CONSTRUCTIVE MATHEMATICAL ANALYSIS
5 (2022), No. 2, pp. 54-59
http://dergipark.org.tr/en/pub/cma
ISSN 2651 - 2939

Research Article

Equilibria for abstract economies in Hausdorff topological vector spaces

Dedicated to Professor Anthony To-Ming Lau with much admiration.

DONAL O'REGAN*

ABSTRACT. In this paper using new fixed point results of the author, we establish a variety of existence results for equilibria for abstract economies.

Keywords: Fixed points, equilibria, abstract economies.

2020 Mathematics Subject Classification: 47H04, 47H10, 47N10, 90A14.

1. INTRODUCTION

Using strategy sets with constraint and preference correspondences defined on subsets of Hausdorff topological vector spaces, we present in this paper a variety of equilibrium results for abstract economies. These equilibrium results are deduced from recent fixed point results in the literature (see [8, 9, 10]) and our theory improves and generalizes corresponding results in the literature (see [1, 4, 5, 6, 11, 12] and the references therein).

Now, we recall some fixed point results [8, 9, 10] in the literature. First, we recall the following notions from the literature. For a subset K of a topological space X, we denote by $Cov_X(K)$ the directed set of all coverings of K by open sets of X (usually we write $Cov(K) = Cov_X(K)$). Given two maps $F, G : X \to 2^Y$ (here 2^Y denotes the family of nonempty subsets of Y) and $\alpha \in Cov(Y)$, F and G are said to be α–close if for any $x \in X$ there exists $U_x \in \alpha$, $y \in F(x) \cap U_x$ and $w \in G(x) \cap U_x$.

Let Q be a class of topological spaces. A space Y is an extension space for Q (written $Y \in ES(Q)$) if for any pair (X, K) in Q with $K \subseteq X$ closed, any continuous function $f_0 : K \to Y$ extends to a continuous function $f : X \to Y$. A space Y is an approximate extension space for Q (written $Y \in AES(Q)$) if for any $\alpha \in Cov(Y)$ and any pair (X, K) in Q with $K \subseteq X$ closed, and any continuous function $f_0 : K \to Y$ there exists a continuous function $f : X \to Y$ such that $f|_K$ is α–close to f_0.

Let V be a subset of a Hausdorff topological vector space E. Then, we say V is Schauder admissible if for every compact subset K of V and every covering $\alpha \in Cov_V(K)$ there exists a continuous functions $\pi_\alpha : K \to V$ such that

(i). π_α and $i : K \to V$ are α–close,
(ii). $\pi_\alpha(K)$ is contained in a subset $C \subseteq V$ with $C \in AES$ (compact).

Received: 12.03.2022; Accepted: 24.04.2022; Published Online: 30.04.2022
*Corresponding author: Donal O'Regan; donal.oregan@nuigalway.ie
DOI: 10.33205/cma.1102400

X is said to be q– Schauder admissible if any nonempty compact convex subset Ω of X is Schauder admissible.

An upper semicontinuous map $\phi : X \to CK(Y)$ is said to Kakutani (and we write $\phi \in Kak(X, Y)$); here $CK(Y)$ denotes the family of nonempty, convex, compact subsets of Y.

Theorem 1.1. *Let I be an index set and $\{X_i\}_{i\in I}$ be a family of sets each in a Hausdorff topological vector space E_i. For each $i \in I$, let K_i be a nonempty compact subset of X_i and suppose $F_i : X \equiv \prod_{i\in I} X_i \to K_i$ is upper semicontinuous with nonempty convex compact values (i.e. $F_i \in Kak(X, K_i)$). Also assume $K \equiv \prod_{i\in I} K_i$ is a Schauder admissible subset of the Hausdorff topological vector space $E \equiv \prod_{i\in I} E_i$. Then, there exists a $x \in K$ with $x_i \in F_i(x)$ for $i \in I$ (here x_i is the projection of x on X_i).*

Remark 1.1. *One could repace K a Schauder admissible subset of E in Theorem 1.1 (and the other results in this paper) with other admissible subsets of E described in* [7].

Let Z and W be subsets of Hausdorff topological vector spaces Y_1 and Y_2 and G a multifunction. We say $G \in DKT(Z, W)$ [2] if W is convex and there exists a map $S : Z \to W$ with $co\,(S(x)) \subseteq G(x)$ for $x \in Z$, $S(x) \neq \emptyset$ for each $x \in Z$ and the fibre $S^{-1}(w) = \{z \in Z : w \in S(z)\}$ is open (in Z) for each $w \in W$.

Theorem 1.2. *Let I be an index set and $\{X_i\}_{i\in I}$ be a family of convex sets each in a Hausdorff topological vector space E_i. For each $i \in I$ suppose $F_i : X \equiv \prod_{i\in I} X_i \to X_i$ and $F_i \in DKT(X, X_i)$. In addition assume for each $i \in I$ there exists a convex compact set K_i with $F_i(X) \subseteq K_i \subseteq X_i$. Also suppose X is a q–Schauder admissible subset of the Hausdorff topological vector space $E = \prod_{i\in I} E_i$. Then, there exists a $x \in X$ with $x_i \in F_i(x)$ for $i \in I$.*

Remark 1.2. *If I is a finite set, then the assumption that "X is a q–Schauder admissible subset of the Hausdorff topological vector space E" can be removed. In fact we have: Let $\{X_i\}_{i=1}^N$ be a family of convex sets each in a Hausdorff topological vector space E_i. For each $i \in \{1, ..., N\}$ suppose $F_i : X \equiv \prod_{i=1}^N X_i \to X_i$ and $F_i \in DKT(X, X_i)$. In addition assume for each $i \in \{1, ..., N\}$ there exists a convex compact set K_i with $F_i(X) \subseteq K_i \subseteq X_i$. Then, there exists a $x \in X$ with $x_i \in F_i(x)$ for $i \in \{1, ..., N\}$.*

Let Z and W be subsets of Hausdorff topological vector spaces Y_1 and Y_2 and F a multifunction. We say $F \in HLPY(Z, W)$ [3, 4] if W is convex and there exists a map $S : Z \to W$ with $co\,(S(x)) \subseteq F(x)$ for $x \in Z$, $S(x) \neq \emptyset$ for each $x \in Z$ and $Z = \bigcup \{\, int\, S^{-1}(w) : \; w \in W\}$; here $S^{-1}(w) = \{z \in Z : w \in S(z)\}$.

Theorem 1.3. *Let I be an index set and $\{X_i\}_{i\in I}$ be a family of convex sets each in a Hausdorff topological vector space E_i. For each $i \in I$ suppose $F_i : X \equiv \prod_{i\in I} X_i \to X_i$ and $F_i \in HLPY(X, X_i)$. In addition assume for each $i \in I$ there exists a convex compact set K_i with $F_i(X) \subseteq K_i \subseteq X_i$. Also suppose X is a q–Schauder admissible subset of the Hausdorff topological vector space $E = \prod_{i\in I} E_i$. Then, there exists a $x \in X$ with $x_i \in F_i(x)$ for $i \in I$.*

Remark 1.3. *If I is a finite set, then the assumption that "X is a q–Schauder admissible subset of the Hausdorff topological vector space E" can be removed. In fact we have: Let $\{X_i\}_{i=1}^N$ be a family of convex sets each in a Hausdorff topological vector space E_i. For each $i \in \{1, ..., N\}$ suppose $F_i : X \equiv \prod_{i=1}^N X_i \to X_i$ and $F_i \in HLPY(X, X_i)$. In addition assume for each $i \in \{1, ..., N\}$ there exists a convex compact set K_i with $F_i(X) \subseteq K_i \subseteq X_i$. Then, there exists a $x \in X$ with $x_i \in F_i(x)$ for $i \in \{1, ..., N\}$.*

We now state a result from the literature [11] which will be used in Section 2.

Theorem 1.4. *Let X and Y be two topological spaces and A an open subset of X. Suppose $F_1 : X \to 2^Y$, $F_2 : A \to 2^Y$ (here 2^Y denotes the family of nonempty subsets of Y) are upper semicontinuous such that $F_2(x) \subset F_1(x)$ for all $x \in A$. Then, the map $F : X \to 2^Y$ defined by*

$$F(x) = \begin{cases} F_1(x), & x \notin A \\ F_2(x), & x \in A \end{cases}$$

is upper semicontinuous.

2. Abstract economy results

Let I be the set of agents and we describe the abstract economy as $\Gamma = (X_i, A_i, B_i, P_i)_{i\in I}$, where A_i, $B_i : X \equiv \prod_{i\in I} X_i \to 2^{E_i}$ are constraint correspondences, $P_i : X \to 2^{E_i}$ is a preference correspondence and X_i is a choice (or strategy) set which is a subset of a Hausdorff topological vector space E_i. We are interested in finding an equilibrium point for Γ i.e. a point $x \in X$ with $x_i \in \overline{B_i}(x)$ and $co\, A_i(x) \cap co\, P_i(x) = \emptyset$ (or $x_i \in B_i(x)$ and $A_i(x) \cap P_i(x) = \emptyset$) for $i \in I$.

Theorem 2.5. *Let $\Gamma = (X_i, A_i, B_i, P_i)_{i\in I}$ be an abstract economy with $\{X_i\}_{i\in I}$ a family of nonempty sets each in a Hausdorff topological vector space E_i (here I is an index set). For each $i \in I$, let $A_i, B_i, P_i : X \equiv \prod_{i\in I} X_i \to 2^{E_i}$ and assume the following conditions are satisfied:*

(2.1) $$U_i = \{x \in X : co\, A_i(x) \cap co\, P_i(x) \neq \emptyset\} \text{ is paracompact and open in } X$$

(2.2) $$cl\, B_i (\equiv \overline{B_i}) : X \to CK(E_i) \text{ is upper semicontinuous}$$

(2.3) $$\begin{cases} \text{there exists a nonempty compact subset } K_i \text{ of } X_i \text{ with } \overline{B_i} : X \to CK(K_i) \\ \text{and } K \equiv \prod_{i\in I} K_i \text{ is a Schauder admissible subset of } E \equiv \prod_{i\in I} E_i \end{cases}$$

and

(2.4) $$x_i \notin co\, A_i(x) \cap co\, P_i(x) \text{ if } x \in U_i;\text{ here } x_i \text{ is the projection of } x \text{ on } E_i.$$

For $i \in I$ and $x \in X$, let $H_i(x) = co\, A_i(x) \cap co\, P_i(x)$ and suppose

(2.5) $$H_i(x) \subseteq \overline{B_i}(x) \text{ for } x \in U_i$$

and

(2.6) $$\begin{cases} \text{there exists a } S_i : U_i \to 2^{E_i} \text{ with } co\, S_i(x) \subseteq H_i(x) \text{ for } x \in U_i \\ \text{and } S_i^{-1}(y) \text{ is open (in } U_i) \text{ for each } y \in E_i. \end{cases}$$

Then there exists a $x \in X$ with for each $i \in I$, we have $x_i \in \overline{B_i}(x)$ and $co\, A_i(x) \cap co\, P_i(x) = \emptyset$.

Proof. Note for each $i \in I$ from (2.6), we have $H_i \in DKT(U_i, E_i)$ so from [2] there exists a continuous (single valued) selection $f_i : U_i \to E_i$ of H_i with $f_i(x) \in co\,(S_i(x)) \subseteq H_i(x)$ for $x \in U_i$. For each $i \in I$, let

$$G_i(x) = \begin{cases} f_i(x), & x \in U_i \\ \overline{B_i}(x), & x \notin U_i \end{cases}.$$

Note for each $i \in I$ that $\{f_i(x)\} \subseteq co\,(S_i(x)) \subseteq H_i(x) \subseteq \overline{B_i}(x)$ (see (2.5)) if $x \in U_i$, so Theorem 1.4 guarantees that $G_i : X \to CK(E_i)$ is upper semicontinuous. Also for each $i \in I$, we have $G_i(x) \subseteq \overline{B_i}(x) \subseteq K_i$ for $x \in X$ so $G_i \in Kak(X, K_i)$. Now, Theorem 1.1 guarantees a $x \in K$ with $x_i \in G_i(x)$ for $i \in I$. If $x \in U_i$ for some $i \in I$, then $x_i = f_i(x) \in H_i(x) = co\, A_i(x) \cap co\, P_i(x)$, which contradicts (2.4). Thus for each $i \in I$, we must have $x \notin U_i$ and then we have $x_i \in \overline{B_i}(x)$ and $co\, A_i(x) \cap co\, P_i(x) = \emptyset$. □

Remark 2.4.

(i). *If $i \in I$ and $H_i^{-1}(y)$ is open (in X) for each $y \in E_i$, then U_i in (2.1) is automatically open in X. This is immediate once one notices that $U_i = \cup_{y \in E_i} H_i^{-1}(y)$.*

(ii). *Of course there are other obvious analogues of Theorem 2.5 if the assumptions on $co\, A_i \cap co\, P_i$ are replaced by assumptions on $co\, A_i \cap P_i$ or $\overline{co}\, A_i \cap P_i$ or $\overline{co}\, A_i \cap \overline{co}\, P_i$ or $\overline{co}\, A_i \cap co\, P_i$ or $A_i \cap co\, P_i$ or $A_i \cap \overline{co}\, P_i$ or $A_i \cap P_i$ or $co\, A_i \cap \overline{co}\, P_i$ and the assumptions on $\overline{B_i}$ are replaced by assumptions on B_i.*

Remark 2.5. *For each $i \in I$ suppose there exists a map $S_i : X \to E_i$ (which may have empty values) with $co\, S_i(x) \subseteq H_i(x)$ for $x \in X$, the fibres $S_i^{-1}(y)$ are open (in X) for each $y \in E_i$ and also assume if $x \in U_i$, then $S_i(x) \neq \emptyset$. Then, (2.6) holds with S_i replaced by $S_i|_{U_i}$. Let $S_i^\star$ denote $S_i|_{U_i}$. For $i \in I$ note $S_i^\star : U_i \to 2^{E_i}$, $co\, S_i^\star(x) \subseteq H_i(x)$ for $x \in U_i$ and for $y \in E_i$ note*

$$(S_i^\star)^{-1}(y) = \{x \in U_i : y \in S_i^\star(x)\} = \{x \in X : y \in S_i(x)\} \cap U_i = S_i^{-1}(y) \cap U_i,$$

so $(S_i^\star)^{-1}(y)$ which is open in U_i.

Theorem 2.6. *Let $\Gamma = (X_i, A_i, B_i, P_i)_{i \in I}$ be an abstract economy with $\{X_i\}_{i \in I}$ a family of nonempty sets each in a Hausdorff topological vector space E_i (here I is an index set). For each $i \in I$, let $A_i, B_i, P_i : X \equiv \prod_{i \in I} X_i \to 2^{E_i}$ and assume (2.1), (2.2), (2.3) and (2.4) hold. For $i \in I$ and $x \in X$, let $H_i(x) = co\, A_i(x) \cap co\, P_i(x)$ and suppose (2.5) holds. In addition for each $i \in I$ assume*

$$\text{(2.7)} \qquad \begin{cases} \text{there exists a } \ S_i : U_i \to 2^{E_i} \ \text{ with } \ co\, S_i(x) \subseteq H_i(x) \ \text{ for } x \in U_i \\ \text{and } \ U_i = \bigcup \{ int_{U_i}\, S_i^{-1}(w) : \ w \in E_i \} \end{cases}.$$

Then there exists a $x \in X$ with for each $i \in I$ we have $x_i \in \overline{B_i}(x)$ and $co\, A_i(x) \cap co\, P_i(x) = \emptyset$.

Proof. Note for each $i \in I$ from (2.7), we have $H_i \in HLPY(U_i, E_i)$ so from [4] there exists a continuous (single valued) selection $f_i : U_i \to E_i$ of H_i with $f_i(x) \in co(S_i(x)) \subseteq H_i(x)$ for $x \in U_i$. Let G_i for $i \in I$ be as in Theorem 2.5 and the same reasoning guarantees a $x \in K$ with $x_i \in G_i(x)$ for $i \in I$. □

Remark 2.6. *For each $i \in I$ suppose there exists a map $S_i : X \to E_i$ (which may have empty values) with $co\, S_i(x) \subseteq H_i(x)$ for $x \in X$, $X = \bigcup \{ int_X\, S_i^{-1}(w) : \ w \in E_i \}$ and also assume if $x \in U_i$, then $S_i(x) \neq \emptyset$. Then, (2.7) holds with S_i replaced by $S_i|_{U_i}$. Let $S_i^\star$ denote $S_i|_{U_i}$. For $i \in I$ note $S_i^\star : U_i \to 2^{E_i}$, $co\, S_i^\star(x) \subseteq H_i(x)$ for $x \in U_i$ and now we show $U_i = \bigcup \{ int_{U_i}\, (S_i^\star)^{-1}(w) : \ w \in E_i \}$. To see this notice*

$$U_i = U_i \cap X = U_i \cap \left(\bigcup \{ int_X\, S_i^{-1}(w) : \ w \in E_i \} \right) = \bigcup \{ U_i \cap \, int_X\, S_i^{-1}(w) : \ w \in E_i \},$$

so $U_i \subseteq \bigcup \{ int_{U_i}\, (S_i^\star)^{-1}(w) : \ w \in E_i \}$ since for each $w \in E_i$, we have that $U_i \cap \, int_X\, S_i^{-1}(w)$ is open in U_i. On the other hand clearly $\bigcup \{ int_{U_i}\, (S_i^\star)^{-1}(w) : \ w \in E_i \} \subseteq U_i$ so as a result $U_i = \bigcup \{ int_{U_i}\, (S_i^\star)^{-1}(w) : \ w \in E_i \}$.

Theorem 2.7. *Let $\Gamma = (X_i, A_i, B_i, P_i)_{i \in I}$ be an abstract economy with $\{X_i\}_{i \in I}$ a family of nonempty convex sets each in a Hausdorff topological vector space E_i (here I is an index set). For each $i \in I$, let $A_i, B_i, P_i : X \equiv \prod_{i \in I} X_i \to 2^{E_i}$ and assume the following conditions are satisfied:*

$$\text{(2.8)} \qquad co(A_i(x)) \subseteq B_i(x) \ \text{ for } \ x \in X,$$

$$\text{(2.9)} \qquad x_i \notin B_i(x) \cap co\, P_i(x) \ \text{ if } x \in X \ \text{ and } \ A_i(x) \cap P_i(x) \neq \emptyset,$$

$$\text{(2.10)} \qquad \begin{cases} \text{there exists a nonempty convex compact subset } \ K_i \ \text{ of } \ X_i \\ \text{with } \ B_i(X) \subseteq K_i \subseteq X_i \end{cases}$$

and

$$\text{(2.11)}\qquad \begin{cases} \textit{for each}\ \ y_i \in X_i\ \ \textit{the set}\ \ \big[(co\,P_i)^{-1}(y_i) \cup M_i\big] \cap A_i^{-1}(y_i) \\ \textit{is open in}\ \ X\ \ (\textit{here}\ M_i = \{x \in X:\ A_i(x) \cap P_i(x) = \emptyset\}) \end{cases}.$$

Finally, assume X is a q–Schauder admissible subset of $E = \prod_{i\in I} E_i$. Then there exists a $x \in X$ with for each $i \in I$, we have $x_i \in B_i(x)$ and $A_i(x) \cap P_i(x) = \emptyset$.

Proof. For each $i \in I$, let $N_i = \{x \in X : A_i(x) \cap P_i(x) \neq \emptyset\}$ and for each $x \in X$ let

$$I(x) = \{i \in I : A_i(x) \cap P_i(x) \neq \emptyset\}.$$

For each $i \in I$, let $F_i, G_i : X \to 2^{X_i}$ be given by

$$F_i(x) = \begin{cases} A_i(x) \cap co\,P_i(x), & i \in I(x) \\ A_i(x), & i \notin I(x) \end{cases}$$

and

$$G_i(x) = \begin{cases} B_i(x) \cap co\,P_i(x) & , i \in I(x) \\ B_i(x) & , i \notin I(x) \end{cases}.$$

Fix $i \in I$. Note from (2.8) that $co\,F_i(x) \subseteq G_i(x)$ for $x \in X$ (and note $F_i(x) \neq \emptyset$ for $x \in X$). Also note for each $y_i \in X_i$, we have

$$\begin{aligned} F_i^{-1}(y_i) &= \{x \in X:\ y_i \in F_i(x)\} \\ &= \{x \in N_i:\ y_i \in A_i(x) \cap co\,P_i(x)\} \cup \{x \in M_i:\ y_i \in A_i(x)\} \\ &= \big\{\big[(co\,P_i)^{-1}(y_i) \cap A_i^{-1}(y_i)\big] \cap N_i\big\} \cup \big\{A_i^{-1}(y_i) \cap M_i\big\} \\ &= \big[(co\,P_i)^{-1}(y_i) \cap A_i^{-1}(y_i)\big] \cup \big[A_i^{-1}(y_i) \cap M_i\big] \\ &= \big[(co\,P_i)^{-1}(y_i) \cup M_i\big] \cap A_i^{-1}(y_i) \end{aligned}$$

which (see (2.11)) is open in X. Thus for each $i \in I$, we have $G_i \in DKT(X, X_i)$ and also from (2.10) note $G_i(X) \subseteq K_i \subseteq X_i$. Now, Theorem 1.2 guarantees a $x \in K$ with $x_i \in G_i(x)$ for $i \in I$. Note if $i \in I(x)$ for some $i \in I$ then $A_i(x) \cap P_i(x) \neq \emptyset$ and $x_i \in B_i(x) \cap co\,P_i(x)$, which contradicts (2.9). Thus $i \notin I(x)$ for all $i \in I$. Consequently, $x_i \in B_i(x)$ and $A_i(x) \cap P_i(x) = \emptyset$ for all $i \in I$. □

Remark 2.7. *In Theorem 2.7 if I is a finite set, then the assumption that "X is a q–Schauder admissible subset of the Hausdorff topological vector space E" can be removed (see Remark 1.2).*

Theorem 2.8. *Let $\Gamma = (X_i, A_i, B_i, P_i)_{i\in I}$ be an abstract economy with $\{X_i\}_{i\in I}$ a family of nonempty convex sets each in a Hausdorff topological vector space E_i (here I is an index set). For each $i \in I$, let $A_i, B_i, P_i : X \equiv \prod_{i\in I} X_i \to 2^{E_i}$ and assume (2.8), (2.9) and (2.10) hold. Also suppose X is a q–Schauder admissible subset of $E = \prod_{i\in I} E_i$. For each $x \in X$, let $I(x) = \{i \in I:\ A_i(x) \cap P_i(x) \neq \emptyset\}$ and for each $i \in I$, let*

$$F_i(x) = \begin{cases} A_i(x) \cap co\,P_i(x), & i \in I(x) \\ A_i(x), & i \notin I(x) \end{cases}$$

and assume that

$$\text{(2.12)}\qquad X = \cup\{\,int\,F_i^{-1}(w):\ w \in X_i\}.$$

Then there exists a $x \in X$ with for each $i \in I$, we have $x_i \in B_i(x)$ and $A_i(x) \cap P_i(x) = \emptyset$.

Proof. Let N_i and G_i be as in Theorem 2.7. For $i \in I$ note $F_i(x) \neq \emptyset$ and $co\,F_i(x) \subseteq G_i(x)$ for $x \in X$ and $X = \cup\{\,int\,F_i^{-1}(w):\ w \in X_i\}$. Thus for each $i \in I$, we have $G_i \in HLPY(X, X_i)$ and also from (2.10) note $G_i(X) \subseteq K_i \subseteq X_i$. Now, Theorem 1.3 guarantees a $x \in K$ with $x_i \in G_i(x)$ for $i \in I$ and the reasoning in Theorem 2.7 guarantees the result. □

Remark 2.8. *In Theorem 2.8 if I is a finite set, then the assumption that "X is a q–Schauder admissible subset of the Hausdorff topological vector space E" can be removed (see Remark 1.3).*

References

[1] R. P. Agarwal, D. O'Regan: *A note on equilibria for abstract economies*, Mathematical and Computer Modelling, **34** (2001), 331–343.

[2] X. P. Ding, W. K. Kim and K. K. Tan: *A selection theorem and its applications*, Bulletin Australian Math. Soc., **46** (1992), 205–212.

[3] C. D. Horvath: *Contractibility and generalized convexity*, Jour. Math. Anal. Appl., **156** (1991), 341–357.

[4] L. J. Lim, S. Park and Z. T. Yu: *Remarks on fixed points, maximal elements and equilibria of generalized games*, Jour. Math. Anal. Appl., **233** (1999), 581–596.

[5] G. Mehta, K. K. Tan and X. Y. Yuan: *Fixed points, maximal elements and equilibia of generalized games*, Nonlinear Analysis, **28** (1997), 689–699.

[6] E. Tarafdar: *A fixed point theorem and equilibrium point in abstract economy*, J. Math. Econom., **20** (1991), 211–218.

[7] D. O'Regan: *Deterministic and random fixed points for maps on extension type spaces*, Applicable Analysis, **97** (2018), 1960–1966.

[8] D. O'Regan: *Collectively fixed point theory in the compact and coercive cases*, Analele Stiintifice ale Universitatii Ovidius Constanta, Seria Matematica, **30** (2022), 193–207.

[9] D. O'Regan: *A note on collectively fixed and coincidence points*, submitted.

[10] D. O'Regan: *Maximal elements for Kakutani maps*, Journal of Mathematics and Computer Science, **27** (2022), 77–85.

[11] K. K. Tan, X. Z. Yuan: *Maximal elements and equilibria for $\mathcal{U}$–majorised preferences*, Bull. Austral. Math. Soc., **49** (1994), 47–54.

[12] N. Yannelis, N. Prabhaker: *Existence of maximal elements and equilibria in linear topological spaces*, J. Math. Econom., **12** (1983), 233–246.

Donal O'Regan
National University of Ireland
School of Mathematical and Statistical Sciences
Galway, Ireland
ORCID: 0000-0002-4096-1469
E-mail address: donal.oregan@nuigalway.ie