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PAIR DIFFERENCE CORDIALITY OF MIRROR GRAPH,SHADOW GRAPH AND SPLITTING GRAPH OF CERTAIN GRAPHS

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Abstract. In this paper we dicuss the pair difference cordility of Mirror graph, Splitting graph, Shadow graph of some graphs.

1. Introduction

We consider only finite, undirected and simple graphs. The origin of graph labeling is graceful labeling and introduced this concept by Rosa.A [15].Afterwards many labeling was defined and few of them are harmonious labeling[7], cordial labeling [1], magic labeling [16], mean labeling [19]. Cordial analogous labeling was studied in [2,3,4,5,10,11,12,13,14,17,18]. The notion of pair diference cordial labeling of a graph has been introduced and studied some properties of pair difference cordial labeling in [9].The pair difference cordial labeling behavior of several graphs like path, cycle, star etc have been investigated in [9].In this paper we dicuss the pair difference cordility of Mirror graph,Splitting graph,Shadow graph of some graphs.Term not defined here follow from Harary[8].

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2. Preliminaries

Definition 2.1. [6]. *For a bipartite graph G with partite sets X and Y ,let G' be a copy of G and $X^{'}$ and $Y^{'}$ be copies of X and Y. The mirror graph $M^{'}(G)$, of a graph G as the disjoint union of G and $G^{'}$ with additional edges joining each vertex Y to its corresponding vertex in $Y^{'}$.*

Definition 2.2. [6].

The splitting graph of $G, S^{'}(G)$, is obtained from G by adding for each vertex v of G a new vertex $v^{'}$ so that $v^{'}$ is adjacent to every vertex that is adjacent to v.

Definition 2.3. [6].

The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of $G, G^{'}$ and $G^{''}$ and joining each vertex $u^{'}$ in $G^{'}$ to the neighbours of the corresponding vertex $u^{''}$ in $G^{''}$.

Definition 2.4. [6].

The ladder L_n is the product graph $P_n \times K_2$.

Theorem 2.1. [9].
If G is a (p, q) pair difference cordial graph then

$$q \leq \begin{cases} 2p-3 & \textit{if } p \textit{ is even} \\ 2p-1 & \textit{if } p \textit{ is odd} \end{cases}$$

Theorem 2.2. [9].
The path P_n is pair difference cordial for all values of n except $n \neq 3$.

Corollary 2.3. [9].
The cycle C_n is pair difference cordial if and only if $n > 3$.

Theorem 2.4. [9].
The ladder L_n is pair difference cordial for all values of n.

3. Mirror Graphs

Theorem 3.1. *The mirror graph of the path P_n is pair difference cordial.*

Proof. Since $M'(P_n) \cong L_n$,the proof follows from theorem 2.8. □

Theorem 3.2. *$M^{'}(K_{1,n})$ is pair difference cordial if and only if $n \leq 2$.*

Proof. Let $V(M^{'}(K_{1,n})) = \{x, y, x_i, y_i : 1 \leq i \leq n\}$ and
$E(M^{'}(K_{1,n})) = \{xx_i, yy_i, y_ix_i, xy : 1 \leq i \leq n\}$.Since $S^{'}(K_{1,1}) \cong C_4$. By corollary 2.7, $M^{'}(K_{1,1})$ is pair difference cordial. A pair difference cordial labeling of $M^{'}(K_{1,2})$ is shown in Table 1.

Suppose f is a pair difference cordial labeling of $M^{'}(K_{1,n}), n \geq 3$. Obviously $\Delta_{f_1} \leq 4$. Then $\Delta_{f_1}^c \geq q - 4$. This implies that $\Delta_{f_1}^c \geq 3n + 1 - 4 = 3n - 3$. Hence $\Delta_{f_1}^c - \Delta_{f_1} \geq 3n - 7 > 1$, a contradiction. □

n	x	x_1	x_2	y	y_1	y_2
2	-2	-1	-3	2	1	3

TABLE 1.

Theorem 3.3. *$M^{'}(S(K_{1,n}))$ is pair difference cordial if and only if $n \leq 2$.*

Proof. Let (X_1, Y_1) be bipartition of the first copy of $S(K_{1,n})$ where $X_1 = \{x, y_i : 1 \leq i \leq n\}, Y_1 = \{x_i : 1 \leq i \leq n\}$ and (X_2, Y_2) be bipartition of the second copy of $S(K_{1,n})$ where $X_2 = \{x^{'}, y_i^{'} : 1 \leq i \leq n\}, Y_2 = \{x_i^{'} : 1 \leq i \leq n\}$. Therefore $E(M^{'}(S(K_{1,n}))) = \{xx_i, x^{'}x_i^{'}, y_ix_i, y_i^{'}x_i^{'} : 1 \leq i \leq n\} \cup \{xx^{'}, x_ix_i^{'}, y_iy_i^{'} : 1 \leq i \leq n\} \cup \{xy, x^{'}y^{'}\}$. Clearly there are $4n + 2$ vertices and $6n + 1$ edges in the mirror graph of $S(K_{1,n})$. Since $M^{'}(S(K_{1,1})) \cong L_3$, by theorem 2.8, $M^{'}(S(K_{1,1}))$ is pair difference cordial. A pair difference cordial labeling of $M^{'}(S(K_{1,2}))$ is given in Table 2.

n	x	x_1	x_2	y_1	y_2	$x^{'}$	$x_1^{'}$	$x_2^{'}$	$y_1^{'}$	$y_2^{'}$
2	5	2	4	1	3	-5	-2	-4	-1	-3

TABLE 2.

Suppose f is a pair difference cordial labeling of $M^{'}(S(K_{1,n})), n \geq 3$. Obviously $\Delta_{f_1} \leq 2n+2$. Then $\Delta_{f_1}^c \geq q-2n-2$. This implies that $\Delta_{f_1}^c \geq 6n+1-2n-2 = 4n-1$. Hence $\Delta_{f_1}^c - \Delta_{f_1} \geq 2n - 3 > 1$, a contradiction.

□

Theorem 3.4. *$M^{'}(B_{n,n})$ is pair difference cordial if and only if $n \leq 2$.*

Proof. Let (X_1, Y_1) be bipartition of the first copy of $B_{n,n}$ where $X_1 = \{x, y_i : 1 \leq i \leq n\}, Y_1 = \{y, x_i : 1 \leq i \leq n\}$ and (X_2, Y_2) be bipartition of the second copy of $B_{n,n}$ where $X_2 = \{x^{'}, y_i^{'} : 1 \leq i \leq n\}, Y_2 = \{y^{'}, x_i^{'} : 1 \leq i \leq n\}$. Therefore $E(M^{'}(B_{n,n})) = \{xx_i, x^{'}x_i^{'}, yy_i, y^{'}y_i^{'} : 1 \leq i \leq n\} \cup \{xx^{'}, yy^{'}, x_ix_i^{'}, y_iy_i^{'} : 1 \leq i \leq n\}$. Obviously $M^{'}(B_{n,n})$ has $4n + 2$ vertices and $6n + 1$ edges. Since $M^{'}(B_{1,1}) \cong L_4$,by theorem 2.8, $M^{'}(B_{1,1})$ is pair difference cordial. A pair difference cordial labeling of $M^{'}(B_{1,2})$ is shown in Table 3.

n	x	x_1	x_2	y	y_1	y_2	$x^{'}$	$x_1^{'}$	$x_2^{'}$	$y^{'}$	$y_1^{'}$	$y_2^{'}$
2	2	1	3	-2	-1	-3	5	4	6	-5	-4	-6

TABLE 3.

Suppose f is a pair difference cordial labeling of $M^{'}(B_{n,n}), n \geq 3$. Obviously $\Delta_{f_1} \leq 8$. Then $\Delta_{f_1}^c \geq q - 8$. This implies that $\Delta_{f_1}^c \geq 6n + 4 - 8 = 6n - 4$. Hence $\Delta_{f_1}^c - \Delta_{f_1} \geq 6n - 12 > 1$, a contradiction.

□

4. SHADOW GRAPHS

Theorem 4.1. *Let G be a (p, q) graph with $q \geq p$.Then $D_2(G)$ is not pair difference cordial.*

Proof. Suppose G is a pair difference cordial graph with $q \geq p$. Obviously $|V(D_2(G))| = 2p$ and $|E(D_2(G))| = 4q$. By theorem 2.5, $4q \leq 2(2p) - 3$. This implies that $4q \leq 4q - 3$, a contradiction. □

Theorem 4.2. *$D_2(P_n)$ is pair difference cordial for all values of n.*

Proof. Let $V(D_2(P_n)) = \{x_i, y_i : 1 \leq i \leq n\}$, $E(D_2(P_n)) = \{x_ix_{i+1}, y_iy_{i+1} : 1 \leq i \leq n-1\} \cup \{x_iy_{i+1}, y_ix_{i+1} : 1 \leq i \leq n-1\}$. Clearly $D_2(P_n)$ has $2n$ vertices and $4n-4$ edges.
Define $f : V(D_2(P_n)) \to \{\pm1, \pm2, \pm3, \cdots \pm n\}$ by

$$\begin{aligned} f(x_i) &= i, & 1 \leq i \leq n, \\ f(y_i) &= -i, & 1 \leq i \leq n. \end{aligned}$$

This vertex labeling yields that $D_2(P_n)$ is pair difference cordial for all values of n, since $\Delta_{f_1} = 2n-2 = \Delta_{f_1^c}$. □

Theorem 4.3. *$D_2(C_n)$ is not pair difference cordial for all values of n.*

Proof. Let C_n be the first copy of the cycle $x_1x_2\cdots x_nx_1$ and $y_1y_2\cdots y_ny_1$ be the second copy of the cycle C_n. The maximum number of the edges with the labels 1 among the vertex labels $1, 2, \cdots, n$ is $n-1$. Also the maximum number of the edges with the labels 1 among the vertex labels $-1, -2, \cdots, -n$ is $n-1$. Therefore $\Delta_{f_1} \leq 2n-2$. This implies that $\Delta^c_{f_1} \geq 4n - (2n-2) = 2n+2$. Hence $\Delta_{f_1} - \Delta^c_{f_1} \geq 2n+2-(2n-2) = 4 > 1$, a contradiction. □

Theorem 4.4. *$D_2(K_n)$ is pair difference cordial if and only if $n \leq 2$.*

Proof. Clearly $|V(D_2(K_n))| = 2n$ and $|E(D_2(K_n))| = n(n-1) + 2\binom{n}{2}$. Suppose $D_2(K_n)$ is a pair difference cordial. By theorem 2.5 ,$n(n-1) + 2\binom{n}{2} \leq 2(2n) - 3$, which implies that $2n^2 - 6n + 3 \leq 0$. It gives that $n \leq 2$. Hence $D_2(K_n), n > 3$ is not pair difference cordial. Obviously $D_2(K_1)$ is pair difference cordial. Since $K_2 \cong P_2$, by theorem 2.6, $D_2(K_2)$ is pair difference cordial. □

Theorem 4.5. *$D_2(K_{1,n})$ is pair difference cordial if and only if $n \leq 2$.*

Proof. Clearly $|V(D_2(K_{1,n}))| = 2n+2$ and $|E(D_2(K_{1,n}))| = 4n$. Suppose $D_2(K_{1,n})$ is a pair difference cordial . Obviously $\Delta_{f_1} \leq 2n+1$. Let u be the central vertex of $K_{1,n}$ and $u^{'}$ be the corresponding shadow vertex . Hence $d(u) = d(u^{'}) = 2n$ in $D_2(K_{1,n})$. Now $\Delta f_1 \geq 2n-2+2n-2 \geq 4n-4$. Hence $\Delta_{f_1} - \Delta_{f_1^c} \geq 2n-3$. This implies $n \leq 2$. Since $D_2(K_{1,1}) \cong C_4$, by corollary 2.7, $D_2(K_{1,1})$ is pair difference cordial. The labeling f defined by $f(u) = 2, f(u^{'}) = -2, f(u_1) = -1, f(u_2) = -3, f(u_1^{'}) = 1, f(u_2^{'}) = 3$ is a pair difference cordial labeling of $D_2(K_{1,2})$. □

Theorem 4.6. *$D_2(P_n \odot K_1)$ is not pair difference cordial for all values of n.*

Proof. Let $V(D_2(P_n \odot K_1)) = \{x_i, x_i^{'}, y_i, y_i^{'} : 1 \leq i \leq n\}$. There are $4n$ vertices and $8n-4$ edges.
Suppose $D_2(P_n \odot K_1)$ is pair difference cordial for all values of n. The maximum

number of the edges with the labels 1 among the vertex labels $1, 2, \cdots, n$ is $n-1$ and the maximum number of the edges with the labels 1 among the vertex labels $-1, -2, \cdots, -n$ is $n-1$. Therefore $\Delta_{f_1} \leq 2n - 2 + 2 = 2n$. This implies that $\Delta^c_{f_1} \geq 8n - 4 - 2n = 6n - 4$. Hence $\Delta_{f_1} - \Delta^c_{f_1} \geq 6n - 4 - 2n = 4n - 4 > 1$, a contradiction.

□

5. Spilitting Graphs

Theorem 5.1. *$S^{'}(P_n)$ is pair difference cordial for all n.*

Proof. Let $V(S^{'}(P_n)) = \{x_i, y_i : 1 \leq i \leq n\}$ and $E(S^{'}(P_n)) = \{x_ix_{i+1} : 1 \leq i \leq n-1\} \cup \{x_iy_{i+1}, y_ix_{i+1} : 1 \leq i \leq n-1\}$. There are two cases arises.

Case 1. $n \leq 5$.
A pair difference cordial labeling for this case given in Table 4.

n	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	y_4	y_5
2	1	2				-1	-2			
3	1	-1	3			-2	2	-3		
4	1	2	-1	-2		3	4	-3	-4	
5	1	2	5	-2	-4	3	4	-1	-5	-3

Table 4.

Case 2. $n > 5$.
There are four cases arises.
Subcase 1. $n \equiv 0 \pmod 4$.

Assign the labels $1, 5, 9, \cdots, n-3$ to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-2}{2}}$ respectively and assign the labels $2, 6, 10, \cdots, n-2$ respectively to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n}{2}}$. Now we assign the labels $3, 7, 11, \cdots, n-1$ respectively to the vertices $y_1, y_3, y_5, \cdots,$ $y_{\frac{n-2}{2}}$ and assign the labels $4, 8, 12, \cdots, n$ to the vertices $y_2, y_4, y_6, \cdots, y_{\frac{n}{2}}$ respectively.

Next we assign the labels $-1, -5, -9, \cdots, -(n-3)$ respectively to the vertices $x_{\frac{n+2}{2}}, x_{\frac{n+6}{2}}, x_{\frac{n+10}{2}}, \cdots, x_{n-1}$ and we assign the labels $-2, -6, -10, \cdots, -(n-2)$ respectively to the vertices $x_{\frac{n+4}{2}}, x_{\frac{n+8}{2}}, x_{\frac{n+12}{2}}, \cdots, x_n$. Lastly assign the labels $-3, -7, -11, \cdots, -(n-1)$ respectively to the vertices $y_{\frac{n+2}{2}}, y_{\frac{n+6}{2}}, y_{\frac{n+10}{2}}, \cdots, y_{n-1}$ and assign the labels $-4, -8, -12, \cdots, -n$ to the vertices $y_{\frac{n+4}{2}}, y_{\frac{n+8}{2}}, y_{\frac{n+12}{2}}, \cdots, y_n$ respectively.

Subcase 2. $n \equiv 1 \pmod 4$.

Assign the labels $1, 5, 9, \cdots, n-4$ respectively to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-3}{2}}$ and assign the labels $2, 6, 10, \cdots, n-3$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n-1}{2}}$ respectively. Now we assign the labels $3, 7, 11, \cdots, n-2$ to the vertices $y_1, y_3, y_5, \cdots, y_{\frac{n-3}{2}}$ respectively and assign the labels $4, 8, 12, \cdots, n-1$ respectively to the vertices

$y_2, y_4, y_6, \cdots, y_{\frac{n-1}{2}}$ and assign the label n to the vertex $y_{\frac{n+1}{2}}$.

Now we assign the labels $-1, -3, -5, \cdots, -(\frac{n+1}{2})$ respectively to the vertices $x_{\frac{n+1}{2}}, x_{\frac{n+5}{2}}, x_{\frac{n+9}{2}}, \cdots, x_n$ and we assign the labels $-(\frac{n+3}{2}), -(\frac{n+7}{2}), -(\frac{n+11}{2}), \cdots,$ $-(n-1)$ respectively to the vertices $y_n, y_{n-2}, y_{n-4}, \cdots, y_{\frac{n+5}{2}}$. Next assign the labels $-2, -4, -6, \cdots, -(\frac{n-1}{2})$ respectively to the vertices $y_{\frac{n+3}{2}}, y_{\frac{n+7}{2}}, y_{\frac{n+11}{2}}, \cdots, y_{n-1}$ and assign the labels $-(\frac{n+5}{2}), -(\frac{n+9}{2}), -(\frac{n+13}{2}), \cdots, -(n)$ to the vertices $x_{n-1}, x_{n-3}, x_{n-5},$ $\cdots, x_{\frac{n+3}{2}}$ respectively.

Subcase 3. $n \equiv 2 \pmod 4$.
as in case 1 assign the labels to the vertices $x_i, y_i (1 \leq i \leq n-2)$. Finally we assign the labels $(n-1), n, -n, -(n-1)$ to the vertices $x_{n-1}, x_n, y_{n-1}, y_n$.

Subcase 4. $n \equiv 3 \pmod 4$.

Assign the labels $1, 5, 9, \cdots, n-2$ respectively to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-1}{2}}$ and we assign the labels $2, 6, 10, \cdots, n-5$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n-3}{2}}$ respectively. Now we assign the labels $3, 7, 11, \cdots, n-4$ to the vertices $y_1, y_3, y_5, \cdots, y_{\frac{n-5}{2}}$ respectively and assign the labels $4, 8, 12, \cdots, n-3$ respectively to the vertices $y_2, y_4, y_6, \cdots, y_{\frac{n-3}{2}}$.

Next we assign the labels $-1, -3, -5, \cdots, -(\frac{n-1}{2})$ respectively to the vertices $x_{\frac{n+1}{2}}, x_{\frac{n+5}{2}}, x_{\frac{n+9}{2}}, \cdots, x_{n-1}$ and we assign the labels $-(\frac{n+3}{2}), -(\frac{n+7}{2}), -(\frac{n+11}{2}), \cdots,$ $-(n)$ respectively to the vertices $x_n, x_{n-2}, x_{n-4}, \cdots, x_{\frac{n+3}{2}}$. Next assign the labels $-2, -4, -6, \cdots, -(\frac{n+1}{2})$ respectively to the vertices $y_{\frac{n+3}{2}}, y_{\frac{n+7}{2}}, y_{\frac{n+11}{2}}, \cdots, y_n$ and assign the labels $-(\frac{n+5}{2}), -(\frac{n+9}{2}), -(\frac{n+13}{2}), \cdots, -(n-1)$ to the vertices $y_{n-1}, y_{n-3},$ $y_{n-5}, \cdots, y_{\frac{n+5}{2}}$ respectively.

Finally assign the labels $n-1, n$ to the vertices $y_{\frac{n-1}{2},} y_{\frac{n+1}{2}}$ respectively.

□

Theorem 5.2. *$S'(P_n \odot K_1)$ is pair difference cordial.*

Proof. Let $V(S'(P_n \odot K_1)) = \{x_i, x'_i, y_i, y'_i : 1 \leq i \leq n\}$ and $E(S'(P_n \odot K_1)) = \{x_i x'_{i+1}, x_{i+1} x'_i : 1 \leq i \leq n-1\} \cup \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{y_i x'_i, x_i y'_i : 1 \leq i \leq n\}$. There are $4n$ vertices and $6n-3$ edges.There are two cases arises.

Case 1.n is even.

Assign the labels $1, 5, 9, \cdots, (2n-3)$ to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n}{2}}$ respectively and we assign the labels $-1, -5, -9, \cdots, -(2n-3)$ respectively $x_{\frac{n+2}{2}}, x_{\frac{n+4}{2}}, x_{\frac{n+6}{2}}, \cdots,$ x_n. Next assign the labels $4, 8, 12, \cdots, 2n$ to the vertices $x'_1, x'_2, x'_3, \cdots, x'_{\frac{n}{2}}$ respectively and we assign the labels $-4, -8, -12, \cdots, -(2n)$ respectively $x'_{\frac{n+2}{2}}, x'_{\frac{n+4}{2}}, x'_{\frac{n+6}{2}},$ $\cdots, x'_n$.

Now we assign the labels $3, 7, 11, \cdots, (2n-1)$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n}{2}}$ respectively and we assign the labels $-3, -7, -11, \cdots, -(2n-1)$ respectively $y_{\frac{n+2}{2}}, y_{\frac{n+4}{2}}, y_{\frac{n+6}{2}}, \cdots, y_n$. Next assign the labels $2, 6, 10, \cdots, (2n-2)$ to the vertices $y_1^{'}, y_2^{'}, y_3^{'}, \cdots, y_{\frac{n}{2}}^{'}$ respectively and assign the labels $-2, -6, -10, \cdots, -(2n-2)$ respectively $y_{\frac{n+2}{2}}^{'}, y_{\frac{n+4}{2}}^{'}, y_{\frac{n+6}{2}}^{'}, \cdots, y_n^{'}$.

Clearly $\Delta_{f_1} = 3n-2, \Delta_{f_1^c} = 3n-1$. This vertex labeling gives that $S^{'}(P_n \odot K_1)$ is pair difference cordial for all even values of n.

Case 2.n is odd.

As in case 1 , assign the labels to the vertices $x_i, y_i, x_i^{'}, y_i{'}(1 \leq i \leq n-1)$. Finally we assign the labels $2n-1, -(2n-1), 2n, -2n$ to the vertices $x_n^{'}, x_n, y_n, y_n^{'}$.

Clearly $\Delta_{f_1} = 3n-2, \Delta_{f_1^c} = 3n-1$. This vertex labeling gives that $S^{'}(P_n \odot K_1)$ is pair difference cordial for all odd values of n.

□

Theorem 5.3. *$S^{'}(K_n)$ is pair difference cordial if and only if $n \leq 3$.*

Proof. Clearly $\left|V(S^{'}(K_n))\right| = 2n$ and $\left|E(S^{'}(K_n))\right| = \frac{3n(n-1)}{2}$.
Case 1. $n \leq 3$.
Obviously $S^{'}(K_1)$ is pair difference cordial.Since $S^{'}(K_n) \cong C_4$, then $S^{'}(K_2)$ is pair difference cordial. By theorem 5.2, $S^{'}(K_3)$ is pair difference cordial.

Case 2. $n > 3$.
Suppose $S^{'}(K_n)$ is pair difference cordial. By theorem 2.5 ,

$$\frac{3n(n-1)}{2} \leq 2(2n) - 3,$$
$$\Rightarrow 3n^2 - 3n \leq 4(2n) - 6,$$
$$\Rightarrow 3n^2 - 11n \leq -6,$$
$$\Rightarrow -3n^2 + 11n \geq 6, \text{a contradiction}$$

□

Theorem 5.4. *$S^{'}(K_{1,n})$ is pair difference cordial if and only if $n \leq 3$.*

Proof. Let $V(S^{'}(K_{1,n})) = \{x, y, x_i, y_i : 1 \leq i \leq n\}$ and $E(S^{'}(K_{1,n})) = \{xx_i, yy_i, y_ix : 1 \leq i \leq n\}$.Since $S^{'}(K_{1,1}) \cong P_4$. By theorem 2.6, $S^{'}(K_{1,1})$ is pair difference cordial. A pair difference cordial labeling of $S^{'}(K_{1,2})$ and $S^{'}(K_{1,3})$ is shown in Table 5.

n	x	x_1	x_2	x_3	y	y_1	y_2	y_3
2	-1	-2	-3		2	1	3	
3	-1	-2	-3	-4	2	1	3	4

TABLE 5.

Suppose f is a pair difference cordial labeling of $S^{'}(K_{1,n}), n > 3$. Obviously $\Delta_{f_1} \leq 4$. Then $\Delta_{f_1}^c \geq q - 4$. This implies that $\Delta_{f_1}^c \geq 3n - 4$. Hence $\Delta_{f_1}^c - \Delta_{f_1} \geq 3n - 8 > 1$, a contradiction.

□

6. Conclusions

In this paper, we have studied about the pair difference cordility of Mirror graph,Splitting graph,Shadow graph of some graphs.Investigation of the pair difference cordility of Mirror graph,Splitting graph,Shadow graph of some special graphs are the open problems.

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