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ARAŞTIRMA MAKALESİ / RESEARCH ARTICLE

PORTFOLIO OPTIMIZATION WITH GRAPHICAL LASSO AND AN APPLICATION IN BORSA ISTANBUL

GRAFİKSEL LASSO İLE PORTFÖY OPTİMİZASYONU VE BORSA İSTANBUL'DA BİR UYGULAMA

Erhan USTAOĞLU[*]

Abstract

Graphical Lasso (Least absolute shrinkage and selection operator) has become a popular tool in the field of machine learning in recent years. Although it has been deployed mainly for feature selection in classification problems, it is also used for covariance matrix estimation. Mean-variance portfolio optimization relies on sample covariance matrix for the calculation of the portfolio's risk, whereas it has been most hardly criticized. The aim of this study is to demonstrate the effect of the covariance matrix estimation by Graphical Lasso algorithm with varying L_1 penalty factors. Mean-variance portfolio optimization using empirical and estimated covariance matrices are applied to BIST 30 index and the results are compared.

Keywords: Graphical Lasso, Portfolio Optimization, Covariance Matrix

Öz

Grafiksel Lasso (least absolute shrinkage and selection operator) algoritması son yıllarda makine öğrenmesi alanında popüler bir araç oldu. Genel olarak sınıflandırma problemlerinin özellik seçimi için kullanılıyor olsa da aynı zamanda kovaryans matris tahmininde de başvurulur hale geldi. Ortalama-varyans portföy optimizasyonu, portföy riskinin hesaplanmasında tarihi verilerden yararlanılarak oluşturulan kovaryans matrisini kullanmaktadır. Bu aynı zamanda ortalama-varyans portföy optimizasyonu metodunun en çok eleştiri aldığı konudur. Bu çalışmanın amacı farklı L_1 ceza faktörleri kullanarak grafiksel Lasso algoritmasının kovaryans matris tahminine ve bunun portföy optimizasyon performansına olan etkilerini göstermektir. Çalışmada ortalama-varyans portföy optimizasyonu amprik ve tahmini kovaryans matrisleri kullanılarak BIST 30 endeksine uygulanmakta ve sonuçlar karşılaştırılmaktadır.

Anahtar Kelimeler: Grafiksel Lasso, Portföy Optimizasyonu, Kovaryans Matrisi

* Marmara University, Management Information Systems, erhan.ustaoglu@marmara.edu.tr, ORCID ID: 0000-0002-9077-4370

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1. Introduction

Mean-variance portfolio optimization, though nearly 70 seventy years old, is still widely used by finance professionals and academics throughout the world (Markowitz, 1952). Yet it is one of the most criticized financial methods from many aspects (Kolm, Tütüncü, & Fabozzi, 2014). Mean-variance portfolio optimization is a trade-off between portfolio risk and portfolio return. One of the most criticized points in the method is the estimation of the covariance matrix, which the method uses for the calculation of the portfolio risk. The true relationships between the assets, which make up the portfolio, should be known or somehow guessed for the exact calculation of the portfolio risk. However, this is practically almost impossible when the number of assets in a portfolio are high, especially in volatile markets. Instead, historical price movements are employed for the calculation of covariance matrix hence, the portfolio risk. But this leads to severe estimation errors especially when the number of sample data are not at least an order greater than the number of assets in the portfolio (Ledoit & Wolf, 2003). Moreover, estimation errors are maximized by the mean-variance method (Michaud, 1989). As a result, some of the assets in the portfolio may be overweighted, and some other may be underweighted which leads to poor out-of-sample performances. De Miguel et al. (2009) showed that even equally diversified portfolios can outperform mean-variance optimized portfolios in practical applications. Some methods have been proposed to overcome the errors incurred in the estimation of the portfolio risk. Shrinkage estimators have been widely used for reducing the estimation errors in covariance matrix. Ledoit and Wolf (2004) suggested an estimation method using a weighted average of the sample covariance matrix and a structured estimator, which assumes all the variances to be the same and all the covariances to be zero. Bai et al. (2009) used large-dimensional random matrix theory for combining sample covariance and the estimators. Bodnar et al. (2014) proposed a more general linear shrinkage estimator utilizing a symmetric positive definite target matrix. An optimal expected gain/loss estimator was also proposed using shrinkage covariance matrices (Liu et. al, 2016).

Lasso was proposed to improve the prediction accuracy and the interpretation of the ordinary least square regression (Tibshirani, 1996) and became one of the widely applied models in the field of machine learning in 1990s together with neural networks and support vector machines. Since it causes the solution to be sparse, it is used for mainly feature selection (Perrin and Roncalli, 2019). The model has been used in numerous applications ranging from NLP (Loukina et. al, 2015) to seismology (Kiani et. al, 2019). It was also shown that Lasso worked better compared to Ridge and ordinary least square regression with simulated and real data (Muthukrishnan and R. Rohini, 2016). Friedman et. al (2008) proposed a fast algorithm for the solution of sparse graphical models. The algorithm used L_1 (Lasso) penalty for the estimation of sparse inverse covariance matrix multivariate Gaussian distribution. This simple and fast algorithm facilitated the solution of the large data problems which involve sparse covariance or sparse inverse covariance procedures. As mean-variance model calculates portfolio risk through covariance of assets included in the portfolio, graphical Lasso has found applications in this field recently (Avagyan et. al, 2017; Yuan et. al, 2020). However, deployment of graphical Lasso algorithm in Turkish financial markets for portfolio optimization has not been studied yet. This study aims to demonstrate the effectiveness of the Lasso algorithm in portfolio optimization for Turkish financial markets and to provide academicians and financial professionals with an insight into the subject. For the application, a portfolio optimization problem in BIST 30 is considered. The covariance matrix is estimated by graphical Lasso algorithm and the results are compared with that of using the sample covariance.

2. Methodology

Mean-variance portfolio optimization, Sharpe ratio which is used as the optimization objective function and the graphical Lasso algorithm are defined in the following sections.

2.1. Mean-Variance Optimization

Mean-Variance optimization proposed by Markowitz (1952), seeks the minimization of the risk of a portfolio given a level of expected return or the maximization of the expected return given a level of risk. The expected return and the risk of a portfolio are defined as:

$$R_p = \boldsymbol{w}^{\boldsymbol{T}} \boldsymbol{r} \tag{1}$$

$$Var_p = \boldsymbol{w}^{\boldsymbol{T}}\, \boldsymbol{\Sigma}\, \boldsymbol{w} \tag{2}$$

respectively, where $\boldsymbol{r}$ is the return vector of assets, $\boldsymbol{w}$ is the vector, which gives the weights of the assets and $\boldsymbol{\Sigma}$ is the covariance matrix of the returns in the portfolio. The mean-variance optimization problem is given as

$$\max_{\boldsymbol{w}} \boldsymbol{w}^{\boldsymbol{T}} \boldsymbol{r} \quad \text{subject to} \quad \boldsymbol{w}^{\boldsymbol{T}}\, \boldsymbol{\Sigma}\, \boldsymbol{w} \le Var_{pmax} \quad \text{and} \quad \boldsymbol{w}^{\boldsymbol{T}} \mathbf{1}_N = 1 \tag{3}$$

or

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\boldsymbol{T}}\, \boldsymbol{\Sigma}\, \boldsymbol{w} \quad \text{subject to} \quad \boldsymbol{w}^{\boldsymbol{T}} \boldsymbol{r} \ge R_{pmin} \quad \text{and} \quad \boldsymbol{w}^{\boldsymbol{T}} \mathbf{1}_N = 1 \tag{4}$$

where $\boldsymbol{w}^{\boldsymbol{T}} \mathbf{1}_N = 1$ reflects the no short selling condition which is a common practice in theoretical portfolio analysis. Another alternative for the optimization of the mean-variance problem is using the Sharpe ratio (Sharpe, 1966) in the objective function as

$$\max_{\boldsymbol{w}} \frac{\boldsymbol{w}^{\boldsymbol{T}} \boldsymbol{r}}{\boldsymbol{w}^{\boldsymbol{T}}\, \boldsymbol{\Sigma}\, \boldsymbol{w}} \quad \text{subject to} \quad \boldsymbol{w}^{\boldsymbol{T}} \mathbf{1}_N = 1 \tag{5}$$

which is used in this study.

2.2 Graphical Lasso Algorithm

Assuming we have N samples from p multivariate Gaussian distributions with mean μ and covariance Σ, then the inverse covariance matrix Σ^{-1} can be estimated by maximizing the multivariate loglikelihood function as

$$\max_{\boldsymbol{\Sigma}^{-1}} \log \{\det \boldsymbol{\Sigma}^{-1} - tr(\boldsymbol{S}\boldsymbol{\Sigma}^{-1}) - \rho \|\boldsymbol{\Sigma}^{-1}\|_1\} \tag{6}$$

where $tr(\boldsymbol{S}\boldsymbol{\Sigma}^{-1})$ denotes trace of the product of the matrices, $\boldsymbol{S}$ is the sample covariance matrix and $\|\boldsymbol{\Sigma}^{-1}\|_1$ is the sum of the products of the absolute values of the elements $\boldsymbol{\Sigma}^{-1}$ and ρ is the L_1 penalty factor (Friedman et. al, 2008).

3. Application in BIST 30

Daily return values BIST 30 stocks between 01.01.2018 and 31.12.2019 (512 daily returns for 30 stocks) are considered for the application. Employing mean-variance optimization seven different portfolios are obtained for which the Sharpe ratio is used in the objective function for the maximization. The first portfolio is constructed using the sample covariance matrix. The other six portfolios are constructed using estimated covariance matrices obtained through the graphical Lasso algorithm for six different L_1 penalty values ρ from 0.1 to 0.6.

As the L_1 penalty value ρ increases, the sparsity of the covariance matrix increases. The sparsity diagrams are displayed for the sample covariance matrix and the estimated covariance matrices for $\rho = 0.1, 0.3,$ and 0.6 (Figure 1).

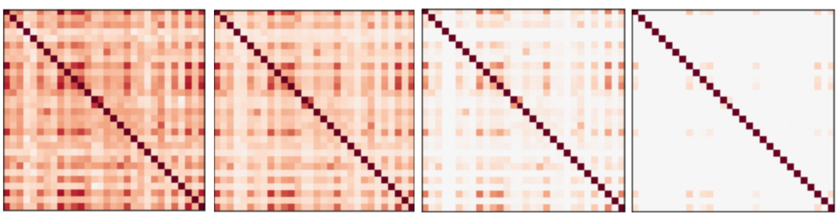

Figure 1: Sparsity Diagrams

Sharpe ratios, mean returns, standard deviations, sparsity ratio of the covariance matrices, and the number of stocks with portfolio shares different than zero are listed in Table 1. All of the portfolios with Lasso penalties, which are higher than zero, have better Sharpe ratios than that of sample covariance matrix. Moreover, as the sparsity ratio decreases, Sharp ratio increases that means a higher expected return for the risk incurred. Meanwhile the number of stocks in the portfolio increases too, meaning a better diversification. This behavior addresses the criticism against the mean-variance portfolio model that the model overweights some of the assets in the portfolio, which in turn causes poor out-of-sample results. These results indicate a concrete improvement in the portfolio optimization performance when the graphical Lasso is used for covariance matrix estimation.

Table 1. Portfolio Performances

Covariance	Sharpe Ratio	Sparsity Ratio	Number of Stocks
Sample Covariance	0.00236	1	7
Est. Covariance $(\rho = 0.1)$	0.00260	1	9
Est. Covariance $(\rho = 0.2)$	0.00301	1	12
Est. Covariance $(\rho = 0.3)$	0.00346	0.94	14
Est. Covariance $(\rho = 0.4)$	0.00365	0.60	17
Est. Covariance $(\rho = 0.5)$	0.00368	0.41	19
Est. Covariance $(\rho = 0.6)$	0.00370	0.15	19

4. Conclusion

Covariance matrix estimation has been one of the points for which the mean-variance portfolio optimization has been criticized. Many methods have been proposed to overcome the estimation errors. Recently Lasso algorithm has also been used for the estimation of covariance matrix, which depicts the relationships between asset returns. In this study, Graphical Lasso algorithm is applied to calculate the estimated covariance matrix in the mean-variance portfolio optimization for BIST-30 stocks and the results are compared with that of sample covariance matrix. Sharpe ratios achieved for Lasso estimated portfolio optimizations are greater than that of the sample covariance matrix which means greater return-risk ratios. As the L_1 penalty increases covariance matrix gets sparser and the number of stocks in the portfolio increases. This indicates that the portfolios are more diversified and the problem of overweighting of some assets are overcome. These results are inline with the findings of the studies, which also reported improved Sharpe ratios and decreased risks for portfolio optimizations. As the results indicate, Sharpe ratios increase monotonically as L_1 penalty factor increases. However, this does not imply that the higher the L_1, the better the portfolio performance. L_1 values should also be optimized for better portfolio performances regarding the assets and time span under investigation. As a further study subject, out of sample performances of the optimized portfolios can be observed using a rolling window approach and the L_1 penalty value can be optimized for BIST 30.

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