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## First Theoretical Identification of the Magnetic Dipole Moment of the 97.43 keV State in $^{153}\text{Eu}$

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### Abstract

Two alternative values,  $+3.21 \pm 0.22 \mu_N$  and  $-0.52 \pm 0.22 \mu_N$ , for the magnetic dipole (M1) moment of the excited [532]  $5/2^-$  state at 97.43 keV in  $^{153}\text{Eu}$  were reported in the Mossbauer-effect study. The Quasiparticle Phonon Nuclear Model (QPNM) has been used to determine the correct value of the magnetic moment of this state. According to the QPNM calculations, the experimental 97.43 keV level is the [532]  $5/2^-$  Nilsson state occurring at 79 keV. The QPNM predicted the magnetic moment of this state to be  $+3.2162 \mu_N$ , which agrees well with one of the experimental values, i.e.,  $+3.21 \pm 0.22 \mu_N$ . Therefore, the correct value for the magnetic moment of the 97.43 keV level of  $^{153}\text{Eu}$  is most probably  $+3.21 \pm 0.22 \mu_N$ . The measured value ( $+3.4717 \pm 0.006 \mu_N$ ) of the magnetic moment of  $5/2^-$  ground-state, which is probably a [532] Nilsson state according to our QPNM calculations, supports our prediction.

**Keywords:** Odd-mass nuclei,  $^{153}\text{Eu}$ , magnetic dipole, M1, qpnm

### 1. INTRODUCTION

The deformed nuclei in the rare-earth mass region of the periodic table exhibit a rich variety of nuclear structure phenomena, such as multipole excitations, shape changes and shape coexistence [1]. Determining the magnetic dipole moments of odd-mass nuclei in this

region allows us to obtain directly or indirectly information on the underlying structure of these nuclear phenomena due to the direct relationship between the magnetic moments and the single particle configurations [2]. Therefore, magnetic moments are crucial to clarify whether the single particle or the collective motion is responsible for the observed nuclear features [3].

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Considerable experimental data are available on the magnetic moment of the ground- and some low-lying excited-states of the rare-earth odd-mass nuclei [4]. Extensive theoretical investigations have also been carried out on the ground-state magnetic moments of these nuclei [5-10]. However, a limited number of theoretical studies are available for the magnetic moments of the excited-states in odd-mass deformed nuclei [11].

The problem of correctly describing the observed magnetic moments of the odd-mass nuclei is required proper interpretation of the core-polarization which is directly affect the magnetic moments [1]. The core polarization in the odd-mass nuclei arises as a result of interaction between the valance nucleon and the M1 excitations of the core [1, 12, 13]. The core polarization effect has been subject of several theoretical works in which the importance of the requirement of considering core-polarization in the calculation of magnetic moments has been pointed out [11-17]. Recently, a microscopic method [14-17] within the framework of the Quasiparticle Phonon Nuclear Model (QPNM) [18] has been introduced for the magnetic moment calculation of odd-mass deformed nuclei. It has been demonstrated with numerical results that this method not only reproduce the experimental magnetic moments and intrinsic factors but also gives a theoretical explanation for the effective spin gyromagnetic factors, which are assumed to be 0.6-0.7 in several calculations [14-17].

Undoubtedly, investigation of the magnetic moment of an excited-state would be an important challenge for this model to test whether it retains its success in calculating the ground-state magnetic properties for excited states. In light of the above-mentioned fact, in the present paper, the magnetic dipole (*M1*) moment of the excited [532] 5/2<sup>-</sup> state at 97.43

keV in <sup>153</sup>Eu nucleus is studied in detail using the method based on the QPNM [14-18]. It is an intriguing nucleus to study since there are two alternative experimental values, +3.21±0.22  $\mu_N$  and -0.52±0.22  $\mu_N$ , for the 97.43 keV state [19]. Therefore, it constitutes an invaluable ground to test the possibility of describing the magnetic properties of the excited-states in odd-mass deformed nuclei within the framework of the QPNM. Naturally, determining the correct value of the M1 moment of this state is another purpose of the current work.

In Sec. II, the way of treating the magnetic properties within the QPNM is described, and related analytical expressions are presented. In Sec. III, the results of the calculation are given and discussed. Sec. IV is a summary of the obtained results.

## 2. THEORY

In order to investigate the magnetic properties of an odd-mass nucleus, a model nuclear Hamiltonian taking into account the pairing and residual spin-spin interactions is constructed as follows [14-17]:

$$H \approx H_{spp} + H_{coll.} + H_{int.} \quad (1)$$

where the  $H_{spp}$  and  $H_{coll.}$  described the single-quasiparticle and collective motion in axially-symmetric mean-field, respectively, and  $H_{int.}$  represents the interaction between single quasiparticles and phonons of collective motion. The explicit expressions of  $H_{spp}$ ,  $H_{coll.}$  and  $H_{int.}$ , and the detail of the notations used in this study can be found in refs [14-17].

For the states with  $K > 1/2$ , the wave function of odd-mass deformed nuclei includes the one-

quasiparticle term as well as the single-quasiparticles $\otimes$ phonon mixing term:

$$\Psi_K^j(\tau) = \left\{ N_K^j \alpha_K^+(\tau) + \sum_{iv} G_{ij}^{Kv} \alpha_v^+(\tau) Q_i^+ \right\} |\psi_0\rangle \quad (2)$$

Here the index j defines the number of the states for a given  $K^\pi$ .  $N_K^j$  and  $G_{ij}^{Kv}$  are the one-quasiparticle and the quasiparticle $\otimes$ phonon amplitudes of the wave function, respectively [14-17].

Within the framework of the well-known variational method, the analytical expression of the secular equation can be found as follows:

$$-P(\eta) \equiv \varepsilon_K - \eta_K - \sum_{iv} \frac{1}{Z(\omega_i)} \frac{q^2 \sigma_{Kv}^2 M_{Kv}^2}{(1 + \chi F_p^i)^2 (\omega_i + \varepsilon_v - \eta_K)} = 0 \quad (3)$$

where

$$Z(\omega_i) = \frac{1}{(-\chi F_n)^2} Y_n(\omega_i) + \frac{q^2}{(1 + \chi F_p)^2} Y_p(\omega_i) \\ Y_\tau = 4\omega_i \sum_{ss'} \frac{\varepsilon_{ss'} \sigma_{ss'}^2 L_{ss'}^2}{(\varepsilon_{ss'}^2 - \omega_i^2)^2}; \quad F_\tau = 2 \sum_{ss'} \frac{\varepsilon_{ss'} \sigma_{ss'}^2 L_{ss'}^2}{\varepsilon_{ss'}^2 - \omega_i^2} \quad (4)$$

here,  $\omega_i$  is the energies of the collective  $1^+$  states of the even-even core. The roots ( $\eta_K$ ) of Eq.(3) are the ground- and excited-states energies of the non-rotational states of the odd-mass nucleus under investigation. The analytical expressions for  $G_{ij}^{Kv}$  and  $N_K^j$  can be easily obtained by using the secular Eq. (3) and the normalization condition of the wave function. The explicit form of the  $G_{ij}^{Kv}$  and  $N_K^j$  can be found in refs [14-17].

For a  $K > 1/2$  state of an odd-mass nucleus, the intrinsic magnetic moment i.e.  $\mu_K = g_K K$  is the expectation value of  $\mu_z$  [14-17]:

$$\mu_K = \left\{ \begin{aligned} & (g_s^p - g_s^r) \left( 1 + N_K^2 \frac{1}{Z(\omega_i)} \frac{2M_{Kv}(1 + \chi F_p)}{\chi^2 F_n (1 + \chi F_p) (\varepsilon_v + \omega_i - \eta_K)} \right) - \\ & - g_s^r N_K^2 \frac{1}{Z(\omega_i)} \frac{2qM_{Kv}}{\chi(1 + \chi F_p) (\varepsilon_v + \omega_i - \eta_K)} \end{aligned} \right\} \langle K | s_z | K \rangle + g_s^l K \quad (5)$$

Omitting the details and considering the  $K = \nu$  for  $\mu = 0$  following analytical expression for  $g_s^{eff}$  can be easily found from the comparison of Eq. (5) and the well-known Nilsson formula [14-17].

$$g_s^{eff} - g_s^p = (g_s^p - g_s^r) \left( 1 + 2N_K^2 \sum_i \frac{(1 + \chi F_p)}{\chi^2 Z(\omega_i) F_n (1 + \chi F_p) (\varepsilon_K + \omega_i - \eta_K)} \frac{1}{\chi Z(\omega_i) (1 + \chi F_p) (\varepsilon_K + \omega_i - \eta_K)} \right) - g_s^r N_K^2 \sum_i \frac{2q}{\chi Z(\omega_i) (1 + \chi F_p) (\varepsilon_K + \omega_i - \eta_K)} \quad (6)$$

As can be seen from Eq. (6), the mixing of one-quasiparticle and phonon terms caused the quenching of the spin gyromagnetic factor in odd-Z nuclei. According to the Unified Model [1], the magnetic moment of an odd-mass nucleus for a  $K > 1/2$  state includes contribution both from the rotational and intrinsic motion of the nucleus [14- 17]:

$$\mu = \frac{K}{I + 1} (g_K K + g_R) \quad (7)$$

In order to take into account the rotational contribution in our calculations, the rotational gyromagnetic factors ( $g_R$ ) calculated within the Inglis-Belyaev cranking model [20] is used.

### 3. RESULT AND DISCUSSIONS

The calculations were performed with the single particle scheme of levels for an axially symmetric Woods-Saxon potential [21]. The quadrupole deformation parameter  $\beta_2 = 0.3064$  taken from the work of Raman [22]. The neutron and proton pairing constants are  $\Delta_n = 0.986$  MeV and  $\Delta_p = 1.053$  MeV, respectively from the odd-even mass

differences [23]. The chemical potentials for neutrons and protons were calculated to be  $\lambda_n = -7.159$  and  $\lambda_p = -7.993$  following the same procedure of Ref [18].

The calculations show that the interaction between the odd-particle and the  $M1$  excitations of the core causes shifting of the energy of the single quasiparticle state. However, this shift is small and usually does not exceed 0.006 MeV.

Figure 1(a) compares the theoretical and experimental structure of the ground- and 97.43 keV excited-state. As shown in Figure 1(a), the calculated and experimental energies and the Nilsson quantum numbers are in good agreement. It is important to state that the QPNM results for the structure of these two states are consistent with the results reported by Soloviev et al. [24]. The  $5/2^-$  ground- and  $5/2^+$  excited-state in  $^{153}\text{Eu}$  can be assumed to have one-quasiparticle character since the single-quasiparticle contributions to the wave function of these two states exceed 99%. On the other hand, the structure of these states calculated within QPNM contains quasiparticle $\otimes$ phonon admixtures. Although the quasiparticle $\otimes$ phonon terms in the wave functions do not exceed the 0.05%, the coherent contribution of these small admixtures leads to a quenching of spin gyromagnetic factors. The effective gyromagnetic ratio for the excited-state at 97.43 keV is found to be

$$g_s^{\text{eff}} = 0.798 g_s^{\text{free}}$$

As seen from Figure 1(b) when the spin-spin interaction strength equals to  $\chi = 20$ , the  $g_K$  value for the excited-state at 97.43 keV is in excellent agreement with the semi-empirical  $g_K$  value calculated using  $\mu = +3.21 \pm 0.22 \mu_N$  experimental magnetic moment. On the other hand, the semi-empirical value of  $g_K$  calculated using the other experimental value, i.e.  $\mu = -0.522 \pm 0.22 \mu_N$  is considerably smaller than the

theoretical results. Therefore, the actual value of the experimental magnetic moment of the states at 97.43 keV in  $^{153}\text{Eu}$  is most probably  $\mu = +3.21 \pm 0.22 \mu_N$ .

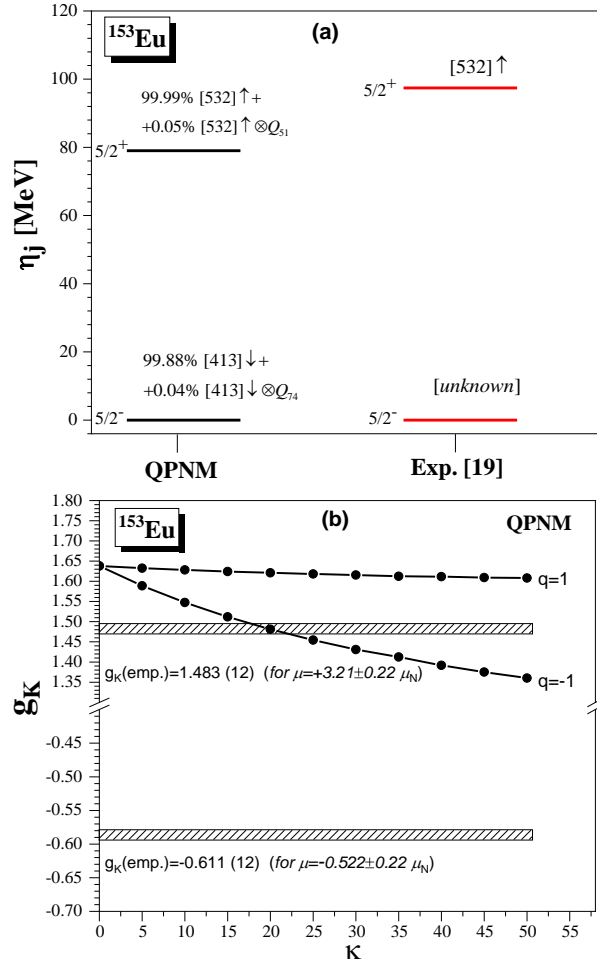


Figure 1 (a) Comparison of the theoretical and experimental structure of the ground- and 97.43 keV excited-state in  $^{153}\text{Eu}$ . (b) Intrinsic magnetic moment ( $g_K$ ) of the excited-state at 97.43 keV as a function of  $q$  and  $K$  where  $q=1$  and  $q=-1$  correspond to the isovector and isoscalar part, respectively

Figure 1(b) shows the intrinsic magnetic moment ( $g_K$ ) of the excited-state at 97.43 keV as a function of  $q$  and  $K$ .

Using the  $g_R = 0.42$  determined from the cranking model and taking into account the  $g_K$  factor calculated within QPNM, the magnetic

moment of this excited state is predicted to be  $\mu=+3.2162 \mu_N$ . which agrees well with one of the experimental values,  $\mu=+3.21\pm 0.22 \mu_N$ . According to the single particle model calculations,  $\mu=+3.493 \mu_N$  which is also consistent with our result. Besides, the measured value ( $\mu=+3.4717 \pm 0.006 \mu_N$ ) of the magnetic moment of  $5/2^-$  ground-state, which is probably a [532]  $5/2^-$  Nilsson state according to our QPNM calculations, support our prediction.

#### 4. CONCLUSION

The magnetic moment of the state at 97.43 keV in  $^{153}\text{Eu}$  has been theoretically investigated using the QPNM. There are two different experimental magnetic moments values ( $+3.21\pm 0.22 \mu_N$  and  $-0.52\pm 0.22 \mu_N$ ) for these state. According to QPNM calculations the true value of the magnetic moment of this excited states is likely  $+3.21\pm 0.22 \mu_N$ . This prediction is supported by the result of the single particle model calculation and the magnetic moment values of the ground-state of  $^{151}\text{Eu}$  which has the same Nilsson quantum numbers as the state at 97.43 keV in  $^{153}\text{Eu}$ .

In the light of the above mentioned results it is possible to state that the method based on the QPNM is successful not only in the determination of the magnetic properties of ground-states, but also in description of the magnetic properties of low-lying states in odd-mass nuclei.

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#### *Authors' Contribution*

The authors contributed equally to the study.

#### *The Declaration of Conflict of Interest/ Common Interest*

No conflict of interest or common interest has been declared by the authors.

#### *The Declaration of Ethics Committee Approval*

This study does not require ethics committee permission or any special permission.

#### *The Declaration of Research and Publication Ethics*

The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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