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Geometrical Models Some of Microstructure Using Tessellation

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Abstract

The physical, biological and functional properties of many materials in nature depend on the size, shape, and spatial distribution of their microstructures, as well as their location. This distribution of layout corresponds to the concept of tessellation, which is expressed by formulas in geometry. This concept has been defined in botany by the term "mosaic" for morphological structures such as a flower petal, bark, or fruit. It is easier to determine the productivity of materials with known microstructures at the macro scale. In many areas, the geometry of the microstructures of materials can be clarified and their usability can be increased. Similar research has been done in the food field. The stability, transport properties, structural integrity or nutritional quality of food materials are reflections of microstructure communities, and when they are embedded, they reveal the engineering structures we call macrostructure (tissue and organ). As a result, when the micro-geometrical structures of the materials are known and considered, the productivity parameters at the macro scale become determinable. The transition from microstructure to macrostructure can then be achieved by appropriate homogenization procedures. Since the microstructures that make up the materials make up the whole of that material, the shape of the connection between them is important. Plant tissues also have a micromorphological structure consisting of many cells. Therefore, the characteristics of the plant depend not only on the characteristics of individual cells, but also on the connection, location, and interactions between the cellular components. In this study, it has been tried to determine the geometric models of the cells that make up the microscopic structures of some plants by using the geometric definitions of tessellation, which is a mathematical concept.

Keywords: Geometric model, microstructure, tesellation, plant

Bazı Mikroyapıların Mozaiklenmeye (tessellation) Dayalı Geometrik Modelleri

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Özet

Tabiatta yer alan birçok materyallerin fiziksel, biyolojik ve işlevsel özellikleri, onların mikro yapılarının boyutuna, şekline, uzaysal dağılımlarına ve ayrıca bunların yerleşim biçimlerine bağlıdır. Bu yerleşim biçimi geometride formüller ile ifade edilen tessellation kavramı ile karşılık bulur. Bu kavram botanikte, bir çiçek yaprağı, ağaç kabuğu veya meyve gibi morfolojik yapılar için "mozaik" terimi ile tanımlanmıştır. Mikroyapıları bilinen materyallerin makro ölçekte verimliliklerinin

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belirlenmesi kolaylaşır. Birçok alanda materyallerin mikroyapılarının geometrileri açığa kavuşturularak onların kullanılabilirlikleri artırılabilir. Gıda alanındaki benzer araştırmalar yapılmıştır. Gıda malzemelerinin stabilitesi, taşıma özellikleri, yapısal bütünlüğü veya besinsel kalitesi, mikro yapı topluluklarının yansımaları olup, içine gömüldüklerinde, makro yapı (doku ve organ) dediğimiz mühendislik yapılarını ortaya çıkarmaktadır (Aguilera, 2005). Sonuç olarak, materyallerin mikro geometrik yapıları bilinip, göz önüne alındığında, makro ölçekteki verimlilik parametreleri belirlenebilir hale gelir. Mikroyapıdan makroyapıya geçiş daha sonra uygun homojenleştirme prosedürleriyle sağlanabilir. Materyalleri oluşturan mikroyapılar o materyalin bütününü oluşturduğundan aralarındaki bağlantının şekli önemlidir. Bitki dokuları da çok sayıda hücreden oluşan mikromorfolojik yapıya sahiptir. Dolayısıyla bitkinin özellikleri sadece tek tek hücrelerin özelliklerine değil, aynı zamanda hücresel bileşenler arasındaki bağlantı, yerleşme biçimi ve etkileşimlere de bağlıdır. Bu çalışmada bazı bitkilerin mikroskobik yapılarını oluşturan hücrelerin yerleşim biçimleri dolayısıyla geometrik modelleri matematiksel bir kavram olan tessellation (mozaikleme) geometrik tanımları kullanılarak belirlenmeye çalışılmıştır.

Anahtar Kelimeler: Geometri model, mikroyapı, mozaiklenme, bitki.

1. Introduction

Geometric modeling of plant microstructure, plant tissues at cellular level; It is very important in terms of the quality of its physical properties such as durability, smallest area coverage and mass transfer. Evaluation of micromorphological structures from this point of view attracts great attention in many fields such as food engineering and so on. Material stability, transport properties, structural integrity or nutritional quality of food materials are the reflections of the assemblies of microstructures, when embedded in, give rise to an engineered structure, which we call macrostructure [1]. As a result, when the micro-geometrical structures of the materials are known and considered, the productivity parameters at the macro scale become determinable. The transition from microstructure to macrostructure can then be achieved by appropriate homogenization procedures. Since the microstructures that make up the materials make up the whole of that material, the shape of the connection between them is important. Generating geometric models of microscopic structures is considered a fundamental requirement for developing microscale models to study and describe these properties [2]. It has been demonstrated that the shape, size, and spatial distribution of cells governs the physical, biological, and structural properties of a cellular (tissue) material [3]. Geometric modeling studies of morphological structures have been carried out by some researchers in the literature [4-5]. However, geometry modeling studies of plant micromorphological structures are limited [6-7].

A plant organ (e.g.petiole) is generally composed of an assembly of cellular tissues, which make up its microstructure and largely govern its physical properties. Each tissue grows to meet specific functional requirements that guarantee plant survival in a given environment. The way in which multiple tissues are geometrically assembled within an organ helps determine mechanical performance, important for structural support. Plant structures often exhibit excellent mechanical properties. This feature is largely controlled by the geometrical structures of their micromorphology. [3]. The concept of tessellation, which is expressed by formulas in geometry, has been defined in botany with the term "mosaic" for morphological structures such as a flower leaf, bark or fruit [7].

Geometric tessellation is one of the foundations of computer graphics. The application of geometric tessellation in computer technology is based on the algorithm of surface, from the input of the surface to the output of the geometry. In this study, examples of mosaicism in some micromorphological structures, corresponding to geometry and expressed by formulas, were examined.

2. Materials and Methods

The sections of 10-20 µm were taken from the samples with the help of a microtome to obtain the microstructures to be used in defining the geometric models. For this, the paraffin method was applied, and the sections were stained with safranin and fast green [8]. In addition, hand sections were taken for the microstructure of the samples and colored with Sartur reagent [9]. The preparations prepared from the sections were examined and photographed using Leica DM3000 motorized microscope objectives.

In the mathematical evaluations, the tessellation (tiling), which is mathematical concept, were used to describe the microstructures' geometric model. In the study, literature information on geometric structures and their mathematical formulas were evaluated.

3. Results and Discussion

In this study, microscopic observations of some parts of the plants were made. It has been determined that the microstructures of the plants obtained from these observations have different arrangements. These different settlements of the cells belonging to the microstructures were evaluated according to the geometric concept tessellation (tiling). A tessellation or tiling is the covering of a surface, often a plane, using one or more geometric shapes, called tiles, with no overlaps or no gaps. In mathematics, tessellation can be generalized to higher dimensions and a variety of geometries. A tiling of regular polygons (in two dimensions), polyhedra (three dimensions), or polytopes (n dimensions) is called a tessellation. Tessellations can be specified using a Schläfli symbol. Tessellation in two dimensions, also called planar tiling, is a topic in geometry that studies how shapes, known as tiles, can be arranged to fill a plane without any gaps, according to a given set of rules. These rules can be varied. Common ones are that there must be no gaps between tiles, and that no corner of one tile can lie along the edge of another [10].

Geometric tessellation types are primarily considered in two main groups.

1. Two-Dimensional Tessellation

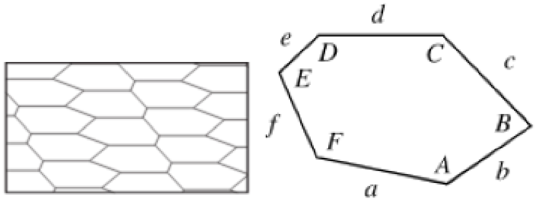
Tessellation in two-dimensional space refers to the tessellation of a plane or of two-dimensional surfaces, and can be simplified into some basic geometries, which are called "prototypes".

2. Three-Dimensional Tessellation

Three-dimensional tessellation, also called spatial tessellation, has more dimensional variations than two dimensional ones.

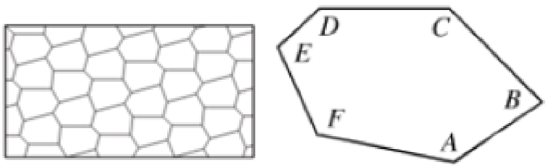
The microstructures we examined in our study show Two-Dimensional Tessellation feature. Hexagon tessellation is one of the types of Two-Dimensional Tessellation. A hexagon tessellation is a tiling of the plane by identical hexagons. In geometry, the hexagonal tiling or hexagonal tessellation is a regular tiling of the Euclidean plane, in which exactly three hexagons meet at each vertex. Hexagon tessellation is divided into two main groups as regular and irregular. The regular hexagon forms a regular tessellation, also called a hexagonal grid.

On the other hand, there are at least three tiling of irregular hexagons, their types and general formulas are shown follow (Figure1-3). It should be noted that periodic hexagonal tessellation is a degenerate case of all three tiling. [11-12].



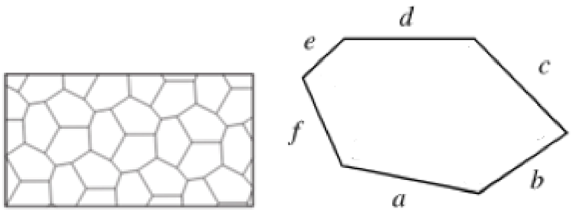
$$A + B + C = 360^\circ \qquad a = d$$
$$A + B + D = 360^\circ \qquad a = d,\ c = e$$
$$A = C = E = 120^\circ \quad a = b,\ c = d,\ e = f \tag{1}$$

Figure 1. General formulas and tessellation representations of the first type of irregular hexagons [12].



$$A = B = C = D = E = F \tag{2}$$

Figure 2. General formulas and tessellation representations of the second type of irregular hexagons[12].



$$a = b = c = d = e = f. \tag{3}$$

Figure 3. General formulas and tessellation representations of the third type of irregular hexagons [12].

Present study revealed that the microscopic structures some of the studied plants showed the irregular hexagon tessellations. Their types were determined as 1st and 3rd types. Their micromorphological appearances and tessellation types are shown in figure 4 and 5 (Figure 4,5).

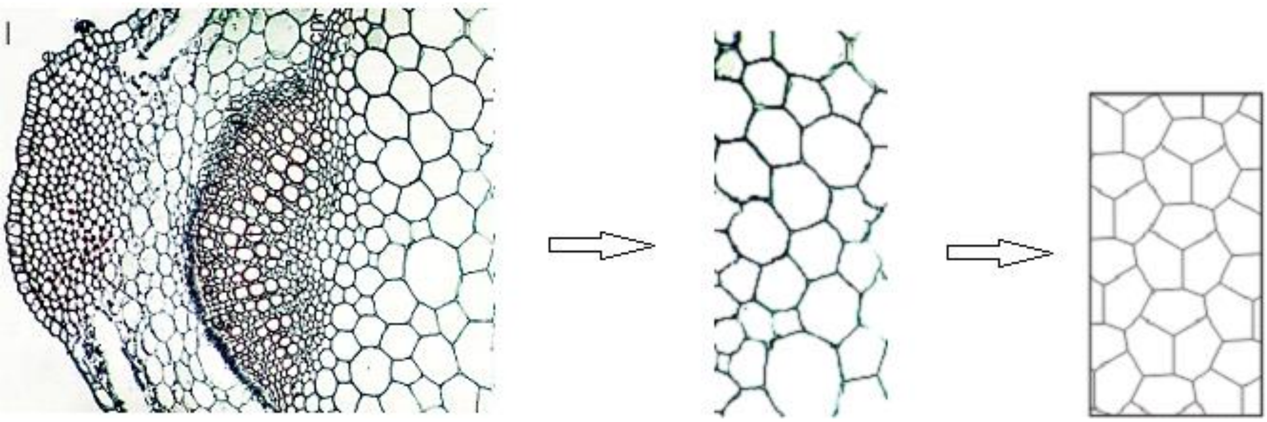


Figure 4. (a) Stem croos-section of *Lamium pisidicum* on with multiple tissue(Scale Bars: 25µm). **(b)** Tissue microstructere **(c)** Model of geometric

(a), (b): Micromorphological photographs of plants taken under a microscope.
(c) : Determined geometric tessellation type of microstructure of plant

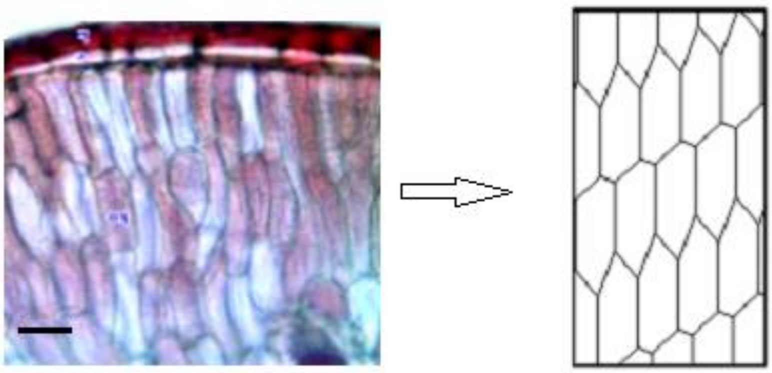
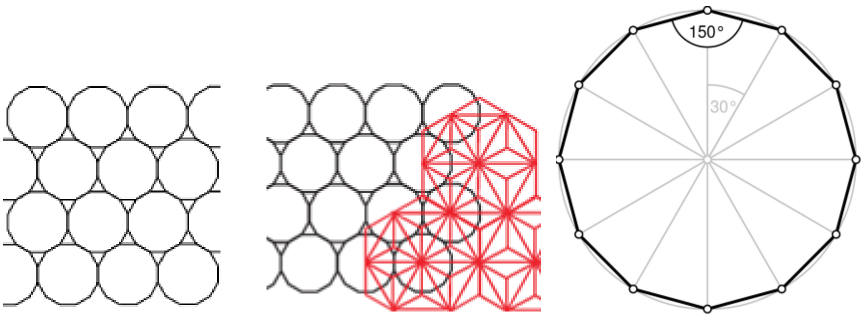


Figure 5. (a) Stem croos-section of *Limonium.effusum* on with multiple tissue(Scale Bars: 40µm)

(b)Tissue microstructere

(a): Micromorphological photographs of plants taken under a microscope.
(b): Determined geometric tessellation type of microstructure of plant

On the other hand, we found that some of the microstructures we examined showed tessellation, which is called semiregular tessellation in geometry. Regular tessellations of the plane by two or more convex regular polygons such that the same polygons in the same order surround each polygon vertex are called semiregular tessellations, or sometimes Archimedean tessellations [13-15].
In the plane, there are eight such tessellations One of them is dodecagon semiregular tessellation that we observed in our study. It has twelve lines of reflective symmetry and rotational symmetry of order 12. The internal angle at each vertex of a regular dodecagon is 150°. It's type and general formulas are shown follow (Figure 6).



$$A = 6sin\left(\frac{\pi}{6}\right)R^2 = 3R^2$$

$$p = 24Rtan\left(\frac{\pi}{12}\right) = 12R\sqrt{2-\sqrt{3}}$$

Figure 6. General formulas and tessellation representations of dodecagon semiregular tessellation [15].
(A) : In terms of the circumradius R, the area
(R) : Circumradius
(p) : The perimeter of a regular dodecagon in terms of circumradius

Present study revealed that the microscopic structures some of the investigated studied plants showed the dodecagon semiregular tessellation. Their micromorphological appearances and tessellation types are shown in figure 4 and 5. (Figure 7).

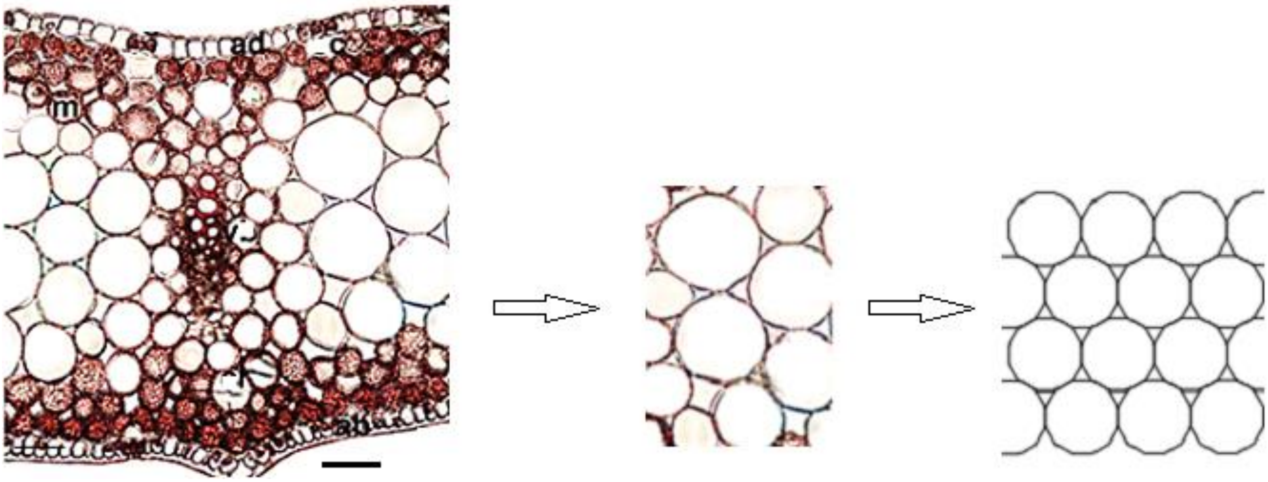


Figure 7. (a) Leaf croos-section of *Puschkinia bilgineri* on with multiple tissue(Scale Bars: 35μm) (b)Tissue microstructere (c)Model of geometric tesellation
(a), (b): Micromorphological photographs of plants taken under a microscope.
(c) : Determined geometric tessellation type of microstructure of plant

Plant tissues also have a micromorphological structure consisting of many cells. Therefore, the characteristics of the plant depend not only on the characteristics of individual cells, but also on the connection, location, and interactions between the cellular components.
In this study, along with the micromorphological structure of plant parts, their geometric tessellation expressed by photographs taken under a microscope were provided. We tried to show that some micromorphological structures of plants have geometric properties that can be expressed with geometric concepts.

4. Conclusions

Present study, it has been tried to determine the geometric tessellation models of the cells that make up the microscopic structures of some plants by using the geometric definitions of tessellation.
The feature takes an essential place in geometry. The tessellation feature provides the advantages of resistance and minimal space occupation to the cells.

The geometrical representation of a tissue can helps enhance our understanding of how microstructure determines mechanical properties. It can also help us to develop predictive models of known mechanical behavior. Geometric tessellation properties such as the "irregular hexagon tessellations, dodecagon semiregular tessellation" observed in the investigated plant microstructure are also possible in other plant micromorphological structures.
With the study a different perspective was tried to be brought in plant anatomy studies. As a result, we believe that the study provides a new comparing opportunity for future researchers on the related subjects.

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