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Calculating With the Theoretical Approach of The Settling Velocity of Fish Feed Pellets

Huriye ARIMAN KARABULUT[1], Hasan KARABULUT[2] , Ilhan YANDI[3]

[1]Department of Aquaculture, Faculty of Fisheries, Recep Tayyip Erdogan University, Rize, 53100, Turkey.
[2]Physics Department, Recep Tayyip Erdogan University, Faculty of Arts and Sciences, Rize, 53100, Turkey.
[3]Maritime Faculty, Recep Tayyip Erdogan University, Derepazarı, Rize 53900 Turkey.

Abstract: This study, settling velocity of fish feed pellets was investigated using the formula given by Isaacs and Thodos [1]. This formula is accurate for large pellets (larger the 3.5-4 mm diameter). The settling velocity data in the literature was compared to the predictions of this formula and good agreement was found. The formula also has implications on how settling velocity depends on various parameters such as water temperature, salinity, pellet diameter, length, density of pellet etc. Settling velocity was found to be extremely sensitive to errors in density measurements of pellets according to the Isaacs and Thodos [1] formula. A few percent errors in particle density found to cause large deviations in settling velocity. Settling velocity is independent of viscosity according to Isaacs and Thodos formula. The settling velocity was also found to be largely independent of temperature for large pellets. The settling velocity is proportional to square root of diameter according to Isaacs and Thodos [1] formula. This dependence on square root of diameter was demonstrated using the data from Sutherland et al. [2] paper. Salinity was also found to be important parameter. Salinity affects settling velocity by increasing density of water and tends to decrease settling velocity.

Keywords: Fish feed pellets, settling velocity, drag force, drag coefficient

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Balık Yemlerinin Batma Hızının Teorik Yaklaşımla Hesaplanması

Özet: Bu çalışmada balık yemi batma hızları Isaacs ve Thodos [1] tarafından verilen formüllere göre araştırıldı. Bu formül büyük peletler (3.5-4 mm'den büyük) için hassastır. Literatürdeki batma hızı verileri, bu formülün sonuçları ile karşılaştırıldı ve iyi uyum sağladığı tespit edildi. Formül aynı zamanda batma hızının birçok parametrelere (su sıcaklığı, yem yoğunluğu, tuzluluk, pellet yarıçapı v.s.) nasıl bağlı olduğunu da açıklamaktadır. Isaacs ve Thodos [1] formülüne göre, batma hızının yoğunluk ölçümündeki hatalara karşı son derece duyarlı olduğu bulunmuştur. Yoğunluktaki yüzde birkaç hatanın batma hızında büyük sapmalara sebep olduğu bulunmuştur. Isaacs ve Thodos [1] formülüne göre, batma hızının hem vizkositeden bağımsız hem de büyük peletler için suyun sıcaklığından büyük ölçüde bağımsız olduğu bulundu. Isaacs ve Thodos [1] formülüne göre, batma hızı çapın kareköküyle orantılıdır. Batma hızında çapın kareköküyle bağıntısı, Sutherland vd. [2]'nin makalesinden alınan veriler kullanılarak gösterilmiştir. Aynı zamanda tuzluluğun da önemli bir parametre olduğu belirlendi. Tuzluluk; suyun yoğunluğu arttırmak suretiyle, batma hızını yavaşlatma eğiliminde etki göstermektedir.

Anahtar kelimeler: Balık yemi, batma hızı, sürtünme kuvveti, sürtünme katsayısı

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1. Introduction

One of the main issues in marine fish farming is to correctly predict the impact of uneaten feed pellets and fish faeces on the benthic environment. There are computer programs and models to predict these effects [3, 4]. There are many papers studying these effects. To see some recent ones see Piedecausa et al. [5] and Pérez et al. [6]. One of the data required by these models and computer programs is the settling velocity of uneaten fish pellets and fish faeces.

The problem of settling velocities of objects is old one and there is a considerable literature on the subject. It is possible to predict settling velocities of fish pellets using the formulae available in the literature. There are papers in aquaculture literature on measuring settling velocities of feed pellets [2, 7, 8, 9, 10, 11]. A study of them shows that most of these papers have not taken advantage of the formulae and the theoretical knowledge in engineering literature to interpret their data. The goal of this paper is to show how useful the information and data in engineering literature to predict settling velocities of fish feed pellets.

There are many formulae in the engineering literature to calculate settling velocities. For a general evaluation and comparison of them see the review papers [12, 13, 14] and the book by Clift et al [15]. The study is limited to 'large pellets' which settle in high Reynolds number regime. Since most fish feeds are cylindrical pellets, cylindrical pellets is only studied here. Drag formula of Isaacs and Thodos [1] from literature to discuss settling velocities of feed pellets is chosen. This particular formula is the only one derived from cylindrical particle data exclusively in the literature to our knowledge. Moreover it is accurate and easy to use. Its shortcoming is that it is valid for high Reynolds number regime. But this is sufficient to discuss settling velocities of large pellets studied in this paper

2. Material and Method

For a particle moving with velocity v in a fluid with the density ρ_f, The drag force applied to the particle is

$$F_D = \frac{1}{2}\rho_f A v^2 C_D,$$ (1)

Where, A is some suitably defined cross-sectional area and C_D is a dimensionless number. From dimensional analysis it can be shown that C_D is a function of the Reynolds number $R = \frac{\rho_f d_C}{\mu} v$ and the dimensionless numbers characterizing the geometry. In the case of cylinders the only relevant dimensionless number is aspect ratio E. The aspect ratio is defined as $E = H/d_C$, where H is the height of cylinder and d_C is the diameter of the cylinder. Therefore drag coefficient is a function of Reynolds number and aspect ratio $C_D(R, E)$ for cylinders.

Parameters entering the definition of Reynolds number are density of fluid flow ρ_f, the velocity v, a characteristic length (here the diameter d_C) and kinematic viscosity μ. The Reynolds number is a dimensionless number characterizing the flow. It is perhaps the most important dimensionless number in fluid mechanics. Reynolds number can be thought of as a measure of ratio of inertial forces to viscous forces in the flow. Hence, there is density and velocity in the nominator and viscosity in the denominator. For high Reynolds numbers the effect of viscous forces on the flow is negligible and therefore it will be seen that settling velocity does not depend on viscosity for high Reynolds numbers.

The cylinder has a weight mg and buoyancy force $-\rho_f V g$ (here V is the volume of the particle). So the net force downward is $mg - \rho_f V g = V g(\rho_P - \rho_f)$ where ρ_P the particle's average

density and V is the volume of the particle. To find the settling velocity, this force equal to drag force is set.

$$\frac{1}{2}\rho_f A v^2 C_D = V g (\rho_P - \rho_f). \tag{2}$$

Choosing $A = d_C H$ and $V = \pi d_C^2 H / 4$ this equation is written as $R^2 C_D(R) = G$ where G is another dimensionless number called Galileo number:

$$G = \frac{\pi d_C^3 \rho_f g (\rho_P - \rho_f)}{2\mu^2}. \tag{3}$$

Usually $C_D(R)$ as a function of Reynolds number is tabulated, or, graphed using experimental data. As a convenience they are also given as empirical formulae obtained from experimental data. Such empirical $C_D(R)$ formulae are called correlations. Therefore when G is known $R^2 C_D(R,E) = G$ is a nonlinear equation for Reynolds number and can be solved numerically. From Reynolds number the settling velocity is easily obtained.

$C_D(R,E)$ Versus R curves has universal limit shapes for low and high Reynolds numbers. For low Reynolds numbers ($R < 1$) $C_D \approx c(E)/R$ is observed for all shapes where c is a constant. For high Reynolds numbers $C_D(R)$ is a constant and independent of Reynolds number.

Isaacs and Thodos [1] studied free settling of cylindrical particles and they gave a correlation valid for high Reynolds number ($R > 200$). They observed that the motion of cylinders for $E > 1$ and $E < 1$ are different. Therefore they found two different expressions for the drag coefficient. Furthermore they observed that the drag coefficient for $R > 200$ is constant and independent of Reynolds number. Drag coefficients they found are

$$C_D = 0.99(\frac{\rho_P}{\rho_f})^{-0.12} E^{-0.08} \quad (E > 1)$$
$$C_D = 1.25(\frac{\rho_P}{\rho_f})^{-0.05} E^{-0.18} \quad (E < 1) \tag{4}$$

This formula is the actual formula given by Isaacs and Thodos [1] and this will be called Isaacs and Thodos [1] formula here. A few formulae which are direct consequence of this formula are derived, and since they all come from this their consequences will be also mentioned as consequences of Isaacs and Thodos [1] formula.

The area in the definition of C_D for $E > 1$ is $A = d_C H$ whereas the area in the definition of C_D for $E < 1$ is $A = \pi d_C^2 / 4$. From these drag coefficients one can derive the following Reynolds number-Galileo number relations

$$R = 1.005(\frac{\rho_P}{\rho_f})^{0.06} E^{0.04} \sqrt{G} \quad (E > 1)$$
$$R = 1.0093(\frac{\rho_P}{\rho_f})^{0.025} E^{0.59} \sqrt{G} \quad (E < 1) \tag{5}$$

For $E = 1$ both equations give almost equal values within one percent difference. The most important difference is that for $E > 1$ the Reynolds number (hence the settling velocity) depends on E very weakly whereas for $E < 1$ the settling velocity is roughly proportional to $\sqrt{E}$

3. Results and Discussion

3.1. Density of seawater depending on temperature and salinity

Siedler and Peters [16] gives a large collection of information on fresh and seawater properties at one atmosphere. In particular, density of fresh water and seawater as a function of temperature and salinity are needed.

The pure water density is given by the formula

$$\rho_W(T) = 999.842594 + 6.793952 \times 10^{-2}T$$
$$-9.095290 \times 10^{-3}T^2 + 1.001685 \times 10^{-4}T^3,$$
$$-1.120083 \times 10^{-6}T^4 + 6.536332 \times 10^{-9}T^5$$

(6)

Where density is in kg/m^3 and temperature is in Celsius degrees ($^{\circ}C$). The density of sea water as a function of temperature and salinity is

$$\rho_W(S,T) = \rho_W(T) + a(T)S + b(T)S^{3/2} + c(T)S^2$$

(7)

Where,

$$a(T) = 8.24493 \times 10^{-1} - 4.0899 \times 10^{-3}T + 7.6438 \times 10^{-5}T^2$$
$$-8.2467 \times 10^{-7}T^3 + 5.3875 \times 10^{-9}T^4$$

(8)

$$b(T) = -5.7246610^{-3} + 1.0227 \times 10^{-5}T - 1.6546 \times 10^{-6}T^2$$

$$c(T) = 4.8314 \times 10^{-4}$$

Here S is the salinity which is usually given as mass of salt (in grams) in one liter of seawater. For a more technical and precise definition see Siedler and Peters [16]. This formula is valid for $0 \le S \le 42$ and $-2^{\circ}C \le T \le 40^{\circ}C$.

Let us examine this equation for $T = 15^{\circ}C$. Then the formula becomes

$$\rho(15^{\circ}C,S) - \rho_W(15^{\circ}C) = 31.113(S/40) - 1.154(S/40)^{3/2} + 0.773(S/40)^2 .$$

(9)

Obviously, since $(S/40) > (S/40)^{3/2} > (S/40)^2$ for $S < 40$, the dominant term is the first term which is linear in salinity S.

3.2. How large is a large pellet?

The correlation given by Isaacs and Thodos [1] (eq.4) is among the simplest in the literature. But unfortunately it is valid for high Reynolds numbers. Isaacs and Thodos [1] state that their drag coefficient formula is valid for R>200. This roughly corresponds to $G \approx 40,000$. In their review paper Gabitto and Tsouris [13] state that Isaacs and Thodos [1] formula is valid for $G \ge 100,000$ which corresponds to $R \ge 300$ roughly. It will be started by estimating the diameter of pellets satisfying either criterion.

$\rho_P = 1100 kg/m^3$ which is a reasonable pellet density is taken. $\rho_f = 1000 kg/m^3$ is taken as water density and $\mu \approx 1.3 \times 10^{-3} Pa \cdot s$ as water viscosity. The gravitational acceleration is taken as $g \approx 10 m/s^2$. The precise values of these quantities are not necessary since this is only an estimation of order of magnitude. Then the following equation will be solved:

$$G = 40 \times 10^3 = \frac{\pi d_C^3 \rho_f g(\rho_P - \rho_f)}{2\mu^2} = 0.93 \times 10^{12} d_C^3 ,$$

(10)

For the pellet diameter d_C. From this $d_C \approx 3.50 mm$ is found. If $G = 100 \times 10^3$ is set then $d_C \approx 4.75 mm$ is found.

Assuming pellet density $\rho_P = 1200 kg/m^3$ these estimates change. This time taking $G = 40 \times 10^3$ gives $d_C \approx 2.78 mm$ and taking $G = 100 \times 10^3$ gives $d_C \approx 3.77 mm$. Therefore it can be said that formula of Isaacs and Thodos [1] is reliable for $d_C > 3.5 - 4.0 mm$. These pellets will be said as large pellets.

3.3. Dependence of settling velocity on diameter

Most pellets have slightly higher lengths than diameters. That means E>1 and E is approximately within 1.0-1.2 range. Such pellets are termed isometric. The pellets with density data discussed in section 3.8 are all isometric E>1 pellets. The Isaacs and Thodos [1] formula for E>1 gives

$$R = 1.005 \left(\frac{\rho_P}{\rho_f}\right)^{0.06} E^{0.04} \sqrt{G} \cdot \tag{11}$$

Notice how small the exponents are in the first two terms. For most feed pellets $\frac{\rho_P}{\rho_f} < 1.2$ and even

for $\frac{\rho_P}{\rho_f} = 1.2$ the term $\left(\frac{\rho_P}{\rho_f}\right)^{0.06} = (1.2)^{0.06} = 1.011$ is very close to unity and the $\left(\frac{\rho_P}{\rho_f}\right)^{0.06}$ term in Isaacs and

Thodos [1] equation can be replaced by 1.01 with an error of order 1% or less. Similarly, for isometric particles $E \approx 1$. Even for $E = 1.4$ the second term is $E^{0.04} = 1.4^{0.04} = 1.013$ and can be replaced with unity with 1% error. Then the equation becomes $R = 1.016\sqrt{G}$. Inserting G from Eq. (3) and setting $v = \frac{\mu}{d_C \rho_f} R$ the following are obtained:

$$v = \frac{\mu}{d_C \rho_f} 1.016 \sqrt{\frac{\pi d_C^3 \rho_f g (\rho_P - \rho_f)}{2\mu^2}} = 4.13 \sqrt{\frac{(\rho_P - \rho_f)}{\rho_f}} d_C^{1/2} \cdot \tag{12}$$

$$v(cm/s) = 41.3 \sqrt{\frac{(\rho_P - \rho_f)}{\rho_f}} [d_C(cm)]^{1/2}$$

A plot of settling velocity versus square root of diameter will be a straight line with zero constant term. This formula is derived from Isaacs and Thodos [1] formula (eq.4) and a direct consequence of it.

3.4. Dependence of settling velocity on aspect ratio (E)

For E>1,

$$R = 1.005 \left(\frac{\rho_P}{\rho_f}\right)^{0.06} E^{0.04} \sqrt{G} \quad (E > 1) \cdot \tag{13}$$

The dependence on E is rather weak ($E^{0.04}$ term) because of small exponent of E. But For E<1 the situation changes dramatically

$$R = 1.0093 \left(\frac{\rho_P}{\rho_f}\right)^{0.025} E^{0.59} \sqrt{G'} \quad (E < 1) \cdot \tag{14}$$

The dependence on E ($E^{0.59}$ term) is rather strong. Fortunately pellets with E<1 are rare. Therefore it is needless to worry about the complications it brings in.

3.5. Dependence of settling velocity on temperature

Studies on settling velocity of feed pellets usually measure the settling velocities for different water temperature and salinities. The Isaacs and Thodos [1] formula used helps us clarify dependence of settling velocity on these parameters.

It is started by studying temperature effects. The temperature can change settling velocity in two ways. Either it changes density of water and particle or it changes viscosity of water. Therefore both effects should be considered.

Density depends on temperature weakly. For example density of pure water changes from $999.102 kg/m^3$ for $T = 15°C$, to, $997.048 kg/m^3$ for $T = 25°C$. Density of pure water changes by $2.05 kg/m^3$ only for $10°C$ temperature increase. Since typically density difference between the pellet and water is about $100 kg/m^3$ this isn't a significant change. Moreover density of the pellet can change by a comparable amount (the pellet undergoes thermal expansion) to compensate the change in density of water and the net effect might become totally negligible. Since there is no information on how density of pellets change with temperature it cannot be said much about the net effect. But it can be concluded that density change due to temperature variation is small and its net effect on settling velocity is negligibly smal1.

As a practical estimation of the effect consider a typical density difference $\rho_P - \rho_f = 100 kg/m^3$. From (eq.12) the ratio settling velocities as a consequence of $10^{\circ}C$ temperature change ($2.05 kg/m^3$ density change) of water is $\sqrt{\dfrac{100 - 2.05}{100}} = 0.99$. This is only one percent difference.

This calculation is based on the assumption that density of the pellet does not change. Since pellet also undergoes thermal expansion by a comparable amount the change in density difference is reduced further and the resulting net effect of temperature change on settling velocity will be even smaller. It is estimated that it is fraction of %1 at most which is indistinguishable from much larger other experimental uncertainties.

The viscosity is quite different however. It strongly depends on temperature. Viscosity of pure water goes from $1.138 \times 10^{-3} Pa \cdot s$ for $T = 15^{\circ}C$, to, $0.89 \times 10^{-3} Pa \cdot s$ for $T = 25^{\circ}C$ which means that it decreases by %22. Therefore the main effect in viscosity dependence must be looked for.

For large pellets eq. (12) shows that settling velocity does not depend on viscosity according to Isaacs and Thodos [1] formula. This is only true for high Reynolds numbers since cancellation of viscosity term form the settling velocity formula depends on the fact that drag coefficient is independent of Reynolds number and it is well known that this happens at high Reynolds numbers. Large pellets settle at high Reynolds number and consequently the settling velocity is independent of the viscosity. For smaller pellets settling velocity does depend on viscosity. As Reynolds number increase the settling velocity depends on viscosity less and less. In order to see how this transition happens one must study correlations given for low Reynolds numbers but it s not expressible by a simple behavior (e.g. linear, exponential etc.) The point of this discussion is that for large enough Reynolds numbers (200-300 for Isaacs and Thodos [1] formula) settling velocity is practically independent of viscosity. The prediction of Isaacs and Thodos [1] formula is that for large pellets settling velocity does not depend on viscosity and hence does not depend on temperature. For smaller pellets settling velocity depends on viscosity and it is expected that settling velocity for smaller pellets will show strong temperature dependence.

3.6. Dependence of settling velocity on salinity

Now the dependence of settling velocity on salinity of water is considered. Salinity changes viscosity and density of water. For larger pellets the settling velocity is independent of viscosity according to Isaacs and Thodos [1] formula. Therefore only density change has an effect.

Density effects can be quite large. If the settling velocities of a pellet in water with salinities 0 and S are $V(0)$ and $V(S)$ then from Isaacs and Thodos [1] formula their ratio is

$$\frac{V(S)}{V(0)} = \sqrt{\frac{\rho_f(S)}{\rho_f(0)}\left(\frac{\rho_P - \rho_f(S)}{\rho_P - \rho_f(0)}\right)} \tag{15}$$

The ratio of densities $\dfrac{\rho_f(S)}{\rho_f(0)}$ changes very little from unity for the usual seawater salinities (0-40 g/L) and can be neglected with a one or two percent error:

$$\frac{V(S)}{V(0)} \approx \sqrt{\frac{\rho_P - \rho_f(S)}{\rho_P - \rho_f(0)}} . \tag{16}$$

But the ratio of density differences can change significantly. The density of fresh water is approximately $\rho_f(0) \approx 1000 kg/m^3$ and density of seawater at $S = 35$ g/L salinity and $T = 15^{\circ}C$ temperature is $\rho_f \approx 1025 kg/m^3$. For a typical pellet density $1080 kg/m^3$ the ratio of velocities (for $T = 15^{\circ}C$) is

$$\frac{V(35g/L)}{V(0)} \approx \sqrt{\frac{1080 - 1025}{1080 - 1000}} = 0.90 . \tag{17}$$

Therefore settling velocity decreases by 10%. The term $\sqrt{\frac{\rho_f(S)}{\rho_f(0)}}$ neglected is $\sqrt{\frac{1025}{1000}} = 1.0124$ which means that by neglecting it 1% error is made. The main part of density effect is due to ratio of density differences $\sqrt{\frac{\rho_P - \rho_f(S)}{\rho_P - \rho_f(0)}}$. The term $\sqrt{\frac{\rho_f(S)}{\rho_f(0)}}$ has a much smaller secondary effect.

Consider the formula,

$$\rho_W(S,T) = \rho_W(T) + a(T)S + b(T)S^{3/2} + c(T)S^2 . \tag{18}$$

It is known from example for $T = 15°C$ in Eq. (9) that the linear term is dominant. Neglecting other terms and using this in eq. (16) The following equation is obtained:

$$\left(\frac{V(S)}{V(0)}\right)^2 \approx \frac{\rho_P - \rho_W(S,T)}{\rho_P - \rho_W(T)} \approx 1 - \frac{a(T)}{\rho_P - \rho_W(T)}S . \tag{19}$$

Therefore a plot of settling velocity square versus salinity will be approximately linear with a negative slope. But the slope of the line will be temperature dependent. Also slope depends on density difference $\rho_P - \rho_W(T)$. If density difference $\rho_P - \rho_W(T)$ is small the effect of salinity on settling velocity can be quite large.

3.7. Sensitivity of settling velocities to errors in density data

Let us explain this with an example. Let's take water density as $\rho_W = 1000 kg/m^3$. Let us also assume that our particle density is $\rho_P = 1080 kg/m^3$. Instead of measuring the true value $1080 kg/m^3$ it is measured as $1050 kg/m^3$ which introduces about %3 error in the density. The formulae for settling velocity involves $\rho_P - \rho_W$ and the $\rho_P - \rho_W$ value is measured as $\rho_P - \rho_W = 50 kg/m^3$ instead of its true value $\rho_P - \rho_W = 80 kg/m^3$. The $\rho_P - \rho_W$ is determined with a 37.5% error. This would introduce about 21% error in the settling velocity for a large pellet. The lesson from this example is that density data must be accurate to get reliable results from the settling velocity formulae. A 3% error in density may look small but it is unacceptable for settling velocity formulae. This problem is not unique to Isaacs and Thodos [1] formula. Any formula from the literature will show this sensitivity to errors of density data.

3.8. A critical comparison with literature

There are several papers in the literature reporting settling velocity measurements. In order to compare their measurements with Isaacs and Thodos [1] formulae, diameter, length and most importantly density of these pellets are needed. Unfortunately papers that report densities of the pellets are few. Only three papers containing the density information could be found. They are Elberizion and Kelly [8], Sutherland et al [2], Piedecausa et al [11]. These papers will be discussed one by one. These papers actually report more pellets than it is shown here. But some of these pellets were not large pellets ($d_C < 3.5mm$). They are excluded from this discussion.

To be precise about the errors absolute percent error (APE) is defined, Let us define

$x(i) =$ Experimental settling velocity for the ith pellet,

$y(i) =$ calculated settling velocity for the ith pellet,

$$APE(i) = \frac{|x(i) - y(i)|}{x(i)} \times 100 . \tag{20}$$

Average of APE over all pellets is called 'Average absolute percent error', abbreviated as AAPE,

$$AAPE = \frac{1}{N}\sum_{i=1}^{N} APE(i) . \tag{21}$$

Roughly, AAPE is the average percent error of the calculations. Here N is the number of pellets. One can define various measures for the error. Above definitions are found useful and instructive.

3.8.1. Sutherland et al [2] paper

Table 1 contains data of the pellets they used, the settling velocities they measured and the settling velocity calculated with Isaacs and Thodos [1] formula.

Table 1. Data and calculated and measured settling velocities for the pellets in Sutherland et al [2] paper. Calculations are done by Isaacs and Thodos [1] formula

Description	Orion 3.5mm	Orion 5mm	Orion 6.5mm	Orion 8.5mm	Orion 11mm
Diameter (mm)	3.5 ± 0.1	5.3 ± 0.1	6.4 ± 0.2	8.5 ± 0.2	10.9 ± 0.2
Length (mm)	4.4 ± 0.5	7.1 ± 0.7	9.5 ± 0.5	11.8 ± 1.0	13.6 ± 1.0
Mass (mg)	44.38 ± 6.0	167.4 ± 29	302.1 ± 29	693.6 ± 53	1291 ± 90
Density (kg/m^3)	1180	1180	1180	1180	1180
Galileo number	70.000	242.000	427.000	1.000.000	2.100.000
Meas. Vel.(cm/s)	10.5 ± 1.36	14.5 ± 1.42	14.0 ± 2.79	16.1 ± 1.29	20.1 ± 0.8
Calc. Vel. (cm/s)	10.18	12.57	13.87	15.94	17.97
Abs. Percent error (APE)(%)	3.0	7.9	0.9	1.0	10.6

AAPE is 4.68%

They used fresh water at 24ºC (density 997.2 kg/m^3, viscosity $0.9108 \times 10^{-3} Pa \cdot s$). The first thing that can be noticed from the table is that densities are all equal to $1180 kg/m^3$. A second thing is that except Orion 3.5, all pellets have Galileo numbers above 100,000 and Isaacs and Thodos [1] formula is valid. Even Orion 3.5 has G$=70 \times 10^3$ and it is above the limit given by Isaacs and Thodos [1] themselves. From the table it is clear that calculations agree with experiment within 5-10% error at most. The AAPE is 4.68%. This is well within experimental error limits. Hence the agreement with Isaacs and Thodos [1] formula and experiment is very satisfactory for the data in this paper.

Since they all can be considered large pellets and they all have the same density our prediction may prove that for large pellets the settling velocity is proportional to square root of diameter (cf. eq. (12)).

Setting $\rho_P = 1180 kg/m^3$ (from Table 1) and $\rho_f = 997.2 kg/m^3$ in eq. (12) $v(cm/s) = 17.68\sqrt{d_C (cm)}$ is found.
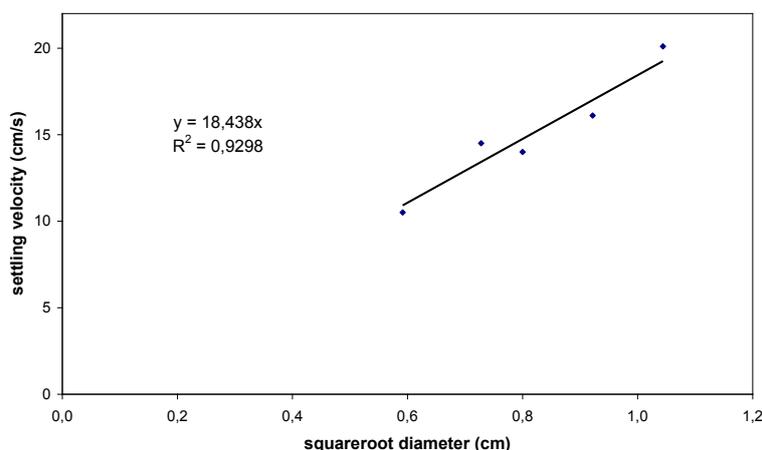
Figure 1. Settling velocity versus square root of diameter for the pellets of Sutherland et al [2] paper

Figure 1 shows the settling velocity vs. $\sqrt{d_C}$ graph. There are five points on the graph corresponding to five different pellets they measured. (See table-1) The $y = mx$ line is fitted to the

curve and $R^2 = 0.93$ indicates that it is a good fit. The slope of the fitted line is 18.43 which are close to the predicted value 17.68.

$y = mx$ is used here instead of more general $y = mx + n$. The $y = mx + n$ line fit imply that at zero radius the settling velocity is nonzero. From physics it is easy to argue that settling velocity must go to zero as the diameter goes to zero.

One might think that dependence of settling velocity on diameter may be linear as it is usual in papers in the literature to look at linearity assumption first. This assumption is also tested.
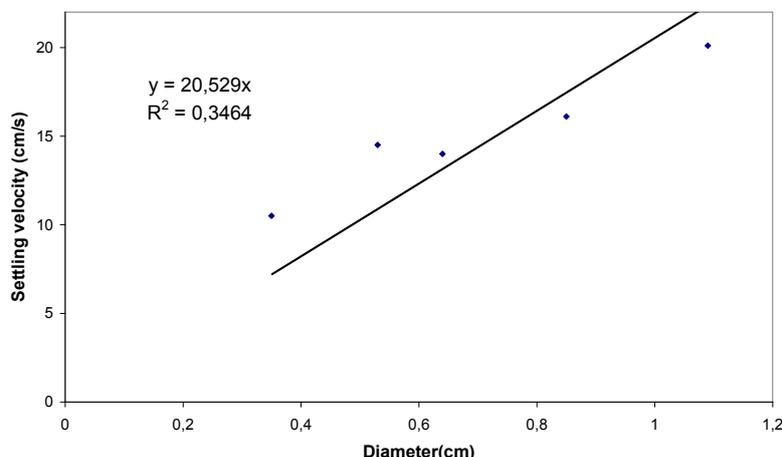
Figure 2. Settling velocity versus diameter for the pellets of Sutherland et al [2] paper

Figure 2 shows settling velocity vs. diameter (d_C) graph and $y = mx$ line fit gives $R^2 = 0.35$ which indicates that it is a very poor fit. In contrast the $y = mx$ line fit to settling velocity vs. $\sqrt{d_C}$ graph gives a good fit ($R^2 = 0.93$) and moreover the proportionality constant is predicted correctly from Isaacs and Thodos formula within a few percent error.

3.8.2. Elberizion and Kelly [8] paper

They used deionizer fresh water at 2ºC, 10ºC, 13ºC. 10ºC results (density $999.7 kg/m^3$, viscosity $1.306 \times 10^{-3} Pa \cdot s$) were only taken. Table 2 gives physical data about the pellets they used.

Table 2. Data for the pellets used in Elberizion and Kelly [8] paper. Only large pellets included. Measured and calculated velocities and percent error is given. Percent error APE and AAPE are defined in the beginning of section 3.8.

Diet	Code	Diameter (mm)	Length (mm)	Density kg/m^3	Galileo number	Meas. velocity	Calc. Velocity (cm/s)	Percent error APE(%)
Trout Rapid 3.5	TR3.5	3.15 ± 0.19	3.70 ± 0.39	1100 ± 20	28.000	8 ± 0.5	7.1	11.25
Trout Rapid 5.0	TR5.0	5.07 ± 0.21	5.55 ± 0.27	1130 ± 10	153.000	11 ± 1	10.25	6.8
Trout Rapid 6.5	TR6.5	6.22 ± 0.25	6.64 ± 0.58	1200 ± 10	435.000	11.9 ± 1	14.12	-19.3
Trout Rapid 8.0	TR8.0	8.15 ± 0.25	7.08 ± 0.36	1000 ± 20	*	12 ± 1	*	*

AAPE is 12.45%

Our first comment about the data is that the error ranges are rather wide. Settling velocities have about %10-20 error on the average and densities have about 2% errors. The % 10 errors in the settling velocities are normal for such measurements. The errors on the densities are seemingly

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small but, as explained before, a 2-3% error in the density is too much error for the settling velocity formulae.

BP1.8, BP2.3, BP0.2 reported in their paper were eliminated as they were not large. Table 3 gives physical as well as the measured and calculated settling velocity data for the remaining pellets. The results are mixed. The agreement is satisfactory for some pellets and not satisfactory for others. It is thought that the density data has too much uncertainty to determine $\rho_P - \rho_W$ accurately. Despite relatively large uncertainties in the density data the settling velocity calculations turn out to be not bad. AAPE is 12.45%. So the errors in calculated settling velocities are within 10-15 %. The highest APE is 19.3%. The Isaacs and Thodos [1] formula seems to work well once more.

3.8.3. Piedecausa et al [11] paper

Piedecausa et al (2009) measured settling velocity of five different pellets in sea water. Table 3 gives the data about these pellets.

Table 3. Data for the pellets used in Piedecausa et al [11] paper. The row "calculated density" is added by the author based on size and weight data of Piedecausa et al [11] paper

Description	FP4a	FP4b	FP6	FP8
Diameter (mm)	4.22 ± 0.09	4.07 ± 0.46	5.42 ± 0.17	8.07 ± 0.02
Length (mm)	4.59 ± 0.09	4.21 ± 0.16	7.07 ± 0.16	8.19 ± 0.11
Mass (mg)	72.10 ± 1.90	68.20 ± 1.90	187.30 ± 5.60	360.10 ± 7.60
Stated Density kg/m^3	1119.42 ± 15.06	1125.36 ± 16.49	1069.85 ± 25.36	1069.77 ± 15.06
Calculated density kg/m^3	1123.1	1245.2	1148.2	859.6

They used seawater with salinity 37g/L and they measured settling velocities at two different temperatures 15ºC and 25ºC. Only 15ºC results will be discussed. The density and viscosity at this temperature and salinity are $1027.5 kg/m^3$ and $1.242 \times 10^{-3} Pa \cdot s$.

Table 3 gives physical data about the four different pellets they measured. The fifth pellet (FP2) is not large enough so it was eliminated from discussion.

They gave diameter, length and mass data as well as density data about the pellets. Mean density was given with six significant figures. From the diameter-length-mass data it is also possible to estimate density with the formula

$$\rho(calculated) = \frac{mass}{\left(\pi d_C^{\ 2} H / 4\right)}. \tag{22}$$

The line (calculated density) does not exist in their paper and this line, which contains density calculated from diameter-length-mass data, was added. The two densities (calculated and stated in the paper) are very different.

Settling velocity from both the stated densities and the calculated densities was calculated. Table 4 contains results of these calculations.

Table 4. Settling Velocities calculated with stated densities for the pellets used in Piedecausa et al [11] paper and measured settling velocities

Description	FP4a	FP4b	FP6	FP8
Measured Velocities (cm/s)	10.8	10.75	12.7	13.5
Calc. velocity With stated densities	7.73	7.82	5.98	7.23
Percent error APE (%) with stated densities	28.4	27.2	52.9	46.4
Calc. velocity With calculated densities	7.88	11.73	10.12	*
Percent error APE (%) with calculated densities	27.0	-9.1	20.3	*

The velocity calculations with stated densities are very different from the velocity measurements. AAPE is 38.7% and the highest APE is as high as 52.9%. The calculated settling velocities for

calculated densities are somewhat better. The AAPE is 18.8% and the highest APE is 27%. The only exception is the FP8 pellet. The stated density for this pellet is $1069.77 kg/m^3$ whereas the calculated density is $859.6 kg/m^3$ and according to calculated density the pellet should not sink at all. The density data in the paper is in doubt in our opinion. The difference between stated and calculated densities should not be this much. Also the formulae for settling velocities and drag coefficients can have at most 15-20% errors at most but 50% errors indicate that there are some problems with the data.

4. Conclusions

In this paper our purpose was to use theoretical information available in the literature to interpret settling velocity measurements on feed pellets present in aquaculture literature. This paper is written to emphasize how theory can help calculate settling velocities of pellets and how it clarifies dependence of settling velocity on various parameters such as salinity and temperature of water, particle diameter, particle density etc.

Isaacs and Thodos [1] formula has been chosen from chemical engineering literature and discussed dependence of settling velocity on different parameters such as temperature, salinity, diameter etc. for large pellets. It is enough to have size and density data of the pellet (information on water properties are readily available from tables) to estimate settling velocity from the formulae in the literature. One drawback of this is that the density information must be very accurate. All the formulae for the settling velocity do not depend on other parameters with such sensitivity. As explained in the paper a 3% error in the density can introduce more than 20% errors in settling velocity. But an error of 3% in diameter will introduce about 1.5% errors (for large pellets) in settling velocity.

When you need settling velocity for a large pellet, you can take a few pellets with nice geometrical shapes from the batch. You can measure their size (diameter and length) and mass and you can calculate their average density from them. Then you can use Isaacs and Thodos [1] formula to estimate the settling velocity. You will have a good estimate with 10-15% error at most. To do a better job you need to measure density with an error less than 1%. Probably the geometric measurement method described above will not give such a high precision. If the feed producer companies supply the average size and density data on their products, then estimating settling velocities will be a minute long calculation.

There are many other formulae in chemical engineering literature and using them one can also discuss small pellets and the transition between small and large pellets. The large pellets in this paper were treated because they are simpler and an accurate formula for cylindrical particles available in the literature for them. A second reason is that settling velocity and density data on smaller pellets are rare and not very reliable. But there exists formulae for settling of smaller objects in the literature and waiting for data on small pellets to interpret feed pellets experiments.

A serious issue is that densities of pellets are not the same but has a statistical distribution. It is already known that diameter and the lengths of the pellets have a statistical distribution but the standard deviations are usually small. Because settling velocity does not depends on size so sensitively, it is enough to use just the averages. But the settling velocity sensitively depends on density of pellets. A statistical distribution of densities with a small standard deviation will produce a statistical distribution for settling velocity of pellets with rather large standard deviation. Not all pellets will sink with a velocity close to the average and distribution of settling velocities must be taking into consideration instead. It is easy to incorporate this effect into the Isaacs and Thodos formula [1] and other formulae in the literature but the data on density distributions virtually nonexistent in the current literature. Therefore it should be treated later when data on density distributions are available.

In conclusion the formula selected from the chemical engineering literature explains much about settling velocities of large pellets and should be incorporated to interpret settling velocity

measurements and interpretation of data. The range of Reynolds numbers relevant to feed pellets is rather limited (less than a few hundred) and therefore it is also possible to develop more accurate formulae for this limited range specially designed for feed pellets. Therefore the problem is open to further development.

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