Bartın Üniversitesi
Eğitim Fakültesi Dergisi
Cilt 5, Sayı 2, s. 577-597, Haziran 2016
BARTIN – TÜRKİYE

ISSN: 1308-7177

Bartin University
Journal of Faculty of Education
Volume 5, Issue 2, p. 577-597, June 2016
BARTIN – TURKEY

Doi: 10.14686/buefad.v5i2.5000180307

Pre-Service Mathematics Teachers' Problem Solving Processes with Geometer's Sketchpad: Mirror Problem

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Abstract: Problem solving skill is the core of mathematics education and its importance cannot be denied. This study examined 56 freshmen pre-service mathematics teachers' problem solving processes on a particular problem by means of Geometer's Sketchpad (GSP). They were grouped into pairs to solve a problem called "the mirror problem". According to their works on GSP and related reflections, it was observed that there were two different solution methods for the problem. The results of the study revealed that pre-service teachers could not visualize the problem in their mind and apply it to GSP. In general, they found the problem hard to solve. They experienced difficulty in transferring the static drawings into dynamic environment and in observing the manipulations on GSP.

Key words: problem solving processes, pre-service mathematics teachers, Geometer's Sketchpad, symmetry, technology

Matematik Öğretmen Adaylarının Geometer's Sketchpad ile Problem Çözme Süreçleri: Ayna Problemi

Özet: Problem çözme becerileri matematik eğitiminin özünü oluşturmaktadır ve matematik eğitiminde önemi yadsınamaz. Bu çalışma, Geometer's Sketchpad (GSP) yardımıyla özel bir problem üzerinden 56 birinci sınıf öğretmen adayının problem çözme becerilerini incelemektedir. Öğretmen adayları, "ayna problemi" adında bir problemi çözmeleri için ikişerli gruplara ayrılmışlardır. GSP üzerinde yaptıkları çalışmalar ve verdikleri ilgili yorumlarına göre, problem için iki farklı çözüm metodu ortaya çıkmıştır. Yalnız, çalışmanın sonuçları öğretmen adaylarının problem zihinlerinde canlandıramadıkları ve bunu GSP'de uygulayamadıklarını ortaya çıkarmıştır. Genel olarak, öğretmen adayları problemin çözümünü zor bulmuşladır. Öğretmen adayları statik çizimleri dinamik ortama aktarmada ve GSP'deki sürükleme özelliğini gözlemlemede zorluklar yaşamışlardır.

Anahtar Kelimeler: problem çözme süreçleri, matematik öğretmen adayları, Geometer's Sketchpad, simetri, teknoloji

1. INTRODUCTION

Problem and problem solving have been trend topics in mathematics education for many years (e.g., Awofala, 2014; Schoenfeld, 1985; 1992; Silver, 1994; Yavuz, Arslan, & Gülten, 2010). Main reasons lie behind the significance of the definitions of the terms explained. As it is well known that problems are specific tasks that necessitate steps to get the solution (Polya, 1949). Problem solving is core of any ideas of functionality with mathematics and has a special importance in the study of mathematics as well. About this issue, Lesh and Zawojewski (2007, p. 782) defined mathematical problem solving as:

> "The process of interpreting a situation mathematically, which usually involves several iterative cycles of expressing, testing, and revising mathematical interpretation – and of sorting out, integrating, modifying, revising or refining clusters of mathematical concepts from various topics within and beyond mathematics"

Besides, Wilson, Fernandez, and Hadaway (1993) mentioned that the art of problem solving is the heart of mathematics. Specifically, it means engaging in a task for which the solution is not known in advance. Without the ability to solve problems, the advantage and power of mathematical ideas, knowledge and skills are severely limited (NCTM, 2000).

Given a task, problem solvers need to follow some processes. In this study, Lesh and Zawojewski's (2007) definition of problem solving processes was taken as bases to investigate pre-service mathematics teachers' problem solving processes on a real-life problem. In parallel with Lesh and Zawojewski's (2007) definition, Iranzo and Fortuny (2011) stated that problem solvers firstly notice the possible solution strategies, choose the optimum one, test and revise the strategies and scaffold a solution at the end. Based on the solution constructed, it requires to direct the problem solvers from mathematical argumentation to logical deduction. This process also gives problem solvers opportunities to diagnose their mistakes and difficulties in solving problem (Healy & Hoyles, 2001).

With the particular characteristics, some problems are separated from each other. In addition to arithmetic or algebraic operations, the real-life problems necessitate daily life experiences or their simulations. Moreover, there is a need to look from different perspectives for solving such kind of problems. (Baki, 2008; Channon & Crawford, 1999). With this respect, mirror problem can also be considered as real-life problem because it cannot be solved only with arithmetic and algebraic operations. In addition, the problem solvers consider their life experiences to produce solution strategies and they can reach solution with alternative ways.

This study particularly draws attention to the analysis of pre-service elementary mathematics teachers' problem solving processes on a given real-life problem by means of GSP. The problem given as a task to pre-service mathematics teachers was the mirror problem.

1.1. GSP as Educational Tool

In parallel with the developing technology, problem solving tools have become diversified. One of these tools is GSP software giving chance to visualize modeling variations (Idris, 2009; Leong, 2001). It is a dynamic mathematics environment used for exploring algebra, geometry, calculus, and other areas of mathematics and sciences (NCTM, 2005). Dynamic geometry systems, as Goldenberg (2000) stated, offer students a richer and deeper understanding of mathematical topics and also help students improve their problem solving skills. GSP provides students to visualize the geometric shapes and solids in their mind. Besides, GSP fosters the learners' constructions and ways of thinking. According to Kerrigan (2002), using mathematics software promotes students' higher order thinking skills, and also develops and maintains their computational skills.

There are some advantages of using technology in education such as visualizing mathematical contents, its being fast and practical, storage of information, avoiding loss of time (Halat, 2007; Kağızmanlı & Tatar, 2012; Saltan, Arslan & Gök, 2010). The studies related to GSP in the literature validate the educational impacts of these advantages. In the conducted studies, using technology properly (Yiğit, 2014) and especially using GSP have affected students' attitude towards mathematic lessons positively (O'Donnell, 2011). Likewise, using technology in education and using GSP influence students' success positively on some subjects in mathematics like functions (Eu, 2013) and geometry (Dimakos & Zaranis, 2010). Furthermore, studies conducted reveal that there is a high positive correlation between GSP applications and students' problem-solving skills (Koyuncu, Akyüz, & Çakıroğlu, 2015). On the other hand, there are also some disadvantages. For example, technical problems and problems related to using technology of teachers and students (Engin, Tösten & Kaya, 2010) are some of them. Those restrictions can be prevented considerably by taking only essential precautions.

1.2. Problem Solving and GSP

Problem solving is thought as the basis of mathematics and the main purposes of mathematics courses are to make students well-informed about mathematical knowledge and skills and also help them solve real-life problems (Baki, 2008). The main characteristic of a problem is that students should not know the solution, so its solution should not be obvious for students (MoNE, 2013). There is a need for students to construct strategies for correct solution. Therefore, undergoing higher order thinking skills are needed to be activated while studying on a problem (Ersoy & Başer, 2013), because real-life problems requires to look it from different perspectives and necessitates different solution strategies (Artigue & Blomhøj, 2013). As the GSP has opportunities of retrying (trial and error), visualizing and property of dragging (Hollebrands, 2007), real-life situations (or problems) can be simulated in such dynamic environment. Especially, the dragging property gives students opportunities to see the different conditions of any real-life situation (or problem) modeled after dragging a point, a line or combination of them. Therefore, students can investigate different conditions of it and if there are mistakes on their constructions, they can observe the new conditions (Hollebrands, 2007). Considering the educational purpose of solving real-life problems, GSP is appropriate environment for these activities. By using dynamic geometry environments, the students can seek for additional solutions for the problems imposed and they get deeper understanding of the tasks that they work (Koyuncu, Akyüz, & Çakıroğlu, 2015).

Integrating technology into problem solving tasks, Laborde, Kynigos, Hollebrands, & Strässer (2006) mentioned that four different geometric tasks can be used by means of dynamic geometry environments including GSP. These tasks also cover the mathematical problems asked in the classrooms. These were as follow:

> • "Tasks in which the environment facilitates the material actions but does not change the task for the students, for example, producing figures and measuring their elements.
>
> • Tasks in which the environment facilitates students' exploration and analysis, for example, identifying relations within a figure through dragging,
>
> • Tasks that have a paper and pencil counterpart but can be solved differently in the environment, for example a construction task may be solved in dynamic geometry environment by using a geometric transformation,
>
> • Tasks that cannot be posed without the mediation of the environment, for example, reconstructing a dynamic diagram through experimenting with it in order to identify its properties" (p. 293).

In general, the real-life problem solving activities by using dynamic geometry environments are similar to second and third tasks that Laborde, Kynigos, Hollebrands, & Strässer (2006) proposed. For example, students need to use dragging property to see the relations among shapes and figures while studying on a real-life problem. In mirror problem, specifically, students need to know symmetry property which can be applied by using a geometric transformation in dynamic geometry environment.

1.3. Teachers' Role in Problem Solving and GSP

Although there are various advantages of using GSP in problem solving processes, it is meaningless without influential practitioners due to possible disadvantages and ineffective usage of the software (Çıldır, 2012). Teachers should take this position by appropriately mediating it in classrooms. According to NCTM (2000), teachers are expected to provide well-designed activities, appropriate tools, and support to the students. Besides, NCTM (2000) seeks teachers to prepare students for the challenges of a new technological world by becoming mathematical problem solvers through developing their own ability to think mathematically and to acquire mathematical power. In addition, teachers play a critical role in the success of the students and in developing students' problem solving dispositions, so they should choose problems that engage students (NCTM, 2000). In an attempt to decrease students' difficulties during problem solving processes, first of all, the level of teachers' problem solving skills should be revealed. When education is thought as a problem solving process, teachers should also have strong problem solving skills (Yavuz, Arslan, & Gülten, 2010). However, many teachers don't have different thinking strategies that require problem solving experiences; they generally have experience in algorithmic problem solving. This causes students' problem solving process to be difficult, so teachers' experiences should be enriched and improved (Chapman, 1999). Due to the increased emphasis on problem solving in mathematics education, the analysis of problem solving processes and pre-service teachers' proficiencies in this subject have become significant (Kayan & Çakıroğlu, 2008). Christou, Mousoulides, Pittalis, & Pitta-Pantezi (2005) emphasized the importance of use of dynamic geometry environment in using problem strategies and developing high level problem solving skills. Beside, teachers' problem solving processes through dynamic geometry software should be examined. For example, Özen and Yavuzsoy-Köse (2013) mentioned teachers' role in dynamic geometry environment by stating that

> "...to support and encourage students to discover the mathematical concepts and relations, to construct their own conjectures and justify them with reasoning. In this way, they can choose appropriate problems and develop a variety of strategies and perspectives so that their future students can learn mathematical concepts (p. 62)."

Goldenberg, Harvey, Lewis, Unniker, West and Zodhiates (1988) argue that providing the opportunities and dynamic tools for students' explorations promote the habits of mind that constitute true mathematical power. Therefore, teachers can use dynamic geometry software to enhance students' learning opportunities by creating mathematical tasks such as problem solving activities (Furner & Marinas, 2007).

The teachers play key role in designing problem solving activities in classroom (MoNE, 2013). Keeping Özen and Yavuzson-Köse's (2013) statements in mind, how teachers' problem solving processes and skills through dynamic geometry environment are improved influence growth in students' problem solving skills. In fact, the studies related to teachers' problem solving processes in such environments are limited (e.g., Özen & Yavuzsoy-Köse, 2013), there is a need for more studies related to such issue. With this regard, this study may contribute to the literature in the way of investigating the future mathematics teachers' problem solving processes.

1.4. Related Literature

In the literature, there were many studies investigating the problem solving activities in dynamic geometry environment. Based on Polya's (1945) problem solving process, Poon and Wong (2011) analyzed an optimization problem in geometry via Geogebra. In addition to its being real-life problem, solving such problem requires to make an association among different mathematical and interdisciplinary subjects and abilities. This study indicated that dynamic geometry software is appropriate as a problem solving tool to sustain students' problem solving processes.

Problem solving in dynamic geometry environment also influence students' problem solving preferences during solution processes. Although dynamic geometry software could be used as a problem solving tool, students' preferences were dominant on using paper-pencils as solution preferences than using visual environment (Coşkun, 2011) such as GSP. However, regardless of students' visual or non-visual preferences in solving problems, dynamic geometry environment forced students to construct more visual solution strategies (Coşkun, 2011).

In the experimental study conducted by Köse, Tanışlı, Erdoğan and Ada (2012), the uses of dynamic geometry software and paper-pencil on geometric construction problems were compared. The study showed that the use dynamic geometry software improved students' reasoning skills and the solution strategies. In addition, the former method was found more effective than using paper-pencil on the processes of solving geometric construction problem. They attributed these results to software's property of dragging, which allows students to do many trials (trial and error) to reach correct solution.

In addition to studies conducted to K-12 students, mentioning about the studies with pre-service mathematics teachers as participants plays crucial role in emphasizing the significance of the present study. For example, Özdemir and Reis (2013) also indicated that using dynamic software allowed to present multiple representations in solving problems. This influenced mathematics pre-service teachers' perceptions about problem solving processes in positive way (Özdemir & Reis, 2013). Moreover, Çiftçi and Tatar (2014) compared the achievement levels of pre-service mathematics teachers' fundamental geometry constructions. According to the findings in this study, the achievement levels of those who used dynamic geometry environments were significantly higher than that of those who used traditional construction materials. In addition, the former pre-service teachers stated that their constructions were more visual and they had opportunity to see the different orientations of their constructions by means of dragging property.

Despite its widespread applications including problem solving processes, relation with multiple representations (Özdemir & Reis, 2013) and basic geometry constructions (Köse, Tanışlı, Erdoğan, & Ada, 2012), there were also many applications relating the real-life problems and dynamic geometry environments in the literature (e.g., Christou, Mousoulides, Pittalis, & Pitta-Pantezi, 2005; Poon & Wong, 2011). For example, Christou, Mousoulides, Pittalis and Pitta-Pantazi (2005) stated in their study that dynamic geometry software is helpful tool for solving real-life problems, construction models for real-life situations, reaching logical reasoning and doing generalization. In addition, they expressed the mediating role of dynamic geometry environments to take students' attentions with their features of dragging and measuring while solving real-life problems.

Considering the pre-service mathematics teachers' problem solving processes, this study looks from different perspective of using GSP in mathematics education. The previous studies mentioned were related to the effect of dynamic geometry environment on students' or pre-service teachers' achievements or their perceptions about it. On the other hand, the perspective that this study follow was to investigate the ongoing processes that pre-service teachers followed when they were working on a real-life problem.

1.5. Purpose of the Study

Understanding pre-service teachers' problem solving processes may help to improve their students' abilities and their approaches to the solution of problems (Demircioğlu, Argün, & Bulut, 2010). In addition, this study presents a practical application for problem solving activities by using dynamic geometry environments in classrooms. At this point the purpose of the study coincides with it. In this study, the purpose was to investigate pre-service elementary mathematics teachers' problem solving processes in line with "the mirror problem" by means of GSP. For this study, the reason of choosing this problem is two folds. Firstly, this problem is a kind of real-life problem. The national and international curricula impose using real-life and multidisciplinary problems in classrooms (MoNE, 2013; NCTM, 2000) and the pre-service teachers are the future's first practitioner of the curriculumin real classroom environments, so students can use their mathematical knowledge and abilities while solving them (Blum & Leiß, 2007). Secondly, this problem necessitates using the math and physics knowledge together. One of the characteristics of effective mathematical problem is being multidisciplinary, so it helps students to construct comprehensive and flexible knowledge (Baki & Şahin, 2004). Based on this purpose, the sub-research questions are given below:

- What solution ways the pre-service teachers come up with while solving the mirror problem?
- What were their mistakes and possible reasons while solving the mirror problem?
- What were the participants' reflections about their solutions?

2. METHOD

This study follows qualitative paradigm. Single case study design was used in this study. Case study examines one or more than one fact, environment or other systems which are connected deeply (McMillan & Schumacher, 2006) and its aim is to analyze one or more than one situation in its own holistic limits (Yıldırım & Şimşek, 2008). Creswell (2007) states that "a case study research is a qualitative approach in which the researcher explores a real-life, contemporary bounded system (case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information and reports a case description and case-based themes" (p.97). With this respect, the present study is a single case study, the purpose of which is to do in-depth investigation of pre-service teachers' problem solving processes on a real-life problem (mirror problem) in dynamic geometry environment, GSP.

2.1. Participants

This study was carried out in spring term during 2011-2012 academic year with 56 freshmen (21 males and 35 females) pre-service elementary mathematics teachers in a university located in the east part of Turkey. That was their second semester in their university education. Participant selection was done with convenience sampling method which is non-probability sampling method. Convenience sampling method is used when the sample is close and easy to come by (Yıldırım & Şimşek, 2008).

In this study, one bounded system (one case) was investigated. The case was all freshmen pre-service elementary mathematics teachers enrolled in this course. Pre-service teachers were randomly paired in groups. Therefore, there were 28 pairs in this study.

2.2. Data Collection Tools and Procedures

Pre-service teachers' problem solving processes on the mirror problem were investigated through their works in GSP platform and their reflections to the solution of this problem. Within the scope of Computer course, participants learnt how to use Geometers Sketchpad (GSP) software and achieved problem solving with GSP and they prepared mathematics activities with GSP. In this regard, it is accepted that participants learnt about GSP software and its basic features in the first two weeks. Then, during six weeks, participants were given problems and were asked to solve these problems with GSP. In general, the problems to be solved began easier ones to harder ones which required visualizing what students had in their minds about the solution. Before solving these problems, students studied on (for example) the proof of Pythagoras theorem and working principles of analog clocks by using GSP. So, they needed to use several features of GSP. Besides, at the end of each lesson participants were asked to write their solutions and reflections including how they organized the solution and their ideas about the problem. In the following lessons, participants were given feedback for the previous problems. The problem analyzed in this study was one of the problems used during the course.

In this study, mirror problem (a kind of real-life problem) that can be solved through different problem solving approaches was chosen. Moreover, this mirror problem addresses various disciplines. In addition to other problems asked during the course, this problem required the knowledge of physics and mathematics, together. Different solution methods can be applied to solution. In this problem, students needed to think deeply about the solution, to choose the most appropriate perspective and to visualize it by using GSP. Moreover, they needed to simulate the real-life situation into GSP environment. With the perspective chosen in the solutions, the results showed students' problem solving processes in this problem.

After participants were grouped into pairs, they were asked to solve "the mirror problem" in computer lab within 60 minutes. There were 28 pairs in the application. The groups were expected to solve the following question on GSP environment.

For a person who wants to see himself completely in front of the mirror,

a) What should a mirror's height be?

b) What should be the distance between the mirror and the person?

c) What should be the altitude of the mirror from the ground?

Since this problem requires considering different situations such as the length of mirror and the distance between the mirror and the person, there was a need for the participants to use dynamic environment like GSP. So, the participants could manipulate the variables in the questions and observe the differences. Pre-service teachers' problem solving processes can be investigated with this problem.

At the end, pairs delivered the GSP files to researchers. Moreover, they were supposed to write their reflections about the activity. The reflection sheet should include their general ideas as well as procedures that they follow to solve the problem, their possible mistakes, usage of GSP.

2.3. Data Analysis

The data gathered from GSP files and reflection sheets were analyzed according to descriptive analysis method in order to make the data more meaningful. First of all, each pair's works on GSP and their reflection sheets about the problem solving processes were analyzed by at least two researchers. According to researchers' analysis, two solution methods were found. In order to ensure the internal validity, an external researcher who had experience in dynamic geometry software also analyzed the data. After the consensus was provided among the researchers, the findings were reported. In addition, Miles and Humerman's (1994) inter-rater reliability coefficient was utilized to ensure reliability. Considering Lesh and Zawojewski's (2007) and Iranzo and Fortuny's (2011) definitions of problem solving processes, the present study investigated the pre-service teachers' alternative solution ways to the problem as a whole and to each part, the methods that they use during solving it and diagnosis of their mistakes and the possible reasons. In the mirror problem, there were three sub-problems which give information about teachers' problem solving processes. These sub-problems were related to each other. Pre-service teachers need to observe them and give answers accordingly. Based on each step, pre-service teachers' answers were compared and common answers were reported. Therefore, pre-service teachers' problem solving processes were analyzed according to the alternative ways of solutions, their mistakes and the possible reasons for them. Having inspiration from and adopting Lesh and Zawojewski's (2007) and Iranzo and Fortuny's (2011) definitions of problem solving processes, what processes the pre-service mathematics teachers' problem solving processes followed was analyzed according to the chart shown in the Figure 1 below.

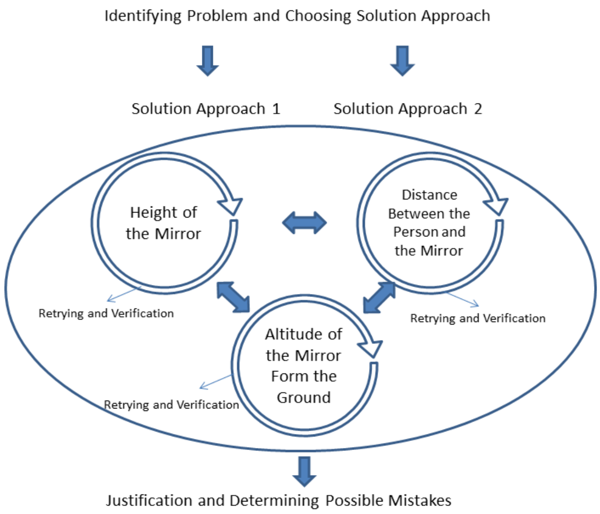


Figure 1. Problem solving processes adopted from Lesh and Zawojewski (2007) and Iranzo and Fortuny (2011)

Operationally, it was expected that participants begin with the phase of identifying and understanding the problem given at the beginning of the problem solving process adopted. Then, they were expected to design an appropriate approach to solve the problem. In the data analysis, what solution approaches that participants followed were identified from their GSP files and how they proceeded in such approaches were explained. In the cycle of interrelated sub-problems, the frequencies of correct and incorrect answers were determined for each part. By means of descriptive analysis, what processes they followed during reaching their correct answers and mistakes during retrying and verifying their works were presented in line with their GSP files and statements they proposed in them. Analysis of problem solving processes continued with whether they were confident about the correctness of their solutions descriptively. This was the justification phase of the process. Their ideas were supported with their statements about their answers.

For each question in the problem, the pre-service teachers' answers were compared. Based on agreements and disagreements, the inter-rater reliability coefficients were found to be 90%, 92%, 92% for a, b, and c parts of the problem, respectively. These values were sufficient for the reliability of study (Miles & Huberman, 1994). At the end, the frequency and percentage tables were constructed to present the findings visually.

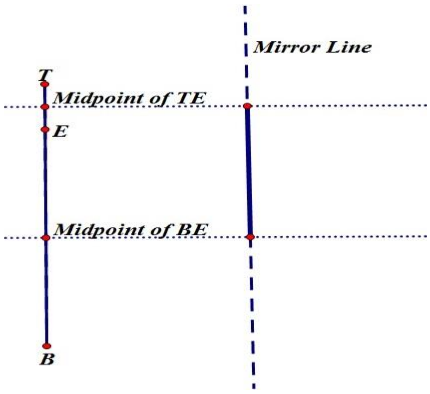
3. RESULTS

In this part of the study, the findings from the participants' solutions to "the mirror problem" and their reflections about their solutions are presented. In this study, two different approaches were observed for solving the mirror problem. After adopting a solution approach, participants entered the cycle of solving sub-problems. Each part of the problem was interrelated with each other. The process continued with cycle of retrying and verifying what was done in participants' solutions for each sub-problem. Therefore, according to the solution approach adopted, participants retried and verified their solutions for the height of the mirror, the distance between the person and the mirror, and the altitude of the mirror, separately. At the end, they justified their solution and determined their difficulties and mistakes.

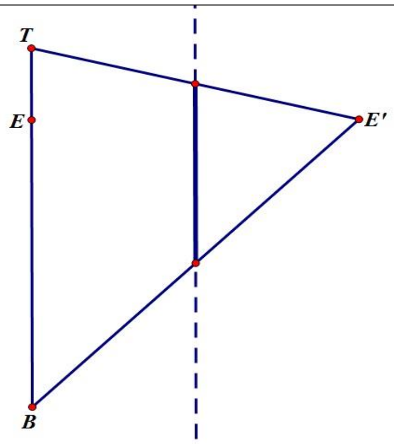
Students were expected to make connections with this case during problem solving processes since they had already learned concepts like optics in physics lesson and lines, rays in mathematics lesson in high school. Correspondingly, they were also expected to make drawings in problem solving.

Mainly, participants solved the problem in two different ways. Participants adopted such solution approaches and constructed three questions asked for mirror problem according to the approach they followed. In either approach, it was possible for participants to find the correct answers for such questions. Explanations regarding these two different solutions are presented in following Table 1.

Table 1
Two Solution Approaches followed by Groups for "the Mirror Problem"



Solution 1: T is the upper point, B is the lower point and E is the eye level, so three points representing human are determined. Mid-points of line segments $|TE|$ and $|EB|$ are determined and then these points are determined on the mirror line. At last, position of mirror and its dimension are emerged.



Solution 2: T is the upper point, B is the lower point and E is the eye level, so these three points representing human are determined. The reflection of point E with respect to mirror is obtained and point E' is obtained. Intersection points on line segments TE' and BE' specified the dimension and position of the mirror.

In both solutions, the participants found out that when the distance between the human and the line was changed, there was not any change in the dimension and the position of the mirror and as a result, this distance was insignificant.

First of all, the correct answers to the problem were needed. As it was seen in the two types of the answer, the necessary length of the mirror should be –if appropriately placed- the half of height of the person looking to the mirror. In the second question, the distance was insignificant. This was because the angle of the rays from the eyes of the person to edges of the mirror changes according to the distance between the person and the mirror. Last part of the question was the hardest one. The correct answer was that the altitude of the mirror from the ground was half of the distance between the person's eyes and the ground.

The participants' answers to each question were presented with the help of descriptive statistics and their reflections to their answers. The frequencies for the answers to the part (a) were given in the Table 2 below.

Table 2
Frequency Table for the Part (a) of "the Mirror Problem"

	The half of the person's height	**The half of the distance between person's eyes to ground**	**Insufficient knowledge**
Height of the mirror	21	6	1

As it was shown in Table 2, most of the groups found the correct answer. From the solutions, 21 participant teams stated that the height of the mirror should be half of the person's height. In general, they used trial and error method. After realizing that they could see the person as a whole while the height of mirror was half of the person's height, they stated the answer by assigning static value to height of mirror. Some answers were as follows.

…We tried different values for the height of mirror. We observed the changes in values of both the heights of the mirror and the person. When the ratio between the heights of mirror and the person became $\frac{1}{2}$*, we could see the person as a whole. So, we realized that what ratio between the heights of mirror and the person must be* $\frac{1}{2}$ *…*

…In our solution, we assigned the height of person as 6h. Then, we chose a mirror with 3h height. Then, we put the person in front of the mirror with a

> *certain distance. We constructed rays from person's eyes to mirror. The reflection of the person could totally be seen in GSP by using reflect property…*

On the other hand, 6 pairs neglected the distance from the person's eyes to the top of his head. While trying to solve the question, these group members ignored where the person saw the mirror from. Therefore, the person could not see some part of his head. One reason for their incorrect answer was just assigning arbitrary value and not trying other values. One answer was as follows.

> *… First, we draw the person and the eyes of person on GSP. Then, we assigned the height of the person as 3h while the distance between the top of the person and the eyes is h and the distance between eyes to feet as 2h… Then, we assigned the height of the mirror as half of the distance between person's eyes and the ground. Our construction showed that the person could be seen in the mirror…*

Table 3
Frequency Table for the Part (b) of "the Mirror Problem"

	Insignificant	**Not mentioned**	**Different answers**
Distance between person and mirror	13	10	5

Table 3 indicated that 13 pairs found the correct answer. However, 10 pairs did not mention about whether the distance was significant or not. Actually, participants' drawings showed different distances. However, this did not give any evidence whether the members of these groups thought the significance of the distance between the mirror and the person or not. In general, students tried to use the dragging properties of the GSP to reach correct answer. The explanation of one group member for their answers for the part (a) and (b) of the problem was as follows.

> *…When we dragged the mirror from left to right on the plane we worked, we realized that the distance between Cinali (*this is what they called to the person in their works on GSP – kind of a superficial book character*) and the mirror was insignificant. What was important was that the height of mirror must be at least half of Cinali's height. We checked the answer by moving each ray constructed on GSP...*

Another answer was about the reflection property of GSP. One group stated as follows.

> *… The distance* (between the person and the mirror*) was insignificant, because when we used the reflect* (property in GSP), *the arrangement in the mirror was easily seen in our construction.*

One interesting finding was that some groups considered the problem as a whole. For example, one group stated that without satisfying the requirements for part (a) and (c), the solution of the part (b) would be nonsense. After stating the correct answers for parts (a) and (b), one group stated as follows.

> *… If the first two conditions were satisfied, the distance between the person and the mirror would be no important. The ratio between the line segments was* $\frac{1}{2}$. *This is the similarity ratio between two triangles in the construction.* (They

referred to their work on GSP). *With the conditions stated, these triangles showed that the distance* (between the person and mirror) *was not important…*

To explain their correct answer, one group stated what would change if the distance between the person and mirror changed as follows.

…No matter whether the person was far from or close to the mirror, the person saw himself in the mirror. The only thing that changes was the field of view…

On the other hand, four groups among these 10 groups did not specifically stated the correct answer, although their drawings correctly showed that the distance was not significant. In their drawings, while the figure of the person was moved in the GSP from left to right or right to left, the rays coming from the top and the bottom of the figure, which presented the person, generated the image of the person at the other side of the mirror. However, the students were not aware of this situation.

Lastly, five groups gave specific value for the distance between the person and the mirror. One group said that the distance should have been $\frac{2}{5}$ of person's height, while one other group found this distance as one half of person's height. From their drawings, it was understood that these groups did trial and error method. They gave a value and that value satisfied that the person saw the complete body of himself. However, they did not consider whether other values satisfied the situation or not. Some of the answers are as follows.

…First of all, we constructed a line segment with 10 cm height… the distance between the person and the mirror was constructed as two fifth of person's height, which was 4 cm… Then, by considering the rays from person's eyes to mirror and symmetry of line segment (person), we saw that the person could be totally seen in the mirror…

For the part (c), there were also different answers. The table indicates frequency of the groups for different answers.

Table 4
Frequency Table for the Part (c) of "the Mirror Problem"

	Half of the distance between eyes and the ground	**Not mentioned**	**Incorrect answers**	**Insignificant**
The altitude of the mirror from the ground	14	10	3	1

In this question, most of the groups in the application found the correct answer. Those who found the correct answer explained it by referring to their constructions on GSP. The rays constructed from the person's eyes to the mirror were considered as reference point for them. Therefore, they gave correct answer. One explanation from students' works is as follows.

…We constructed a line segment representing the person. We put a point, as eyes, on it. Then, we constructed rays from the eyes to the mirror by using symmetry property of GSP. As a result, the altitude of the mirror must be half distance between the eyes and feet…

However, 10 groups did not mention about the answer of the question in their statements, again. Although four groups among those who did not mention about the solution of problem in their reflections correctly solved it in their drawings, it was seen from their statements that they were not aware of whether they found the correct answer or not. For instance, one group said that the altitude was insignificant. On the other hand, three of them gave specific answers for the question.

Other than the answers to the questions of "the mirror problem", it was found that nine groups ignored the distance above the eyes in their drawings. Therefore, their answers were given accordingly.

Secondly, their statements for the solution of the questions and their reflections indicated that 16 groups were confident about the correctness of their answers, while 11 groups were not confident about their answers. At this point, correctness of their answers was not considered while evaluating their confidence. From the reflections, it was observed that participants were either confident about the correctness of their solutions or they had slightly confidence about it. Two examples from their reflections to solutions about confidence and in-confidence are given below. One of the students group asserted that

> *…First of all, it seemed too difficult for us, but at last we reached to the correct solution…*

On the other hand, another group stated that

> *…We tried to solve it. We hope we found the correct answer…*

One crucial finding about their drawings was that most of the participants could not use the GSP correctly. While trying to solve the questions on GSP by visualizing the situation, 18 groups' drawings included minor or major mistakes. Most of the mistakes were done while drawing the reflections of the rays on mirror. They had made mistakes in joining the points on the mirrors and the figure representing the person. For example, while moving the person on GSP, the ray disconnected from the mirror line. Another example was that the drawings did not allow dragging property of the GSP. This means their models were static and were not appropriate to be manipulated.

During drawing the visual representation of "the mirror problem", the group members used translation and reflection properties of the software. However, their use of these properties was very limited and included mistakes. Moreover, some of the participants showed the correct answer in their drawings for the part (a). By looking to the quantitative values of the mirror's and person's heights, it was easy to see the ratio as $\frac{1}{2}$. However, it was only shown as the quantitative values. They did not state that the ratio was $\frac{1}{2}$. Nevertheless, their drawings were static. Namely, it was observed that GSP models which were formed by the students were inappropriate to manipulate just like the ones made on paper.

In addition, participants' reflections to solution indicated that they had knowledge about the mirror problem situation in their previous educational background. For example, they learnt the properties of the mirror in the physics lessons in the high school; however, they forgot the properties. They also asserted that they did not know the relation between the GSP and physics, and they could not connect the relation between the usage of GSP and physics. Some of the students' evaluations included the following statements.

> *…This problem was hard for those, like me, who suffered from physics course. … Although I know the close relationship between physics and geometry, I did not think that we would encounter with such kind of problem…*

> *…This problem required to use related previous knowledge, such as physics…*

...Before starting to the solvethe problem, I discussed properties of mirrors. Then, I realized that this problem was somehow related to physics course. However, we were stuck to relate the knowledge of physics into the solution of this problem…

These findings gave evidences about students' thoughts about GSP. They think that GSP can be used only in mathematics lessons. Instead of just trying to solve the question, some of the students found the answers by considering the symmetry properties of the mirrors and they tried to explain the solution according to the correct answer. In their drawings, however, they could not correctly explain the answers and they could not give satisfactory justifications. At last, students stated that this problem was the most difficult question among the problems they had struggled so far. Their recommendation was that they should get prepared for the lesson before they attend the lesson for this problem.

4. DISCUSSION and CONCLUSION

Pre-service teachers' previous educational practices can inhibit the use of GSP's static images. In their previous educational background, they were more prone to use paper-pencil type of problem solving activities. As Coşkun (2011) said students generally prefer to use paper-pencil while solving problems, similar situation was observed in pre-service teachers' GSP files. They were in an attempt to transfer what they prefer, which is paper-pencil, in dynamic geometry environment. From this point, Coşkun's (2011) results were parallel with the results of this study.

As the paper-pencil activities did not allow students to manipulate the shapes or the situation, they might think that the mirror-problem can also be solved by using the static image of the problem. Based on this issue, this problem was an example for third type of tasks that Laborde, Kynigos, Hollebrands, & Strässer (2006) proposed. With this task, pre-service teachers could manipulate the figures on GSP to reach the correct solution. In addition, their drawings in papers about the problem helped students to construct dynamic models to solve it in GSP. Therefore, they could observe the effects of possible manipulations, which cannot be done with paper-pencil activities. In this process, therefore, pre-service teachers had opportunity to explore the effects of manipulations on GSP and tried to verify their answers (Stylianides &Stylianides, 2005). Based on Lesh and Zawojewski's (2007) definition, verifying what was done during working on a task given was one of the most crucial parts of problem solving process. Especially in dynamic geometry environment, participants can easily retry their works and reach the results. They can retry and verify their solutions continuously and simultaneously. This process was included in the Iranzo & Fortuny's (2011) definition of processes of problem solving.

Findings indicated that, pre-service teachers used static images instead of using the dynamic environment or properties of GSP. Although Köse, Tanışlı, Erdoğan and Ada (2012) found that dynamic geometry software was advantageous about problem solving with the property of dragging. However, the findings showed that pre-service teachers tried to transfer their ways of solving problems in paper-pencil tasks into GSP environment. In such process, the property of dragging was meaningless for them. During solving this problem, the dragging property gives the participants opportunities to adopt *alternative solution paths* (Iranzo & Fortuny, 2011, p. 91), but they could not utilize it properly.

In addition, students tried to construct a design to solve the problem. As it was expected, in this design, they used previous physics knowledge and reflection properties of rays learned in their high school education. They tried to imagine how the person could be seen from feet to head completely in front of a mirror. However, some of them had incorrect answers. This solution indicated that some of the participants failed to reach a solution due to lack of problem solving skills, not being able to relate previous knowledge or not being able to use basic skills in sketchpad (Kin, 2003). However, participants noticed their difficulties and mistakes at the end of problem solving processes. In fact, diagnosis of difficulties and mistakes in problem solving was one of the expected goals of this process (Lesh & Zawojewski, 2007).

During the data analysis, pre-service mathematics teachers chose different approaches to solve the problem. With the approaches chosen, it appeared that the students' thoughts varied during their problem solving processes. Poon and Wong (2011) mentioned about the importance of integrating interdisciplinary knowledge into dynamic geometry environment. The findings were also showed how important that the participants need to combine their mathematics and physics knowledge to reach correct solution during solution processes. For example, those who chose the first solution relied on the physical properties of the rays; on the other hand, those who chose the second solution basically used the symmetry of the reflection point of the person's eyes. While some students preferred to use their physics knowledge about mirrors and symmetry, the others used only geometric one. From this point, Poon and Wong's (2011) study supports the findings of this study.

Another important point was that some of the pairs used the internet to search for general theories about reflections and symmetry in order to reach the answer. Then, they just tried to explain the correct answer which they found on the internet. However, while they were explaining it, they missed the possible changes. That means GSP platform permits users to join the points and move the shapes according to one another while using reflection, translation, rotation properties of the GSP. However, they just drew static shapes that did not permit the movements. Therefore, the researchers could not understand whether they correctly transfer their thoughts onto the GSP environment from the drawings. Considering their reflections to the solutions, students searched the theories about mirrors and symmetry on the internet, however, they could not construct correct models on GSP environment based on theories that they searched. Their drawings did not allow the user to move the person near or far from the mirror. Their drawings were static. Regarding the necessity of using different solution strategies during solving problems (Artigue & Blomhøj, 2013), the findings showed that types of their solution methods were more than one. Although a static shape also indicates the understanding of a problem, a dynamic shape requires more and deeper understanding of the relationships (Coşkun, 2011; Koyuncu, Akyüz, & Çakıroğlu, 2015).

5. RECOMMENDATIONS

Students' problem solving processes may vary while solving problems. Based on the findings of "mirror problem" situation, it can be recommended that students' problem solving processes can be investigated with mathematical problems and their skills can be compared. This study was based on only one problem and it showed differences of problem solving skills among students. It is possible for students to see various problem solving skills when they experience different problems. Therefore, students' problem solving processes can be analyzed within a long term project that includes several problems and their solutions.

The fact about students' thoughts that dynamic geometry software can be used only in mathematic lessons might show that they cannot comprehend the relation between mathematics and physics, more studies in this field can be useful. Lastly, like physics, mathematical problem solving processes can be investigated through interdisciplinary problems.

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GENİŞ ÖZET

Problem çözme becerileri matematik eğitiminin özünü oluşturmaktadır ve matematik eğitiminde önemi yadsınamaz. Problem, sonucu belirsiz ve sonuca ulaşmak için bir takım işlemler gerektiren durum olarak tanımlanabilir. Bir problem durumunda problem çözümü için bir sürecin takip edilmesi gerekir. Bu çalışmada öğretmen adaylarının gerçek hayat problemlerini çözme süreçlerini incelerken Lesh ve Zawojewski'nin (2007) problem çözme süreci tanımı temel alınmıştır. Tanıma paralel olarak, Iranzo ve Fortuny (2011) problem çözücülerin öncelikle muhtemel çözüm stratejilerini düşünmelerinin, en uygununu seçmelerinin, stratejileri deneyip düzenlemelerinin ve sonucunda çözüme ulaşmalarının problem çözme sürecini oluşturduğunu ifade etmiştir. Bu süreçte problem çözücüler hatalarını tespit etme imkânı da bulurlar. Gelişen teknolojiye paralel olarak, problem çözme araçları çeşitlilik göstermektedir. Bunlardan bir tanesi dinamik geometri yazılımlarından olan Geometer's Sketchpad'dir (GSP). Dinamik geometri yazılımları kavramların görselleştirilmesine imkân verdiği için matematiksel konuların derinlemesine anlaşılmasına ve problem çözme becerilerinin geliştirilmesine yardımcı olur. Bu yazılımların görselleştirme, deneme yanılmaya yardım etmesi ve sürükleme özelliğiyle gerçek hayat durumlarının modellenmesinde kullanılabilirliğini artırmaktadır. Öğretmenlerin öğrencilere rehberlik etmesi yönüyle problem çözme etkinliklerinde dinamik geometri yazılımlarının kullanılması durumunda öğretmenlerin bu yazılımları doğru bir şekilde kullanmaya aşina olmaları önemlidir.

Bu çalışmanın amacı, matematik öğretmen adaylarının GSP ile problem çözme süreçlerinin "Ayna Problemi" örneğinde incelenmesidir. Araştırmanın alt problemleri şunlardır.

1- Öğretmen adayları ayna problemi çözümünde hangi çözüm yollarını kullanmışlardır?
2- Ayna problemini çözerken yaptıkları hatalar ve muhtemel sebepleri nelerdir?
3- Katılımcıların çözümleri hakkındaki görüşleri nelerdir?

Çalışma, nitel paradigmanın benimsendiği, tekli durum çalışmasıdır. Katılımcılar bir devlet üniversitenin birinci sınıfında öğrenim gören 56 ilköğretim matematik öğretmen adayından oluşmaktadır. Katılımcılar, ayna problemini çözmeleri için ikişerli gruplar halinde çalışmıştır. Dolayısıyla, 28 grubun problem çözme süreçleri incelenmiştir. Öğretmen adaylarının problem çözme süreçlerinin incelenmesi için modeli oluşturdukları GSP dosyalarından ve yazılı görüşlerinden faydalanılmıştır. Ayna problemi, öğretmen adaylarının aldıkları ders kapsamında çözdükleri sorulardan birisidir. Ders kapsamında GSP ile problem çözme etkinliklerinden önce GSP'nin kullanımı ile ilgili öğretmen adaylarına eğitim verilmiştir. Daha sonra farklı problemleri GSP'den yardım alarak istenmiştir. Ayna problemi, bir kişi kendisini aynada tamamen görmek istiyorsa şu sorulara cevap aramaktadır.

1- Aynanın boyu ne kadar olmalıdır?
2- Bu kişi ile ayna arasındaki uzaklık ne kadar olmalıdır?
3- Aynanın yerden yüksekliği ne kadar olmalıdır?

Problemin çözümü için farklı değişkenlerin farklı durumlarını dikkate almak gerektiğinden GSP ile oluşturulacak modelin problemin çözümüne önemli katkı sağlayacağı öngörülmüştür.

GSP dosyaları ve adayların görüşlerinden elde edilen veriler, verileri daha anlamlı hale getirmek amacıyla betimsel analiz yöntemiyle incelenmiştir. Araştırmacıların analizlerinde probleme ait iki çözüm yolu ortaya çıkmıştır. Bu çalışma, öğretmen adaylarının problemin bütünü ve parçalarını çözme yollarını, problem çözmede kullandıkları yöntemlerini ve yaptıkları hataların belirlenmesini, Lesh ve Zawojewski'nin (2007) ve Iranzo ve Fortuny'nin (2011) problem çözme süreçleri tanımı dikkate alınarak ortaya çıkarmayı hedeflemektedir. Verilerin analizi, bu doğrultuda yapılmıştır.

Araştırma bulgularına göre, öğretmen adayları problemin çözümünde iki farklı çözüm yaklaşımı kullanmışlardır. Her iki çözüm yaklaşımıyla da alt problemlerde sonuca gidilebildiği görülmüştür. Bu bağlamda alt problemlerin ilki olan aynanın boyu ile ilgili sorunun cevabını katılımcıların çoğunluğu doğru cevaplamıştır. Doğru cevabı veren bazı katılımcıların durağan çizimler kullanarak cevabı bulmaya çalıştıkları gözlenmiştir. Gruplardan altı tanesinin oluşturdukları modelde aynanın karşısındaki kişinin gözü ile başının üstü arasındaki mesafeyi göz ardı ettikleri gözlenmiştir. Kişi ile ayna arasındaki uzaklığın incelendiği ikinci alt problemde doğru cevap olan mesafenin önemsiz olduğu sonucuna katılımcıların yaklaşık yarısı ulaşmıştır. Bu sonuca ulaşırken katılımcılar programın simetri özelliğinden ve gözden çıkan ışınların geometrik çizgiler şeklinde modellenmesinden faydalanmışlardır. Doğru cevaba ulaşamayanlar ise statik çizimlerinde tek bir değer üzerinden genellemeye gitmeye çalıştıklarından başarısız olmuşlardır. Ayna probleminin son parçası olan aynanın yerden yüksekliği ile ilgili kısımda katılımcıların yaklaşık olarak yarısı doğru cevaba ulaşmışlardır. Çözümde önceki kısma benzer yaklaşımlarda bulunmuşlardır. Katılımcıların problemin çözünde kendilerini başarılı bulmalarına yönelik görüşlerinde, 16 grup kendini başarılı bulurken, 11 grup problemin çözümünün yanlış yapmış olabileceklerini düşünmektedir. Bu bulgu katılımcıların problemi çözme oranlarıyla örtüşmektedir. Katılımcıların yaptıkları hatalar incelendiğinde, GSP'nin yansıma ve öteleme özelliklerini eksik veya hatalı kullandıkları, çizimleri statik model üzerine kurgulamaları, disiplinler arası anlamda fizik bilgilerini yeterince kullanamamaları dikkat çekmiştir.

Araştırma sonuçlarına göre, öğretmen adaylarının problem çözmede genellikle kâğıt-kalem kullanmaya alışkın olmaları, onların dinamik geometri ile problem çözme etkinliklerinde kâğıt-kalem ile çözümlerde kullandıklarına benzer durağan modeller oluşturmaya eğilimli olduklarını göstermiştir. GSP ile amacına uygun modeller oluşturabilen öğretmen adaylarının problem çözme süreçleri, sürükleme özelliği ile değişken durumları inceleme ve çözümü tekrar deneme imkânına sahip olma noktasında Lesh ve Zawojewski'nin (2007), katılımcıların alternatif çözüm yollarını deneme noktasında ise Iranzo ve Fortuny'nin (2011) problem çözme süreçleri tanımlarıyla örtüşmektedir. Öğretmen adaylarının yaptıkları hataları ve karşılaştıkları zorlukları tespit etme durumları da bu tanımlarla örtüşmüştür. Ayrıca öğretmen adaylarının çözüme ulaşırken fizik bilgilerinin kullanılması vasıtasıyla disiplinler arası ilişkiler kurma imkânına sahip olmuşlardır. Bu çalışma ışığında birden çok problemle benzer gözlemlerin yapılması daha genellenebilir durumları ortaya koyabilir.