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ANALYTICAL SOLUTIONS OF THE NONLINEAR (2 + 1)-DIMENSIONAL SOLITON EQUATION BY USING SOME METHODS

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Abstract

In this work, it has been applied two methods for solving the (2+1)-dimensional soliton equation, namely, the ansatz method and the F-expansion method. These methods are utilized to provide new accurate periodic and soliton solutions to this problem that are more generic. An appropriate transformation can be used to convert this nonlinear system into another nonlinear ordinary differential equation. In mathematical physics, it is demonstrated that the ansatz method and the F-expansion method give a strong mathematical tool for solving a large number of systems of nonlinear partial differential equations.

Keywords: (2+1)-dimensional soliton equation, the ansatz method, the F-expansion method

1. Introduction

There are numerous complex natural occurrences in this world that have a wide range of mathematical applications. The nonlinear evaluation equations (NLEEs), which have a significant impact on the exploration of nonlinear sciences, are mathematical models of nonlinear physical events. Obtaining accurate soliton solutions to NLEEs using computer programs that make repetitive and monotonous mathematical computations simpler has been a wonderful field for analysts and researchers in recent years. In fluid mechanics, optical fibers, material science, geochemistry, ocean engineering, geophysics, mathematical physics, plasma physical science, and several other logical fields, NLEEs play a crucial role in representing the actual behavior of genuine phenomena and dynamical processes.In this cutting-edge era of science, nonlinear science is one of the most fascinating fields for researchers. Because of its primary devotion to analyze the genuine part of the frameworks, researchers have focused on tracking down analytically or exact results.

One of the fundamental subjects of perpetual interest in mathematics and physics has long been the search for accurate solutions to NLEEs. Many powerful methods for finding exact solutions have been proposed with the development of symbolic computation packages like Maple and Mathematica, such as the trial equation technique [1], the Adomian decomposition method [2], the variation iteration method [3], the direct algebraic method [4], the extended Fan sub-equation method [5], the generalized exponential rational function method [6], the Sine-Gordon expansion method [7], the Jacobi elliptic expansion method [8], the Extended Jacobian elliptic expansion method [9], the (G'/G^2)-expansion method [10], the solitary wave ansatz method [11] and the (G'/G) expansion method [12,13].

The main goal of this research is to use the ansatz method, and the F-expansion method to generate a range of soliton solutions in the (2+1)-dimensional soliton equation. In this study, we will regard the (2+1)-dimensional soliton equation presented by [14]

$$
\begin{aligned}
&i\frac{\partial \phi}{\partial t}+\frac{\partial^2 \phi}{\partial x^2}+\phi\sigma=0,\\
&\frac{\partial \sigma}{\partial t}+\frac{\partial \sigma}{\partial y}+\frac{\partial}{\partial x}(\phi\phi^*)=0
\end{aligned}
\tag{1}
$$

where * denotes the complex conjugate, $i=\sqrt{-1}$, $\phi=\phi(x,y,t)$ is a complex function and $\sigma=\sigma(x,y,t)$ is a real function. The spatial domains and time are represented by *x*, *y*, and *t*, respectively. The governing equation is related to the integrable Zakharov equation in plasma physics, which regulates the behavior of weakly nonlinear ion-acoustic waves in a plasma and plays a significant role in various practical applications. The interaction of Langmuir and ion-acoustic waves in plasmas is the most physically significant example. There have been several investigations of the (2+1)-dimensional soliton in the literature [15-21]. The authors discovered a few solutions to this equation. Two recommended methods will be used to create the more effective, innovative solitary wave solutions of this equation.

The flow of this article is organized as follows: Description of the Ansatz method and the F-Expansion method are discussed in section 2. Finding analytical solutions for both methods (Soliton solutions, trigonometric and hyperbolic solutions) is given in section 3. The main conclusions is captured in Section 4.

2. Description of the methods for partial differential equation

In this section, we give a brief overview of the recommended methods. Let's consider the following type of nonlinear partial differential equation (PDE):

$$
H_1\Big(u,\frac{\partial u}{\partial x_1},\frac{\partial u}{\partial x_2},\frac{\partial u}{\partial x_3}...,\frac{\partial u}{\partial t},\frac{\partial^2 u}{\partial x_1^2},\frac{\partial^2 u}{\partial x_2^2},\frac{\partial^2 u}{\partial x_3^2},...\Big)=0 \tag{2}
$$

where $u=u(t,x_1,x_2,x_3,...)$ and H_1 is a polynomial of u and its partial derivatives. The following wave transform is used first to find the wave solutions of the equations

$$u(t, x_1, x_2, x_3, ...) = U(\varepsilon), \;\; \varepsilon = a_1x_1 + a_2x_2 + a_3x_3 + ... + kt$$
(3)

where $a_1, a_2, a_3, ..., k$ are non-zero constants. Substituting (3) to (2), the following nonlinear ordinary differential equation (ODE) is obtained,

$$H_2\left(U, \frac{dU}{d\varepsilon}, \frac{d^2U}{d\varepsilon^2}, \frac{d^3U}{d\varepsilon^3} ...\right) = 0$$
(4)

2.1 The ansatz method

The ansatz method has been given as follows [22]:

$$u(t, x_1, x_2, x_3, ...) = U(\varepsilon) = \tau_1 \text{sech}^{p_1}(\theta_1\varepsilon),$$
(5)
$$u(t, x_1, x_2, x_3, ...) = U(\varepsilon) = \tau_2 \tanh^{p_2}(\theta_2\varepsilon),$$
(6)
$$u(t, x_1, x_2, x_3, ...) = U(\varepsilon) = \tau_3 \text{csch}^{p_3}(\theta_3\varepsilon)$$
(7)

are solitary wave ansatzes of the forms bright, dark and singular solitons in searched for, respectively and
$$\varepsilon = a_1x_1 + a_2x_2 + a_3x_3 + ... + kt$$
(8)

is the wave transformation where; $\tau_i, \theta_i \;\; (i = 1,2,3)$ and k are, respectively, the amplitude, the inverse width and the velocity of the soliton, $a_1, a_2, a_3, ...$ are non-zero constants, and $p_i \;\; (i = 1,2,3)$ is an unknown exponent that will be found. It is feasible to get required derivatives from the given ansatzes with equalities. We get a set of algebraic equations for τ_i, θ_i and k when we replace the acquired derivatives in equation (4), collect all terms with the same order of required terms, and equal each coefficient of the resultant polynomial to zero. Finally, we can find the exact solution of (4), computing the system of equations.

2.2 The F-expansion method

In this section, the detailed description of the F-expansion method is given below [23,24]:

1. By taking Eq.(3), the traveling wave solutions of Eq.(2) are sought and it is transformed into an ordinary differential equation as in Eq.(4).

2. Suppose that, the solution $U(\varepsilon)$ of (4) can be described as

$$U(\varepsilon) = a_0 + \sum_{i=1}^{N}\left(a_i f^i(\varepsilon) + \frac{b_i}{f^i(\varepsilon)}\right)$$
(9)

where a_0 and a_i, b_i $(i = 1, 2, 3, \ldots, N)$ are constants to be determined, $f(\varepsilon)$ is a solution of ODE

$$[f'(\varepsilon)]^2 = K_2[f(\varepsilon)]^4 + K_1[f(\varepsilon)]^2 + K_0$$
(10)

where K_0, K_1 and K_2 are special values in Table 1. N is positive integer which can be determined from Eq.(10) as follows where $deg(U(\varepsilon)) = N$ is degree of $U(\varepsilon)$

$$deg\Big[\frac{d^qU}{d\varepsilon^q}\Big] = N + q, \quad deg\Big[U^r\Big(\frac{d^qU}{d\varepsilon^q}\Big)^s\Big] = Nr + s(q+N).$$

3. By substituting (9) with (10) into (4) and gathering the coefficients of $f^j(\varepsilon)$ $(j = 0, \pm1, \pm2, ...)$, a set of specified algebraic equations consisting of a_0, a_i, b_i $(i = 1, 2, ..., N)$. These parameters may be clearly identified by solving these algebraic equations. The Jacobi elliptic function solutions for (10) are known to be as Table 1.

4. Eq. (10) have Jacobi elliptic function solutions in Table 1. In Table 1, sn(ε)=sn(ε,m), cd(ε)=cd(ε,m), cn(ε)=cn(ε,m), dn(ε)=dn(ε,m), ns(ε)=ns(ε,m), cs(ε)=cs(ε,m), ds(ε)=ds(ε,m), sc(ε)=sc(ε,m), sd(ε)=sd(ε,m) are the Jacobi elliptic functions with the modulus $0 \le m \le 1$. When $m \to 0$ and $m \to 1$ are used, these functions turn into trigonometric and hyperbolic functions, as shown in Table 2.

5. By using the parameters found in step3 and the known values in step4 into (9), the solutions of (2) are obtained.

Table 1. Jacobi elliptic function solutions.

Case	K_0	K_1	K_2	$f(\varepsilon)$
1	1	$-(1+m^2)$	m^2	sn(ε) or cd(ε)
2	$1-m^2$	$2m^2-1$	$-m^2$	cn(ε)
3	m^2-1	$2-m^2$	-1	dn(ε)
4	m^2	$-(1+m^2)$	1	ns(ε) or dc(ε)
5	$-m^2$	$2m^2-1$	$1-m^2$	nc(ε)
6	-1	$2-m^2$	$-(1-m^2)$	nd(ε)
7	1	$2-m^2$	$1-m^2$	sc(ε)
8	1	$2m^2-1$	$-m^2(1-m^2)$	sd(ε)
9	$1-m^2$	$2-m^2$	1	cs(ε)
10	$-m^2(1-m^2)$	$2m^2-1$	1	ds(ε)
11	$(1-m^2)/4$	$(1+m^2)/2$	$(1-m^2)/4$	nc(ε)±sc(ε) or $\mathrm{cn}(\varepsilon)/1\pm\mathrm{sn}(\varepsilon)$

12	$-(1-m^2)^2/4$	$(1+m^2)/2$	$-1/4$	$m\,\mathrm{cn}(\varepsilon)\pm\mathrm{dn}(\varepsilon)$
13	$1/4$	$(1-2m^2)/2$	$1/4$	$\mathrm{sn}(\varepsilon)/1\pm\mathrm{cn}(\varepsilon)$
14	$1/4$	$(1+m^2)/2$	$(1-m^2)^2/4$	$\mathrm{sn}(\varepsilon)/\mathrm{dn}(\varepsilon)\pm\mathrm{cn}(\varepsilon)$

Table 2. Conversion of Jacobian elliptic functions to trigonometric and hyperbolic functions.

$m\to 0$	$m\to 1$
sn(ε)=sin(ε)	sn(ε)=tanh(ε)
cd(ε)=cos(ε)	cn(ε)=sech(ε)
cn(ε)=cos(ε)	dn(ε)=sech(ε)
ns(ε)=csc(ε)	ns(ε)=coth(ε)
cs(ε)=cot(ε)	cs(ε)=csch(ε)
ds(ε)=csc(ε)	ds(ε)=csch(ε)
sc(ε)=tan(ε)	sc(ε)=sinh(ε)
sd(ε)=sin(ε)	sd(ε)=sinh(ε)
nc(ε)=sec(ε)	nc(ε)=cosh(ε)
dn(ε)=1	cd(ε)=1

3. Mathematical analysis

In this section, analytical solutions will be found for the (2+1)-dimensional soliton equation given in Eq. (1) by applying the two different methods mentioned. Firstly, to search for solutions of Eq. (1), it is supposed that

$$\phi(x,y,t)=G_1(\varepsilon)e^{i\varphi},\ \sigma(x,y,t)=G_2(\varepsilon),$$
$$\varepsilon=A(x+By-2kt),\ \varphi=kx+ny+ct,$$
(11)

where $G_1(\varepsilon)$ and $G_2(\varepsilon)$ are real functions, *A*, *B*, *k*, *n* and *c* are real constants. By substituting (11) into (1), it is attained

$$A^2\frac{d^2G_1(\varepsilon)}{d\varepsilon^2}-(c+k^2)G_1(\varepsilon)+G_1(\varepsilon)G_2(\varepsilon)=0,$$
(12)
$$(B-2k)\frac{dG_2(\varepsilon)}{d\varepsilon}+\frac{d}{d\varepsilon}(G_1(\varepsilon))^2=0.$$
(13)

By integrating once with respect to ε and equaling the integration constants to zero, it is obtained

$$G_2(\varepsilon)=\frac{1}{(B-2k)}(G_1(\varepsilon))^2,\quad (B\neq 2k)$$
(14)

By substituting Eq. (14) into Eq. (12), it is found

$$\frac{d^2G_1(\varepsilon)}{d\varepsilon^2}-\frac{(c+k^2)}{A^2}G_1(\varepsilon)+\frac{1}{A^2(B-2k)}(G_1(\varepsilon))^3=0.$$
(15)

3.1 Application of the ansatz method

Firstly, to solve Eq. (15), let's take $-\frac{(c+k^2)}{A^2}=K$ and $\frac{1}{A^2(B-2k)}=L$. Thus, the equation becomes

$$\frac{d^2G_1(\varepsilon)}{d\varepsilon^2}+K\,G_1(\varepsilon)+L\,(G_1(\varepsilon))^3=0.$$
(16)

For the bright soliton, it is regarded as the ansatz

$$G_1(\varepsilon)=\tau_1\text{sech}^{p_1}(\theta_1\varepsilon) \tag{17}$$

and it is obtained

$$G_1''(\varepsilon)=\tau_1 p_1^2\theta_1^2\text{sech}^{p_1}(\theta_1\varepsilon)-\tau_1 p_1(p_1+1)\theta_1^2\text{sech}^{p_1+2}(\theta_1\varepsilon) \tag{18}$$
and

$$G_1^3(\varepsilon)=\tau_1^3\text{sech}^{3p_1}(\theta_1\varepsilon)\,. \tag{19}$$

Substituting (17)-(19) into Eq. (16), the equation that follows

$$\tau_1 p_1^2\theta_1^2\text{sech}^{p_1}(\theta_1\varepsilon)-\tau_1 p_1(p_1+1)\theta_1^2\text{sech}^{p_1+2}(\theta_1\varepsilon)+K\tau_1\text{sech}^{p_1}(\theta_1\varepsilon)+L\tau_1^3\text{sech}^{3p_1}(\theta_1\varepsilon)=0$$

is found. It is achieved $p_1=1$ by equating the exponents p_1+2 and $3p_1$ in this equation using the balancing principle. It is obtained the algebraic equation system below by comparing the different powers of $\text{sech}(\theta_1\varepsilon)$.

$$\begin{aligned}-2\tau_1\theta_1^2-L\tau_1^3&=0\\ \tau_1\theta_1^2+K\tau_1&=0\end{aligned} \tag{20}$$

By solving this system (20), it is attained $\tau_1=\pm\sqrt{-\frac{2K}{L}}$ and $\theta_1=\pm\sqrt{-K}$. By replacing the previously accepted expressions K and L,

$\tau_1=\pm\sqrt{2(c+k^2)(B-2k)}$ and $\theta_1=\pm\frac{1}{A}\sqrt{c+k^2}$

are gained. Therefore,

$$G_1(\varepsilon)=\pm\sqrt{2(c+k^2)(B-2k)}\operatorname{sech}\left(\pm\frac{1}{A}\sqrt{c+k^2}\,\varepsilon\right)$$

$$G_2(\varepsilon)=\frac{1}{(B-2k)}\left(\pm\sqrt{2(c+k^2)(B-2k)}\operatorname{sech}\left(\pm\frac{1}{A}\sqrt{c+k^2}\,\varepsilon\right)\right)^2, \quad (B\neq 2k)$$

(21)

are written. Finally, the bright soliton solutions of Eq. (1) are as follows

$$\phi(x,y,t)=\pm\sqrt{2(c+k^2)(B-2k)}\operatorname{sech}\left(\pm\sqrt{c+k^2}\,(x+By-2kt)\right)e^{i(kx+ny+ct)},$$

$$\sigma(x,y,t)=2(c+k^2)\operatorname{sech}^2\left(\pm\sqrt{c+k^2}\,(x+By-2kt)\right), \quad (B\neq 2k).$$

(22)

For the singular soliton, it is regarded as the ansatz

$$G_1(\varepsilon)=\tau_2\operatorname{csch}^{p_2}(\theta_2\varepsilon) \tag{23}$$

and it is found

$$G_1''(\varepsilon)=\tau_2 p_2^2\theta_2^2\operatorname{csch}^{p_2}(\theta_2\varepsilon)+\tau_2 p_2(p_2+1)\theta_2^2\operatorname{csch}^{p_2+2}(\theta_2\varepsilon) \tag{24}$$

and

$$G_1^3(\varepsilon)=\tau_2^3\operatorname{csch}^{3p_2}(\theta_2\varepsilon)\,. \tag{25}$$

Substituting (23)-(25) into Eq. (16), the equation that follows

$$\tau_2 p_2^2\theta_2^2\operatorname{csch}^{p_2}(\theta_2\varepsilon)+\tau_2 p_2(p_2+1)\theta_2^2\operatorname{csch}^{p_2+2}(\theta_2\varepsilon)+K\tau_2\operatorname{csch}^{p_2}(\theta_2\varepsilon)+L\tau_2^3\operatorname{sech}^{3p_2}(\theta_2\varepsilon)=0$$

is gained. The balancing principle is used by equating the exponents p_2 + 2 and 3p_2 in this equation. So, It is attained $p_2=1$. By comparing the different powers of $\operatorname{csch}(\theta_2\varepsilon)$, the algebraic equation system below is found.

$$\begin{aligned} \tau_2\theta_2^2+K\tau_2&=0\\ 2\tau_2\theta_2^2+L\tau_2^3&=0 \end{aligned} \tag{26}$$

By solving this system (26), it is gained $\tau_2=\pm\sqrt{\frac{2K}{L}}$ and $\theta_2=\pm\sqrt{-K}$. By replacing the previously accepted expressions K and L,

$\tau_2=\pm\sqrt{-2(c+k^2)(B-2k)}$ and $\theta_2=\pm\frac{1}{A}\sqrt{c+k^2}$

are found. Hence,

$$G_1(\varepsilon)=\pm\sqrt{-2(c+k^2)(B-2k)}\operatorname{csch}\left(\pm\frac{1}{A}\sqrt{c+k^2}\,\varepsilon\right)$$
$$G_2(\varepsilon)=\frac{1}{(B-2k)}\left(\pm\sqrt{-2(c+k^2)(B-2k)}\operatorname{csch}\left(\pm\frac{1}{A}\sqrt{c+k^2}\,\varepsilon\right)\right)^2,\quad (B\neq 2k)$$
(27)

are written. Eventually, the singular soliton solutions of Eq. (1) are as follows

$$\phi(x,y,t)=\pm\sqrt{-2(c+k^2)(B-2k)}\operatorname{csch}\left(\pm\sqrt{c+k^2}\,(x+By-2kt)\right)e^{i(kx+ny+ct)},$$
$$\sigma(x,y,t)=-2(c+k^2)\operatorname{csch}^2\left(\pm\sqrt{c+k^2}\,(x+By-2kt)\right),\quad (B\neq 2k).$$
(28)

For the dark soliton, it is regarded as the ansatz

$$G_1(\varepsilon)=\tau_3\tanh^{p_3}(\theta_3\varepsilon) \tag{29}$$

and it is found

$$G_1''(\varepsilon)=\tau_3 p_3\theta_3^2\left((p_3-1)\tanh^{p_3-2}(\theta_3\varepsilon)-2p_3\tanh^{p_3}(\theta_3\varepsilon)+(p_3+1)\tanh^{p_3+2}(\theta_3\varepsilon)\right) \tag{30}$$
and

$$G_3^3(\varepsilon)=\tau_3^3\operatorname{csch}^{3p_3}(\theta_3\varepsilon)\,. \tag{31}$$

Substituting (29)-(31) into Eq. (16), the equation that follows

$$\tau_3 p_3\theta_3^2(p_3-1)\tanh^{p_3-2}(\theta_3\varepsilon)-2\tau_3 p_3^2\tanh^{p_3}(\theta_3\varepsilon)+\tau_3 p_3\theta_3^2(p_3+1)\tanh^{p_3+2}(\theta_3\varepsilon)+K\tau_3\tanh^{p_3}(\theta_3\varepsilon)$$
$$+L\tau_3^3\operatorname{csch}^{3p_3}(\theta_3\varepsilon)=0$$

is found. The balancing principle is applied in this equation by equating the exponents p_3+2 and $3p_3$. As a result, $p_3=1$ has been achieved. The algebraic equation system below is determined by comparing the different powers of $\tanh(\theta_3\varepsilon)$.

$$\begin{aligned}-2\tau_3\theta_3^2+K\,\tau_3&=0\\ 2\tau_3\theta_3^2+L\,\tau_3^3&=0\end{aligned} \tag{32}$$

By solving this system (32), it is gained $\tau_3=\pm\sqrt{\frac{-K}{L}}$ and $\theta_3=\pm\sqrt{\frac{K}{2}}$. By replacing the previously accepted expressions K and L,

$$\tau_3=\pm\sqrt{(c+k^2)(B-2k)}\ \text{and}\ \ \theta_3=\pm\frac{1}{A}\sqrt{\frac{-(c+k^2)}{2}},\quad (c+k^2<0,\ B<2k)$$
are found. Hence,

$$G_1(\varepsilon) = \pm\sqrt{(c+k^2)(B-2k)}\tanh\left(\pm\frac{1}{A}\sqrt{\frac{-(c+k^2)}{2}}\,\varepsilon\right)$$

$$G_2(\varepsilon) = \frac{1}{(B-2k)}\left(\pm\sqrt{c+k^2)(B-2k)}\tanh\left(\pm\frac{1}{A}\sqrt{\frac{-(c+k^2)}{2}}\varepsilon\right)\right)^2, \quad (c+k^2<0,\ B<2k)$$

(33)

are written. Eventually, the dark soliton solutions of Eq. (1) are as follows

$$\phi(x,y,t) = \pm\sqrt{(c+k^2)(B-2k)}\operatorname{csch}\left(\pm\sqrt{\frac{-(c+k^2)}{2}}(x+By-2kt)\right)e^{i(kx+ny+ct)},$$

$$\sigma(x,y,t) = (c+k^2)\tanh^2\left(\pm\sqrt{\frac{-(c+k^2)}{2}}(x+By-2kt)\right), \quad (c+k^2<0,\ B<2k).$$

(34)

3.2 Application of the F-expansion method

In this section, the solution of equation (1) with the F-expansion method will be investigated by using equations (14) and (15). Using the balancing principle between $\frac{d^2G_1(\varepsilon)}{d\varepsilon^2}$ and $(G_1(\varepsilon))^3$ in (15) gives $N = 1$. Hence, from (9), the solution of (15) can be written

$$G_1(\varepsilon) = a_0 + a_1 f(\varepsilon) + \frac{b_1}{f(\varepsilon)}.$$

(35)

where a_0, a_1, b_1 are constants to be determined, $f(\varepsilon)$ meets the elliptic equation (10). The sixth order polynomial in $f(\varepsilon)$ is obtained by substituting (35) and (10) into (15). The following nonlinear system of equations is found by equating all coefficients of $f(\varepsilon)$ to zero.

$$2a_1K_2 + \frac{1}{A^2(B-2k)}a_1^{\ 3} = 0$$

$$\frac{3}{A^2(B-2k)}a_0a_1^{\ 2} = 0$$

$$a_1K_1 - \frac{c+k^2}{A^2}a_1 + \frac{3}{A^2(B-2k)}a_1a_0^{\ 2} + \frac{3}{A^2(B-2k)}b_1a_1^{\ 2} = 0$$

$$-\frac{c+k^2}{A^2}a_0 + \frac{1}{A^2(B-2k)}a_0^{\ 3} + \frac{6}{A^2(B-2k)}a_0a_1b_1 = 0$$

$$b_1K_1 - \frac{c+k^2}{A^2}b_1 + \frac{3}{A^2(B-2k)}b_1a_0^{\ 2} + \frac{3}{A^2(B-2k)}a_1b_1^{\ 2} = 0$$

$$\frac{3}{A^2(B-2k)}a_0b_1^{\ 2}=0$$

$$2b_1K_0+\frac{1}{A^2(B-2k)}b_1^{\ 3}=0$$

It is acquired the following two sets of solutions for unknown coefficients by solving this system with Maple.

Set 1	Set 2
$k=\pm\sqrt{K_1A^2-c}$	$k=\pm\sqrt{K_1A^2-c}$
$a_0=0$	$a_0=0$
$a_1=0$	$a_1=\pm\sqrt{-2K_2B\pm 4K_2\sqrt{K_1A^2-c}}$
$b_1=\pm\sqrt{-2K_0B\pm 4K_0\sqrt{K_1A^2-c}}$	$b_1=0$

Substituting (Set 1)–(Set 2) into (35), it is obtained the following solutions for Equation (15):

Set 1:

$$G_1(\varepsilon)=\frac{1}{f(\varepsilon)}\left(\pm\sqrt{-2K_0B\pm 4K_0\sqrt{K_1A^2-c}}\right)$$

and from Eq. (14),

$$G_2(\varepsilon)=\frac{1}{(B-2k)}\left(\frac{1}{f(\varepsilon)}\left(\pm\sqrt{-2K_0B\pm 4K_0\sqrt{K_1A^2-c}}\right)\right)^2$$

are gained.

Eventually, the exact solutions of equation (1) have the form below.

$$\phi(x,y,t)=\left(\pm\sqrt{-2K_0B\pm 4K_0\sqrt{K_1A^2-c}}\right)\frac{1}{f(\varepsilon)}e^{i\varphi},$$

$$\sigma(x,y,t)=\frac{K_0\left(-2B\pm 4\sqrt{K_1A^2-c}\right)}{B-2\left(\pm\sqrt{K_1A^2-c}\right)}\frac{1}{(f(\varepsilon))^2},$$

(35)

where $\varepsilon=A(x+By-2\left(\pm\sqrt{K_1A^2-c}\right)t),\ \varphi=\left(\pm\sqrt{K_1A^2-c}\right)x+ny+ct$.

Set 2:

$$G_1(\varepsilon)=f(\varepsilon)\left(\pm\sqrt{-2K_2B\pm 4K_2\sqrt{K_1A^2-c}}\right)$$

and from Eq. (14),

$$G_2(\varepsilon)=\frac{1}{(B-2k)}\left(f(\varepsilon)\left(\pm\sqrt{-2K_2B\pm 4K_2\sqrt{K_1A^2-c}}\right)\right)^2$$

are gained.

Eventually, the exact solutions of equation (1) have the form below.

$$\phi(x,y,t)=\left(\pm\sqrt{-2K_2B\pm 4K_2\sqrt{K_1A^2-c}}\right)f(\varepsilon)e^{i\varphi},$$

$$\sigma(x,y,t)=\frac{K_2\left(-2B\pm 4\sqrt{K_1A^2-c}\right)}{B-2\left(\pm\sqrt{K_1A^2-c}\right)}\left[f(\varepsilon)\right]^2,$$

(36)

where $\varepsilon=A(x+By-2\left(\pm\sqrt{K_1A^2-c}\right)t),\ \varphi=\left(\pm\sqrt{K_1A^2-c}\right)x+ny+ct$.

The exact solutions of (1) are obtained by combining (35) and (36) with Table 1 and Table 2. Some of them can be expressed for Set 1 and Set2 as follows:

For Set 1:

Case 1. $K_2=m^2, K_1=-(1+m^2),\ K_0=1,\ f(\varepsilon)=sn(\varepsilon);$

When $m\to 0$, the solution of (1) is

$$\phi(x,y,t)=\left(\pm\sqrt{-2B\pm 4\sqrt{-\left(A^2+c\right)}}\right)\csc(\varepsilon)e^{i\varphi},$$

$$\sigma(x,y,t)=\frac{\left(-2B\pm 4\sqrt{-\left(A^2+c\right)}\right)}{B-2\left(\pm\sqrt{-\left(A^2+c\right)}\right)}\csc^2(\varepsilon)\ ,\quad \left(A^2+c<0\right)$$

(37)

where $\varepsilon=A(x+By-2\left(\pm\sqrt{-\left(A^2+c\right)}\right)t),\ \varphi=\left(\pm\sqrt{-\left(A^2+c\right)}\right)x+ny+ct$.

When $m\to 1$, the solution of (1) is

$$\phi(x,y,t)=\left(\pm\sqrt{-2B\pm 4\sqrt{-\left(2A^2+c\right)}}\right)\coth(\varepsilon)e^{i\varphi},$$

$$\sigma(x,y,t)=\frac{\left(-2B\pm 4\sqrt{-\left(2A^2+c\right)}\right)}{B-2\left(\pm\sqrt{-\left(2A^2+c\right)}\right)}\coth^2(\varepsilon)\ ,\quad \left(2A^2+c<0\right)$$

(38)

where $\varepsilon=A(x+By-2\left(\pm\sqrt{-\left(2A^2+c\right)}\right)t),\ \varphi=\left(\pm\sqrt{-\left(2A^2+c\right)}\right)x+ny+ct$.

Case 7. $K_2=1-m^2, K_1=2-m^2,\ K_0=1,\ f(\varepsilon)=sc(\varepsilon);$

When $m\to 0$, the solution of (1) is

$$\phi(x,y,t)=\left(\pm\sqrt{-2B\pm 4\sqrt{2A^2-c}}\right)\cot(\varepsilon)e^{i\varphi},$$

$$\sigma(x,y,t)=\frac{\left(-2B\pm 4\sqrt{2A^2-c}\right)}{B-2\left(\pm\sqrt{2A^2-c}\right)}\cot^2(\varepsilon)\ ,\quad \left(2A^2-c>0\right)$$

(38)

where $\varepsilon = A(x+By-2\left(\pm\sqrt{2A^2-c}\right)t),\ \varphi=\left(\pm\sqrt{2A^2-c}\right)x+ny+ct$.

When $m \to 1$, the solution of (1) is

$$\phi(x,y,t)=\left(\pm\sqrt{-2B\pm 4\sqrt{A^2-c}}\right)\operatorname{csch}(\varepsilon)e^{i\varphi},$$

$$\sigma(x,y,t)=\frac{\left(-2B\pm 4\sqrt{A^2-c}\right)}{B-2\left(\pm\sqrt{A^2-c}\right)}\operatorname{csch}^2(\varepsilon)\ ,\quad \left(A^2-c<0\right)$$

(39)

where $\varepsilon = A(x+By-2\left(\pm\sqrt{A^2-c}\right)t),\ \varphi=\left(\pm\sqrt{A^2-c}\right)x+ny+ct$.

For Set 2:

Case 3. $K_2=-1, K_1=2-m^2,\ K_0=m^2-1,\ f(\varepsilon)=dn(\varepsilon);$

When $m \to 0$, the solution of (1) is

$$\phi(x,y,t)=\left(\pm\sqrt{2B\pm 4\sqrt{2A^2-c}}\right)e^{i\varphi},$$

$$\sigma(x,y,t)=\frac{\left(2B\pm 4\sqrt{2A^2-c}\right)}{B-2\left(\pm\sqrt{2A^2-c}\right)}\ ,\quad \left(2A^2-c>0\right)$$

(40)

where $\varepsilon = A(x+By-2\left(\pm\sqrt{2A^2-c}\right)t),\ \varphi=\left(\pm\sqrt{2A^2-c}\right)x+ny+ct$.

When $m \to 1$, the solution of (1) is

$$\phi(x,y,t)=\left(\pm\sqrt{2B\pm 4\sqrt{A^2-c}}\right)\operatorname{sech}(\varepsilon)e^{i\varphi},$$

$$\sigma(x,y,t)=\frac{\left(2B\pm 4\sqrt{A^2-c}\right)}{B-2\left(\pm\sqrt{A^2-c}\right)}\operatorname{sech}^2(\varepsilon)\ ,\quad \left(A^2-c>0\right)$$

(41)

where $\varepsilon = A(x+By-2\left(\pm\sqrt{A^2-c}\right)t),\ \varphi=\left(\pm\sqrt{A^2-c}\right)x+ny+ct$.

Case 2. $K_2=-m^2, K_1=2m^2-1,\ K_0=1-m^2,\ f(\varepsilon)=cn(\varepsilon);$

When $m \to 1$, the solution of (1) is

$$\phi(x,y,t)=\left(\pm\sqrt{2B\pm 4\sqrt{A^2-c}}\right)\operatorname{sech}(\varepsilon)e^{i\varphi},$$

$$\sigma(x,y,t)=\frac{\left(2B\pm 4\sqrt{A^2-c}\right)}{B-2\left(\pm\sqrt{A^2-c}\right)}\operatorname{sech}^2(\varepsilon)\ ,\quad \left(A^2-c>0\right)$$

(42)

where $\varepsilon=A(x+By-2\left(\pm\sqrt{A^2-c}\right)t),\ \varphi=\left(\pm\sqrt{A^2-c}\right)x+ny+ct$.

Case 8. $K_2=1-m^2, K_1=2m^2-1,\ K_0=1,\ f(\varepsilon)=sd(\varepsilon);$

When $m\to 0$, the solution of (1) is

$$\phi(x,y,t)=\left(\pm\sqrt{2B\pm 4\sqrt{-\left(A^2+c\right)}}\right)\sin(\varepsilon)e^{i\varphi},$$

$$\sigma(x,y,t)=\frac{\left(2B\pm 4\sqrt{-\left(A^2+c\right)}\right)}{B-2\left(\pm\sqrt{-\left(A^2+c\right)}\right)}\sin^2(\varepsilon)\ ,\quad \left(A^2+c<0\right)$$

(43)

where $\varepsilon=A(x+By-2\left(\pm\sqrt{-\left(A^2+c\right)}\right)t),\ \varphi=\left(\pm\sqrt{-\left(A^2+c\right)}\right)x+ny+ct$.

4. Conclusion

The ansatz method and the F-expansion method were utilized to solve the (2+1)-dimensional soliton problem in this study. The symbolic computing system verified these solutions. This study gave that these methods were a reliable and strong strategy to find new exact solutions. When the results of these methods are compared to those of earlier studies, it is evident that they are original. One of the most distinguishing features of the methods we utilize is their variety in comparison to other methods [14- 21].

The findings of this study are useful in describing various nonlinear systems and provide useful additions to the existing literature. To the best of our knowledge, the novel type of solutions discovered in this work have never been achieved previously. Furthermore, the results show that the suggested methods are an effective mathematical tool and that they are easier, stronger, and quicker thanks to the symbolic program computation system. These methods might be used to address a lot more nonlinear evolution problems. It's worth mentioning that the proposed method may be used for a variety of nonlinear evolution problems in mathematical physics. The findings of this study might have implications for the meaning of various physical problems.

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