)$. Let $(F,A),(G,C) \in NS(\mathcal{O},B), A,C \subseteq B$. The near soft cartesian product $(F,A) \times (G,C)$ is a NSS on $(\mathcal{O},B)\times(\mathcal{O},B)$ such that $(F,A)\times(G,C) = \{((\phi_1,\phi_2), F(\phi_1)\times G(\phi_2)) : \phi_1,\phi_2 \in B\}$

Definition 3.11. A near soft relation(NSR) from (F,A) to $(G,C)$is a NSS (R,B) , $R \subseteq (F,A)\times(G,C)$ with

$$(R,B) = \{((\phi_1,\phi_2), \mathcal{O}(\phi_1)\times\mathcal{O}(\phi_2)) : \phi_1,\phi_2 \in B, \mathcal{O}(\phi_1) \subseteq F(\phi_1), \mathcal{O}(\phi_2) \subseteq G(\phi_1)\}$$

If $((\phi_1,\phi_2), \mathcal{O}(\phi_1)\times\mathcal{O}(\phi_2)) \in (R,B)$ then $(\phi_1, \mathcal{O}(\phi_1)R(\phi_2,\mathcal{O}(\phi_2))$.

Definition 3.12. Let $(F,A),(G,C) \in NS(\mathcal{O},B)$. If the following conditions satisfied then a NSR $f \subseteq (F,A)\times(G,C)$ is a NSM denoted by $f : (F,A) \to (G,C)$;

(1) For each NSP $\alpha = (x_e, A) \in (F,A)$ there exists only one NSP $\beta = (f(x)_e, A) \in (G,C)$ such that $f(\alpha) = \beta$ or $\alpha f \beta$.
(2) For each empty NSP $\alpha \in (F,A), f(\alpha)$ is an empty NSP of (G,C) .

Definition 3.13. Let $(F,A),(G,C) \in NS(\mathcal{O},B)$ and $f : (F,A) \to (G,C)$ be a NSM.

(1) The view of $X \subseteq F$ under (NSM) f is the NSS of $(f(X),C) = (\cup_{\alpha\in X} f(\alpha), C)$ and for each NSM $(f(\emptyset),B) = (\emptyset,B)$.
(2) The inverse of $Y \subseteq G$ under NSM f is the NSS of $(f^{-1}(Y),A) = (\cup\{\{\alpha\} : \alpha \in (F,A), f(\alpha) \in (Y,C)\}, B)$.

Definition 3.14. Let $(F,B),(G,B) \in NS(\mathcal{O},B)$. $(W,B),(W_1,B),(W_2,B) \subseteq (F,B)$,$(Z,B),(Z_1,B),(Z_2,B) \subseteq (G,B)$ and let $f : (F,B) \to (G,B)$ be a (NSM). Then the following hold:

(1) $W_1 \subseteq W_2 \Rightarrow f(W_1) \subseteq f(W_2)$
(2) $Z_1 \subseteq Z_2 \Rightarrow f^{-1}(Z_1) \subseteq f^{-1}(Z_2)$
(3) $W \subseteq f^{-1}(f(W))$
(4) $f(f^{-1}(Z)) \subseteq Z$
(5) $f(W_1 \cup W_2) = f(W_1) \cup f(W_2)$
(6) $f(W_1 \cap W_2) \subseteq f(W_1) \cap f(W_2)$
(7) $f^{-1}(Z_1 \cup Z_2) = f^{-1}(Z_1) \cup f^{-1}(Z_2)$
(8) $f^{-1}(Z_1 \cap Z_2) = f^{-1}(Z_1) \cap f^{-1}(Z_2)$

Definition 3.15. Let $(F,\tau,B),(G,\upsilon,B)$ be a $NSTS$ and $f : (F,B) \to (G,B)$ be a NSM. If $\forall V \in \upsilon, f^{-1}(V) \in \tau$ then f is a near soft continuous mapping and denoted by $NSCM$.

Definition 3.16. Let $(g,h) : (F,B) \to (G,B')$ be a NSM. A NSM (g,h) is an injective, surjective and bijective if g,h are both injective, surjective and bijective, respectively.

Definition 3.17. Let $(\mathcal{O}_k,\tau,B)$ and $(\mathcal{O}_l,\tau,B)$ be two $NSTS$. $f : (\mathcal{O}_k,\tau,B) \to (\mathcal{O}_l,\tau,B)$ be a mapping. For each near soft neighbourhood (H,B) of $(f(x)_\phi,B)$, if there exists a near soft neighbourhood $f((F,B)) \subset (H,B)$ then f is a $NSCM$ (x_ϕ,B). If f is $NSCM$ for all (x_ϕ,B), then f is called $NSCM$.

Definition 3.18. Let $(\mathcal{O}_1,\tau,B)$ and $(\mathcal{O}_2,\tau,B)$ be two $NSTS$. $f : \mathcal{O}_1 \to \mathcal{O}_2$ be a mapping. $\mathcal{O}_1$ is near soft homeomorphic to $\mathcal{O}_2$ if f is a bijection, NSC and f^{-1} is a near soft homeomorphism.

Example 3.19. $\mathcal{O} = \{x_1, x_2, x_3\}, B = \{\phi_1, \phi_2\} \subseteq \mathcal{F}$. Let (F, B) be a NSS defined by $(F, B) = \{(\phi_2, \{x_1, x_2\}), (\phi_2, \{x_2, x_3\})\}$. Then $\sigma = (F, B)$ is a NSS with

$$\begin{aligned} [x_1]_{\phi_1} &= \{x_1, x_2\}, [x_2]_{\phi_2} = \{x_2\} \\ [x_3]_{\phi_1} &= \{x_3\}, [x_1]_{\phi_2} = \{x_1, x_3\} \end{aligned}$$

. And think τ of $NSSs$ of (F, B);

$$\tau = \{\emptyset, (\phi_2, \{x_2\}), (F, B), \{(\phi_1, \{x_1, x_2\}), (\phi_2, \{x_2\})\}\}$$

Then (F, τ) is a $NSTS$. Now taking a NSM $f : (F, B) \to (F, B)$ as follows:

$$\begin{aligned} f(\phi_1, \emptyset) &= (\phi_2, \emptyset), f(\phi_2, \{x_2\}) = (\phi_2, \{x_2\}) \\ f(\phi_2, \emptyset) &= (\phi_1, \emptyset), f(\phi_1, \{x_1, x_2\}) = (\phi_1, \{x_1, x_2\}) \\ f(\phi_3, \emptyset) &= (\phi_3, \emptyset), f(F, B) = (F, B) \end{aligned}$$

Then $f^{-1}(v) \in \tau, \forall v \in \tau$ then f is a $NSCM$.

Proposition 2. Let (C, τ, B) be a $NSCoTS$ and let $f : (C, B) \to (C, B)$ $(NSCM)$. Then $f(C)$ is a $(NSCo)$ in (C, τ, B).

Proof. Let $\{V_i\}_{i \in I} \subseteq \tau$ which $f(C) \subseteq \cup_{i \in I} V_i$. From the near soft continuity of $f, \{f^{-1}(V_i)\}_{i \in I}$ is a family of $(NSOS)$. Then $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(\cup_{i \in I} V_i) = \cup_{i \in I} f^{-1}(V_i)$ and from $NSCo$ of C there exists $i_1, i_2, ..., i_k \in I, k \in N$ which

$$\begin{aligned} C &\subseteq f^{-1}(V_{i1}) \cup f^{-1}(V_{i2}) \cup ... \cup f^{-1}(V_{ik}) \\ f(C) &\subseteq (V_{i1}) \cup (V_{i2}) \cup ... \cup (V_{ik}) \end{aligned}$$

Thus $f(C)$ is a $NSCo$. □

Definition 3.20. Let (F, B) be a NSS and $f : (F, B) \to (F, B)$ be a NSM. If $f(\alpha) = \alpha$ then a NSP $\alpha \in (F, B)$ is a fixed point of f.

Theorem 3.21. *Let (C, τ) be a $NSCoHTS$ and let $f : (C, B) \to (C, B)$ be a $NSCM$ such that:*

(1) for each nonempty NSP $\alpha \in (C, B)$, $f(\alpha)$ is a nonempty NSP of (C, B),
(2) If $f(X, B) = (X, B)$then only one nonempty NSP $\alpha \in (C, B)$ which $f(\alpha) = \alpha$, for each NSC set $(X, B) \subseteq (C, B)$.

Example 3.22. Let $f : (F, B) \to (F, B)$ be a NSM defined in example 35. Then the NSP $(\phi_2, \{x_2\}), (\phi_3, \emptyset), (\phi_1, \{x_1, x_2\})$ are fixed points of f.

4. Conclusions

In this study, we describe the notion of NSM and its fixed point. In the near soft topological space, we tried to create a fixed point structure with near soft mapping, which we created based on the concept of near soft point. Expressions explaining these concepts and showing the necessity of some assumptions are presented. With a different approach to the near soft cluster, it will facilitate the solution of many problems and will help new studies.

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