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On Travelling Wave Solutions of Dullin-Gottwald-Holm Dynamical Equation and Strain Wave Equation

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Abstract: In this study, extended trial equation method (ETEM) is implemented to obtain exact solutions of the Dullin-Gottwald-Holm Dynamical equation (DGHDE) and the strain wave equation. We constitute some exact solutions such as soliton solutions, rational, Jacobi elliptic, periodic wave solutions and hyperbolic function solutions of these equations via ETEM. Then, we present results that we obtained by using this method.

Dullin-Gottwald-Holm Denklemi ve Gergin Dalga Denkleminin Hareketli Dalga Çözümleri Üzerine

Anahtar Kelimeler
Dullin-Gottwald-Holm Dinamik Denklemi,
Gergin dalga denklemi,
Genişletilmiş deneme denklem metodu,
Soliton çözümler,
Rasyonel Jacobi eliptik ve hiperbolik fonksiyon çözümler.

Öz: Bu çalışmada, Dullin-Gottwald-Holm Dinamik denkleminin ve gergin dalga denkleminin kesin çözümlerini elde etmek için genişletilmiş deneme denklem metodu uygulanmıştır. Bu denklemlerin soliton çözümleri, rasyonel, Jacobi eliptik, periyodik dalga çözümleri ve hiperbolik fonksiyon çözümleri gibi bazı kesin çözümleri genişletilmiş deneme denklem metodu ile elde edilmiştir. Daha sonra bu yöntemi kullanarak elde ettiğimiz sonuçlar sunduk.

1. INTRODUCTION

In recent years, travelling wave solutions are substantially significant subject matter in biophysics, geophysical sciences, chemical kinematics, optical fibers, the technology of space, elastic media and some issues in nonlinear sciences. Recently many scientists have applied various methods to obtain travelling wave solutions of NLEEs (nonlinear evolution equations) such as Hirota's direct method [1], Jacobi elliptic function method [2], new version of the trial equation method [3], (G'/G)–expansion method [4], tanh-coth method [5] etc. In this work, the ETEM [6,7] will be performed to get exact solutions of the DGHDE and the strain wave equation.
Firstly, we tackle the following the DGHDE [8]

$$u_t + h_1 u_x - h_2^2\left(u_{xxt} + u u_{xxx} + 2u_x u_{xx}\right) + 3u u_x + h_3 u_{xxx} = 0,\ t \ge 0, \tag{1}$$

where fluid velocity of system is symbolized by u in spatial direction x.
$h_2^2(h_2>0)$ and $\frac{h_1}{h_3}$ indicate squares of length scales, and $h_1=\sqrt{gh}$ (where $h_1=2\omega$) demonstrates the linear wave speed for undisturbed water at rest at spatial infinity. G. M. Octavian has submitted the analysis of wave-breaking solutions to Eq. (1) [9]. M. H. Raddadi et al. have obtained solitary wave solutions of Eq. (1) by using new extended direct algebraic method [10]. R. K. Gupta and B. Anupma have found exact solutions of Eq. (1) via Lie Classical method [11].
Secondly, we investigate the strain wave equation given below [12]:

$$u_{tt}-u_{xx}-\gamma\left(\alpha_1\left(u^2\right)_{xx}-\alpha_3 u_{xxxx}+\alpha_4 u_{xxtt}\right)=0, \tag{2}$$

where γ shows elastic strain, α_1, α_3 and α_4 are arbitrary constants. Kumar et al. have found new exact solitary wave solutions of Eq. (2) by using generalized exponential rational function method [13]. M. G. Hafez and M. A. Akbar have obtained multiple explicit and exact traveling wave solutions of this equation by using an exponential expansion method [14].
The arrangement of this study was done as follows. In Sec. 2, we perform ETEM on DGHDE and strain wave equation. In Sec. 3, the results acquired using this method are expressed.

2. FUNDAMENTALS OF THE ETEM

Step 1. For a known nonlinear partial differential equation

$$P\left(u,u_t,u_x,u_{xx},\ldots\right)=0 \tag{3}$$

we get the wave transformation as

$$u\left(x_1,x_2,\ldots,x_N,t\right)=u(\eta),\ \eta=\lambda\left(\sum_{j=1}^{N}x_j-ct\right), \tag{4}$$

where $\lambda\neq 0$, $c\neq 0$. Accommodating Eq. (4) into Eq. (3) satisfies a nonlinear ordinary differential equation,

$$N\left(u,u',u'',\ldots\right)=0. \tag{5}$$

Step 2. Presume that the trial equation of Eq. (5) can be indicated as following:

$$u=\sum_{i=0}^{\delta}\tau_i\Gamma^i, \tag{6}$$

where

$$(\Gamma')^2=\Lambda(\Gamma)=\frac{\phi(\Gamma)}{\psi(\Gamma)}=\frac{\xi_\vartheta\Gamma^\vartheta+\ldots+\xi_1\Gamma+\xi_0}{\zeta_\varepsilon\Gamma^\varepsilon+\ldots+\zeta_1\Gamma+\zeta_0}. \tag{7}$$

Considering relations (8) and (9), we can have

$$(u')^2=\frac{\phi(\Gamma)}{\psi(\Gamma)}\left(\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}\right)^2, \tag{8}$$

$$u''=\frac{\phi'(\Gamma)\psi(\Gamma)-\phi(\Gamma)\psi'(\Gamma)}{2\psi^2(\Gamma)}\left(\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}\right)+\frac{\phi(\Gamma)}{\psi(\Gamma)}\left(\sum_{i=0}^{\delta}i(i-1)\tau_i\Gamma^{i-2}\right), \tag{9}$$

where $\phi(\Gamma)$ and $\psi(\Gamma)$ are polynomials. Putting these terms into Eq. (5) ensures an equation of polynomial $\Omega(\Gamma)$ of Γ:

$$\Omega(\Gamma)=\sigma_s\Gamma^s+\ldots+\sigma_1\Gamma+\sigma_0=0. \tag{10}$$

In accordance with balance principle, we can describe a relation of ϑ,ε and δ. We can find some values of ϑ,ε and δ.

Step 3. Letting the coefficients of $\Omega(\Gamma)$ all be zero will satisfy an algebraic equations system:

$$\sigma_i=0,\quad i=0,\ldots,s. \tag{11}$$

Solving equation system (11), we will define the values of $\xi_0,\ldots,\xi_\vartheta;\ \ \zeta_0,\ldots,\zeta_\varepsilon$ and $\tau_0,\ldots,\tau_\delta$.

Step 4. Simplify Eq. (7) to elementary integral shape,

$$\pm(\eta-\eta_0)=\int\frac{d\Gamma}{\sqrt{\Lambda(\Gamma)}}=\int\sqrt{\frac{\psi(\Gamma)}{\phi(\Gamma)}}d\Gamma. \tag{12}$$

Applying a complete discrimination system for polynomial to classify the roots of $\Omega(\Gamma)$, we solve the infinite integral (12) and categorize the exact solutions for Eq. (3).

3. IMPLEMENTATIONS OF THE ETEM

In this chapter, we implement the ETEM to the DGHDE and the strain wave equation, respectively.

3.1. Implementation on the DGHDE

In an attempt to find travelling wave solutions of Eq. (1), we take the transformation

$$u(x,t)=U(\eta),\ \eta=x-vt,\ v\neq 0. \tag{13}$$

Then, we get

$$vh_2^2(U'')'-vU'+h_1U'-h_2^2U(U'')'+h_3(U'')'+3UU'-2h_2^2U'U''=0. \tag{14}$$

Also, integrating Eq. (14) according to η and getting the integration constant to zero, we attain

$$U''\left(h_2^2(-v)-h_3+h_2^2U\right)-(h_1-v)U-\frac{3U^2}{2}+\frac{1}{2}h_2^2(U')^2=0. \tag{15}$$

Embedding Eqs. (8) and (9) into Eq. (15), and utilizing the balance principle, we gain

$$\vartheta=\varepsilon+2. \tag{16}$$

Then, we procure the corollaries as follows:

Case 1: If we choose $\varepsilon=0,\ \delta=1$ and $\vartheta=2$, then,

$$(u')^2=\frac{\tau_1^2\left(\xi_0+\Gamma\xi_1+\Gamma^2\xi_2\right)}{\zeta_0}, \tag{17}$$

$$u''=\frac{\tau_1\left(\xi_1+2\Gamma\xi_2\right)}{2\zeta_0}, \tag{18}$$

where $\xi_2 \neq 0,\ \zeta_0 \neq 0.$ Substituting Eq. (6), Eq. (17) and Eq. (18) into Eq. (15), we get an algebraic equation system. Then, by using Wolfram Mathematica 12, $\xi_0,\ \xi_1,\ \xi_2, \zeta_0$ and v coefficients are obtained as following

$$\begin{aligned}
&\xi_0 = \xi_0\ , \xi_1 = \xi_1,\ \xi_2 = \frac{h_2^2 \xi_1 \tau_1}{h_3 + h_2^2\left(h_1 + 2\tau_0\right)},\\
&\tau_0 = \tau_0,\ \tau_1 = \tau_1,\ \zeta_0 = \frac{h_2^4 \xi_1 \tau_1}{h_3 + h_2^2\left(h_1 + 2\tau_0\right)},\\
&v = \frac{-\xi_1\left(h_3 + h_2^2 h_3 \tau_0 + h_2^4 \tau_0^2 + h_1 h_2^2\left(h_3 + h_2^2 \tau_0\right)\right)}{h_2^2\left(h_1 h_2^2 + h_3\right)\xi_1} + \\
&\quad \frac{h_2^2\, \xi_0\left(h_3 + h_2^2\left(h_1 + 2\tau_0\right)\right)\tau_1}{h_2^2\left(h_1 h_2^2 + h_3\right)\xi_1},
\end{aligned} \tag{19}$$

Embedding Eq. (19) into Eqs. (7) and (12), we acquire

$$\pm\left(\eta - \eta_0\right) = A\int \frac{d\Gamma}{\sqrt{\frac{\xi_0}{\xi_2} + \frac{\xi_1}{\xi_2}\Gamma + \Gamma^2}}, \tag{20}$$

where $A = \sqrt{\frac{\zeta_0}{\xi_2}} = h_2.$

Integrating Eq. (20), we gain the solutions of Eq. (1) as follows

$$\left(\eta - \eta_0\right) = A\ln\left(\Gamma - \alpha_1\right) \tag{21}$$

$$\pm\left(\eta - \eta_0\right) = 2A\ln\left[\sqrt{\Gamma - \alpha_1} + \sqrt{\Gamma - \alpha_2}\right], \alpha_2 > \alpha_1. \tag{22}$$

Moreover, α_1 and α_2 are the roots of the polynomial equation,

$$\Gamma^2 + \frac{\xi_1}{\xi_2}\Gamma + \frac{\xi_0}{\xi_2} = 0. \tag{23}$$

Embedding Eq. (21) and Eq. (22) into Eq. (6), we can find the following exact traveling wave solutions for Eq. (1), respectively:

$$u(x,t) = \tau_0 + \tau_1\left(\alpha_1 + e^{\pm\frac{((x - vt) - \eta_0)}{h_2}}\right), \tag{24}$$

$$u(x,t) = \tau_0 + \frac{\tau_1}{4}\left(\begin{array}{c} 2\left(\alpha_1 + \alpha_2\right) + e^{\pm\frac{((x-vt)-\eta_0)}{h_2}} \\ +\left(\alpha_1 - \alpha_2\right)^2 e^{\pm\frac{((x-vt)-\eta_0)}{h_2}} \end{array}\right), \tag{25}$$

where

$$v = \frac{-\xi_1\left(h_3 + h_2^2 h_3 \tau_0 + h_2^4 \tau_0^2 + h_1 h_2^2\left(h_3 + h_2^2 \tau_0\right)\right)}{h_2^2\left(h_1 h_2^2 + h_3\right)\xi_1} + \frac{h_2^2\, \xi_0\left(h_3 + h_2^2\left(h_1 + 2\tau_0\right)\right)\tau_1}{h_2^2\left(h_1 h_2^2 + h_3\right)\xi_1}.$$

For simplicity, we take $\eta_0 = 0, \tau_0 = -\tau_1\alpha_1,$ then Eq. (24) is reduced to the single king solution,

$$u(x,t) = \left(\tilde{A}e^{\tilde{B}(x-vt)}\right), \tag{26}$$

where $\tilde{A} = \tau_1,\ \tilde{B} = \pm\frac{1}{A}$.

For simplicity, we take $\eta_0 = 0, \alpha_1 = 1,\ \alpha_2 = 0,$ then Eq. (25) is reduced to the hyperbolic function solution,

$$u(x,t) = \tau_0 + \frac{\tau_1}{2}\left(1 + \cosh\left(\tilde{B}(x-vt)\right)\right), \tag{27}$$

where

$$v = \frac{-\xi_1\left(h_3 + h_2^2h_3\tau_0 + h_2^4\tau_0^2 + h_1h_2^2\left(h_3 + h_2^2\tau_0\right)\right)}{h_2^2\left(h_1h_2^2 + h_3\right)\xi_1} + \frac{h_2^2\,\xi_0\left(h_3 + h_2^2\left(h_1 + 2\tau_0\right)\right)\tau_1}{h_2^2\left(h_1h_2^2 + h_3\right)\xi_1}.$$

Case 2: If we choose $\varepsilon = 0,\ \delta = 2$ and $\vartheta = 2,$ then

$$(u')^2 = \frac{\left(\tau_1 + 2\Gamma\tau_2\right)^2\left(\xi_0 + \Gamma\xi_1 + \Gamma^2\xi_2\right)}{\zeta_0}, \tag{28}$$

$$u'' = \frac{4\tau_2\left(\xi_0 + \Gamma\xi_1 + \Gamma^2\xi_2\right) + \left(\xi_1 + 2\Gamma\xi_2\right)\left(\tau_1 + 2\Gamma\tau_2\right)}{2\zeta_0}, \tag{29}$$

where $\xi_2 \neq 0,\ \zeta_0 \neq 0$.
Solving algebraic equation system (11), we find

$$\begin{aligned}&\xi_0 = \xi_0,\quad \xi_1 = \xi_1,\quad \xi_2 = -\frac{h_2^2\xi_1^2\tau_1}{3\left(h_1h_2^2 + h_3\right)\xi_1 - 4h_2^2\xi_0\tau_1}, \zeta_0 = -\frac{h_2^4\xi_1^2\tau_1}{3\left(h_1h_2^2 + h_3\right)\xi_1 - 4h_2^2\xi_0\tau_1},\\ &\tau_0 = -2h_1 - \frac{2h_3}{h_2^2} + \frac{2\xi_0\tau_1}{\xi_1}, \tau_1 = \tau_1,\quad \tau_2 = 0,\ v = -2h_1 - \frac{3h_3}{h_2^2} + \frac{3\xi_0\tau_1}{\xi_1}.\end{aligned} \tag{30}$$

Setting these results into Eqs. (7) and (12), we have

$$\pm(\eta - \eta_0) = A_1\int\frac{d\Gamma}{\sqrt{\frac{\xi_0}{\xi_2} + \frac{\xi_1}{\xi_2}\Gamma + \Gamma^2}}, \tag{31}$$

where $A_1 = \sqrt{\frac{\zeta_0}{\xi_2}} = h_2$.

Integrating Eq. (31), we obtain the solutions of Eq. (1) as following:

$$\pm(\eta - \eta_0) = A_1\ln\left(\Gamma - \alpha_1\right), \tag{32}$$

$$\pm(\eta - \eta_0) = 2A_1\ln\left[\sqrt{\Gamma - \alpha_1} + \sqrt{\Gamma - \alpha_2}\right], \alpha_2 > \alpha_1. \tag{33}$$

Furthermore, α_1 and α_2 are the roots of the polynomial equation,

$$\Gamma^2+\frac{\xi_1}{\xi_2}\Gamma+\frac{\xi_0}{\xi_2}=0. \tag{34}$$

Setting Eqs. (32) and (33) into Eq. (6), we find travelling wave solutions of Eq. (1) as

$$u(x,t)=\left[\begin{array}{l}\tau_0+\tau_1 h_1+\tau_1 e^{\pm\frac{(x-vt-\eta_0)}{h_2}}\\+\tau_2\left(h_1+e^{\pm\frac{(x-vt-\eta_0)}{h_2}}\right)^2\end{array}\right], \tag{35}$$

$$u(x,t)=\left[\begin{array}{l}\tau_0+\frac{\tau_1}{4}\left(\begin{array}{l}2(h_2+h_1)+e^{\pm\frac{(x-vt-\eta_0)}{h_2}}\\+(h_1-h_2)^2 e^{\mp\frac{(x-vt-\eta_0)}{h_2}}\end{array}\right)\\+\frac{\tau_2}{16}\left(\begin{array}{l}2(h_2+h_1)+e^{\pm\frac{(x-vt-\eta_0)}{h_2}}\\+(h_1-h_2)^2 e^{\mp\frac{(x-vt-\eta_0)}{h_2}}\end{array}\right)^2\end{array}\right]. \tag{36}$$

For simplicity, we take $\eta_0=0,$ then Eq. (35) is reduced to the single king solution,

$$u(x,t)=\left[\sum_{i=0}^{2}\tau_i\left(\alpha_1+e^{B\left(x-\left(-2h_1-\frac{3h_3}{h_2^2}+\frac{3\xi_0\tau_1}{\xi_1}\right)t\right)}\right)^i\right], \tag{37}$$

where $B=\pm\frac{1}{A_1}$, $v=-2h_1-\frac{3h_3}{h_2^2}+\frac{3\xi_0\tau_1}{\xi_1}$.

For simplicity, we take $\eta_0=0,\alpha_1=1,\alpha_2=0,$ then Eq. (36) is reduced to the hyperbolic function solution,

$$u(x,t)=\left[\sum_{i=0}^{2}\frac{\tau_i}{2^i}\left(1+\cosh\left(B(x-vt)\right)\right)^i\right], \tag{38}$$

where $v=-2h_1-\frac{3h_3}{h_2^2}+\frac{3\xi_0\tau_1}{\xi_1}$.

Case 3: If we choose $\varepsilon=1,\ \delta=1$ and $\vartheta=3$ then

$$(u')^2=\frac{\tau_1^2\left(\xi_0+\Gamma\xi_1+\Gamma^2\xi_2+\Gamma^3\xi_3\right)}{\zeta_0+\Gamma\zeta_1}, \tag{39}$$

$$u''=\frac{(\zeta_0+\Gamma\zeta_1)(\xi_1+2\Gamma\xi_2+3\Gamma^2\xi_3)\tau_1}{2(\zeta_0+\Gamma\zeta_1)^2}-\frac{\zeta_1\left(\xi_0+\Gamma\xi_1+\Gamma^2\xi_2+\Gamma^3\xi_3\right)\tau_1}{2(\zeta_0+\Gamma\zeta_1)^2}, \tag{40}$$

where $\xi_3 \neq 0,\ \zeta_1 \neq 0$. Consecutively, resolving the algebraic equation system (11) yields

$$\xi_1 = \frac{h_2^4 \xi_2^2 \tau_0 \left(2h_3 + h_2^2\left(2h_1 + \tau_0\right)\right)}{\zeta_1 \left(h_3 + h_2^2\left(h_1 + 2\tau_0\right)\right)^2},\ \xi_2 = \xi_2,\ \xi_3 = \frac{\zeta_1}{h_2^2},\ \zeta_0 = 0, \zeta_1 = \zeta_1,$$
$$\tau_0 = \tau_0,\ \tau_1 = \frac{\zeta_1\left(h_3 + h_2^2\left(h_1 + 2\tau_0\right)\right)}{h_2^4 \xi_2}, v = -\frac{h_3}{h_2^2} + \tau_0. \tag{41}$$

Embedding these corollaries into Eqs. (7) and (12), we gain

$$\pm\left(\eta - \eta_0\right) = A_2 \int \frac{\sqrt{\frac{\zeta_0}{\zeta_1} + \Gamma}}{\sqrt{\frac{\xi_0}{\xi_3} + \frac{\xi_1}{\xi_3}\Gamma + \frac{\xi_2}{\xi_3}\Gamma^2 + \Gamma^3}} d\Gamma, \tag{42}$$

where $A_2 = \sqrt{\frac{\zeta_1}{\xi_3}} = h_2$.

Integrating Eq. (42), we get the solutions of Eq. (1) as following:

$$\pm\left(\eta - \eta_0\right) = 2A_2 \left(\begin{array}{l} \ln\left|\sqrt{\frac{\zeta_0 + \zeta_1\Gamma}{\zeta_1}} + \sqrt{\Gamma - \alpha_1}\right| \\ -\sqrt{\frac{\zeta_0 + \zeta_1\Gamma}{b_1\left(\Gamma - \alpha_1\right)}} \end{array} \right), \tag{43}$$

$$\pm\left(\eta - \eta_0\right) = \frac{2A_2}{\sqrt{\zeta_1\left(\alpha_2 - \alpha_1\right)}} \left\{ \begin{array}{l} \sqrt{\zeta_0 + \zeta_1\alpha_1} \arctan\left[\sqrt{\frac{\left(\zeta_0 + \zeta_1\alpha_1\right)\left(\Gamma - \alpha_2\right)}{\left(\zeta_0 + \zeta_1\Gamma\right)\left(\alpha_2 - \alpha_1\right)}}\right] \\ + \sqrt{\alpha_2 - \alpha_1} \ln\left|\sqrt{\frac{\zeta_0 + \zeta_1\Gamma}{\zeta_1}} + \sqrt{\Gamma - \alpha_2}\right| \end{array} \right\}, \tag{44}$$

$$\pm\left(\eta - \eta_0\right) = \frac{2A_2\left(\left(\zeta_0 + \zeta_1\alpha_1\right)F\left(\varphi, l\right)\right)}{\sqrt{\zeta_1\left(\alpha_1 - \alpha_2\right)\left(\zeta_0 + \zeta_1\alpha_3\right)}} + \frac{2A_2\left(\left(\alpha_3 - \zeta_1\alpha_1\right)\pi\left(\varphi, n, l\right)\right)}{\sqrt{\zeta_1\left(\alpha_1 - \alpha_2\right)\left(\zeta_0 + \zeta_1\alpha_3\right)}}, \tag{45}$$

where

$$F\left(\varphi, l\right) = \int_0^{\varphi} \frac{d\psi}{\sqrt{1 - l^2 \sin^2\psi}},$$
$$\pi\left(\varphi, n, l\right) = \int_0^{\varphi} \frac{d\psi}{\left(1 + n\sin^2\psi\right)\sqrt{1 - l^2\sin^2\psi}}, \tag{46}$$

and

$$\varphi = \arcsin\sqrt{\frac{(\Gamma-\alpha_3)(\alpha_2-\alpha_1)}{(\Gamma-\alpha_1)(\alpha_2-\alpha_3)}},$$

$$n = \frac{\alpha_3-\alpha_2}{\alpha_1-\alpha_2},\ l^2 = \frac{(\zeta_0+\zeta_1\alpha_1)(\alpha_3-\alpha_2)}{(\zeta_0+\zeta_1\alpha_3)(\alpha_1-\alpha_2)}. \tag{47}$$

Also, α_1, α_2 and α_3 are the roots of the polynomial equation,

$$\Gamma^3 + \frac{\xi_2}{\xi_3}\Gamma^2 + \frac{\xi_1}{\xi_3}\Gamma + \frac{\xi_0}{\xi_3} = 0. \tag{48}$$

Remark 1. The solutions of Eq. (1) were attained by using ETEM and these obtained solutions were checked in Wolfram Mathematica 12.

3.2. Implementation of the Strain Wave Equation

In an attempt to find travelling wave solutions of Eq. (2), we take the transformation $u(x,t)=U(\eta),\ \ \eta = x-kt,$ where k is an arbitrary constant. Then, we acquire

$$(k^2-1)U'' - \gamma\alpha_1(U^2)'' + \gamma(\alpha_3-\alpha_4k^2)U^{(4)} = 0, \tag{49}$$

Also, integrating Eq. (49) according to η twice and getting the integration constant to zero, we get

$$\gamma(\alpha_3-\alpha_4k^2)U'' + (k^2-1)U - \gamma\alpha_1U^2 = 0. \tag{50}$$

Embedding Eqs. (8) and (9) into Eq. (50), and using the balance principle, we find

$$\vartheta = \delta+\varepsilon+2. \tag{51}$$

After this solution procedure, we get the results as follows:

Case 1: If we take $\varepsilon=0,\ \delta=1$ and $\vartheta=3$, then

$$(u')^2 = \frac{\tau_1^2(\xi_3\Gamma^3+\xi_2\Gamma^2+\xi_1\Gamma+\xi_0)}{\zeta_0}, \tag{52}$$

$$u'' = \frac{\tau_1(3\xi_3\Gamma^2+2\xi_2\Gamma+\xi_1)}{2\zeta_0}, \tag{53}$$

where $\xi_3\neq 0,\ \zeta_0\neq 0$. Respectively, solving the algebraic equation system (11) yields

$$\xi_0=\xi_0,\ \xi_1=\xi_1,\ \xi_2=\xi_2,\ \xi_3=\frac{\tau_1(2\xi_2\tau_0-\xi_1\tau_1)}{3\tau_0^2},$$

$$\zeta_0 = \frac{-2\xi_2\tau_0(-\alpha_3+\alpha_4(1+\gamma\alpha_1\tau_0))}{2\alpha_1\tau_0^2} + \frac{\xi_1(-\alpha_3+\alpha_4(1+2\gamma\alpha_1\tau_0))\tau_1}{2\alpha_1\tau_0^2}, \tag{54}$$

$$\tau_0=\tau_0,\ \tau_1=\tau_1,\ k=\sqrt{\frac{-2\xi_2\tau_0(1+\gamma\alpha_1\tau_0)+\xi_1(1+2\gamma\alpha_1\tau_0)\tau_1}{-2\xi_2\tau_0+\xi_1\tau_1}}.$$

Embedding these results into Eqs. (7) and (12), we have

$$\pm(\eta-\eta_0)=\sqrt{A_3}\int\frac{d\Gamma}{\sqrt{\Gamma^3+\frac{\xi_2}{\xi_3}\Gamma^2+\frac{\xi_1}{\xi_3}\Gamma+\frac{\xi_0}{\xi_3}}}. \tag{55}$$

Integrating Eq. (55), we get the solutions to the Eq. (2) as follows:

$$\pm(\eta-\eta_0)=-2\sqrt{A_3}\frac{1}{\sqrt{\Gamma-\alpha_1}}, \tag{56}$$

$$\pm(\eta-\eta_0)=2\sqrt{\frac{A_3}{\alpha_2-\alpha_1}}\arctan\sqrt{\frac{\Gamma-\alpha_2}{\alpha_2-\alpha_1}},\alpha_2>\alpha_1, \tag{57}$$

$$\pm(\eta-\eta_0)=\sqrt{\frac{A_3}{\alpha_2-\alpha_1}}\ln\left|\frac{\sqrt{\Gamma-\alpha_2}-\sqrt{\alpha_1-\alpha_2}}{\sqrt{\Gamma-\alpha_2}+\sqrt{\alpha_1-\alpha_2}}\right|,\alpha_1>\alpha_2, \tag{58}$$

$$\pm(\eta-\eta_0)=2\sqrt{\frac{A_3}{\alpha_1-\alpha_3}}F(\varphi,l),\alpha_1>\alpha_2>\alpha_3, \tag{59}$$

where

$$A_3=\frac{\zeta_0}{\xi_3}=\frac{3\left(-2\xi_2\tau_0\left(-\alpha_3+\alpha_4\left(1+\gamma\alpha_1\tau_0\right)\right)\right)}{2\alpha_1\tau_1\left(2\xi_2\tau_0-\xi_1\tau_1\right)}+\frac{3\xi_1\left(-\alpha_3+\alpha_4\left(1+2\gamma\alpha_1\tau_0\right)\right)\tau_1}{2\alpha_1\tau_1\left(2\xi_2\tau_0-\xi_1\tau_1\right)},$$
$$F(\varphi,l)=\int_0^{\varphi}\frac{d\psi}{1-l^2\sin^2\psi}, \tag{60}$$

and

$$\varphi=\arcsin\sqrt{\frac{\Gamma-\alpha_3}{\alpha_2-\alpha_3}},\quad l^2=\frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}. \tag{61}$$

Also α_1, α_2 and α_3 are the roots of the polynomial equation,

$$\Gamma^3+\frac{\xi_2}{\xi_3}\Gamma^2+\frac{\xi_1}{\xi_3}\Gamma+\frac{\xi_0}{\xi_3}=0. \tag{62}$$

Substituting the solutions (56-59) into Eq. (6), we can get the following exact traveling wave solutions such as rational function solution, hyperbolic function solutions and Jacobi elliptic function solutions of Eq. (2), respectively:

$$u(x,t)=\tau_0+\tau_1\alpha_1+\frac{4\tau_1A_3}{(x-kt-\eta_0)^2}, \tag{63}$$

$$u(x,t)=\tau_0+\tau_1\alpha_1+\tau_1(\alpha_2-\alpha_1)\tanh^2\left(\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A_3}}(x-kt-\eta_0)\right), \tag{64}$$

$$u(x,t)=\tau_0+\tau_1\alpha_1+\tau_1(\alpha_1-\alpha_2)\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A_3}}(x-kt)\right), \tag{65}$$

and

$$u(x,t)=\tau_0+\tau_1\alpha_3+\tau_1(\alpha_2-\alpha_3)sn^2\left(\pm\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_3}{A_3}}(x-kt-\eta_0),\frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}\right), \tag{66}$$

where

$$k=\sqrt{\frac{-2\xi_2\tau_0(1+\gamma\alpha_1\tau_0)+\xi_1(1+2\gamma\alpha_1\tau_0)\tau_1}{-2\xi_2\tau_0+\xi_1\tau_1}}.$$

If we take $\tau_0=-\tau_1\alpha_1$ and $\eta_0=0$ for simpleness, then the solutions (63)-(65) can degrade to rational function solution

$$u(x,t)=\left(\frac{2\sqrt{\tilde{A}_1}}{x-kt}\right)^2, \tag{67}$$

1-soliton solution

$$u(x,t)=\frac{A_4}{\cosh^2\left[B_1(x-kt)\right]}, \tag{68}$$

singular soliton solution

$$u(x,t)=\frac{A_5}{\sinh^2\left[B_1(x-kt)\right]}, \tag{69}$$

where

$$k=\sqrt{\frac{-2\xi_2\tau_0(1+\gamma\alpha_1\tau_0)+\xi_1(1+2\gamma\alpha_1\tau_0)\tau_1}{-2\xi_2\tau_0+\xi_1\tau_1}},\tilde{A}_1=\tau_1A_3, A_4=\tau_1(\alpha_2-\alpha_1),$$

$$A_5=\tau_1(\alpha_1-\alpha_2),\ B_1=\pm\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A_3}}.$$

Here, A_4 and A_5 are the amplitudes of the solitons, while k is the velocity and B_1 is the reverse width of the solitons. Thus, we can say that the solitons exist for $\tau_1>0$.

In addition, if we receive $\tau_0=-\tau_1\alpha_3$ and $\eta_0=0$, Eq. (66) is converted into the Jacobi elliptic function solution

$$u_i(x,t)=A_6sn^2\left[B_i(x-kt),\frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}\right], \tag{70}$$

where

$$k=\sqrt{\frac{-2\xi_2\tau_0\left(1+\gamma\alpha_1\tau_0\right)+\xi_1\left(1+2\gamma\alpha_1\tau_0\right)\tau_1}{-2\xi_2\tau_0+\xi_1\tau_1}},\quad A_6=\tau_1\left(\alpha_2-\alpha_3\right),\ B_i=\frac{(-1)^i}{2}\sqrt{\frac{\alpha_1-\alpha_3}{A_3}},\ (i=1,2).$$

Remark 2. When the modulus $l\to 1$, Eq. (70) can be converted into dark soliton solutions

$$u_i(x,t)=A_6\tanh^2\left[B_i\left(x-kt\right)\right], \tag{71}$$

where

$\alpha_1=\alpha_2$

and $k=\sqrt{\frac{-2\xi_2\tau_0\left(1+\gamma\alpha_1\tau_0\right)+\xi_1\left(1+2\gamma\alpha_1\tau_0\right)\tau_1}{-2\xi_2\tau_0+\xi_1\tau_1}}$ represents the velocity of the dark soliton.

Case 2: If we take $\varepsilon=0,\ \delta=2$ and $\vartheta=4$, then

$$(v')^2=\frac{\left(\tau_1+2\tau_2\Gamma\right)^2\left(\xi_4\Gamma^4+\xi_3\Gamma^3+\xi_2\Gamma^2+\xi_1\Gamma+\xi_0\right)}{\zeta_0}, \tag{72}$$

$$v''=\frac{\left(\tau_1+2\tau_1\Gamma\right)\left(4\xi_4\Gamma^3+2\xi_3\Gamma^2+2\xi_2\Gamma+\xi_1\right)}{2\zeta_0}+\frac{2\tau_2\left(\xi_4\Gamma^4+\xi_3\Gamma^3+\xi_2\Gamma^2+\xi_1\Gamma+\xi_0\right)}{\zeta_0}, \tag{73}$$

where $\xi_4\neq 0,\ \zeta_0\neq 0$.

Respectively, solving the algebraic equation system (11) outputs as follows:

$$\begin{gathered}\xi_0=\frac{\dfrac{\alpha_1\zeta_0\tau_1^6}{\alpha_3-k^2\alpha_4}-24\xi_1\tau_1^3\tau_2^2+\dfrac{-36\left(-1+k^2\right)^2\zeta_0^2\tau_1^2\tau_2^2+576\gamma^2\left(\alpha_3-k^2\alpha_4\right)\xi_1^2\tau_2^4}{\gamma^2\alpha_1\left(\alpha_3-k^2\alpha_4\right)\zeta_0}}{288\tau_1^2\tau_2^3},\\ \xi_1=\xi_1,\ \xi_2=\frac{\xi_1\tau_2}{\tau_1}+\frac{\alpha_1\zeta_0\tau_1^2}{6\tau_2\left(\alpha_3-k^2\alpha_4\right)},\ \xi_3=\frac{\alpha_1\zeta_0\tau_1}{3\left(\alpha_3-k^2\alpha_4\right)},\ \xi_4=\frac{\alpha_1\zeta_0\tau_2}{6\left(\alpha_3-k^2\alpha_4\right)},\\ \tau_0=\frac{1}{12}\left(\frac{\tau_1^2}{\tau_2}+\frac{6\left(\dfrac{-1+k^2}{\gamma}+\dfrac{4\left(\alpha_3-k^2\alpha_4\right)\xi_1\tau_2}{\zeta_0\tau_1}\right)}{\alpha_1}\right),\ \tau_1=\tau_1,\ k=k.\end{gathered} \tag{74}$$

Embedding these results into Eqs. (7) and (12), we have

$$\pm\left(\eta-\eta_0\right)=A_7\int\frac{d\Gamma}{\sqrt{\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4}}}, \tag{75}$$

where $A_7=\sqrt{\frac{6\alpha_3-6k^2\alpha_4}{\alpha_1\tau_2}}$.

Integrating Eq. (75), we get the solutions to the eq. (2) as follows

$$\pm(\eta-\eta_0)=-\frac{A_7}{\Gamma-\alpha_1}, \tag{76}$$

$$\pm(\eta-\eta_0)=\frac{2A_7}{\alpha_1-\alpha_2}\sqrt{\frac{\Gamma-\alpha_2}{\Gamma-\alpha_1}},\quad \alpha_1>\alpha_2, \tag{77}$$

$$\pm(\eta-\eta_0)=\frac{A_7}{\alpha_1-\alpha_2}\ln\left|\frac{\Gamma-\alpha_1}{\Gamma-\alpha_2}\right|, \tag{78}$$

$$\pm(\eta-\eta_0)=\frac{2A_7}{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}\ln\left|\frac{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}-\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}+\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}\right|,\ \alpha_1>\alpha_2>\alpha_3, \tag{79}$$

$$\pm(\eta-\eta_0)=\frac{2A_7}{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}F(\varphi,l),\ \alpha_1>\alpha_2>\alpha_3>\alpha_4, \tag{80}$$

where

$$A_7=\sqrt{\frac{6\alpha_3-6k^2\alpha_4}{\alpha_1\tau_2}},\ \varphi=\arcsin\sqrt{\frac{(\Gamma-\alpha_1)(\alpha_2-\alpha_4)}{(\Gamma-\alpha_2)(\alpha_1-\alpha_4)}},\ l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}. \tag{81}$$

Also α_1, α_2, α_3 and α_4 are the roots of the polynomial equation,

$$\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4}=0. \tag{82}$$

Substituting the solutions (76)-(80) into Eq. (6), we have

$$u(x,t)=\tau_0+\tau_1\alpha_1\pm\frac{\tau_1A_7}{x-kt-\eta_0}+\tau_2\left(\alpha_1\pm\frac{A_7}{x-kt-\eta_0}\right)^2, \tag{83}$$

$$u(x,t)=\tau_0+\tau_1\alpha_1+\frac{4A_7^{\,2}(\alpha_2-\alpha_1)\tau_1}{4A_7^{\,2}-\left[(\alpha_1-\alpha_2)x-kt-\eta_0\right]^2}+\tau_2\left(\alpha_1+\frac{4A_7^{\,2}(\alpha_2-\alpha_1)}{4A_7^{\,2}-\left[(\alpha_1-\alpha_2)x-kt-\eta_0\right]^2}\right)^2, \tag{84}$$

$$u(x,t)=\tau_0+\tau_1\alpha_2+\frac{(\alpha_2-\alpha_1)\tau_1}{\exp\left[\frac{(\alpha_1-\alpha_2)}{A_7}(x-kt-\eta_0)\right]-1}+\tau_2\left(\alpha_2+\frac{(\alpha_2-\alpha_1)}{\exp\left[\frac{(\alpha_1-\alpha_2)}{A_7}(x-kt-\eta_0)\right]-1}\right)^2, \tag{85}$$

$$u(x,t)=\tau_0+\tau_1\alpha_1+\frac{(\alpha_1-\alpha_2)\tau_1}{\exp\left[\frac{(\alpha_1-\alpha_2)}{A_7}(x-kt-\eta_0)\right]-1}+\tau_2\left(\alpha_1+\frac{(\alpha_1-\alpha_2)}{\exp\left[\frac{(\alpha_1-\alpha_2)}{A_7}(x-kt-\eta_0)\right]-1}\right)^2, \tag{86}$$

$$u(x,t)=\tau_0+\tau_1\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\tau_1}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh\left[\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A_7}(x-kt-\eta_0)\right]}$$

$$+\tau_2\left(\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh\left[\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A_7}(x-kt-\eta_0)\right]}\right)^2, \tag{87}$$

$$u(x,t)=\tau_0+\tau_1\alpha_2+\frac{\tau_1(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sn^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A_7}(x-kt-\eta_0),\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}\right]}$$

$$+\tau_2\left(\alpha_2+\frac{\tau_1(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sn^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A_7}(x-kt-\eta_0),\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}\right]}\right)^2. \tag{88}$$

For simplicity, if we take $\eta_0=0$, then we can write the solutions (83)-(88) as follows:

$$u(x,t)=\sum_{i=0}^{2}\tau_i\left(\alpha_1\pm\frac{A_7}{x-kt}\right)^i, \tag{89}$$

$$u(x,t)=\sum_{i=0}^{2}\tau_i\left(\alpha_1+\frac{4A_7^2(\alpha_1-\alpha_2)}{4A_7^2-[(\alpha_1-\alpha_2)(x-kt)]^2}\right)^i, \tag{90}$$

$$u(x,t)=\sum_{i=0}^{2}\tau_i\left(\alpha_2+\frac{\alpha_2-\alpha_1}{\exp[B_2(x-kt)]-1}\right)^i, \tag{91}$$

$$u(x,t)=\sum_{i=0}^{2}\tau_i\left(\alpha_1+\frac{\alpha_1-\alpha_2}{\exp[B_2(x-kt)]-1}\right)^i, \tag{92}$$

$$u(x,t)=\sum_{i=0}^{2}\tau_i\left(\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh[C(x-kt)]}\right)^i, \tag{93}$$

$$u(x,t)=\sum_{i=0}^{2}\tau_i\left(\alpha_2+\frac{(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sn^2(\varphi,l)}\right)^i, \tag{94}$$

where

$$A_7=\sqrt{\frac{6\alpha_3-6k^2\alpha_4}{\alpha_1\tau_2}},\ B_2=\frac{k(\alpha_1-\alpha_2)}{A_7},\ C=\frac{k\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A_7},$$
$$\varphi=\frac{k\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A_7}(x-kt),l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}.$$

Here, A_7 is the amplitude of the soliton, while k is the velocity and B_2 and C are the inverse width of the solitons.

Remark 3. The solutions of Eq. (2) were reached by using ETEM and these obtained solutions were checked in Wolfram Mathematica 12.

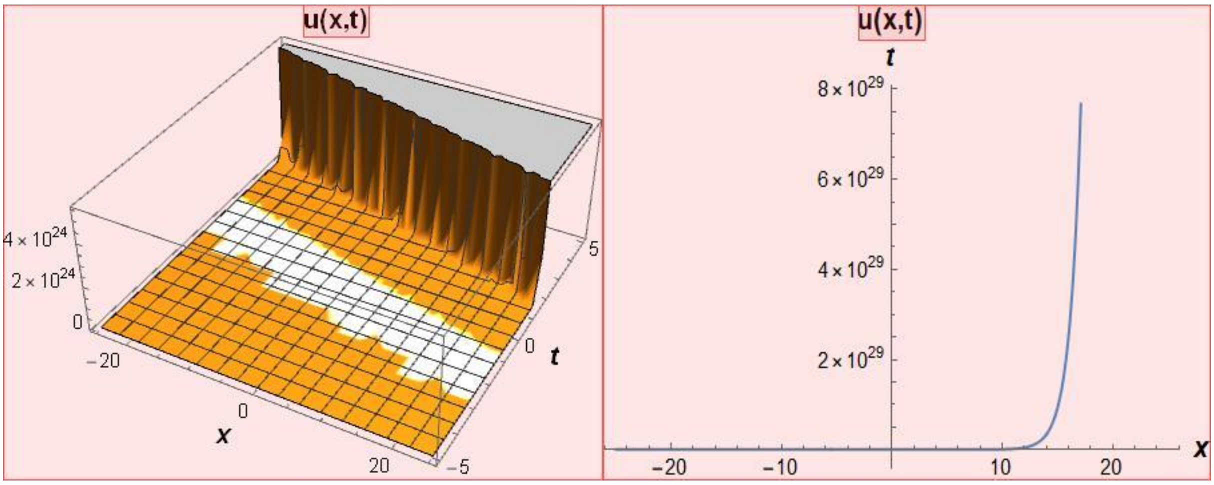

Figure 1. Graph of the solution (26) is indicated at $\tau_0=1,\ \tau_1=2,\ h_1=2,\ h_2=1,\ h_3=-1,\ \xi_0=3,\ \xi_1=-1,\ -25\le x\le 25,$ $-5\le t\le 5$ and the second graph denotes the exact solution of Eq. (26) for $t=3.$

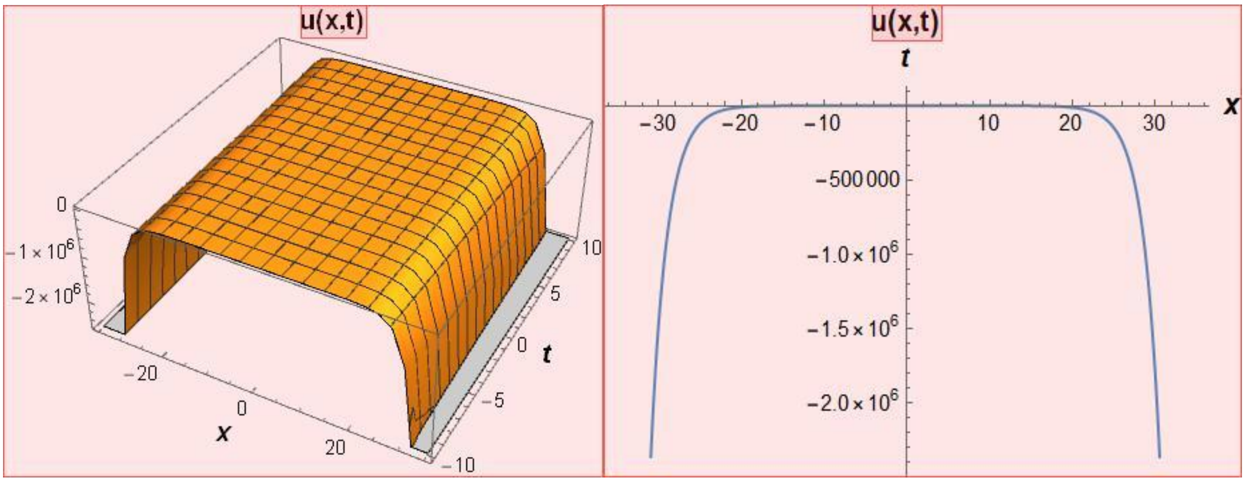

Figure 2. Graph of the solution (27) is indicated at $\tau_0=1,\ \tau_1=-2,\ h_1=-2,\ h_2=2,\ h_3=1,\ \xi_0=-2,\ \xi_1=-1,$ $-35\le x\le 35,\ -10\le t\le 10$ and the second graph denotes the exact solution of Eq. (27) for $t=2$.

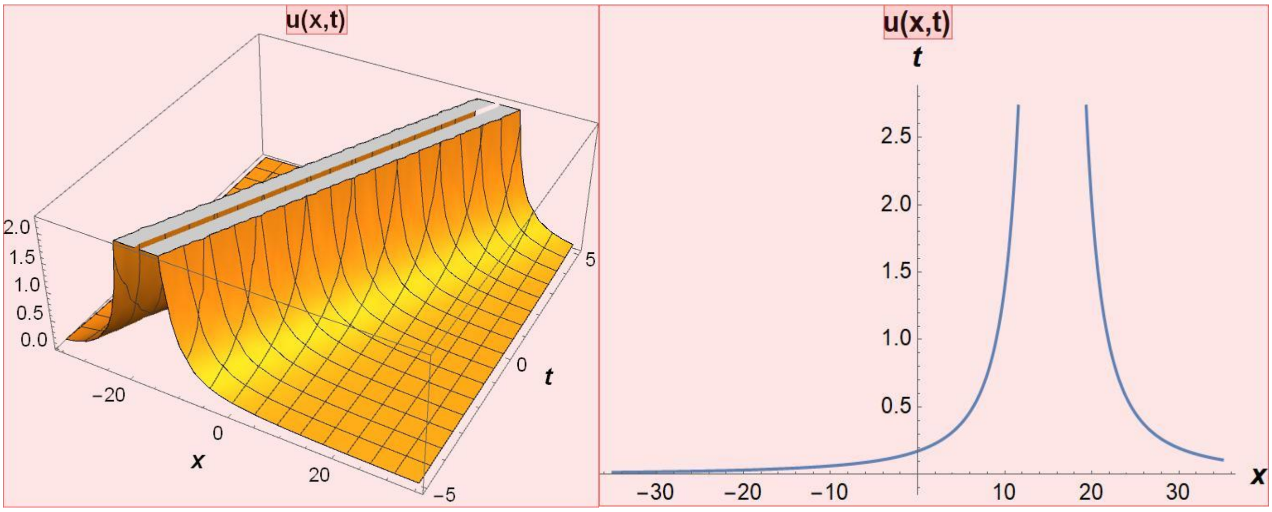

Figure 3. Graph of the solution (67) is indicated at $\tau_0 = -1,\ \tau_1 = 2,\ \xi_1 = 4,\ \xi_2 = 1,\ \alpha_1 = -2,\ \alpha_3 = -2,\ \alpha_4 = 1,\ \gamma = 3,$ $-35 \le x \le 35$, $-5 \le t \le 5$ and the second graph denotes the exact solution of Eq. (67) for $t = 4.5.$

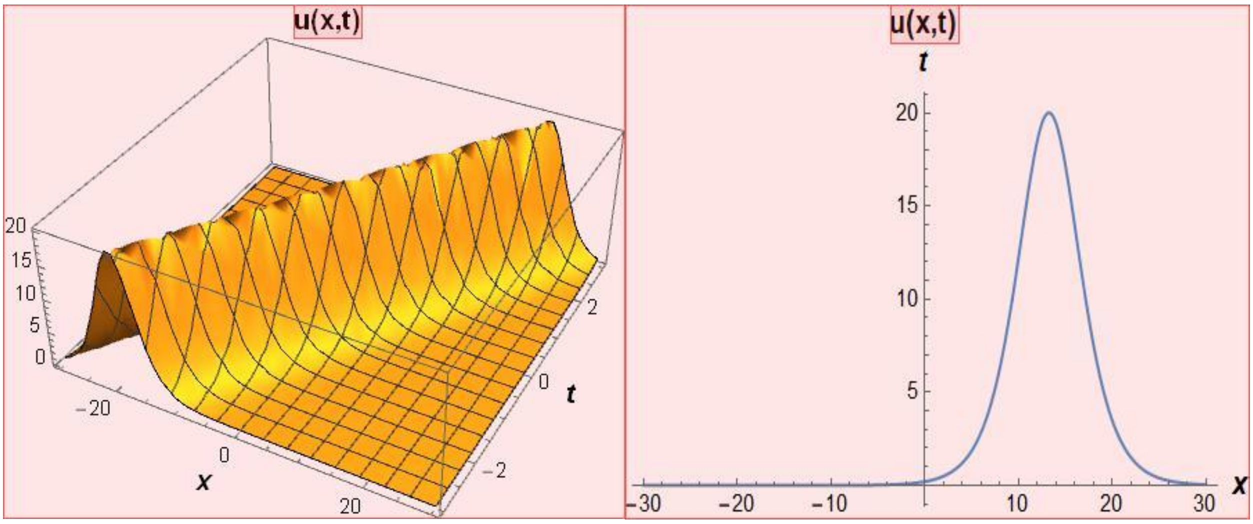

Figure 4. Graph of the solution (68) is indicated at $\tau_0 = -2,\ \tau_1 = 5,\ \xi_1 = 3,\ \xi_2 = 1,\ \alpha_1 = -3,\ \alpha_2 = 1,\ \alpha_3 = -1,\ \alpha_4 = 2,$ $\gamma = 4,\ -30 \le x \le 30,\ -3 \le t \le 3$ and the second graph denotes the exact solution of Eq. (68) for $t = 2.$

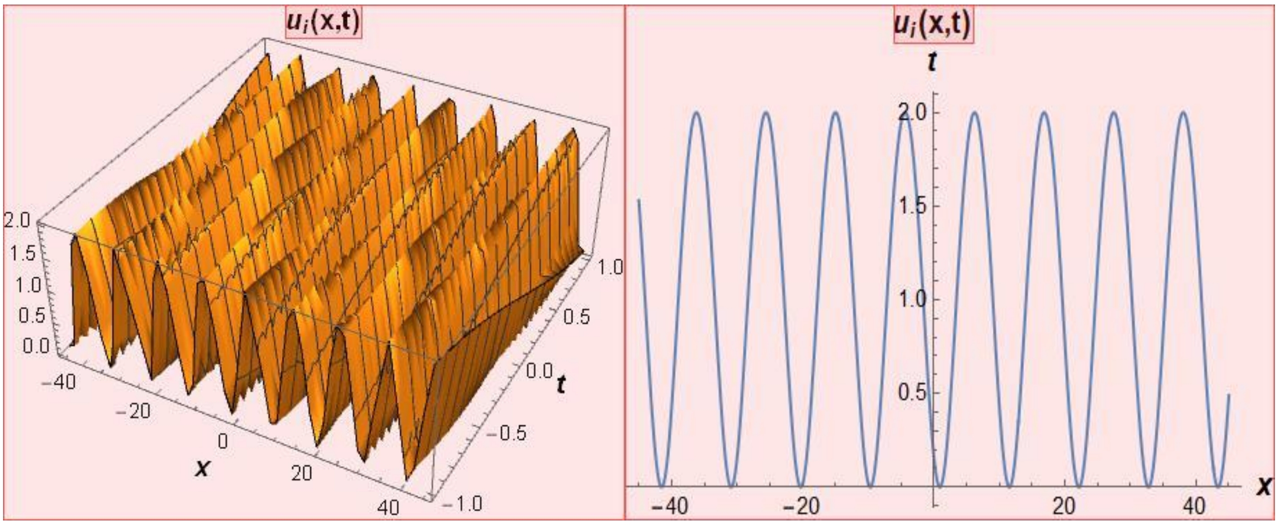

Figure 5. Graph of the solution (70) is indicated at $\tau_0=-1,\ \tau_1=1,\ \xi_1=2,\ \xi_2=4,\ \alpha_1=-1,\ \alpha_2=5,\ \alpha_3=-3,\ \alpha_4=3,$ $\gamma=2,\ -45\le x\le 45,\ -1\le t\le 1$ and the second graph denotes the exact solution of Eq. (70) for $t=0.5$.

4. CONCLUSIONS

In this work, we get travelling wave solutions of the DGHDE and strain wave equation by using ETEM. It is necessary to note that ETEM presents powerful mathematical tool for finding the exact solutions of these equations and this method is highly efficient in the matter of seeking for new solutions such as soliton solutions, rational, Jacobi elliptic, periodic wave solutions and hyperbolic function solutions.

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