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Estimation Study of Multicomponent Stress-Strength Reliability Using Advanced Sampling Approach

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Highlights

- The scope of this study is to estimate reliability in a multicomponent stress-strength model.
- For the estimation issue, we look at three effective sampling procedures.
- Both a simulation investigation and an application to real data are considered.

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Abstract

In this study, we analyze a multicomponent system with v independent and identical strength components $X_1,\ldots, X_v$ and each of these components is exposed to a common random stress Y. The system is considered to be operating only if at least u out of v ($1\leq u \leq v$) strength variables exceed the random stress. The estimate of the system reliability is investigated, assuming the strength and stress random variables follow the exponentiated exponential distribution having different shape parameters. The maximum likelihood estimator for the system reliability is derived from ranked set sampling (RSS), neoteric RSS (NRSS), and median RSS (MRSS). Some accuracy measurements, such as mean squared errors and efficiencies, are used to examine the behaviour of various estimates. Simulation studies demonstrate that the NRSS scheme's reliability estimates are chosen above those of the others under the RSS and MRSS schemes in the majority of situations. Theoretical research is explained through real data analysis.

1. INTRODUCTION

For predicting the population mean, Ref. [1] advised using ranked set sampling (RSS) instead of employing ordinary simple random sampling (SRS) to get a more informative sample. In this context, a limited number of elected units can be ranked by either the research variable or an auxiliary variable. RSS is used in fields like environmental management, ecology, sociology, and agriculture for a given unit when an accurate sample measurement is challenging, costly, or time-consuming. Several authors have proposed a number of RSS enhancements. Reference [2], for example, said that in comparison to SRS, the RSS mean is a more accurate and unbiased estimate of the population mean. The RSS's mathematical underpinnings were created in [3], which demonstrated that this sampling technique yields an effective estimate of the population mean. The procedure for enrolling a sample in RSS is described as follows:

a) Specify the intended population into n units of size n by distributing n^2 randomly picked units. Without any prior knowledge of the variable of interest's values.
b) The units in each set can be ranked visually or through an auxiliary variable.
c) The lowest ranked unit from the first set, the second-smallest ranked unit from the second set, and so on until the biggest ranked unit from the last set should be used to choose a sample for actual quantification.

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d) To create a sample with a size of $n^* = nt$ for the real measurement, iterate the aforementioned procedures t times.

For more references on the RSS method and its mean estimators, see [4-6]. Reference [7] proposed median (MRSS) as an RSS adaptation to minimize RSS efficiency loss owing to ranking errors increase the population mean estimator's effectiveness. The following is an explanation of the MRSS technique.

a) Divide n^2 randomly collected units from the intended population into n sets, each with a size of n, without any prior knowledge of the variable of interest's values.
b) The units in each set can be assumed to be ranked visually or through an auxiliary variable.
c) Elect the $\left[(n+1)/2\right]^{th}$ least rank unit to every for valid measurement if n is odd.
d) If n is even; take $(n/2)^{th}$ least ranked unit from the first $n/2$ samples and the $\left[(n+1)/2\right]^{th}$ smallest ranked unit from the second $n/2$ samples for the measurement.
e) Repeating the preceding procedure t times yields a total sample of size $n^*=nt$.

Reference [8] presented the neoteric RSS (NRSS) technique, which differs from the original RSS approach. In comparison to the SRS and RSS procedures, it has been found that this method produces more precise population mean and variance estimators. Below is a summary of the NRSS system:

a) Specify n^2 randomly elected units of the proposed population and score the sample units using the following principles:
b) Elect the $[J+(p-1)n]^{th}$ ranked unit for $p=1,\ldots,n$, where $J=(n/2)$ and $J=((n+2)/2)$ if p is even and odd respectively for $p=1,\ldots,n$.
c) By repeating the previous process t times, a final sample size of $n^*=nt$ is obtained.

The superiority of RSS and its variations in estimating a wide range of population metrics has been demonstrated in several investigations. Reference [9] used an extreme RSS approach to test for exponentiated Pareto distribution. Reference [10] used RSS to develop parameter estimators for the Zubair Lomax distribution. The inverse Weibull distribution parameters were estimated and the likelihood function (LF)was derived via the NRSS in [11]. Reference [12] presented and explored parameter estimators for the RSS scheme's two-parameter inverted Topp-Leone distribution. The scale family of distributions' maximum likelihood (ML) estimator and some RSS techniques were studied in [13]. Reference [14] considered an application to real data, it was suggested to use RSS to enhance estimate of the inverted Kumaraswamy distribution parameters.

In its most basic form, the stress-strength (SS) model defines component reliability as the probability that the unit's strength (X) exceeds the stress (Y) imposed on it. The expression $R=P(Y<X)$ represents the SS model, having many applications in physics, civil, mechanical, and aeronautical engineering, where R is the reliability parameter. Reference [15] was the first to propose the SS model. It's been widely employed in a variety of fields since then. Several studies, however, have looked at statistical inferences regarding the SS reliability (SSR) model using the RSS approach and its adaptations (see, for example [16-22]).

A multi-component stress-strength (MSS) framework involves v strength elements (ingredients), X_1, $X_2,\ldots,X_v$; where v are identically and independent distributed (iid) random variables, and each component (ingredient) is exposed to a random stress Y. Only if u out of v is such that ($u < v$) strengths withstand the stress, is the system considered alive. Assume X_1, X_2, …,X_v are iid with the common cumulative distribution function (CDF) $F(x)$. Suppose that the CDF of a random stress Y, say $G(y)$. The reliability from MSS presented in [23] is assigned by:

$$\begin{aligned}\mathbb{R}_{u,v} &= P\left[at\,least\ u\ of\ the\ v\ (X_1,X_2,\ldots X_v)\quad exceed\quad Y\right]\\ &=\sum_{i=u}^{v}\binom{v}{i}\int_{-\infty}^{\infty}\left[F(y)\right]^{(v-i)}\left[1-F(y)\right]^{i}dG(y).\end{aligned} \tag{1}$$

Recently, inference from the MSS reliability (MSSR) model has been evaluated by several researchers. For example, [24] discussed the MSSR for the exponentiated Pareto distribution. Reference [25] calculated MSSR using the ML method, assuming a Burr-XII distribution. Reference [26] estimated the parameters of the exponentiated Weibull distribution for the MSSR by using ML estimation. Reference [27] provided the Bayesian and non-Bayesian estimators of MSSR by assuming the Kumaraswamy distribution. Reference [28] studied the MSSR for the Weibull distribution based on records. Reference [29] proposed classical and Bayesian estimation of MSSR for an exponentiated Pareto distribution. The estimation of an MSSR from a generalised inverted exponential distribution using RSS was discussed in [30]. Reference [31] looked at RSS-based Bayesian system reliability estimation for the inverted Topp-Leone distribution.

There are no studies in the literature that deal with estimating $\mathbb{R}_{u,v}$ using MRSS and NRSS. The stress and strengths random variables are assumed to have an exponentiated exponential distribution (EED), this is due to its importance and application in a variety of fields. Expression of $\mathbb{R}_{u,v}$ is derived when the strengths X_1, $X_2,\ldots, X_v$ ~ EED(ϖ,β), and stress Y ~ EED(ϖ,ϑ), where ϖ is a scale parameter, ϑ and β are shape parameters. The ML estimator of $\mathbb{R}_{u,v}$ is derived when the strengths and stress have the same sampling design. Simulation studies are carried out, and real data are offered as an example, in order to evaluate the performances of various estimators.

The following sections are planned as; expression of MSSR model is discussed in section 2. Using RSS, the estimator of $\mathbb{R}_{u,v}$ to be found in section 3. From the NRSS design, section 4 obtains the ML estimator of $\mathbb{R}_{u,v}$. Section 5 provides the ML estimator of $\mathbb{R}_{u,v}$ for odd and even set sizes using MRSS. Section 6 deals with the simulation and analysis of actual data procedures followed by a conclusion in section 7.

2. MODEL SPECIFICATION

In this section, we display the model specification that is used in this study and the reliability of the MSS for the EED.

The EED, a really appealing exponential distribution generalisation, introduced in [32] has received widespread attention. It is interesting and widely utilised in the analysis of lifetime or survival statistics. Given that scale parameter $\varpi > 0$, and shape parameter $\beta > 0$, The EED's CDF and the probability density function (PDF) are, respectively, provided by:

$$F(x) = (1 - e^{-\varpi x})^{\beta}; \qquad x > 0, \tag{2}$$

$$f(x) = \beta\varpi e^{-\varpi x}(1 - e^{-\varpi x})^{\beta-1}; \qquad x > 0. \tag{3}$$

Several characteristics of the EED have been thoroughly investigated, for example [32-34], using several estimating approaches. Based on records, Reference [35] examined expressions and recurrence relation moments for EED. Reference [36] examined the Bayesian and traditional estimators of SSR for EED from SRS. Reference [37] handled the problem of estimating SSR from lower record values in EED. The EED has a wide range of applications (see [38], and [39]). Reference [40] recommended an MSSR estimator of EED that may be used to data out of an airplane's air cooling system. Estimating reliability in progressive-censored samples has been outlined in [41] with application to carbon fiber data. Reference [42] proposed an SSR estimator under the lower record RSS scheme of EED with application to rocket-motor data. Reference [43] handled with estimating SSR for EED using RSS.

Assume $X_1, X_2,\ldots, X_v$ are the structural strength components that are overloaded by the stress Y. Let $X_1, X_2,\ldots, X_v$ are random variables have EED(β,ϖ) and Y another random variable has EED (ϑ,ϖ) are independent. Expression of $\mathbb{R}_{u,v}$ can be obtained using Equation (1) to Equation (3) as:

$$\mathbb{R}_{u,v}=\sum_{i=u}^{v}\binom{v}{i}\int_{0}^{\infty}\left[1-\left(1-e^{-\varpi y}\right)^{\beta}\right]^{i}\left(1-e^{-\varpi y}\right)^{\beta(v-i)+\vartheta-1}\vartheta\varpi e^{-\varpi y}dy.$$

Let $t=\left(1-e^{-\varpi y}\right)^{\vartheta}\rightarrow t^{1/\vartheta}=\left(1-e^{-\varpi y}\right)$. Then $\mathbb{R}_{u,v}$ can be written as

$$\mathbb{R}_{u,v}=\sum_{i=u}^{v}\binom{v}{i}\int_{0}^{1}\left(1-t^{\beta/\vartheta}\right)^{i}\left(t^{\beta/\vartheta}\right)^{(v-i)}dt=\sum_{i=u}^{v}\binom{v}{i}\int_{0}^{1}\left(1-t^{L}\right)^{i}\left(t^{L}\right)^{(v-i)}dt\ ;\quad L=\frac{\beta}{\vartheta}\ .$$

Let $z=t^{L}\rightarrow dz=L\,t^{L-1}dt,\ (z)^{1/L}=t\rightarrow\frac{1}{L}z^{\frac{1}{L}-1}dz=dt$

$$\mathbb{R}_{u,v}=\frac{1}{L}\sum_{i=u}^{v}\binom{v}{i}\int_{0}^{1}(z)^{\left(v-i+\frac{1}{L}-1\right)}(1-z)^{i}dz=\frac{1}{L}\sum_{i=u}^{v}\binom{v}{i}\beta\left(v+\frac{1}{L}-i,i+1\right).$$

Hence, $\mathbb{R}_{u,v}$ takes the following form

$$\mathbb{R}_{u,v}=\frac{1}{L}\sum_{i=u}^{v}\frac{v\,!}{(v-i)!}\left[\prod_{j=0}^{i}\left(v+\frac{1}{L}-j\right)\right]^{-1},\tag{4}$$

where v and i are integers.

3. MSSR ESTIMATOR USING RSS

Herein, the ML approach is used to obtain the MSS system's reliability estimator. The MSSR estimator is proposed, claiming that the chosen random variables, both stresses and strengths are dragged from the RSS design.

Consider $X=\{X_{i(i)c},i=1,\ldots,n,\ c=1,\ldots,t_x\}$ be a chosen RSS from EED~ (β,ϖ), where $n^{*}=nt_x$, is the sample size, n and t_x are the set size and cycles number, respectively.

Given $Y=\{Y_{j(j)b},\ j=1,2,\ldots,m,\ b=1,2,\ldots,t_y\}$ be a chosen RSS from EED~ (ϑ,ϖ) , where $m^{*}=mt_y$, is the sample size, the set size and cycle count are denoted by m and t_y. The LF of MSS model under RSS is

$$l_1=\prod_{c=1}^{t_x}\prod_{i=1}^{n}C_1\left[F(x_{i(i)c})\right]^{i-1}f(x_{i(i)c})\left[1-F(x_{i(i)c})\right]^{n-i}\prod_{b=1}^{t_y}\prod_{j=1}^{m}C_2\left\{\left[1-F(y_{j(j)b})\right]^{m-j}\left[F(y_{j(j)b})\right]^{j-1}f(y_{j(j)b})\right\},$$

where, $C_1=n!\left[(i-1)!(n-i)!\right]^{-1}$ and $C_2=m!\left[(j-1)!(m-j)!\right]^{-1}$. Then the log-LF (LLF) is as below:

$$\begin{aligned}\ln l_1\propto\ & m^{*}\ln\vartheta+n^{*}\ln\beta+\sum_{c=1}^{t_x}\sum_{i=1}^{n}\left[(n-i)\ln(1-(1-e^{-\varpi x_{i(i)c}})^{\beta})-\varpi x_{i(i)c}+(\beta i-1)(\ln(1-e^{-\varpi x_{i(i)c}}))\right]\\ &+(n^{*}+m^{*})\ln\varpi+\sum_{b=1}^{t_y}\sum_{j=1}^{m}\left[(m-j)\ln(1-(1-e^{-\varpi y_{j(j)b}})^{\vartheta})-\varpi y_{j(j)b}+(\vartheta j-1)(\ln(1-e^{-\varpi y_{j(j)b}}))\right].\end{aligned}$$

The first derivatives of l_1 based on ϖ,β and ϑ are:

$$\begin{aligned}\frac{\partial\ln l_1}{\partial\varpi}=\ & \frac{n^{*}+m^{*}}{\varpi}+\sum_{c=1}^{t_x}\sum_{i=1}^{n}\left[\left(\frac{(\beta i-1)x_{i(i)c}}{(e^{\varpi x_{i(i)c}}-1)}\right)-x_{i(i)c}-\frac{\beta(n-i)x_{i(i)c}\left(e^{-\varpi x_{i(i)c}}\right)(1-e^{-\varpi x_{i(i)c}})^{\beta-1}}{(1-(1-e^{-\varpi x_{i(i)c}})^{\beta})}\right]\\ &+\sum_{b=1}^{t_y}\sum_{j=1}^{m}\left[\frac{y_{j(j)b}(\vartheta j-1)}{(e^{\varpi y_{j(j)b}}-1)}-\frac{\vartheta(m-j)y_{j(j)b}e^{-\varpi y_{j(j)b}}(1-e^{-\varpi y_{j(j)b}})^{\vartheta-1}}{(1-(1-e^{-\varpi y_{j(j)b}})^{\vartheta})}-y_{j(j)b}\right],\end{aligned}\tag{5}$$

$$\frac{\partial\ln l_1}{\partial\beta}=\frac{n^{*}}{\beta}+\sum_{c=1}^{t_x}\sum_{i=1}^{n}\left\{i\ln(1-e^{-\varpi x_{i(i)c}})-\left[\frac{\left[\ln(1-e^{-\varpi x_{i(i)c}})\right](n-i)}{(1-e^{-\varpi x_{i(i)c}})^{-\beta}-1}\right]\right\},\tag{6}$$

$$\frac{\partial\ln l_1}{\partial\vartheta}=\frac{m^{*}}{\vartheta}+\sum_{b=1}^{t_y}\sum_{j=1}^{m}\left[j\ln(1-e^{-\varpi y_{j(j)b}})-\left(\frac{(m-j)\ln(1-e^{-\varpi y_{j(j)b}})}{(1-e^{-\varpi y_{j(j)b}})^{-\vartheta}-1}\right)\right].\tag{7}$$

We will have the estimators of ϖ,β and ϑ, say $\hat{\varpi}_1,\hat{\beta}_1$ and $\hat{\vartheta}_1$, after equating Equations (5)–(7) by zero and solving numerically. Then, utilising the ML approach's invariance feature, the ML estimator of $\mathbb{R}_{u,v}$ is produced by plugging $\hat{\varpi}_1,\hat{\beta}_1$ and $\hat{\vartheta}_1$ into Equation (4).

4. MSSR ESTIMATOR USING NRSS

Consider that $X=\{X_{q(i)c}, i=1,2,...,n; c=1,2,...,t_x\}$ is enrolled in the NRSS from EED (β,ϖ), where $n^*=nt_x$ is a sample size. Let $Y=\{Y_{q(j)b}, j=1,2,...,m; b=1,2,...,t_y\}$ be the chosen NRSS from EED (ϑ,ϖ), where $m^*=mt_y$ is a sample size. The LF in this case will be as follows:

$$l_2=\prod_{c=1}^{t_x}\left[\prod_{i=1}^{n}C_3 f(x_{q(i)c})\prod_{i=1}^{n+1}[F(x_{q(i)c})-F(x_{q(i-1)c})]^{q(i)-q(i-1)-1}\right]\prod_{b=1}^{t_y}\left[\prod_{j=1}^{m}C_4 f(y_{q(j)b})\prod_{j=1}^{m+1}[F(y_{q(j)b})-F(y_{q(j-1)b})]^{q(j)-q(j-1)-1}\right]$$

where $C_3=\xi!\left[\prod_{i=1}^{n+1}(q(i)-q(i-1)-1)!\right]^{-1}$, $q(0)=0$, $q(n+1)=\xi+1$ $x_{(q(0))}=-\infty$ and $x_{(q(i+1))}=\infty$, $\xi=n^2$

and, $C_4=\tau!\left[\prod_{j=1}^{m+1}(q(j)-q(j-1)-1)!\right]^{-1}$, $q(0)=0$, $q(m+1)=\tau+1$ $y_{(q(0))}=-\infty$ and $y_{(q(j+1))}=\infty$, $\tau=m^2$.

The LLF, based on NRSS, is:

$$\ln l_2 \propto n^*\left(\ln\varpi+\ln\beta\right)+\sum_{c=1}^{t_x}\left[\sum_{i=1}^{n}\left\{(\beta-1)\ln H_{q(i)c}-\varpi x_{q(i)c}\right\}+\sum_{i=1}^{n+1}\left[A(i)\left\{\ln[(H_{q(i)c})^{\beta}-(H_{q(i-1)c})^{\beta}]\right\}\right]\right]$$

$$+m^*\left(\ln\varpi+\ln\vartheta\right)+\sum_{b=1}^{t_y}\left[\sum_{j=1}^{m+1}B(j)\left\{\ln[(D_{q(j)b})^{\vartheta}-(D_{q(j-1)b})^{\vartheta}]\right\}+\sum_{j=1}^{m}\left\{(\vartheta-1)\ln D_{q(j)b}-\varpi y_{q(j)b}\right\}\right]$$

where $H_{q(\delta)c}=1-e^{-\varpi x_{q(\delta)c}}$, $A(i)=[q(i)-q(i-1)-1]$, $\delta=i,i-1$, $D_{q(\varepsilon)c}=1-e^{-\varpi x_{q(\varepsilon)c}}$, $\varepsilon=j,j-1$; and $B(j)=[q(j)-q(j-1)-1]$. The derivatives of ϖ,β and ϑ are obtained as follows:

$$\frac{\partial\ln l_2}{\partial\varpi}=\frac{n^*+m^*}{\varpi}+\sum_{c=1}^{t_x}\sum_{i=1}^{n}\left[\frac{(\beta-1)x_{q(i)c}}{\left(e^{\varpi x_{q(i)c}}-1\right)}-x_{q(i)c}\right]+\sum_{b=1}^{t_y}\sum_{j=1}^{m}\left[\frac{y_{q(j)b}(\vartheta-1)}{\left(e^{\varpi y_{q(j)b}}-1\right)}-y_{q(j)b}\right]$$
$$+\sum_{c=1}^{t_x}\sum_{i=1}^{n+1}\left[\frac{\beta A(i)\left[(H_{q(i)c})^{\beta-1}x_{q(i)c}e^{-\varpi x_{q(i)c}}-(H_{q(i-1)c})^{\beta-1}x_{q(i-1)c}e^{-\varpi x_{q(i-1)c}}\right]}{(H_{q(i)c})^{\beta}-(H_{q(i-1)c})^{\beta}}\right] \tag{8}$$
$$+\sum_{b=1}^{t_y}\sum_{j=1}^{m+1}\frac{\vartheta B(j)\left[y_{q(j)b}e^{-\varpi y_{q(j)b}}(D_{q(j)b})^{\vartheta-1}-(D_{q(j-1)b})^{\vartheta-1}y_{q(j-1)b}e^{-\varpi y_{q(j-1)b}}\right]}{(D_{q(j)b})^{\vartheta}-(D_{q(j-1)b})^{\vartheta}},$$

$$\frac{\partial\ln l_2}{\partial\beta}=\frac{n^*}{\beta}+\sum_{c=1}^{t_x}\left[\sum_{i=1}^{n}\ln(H_{q(i)c})+\sum_{i=1}^{n+1}\frac{A(i)\left[(H_{q(i)c})^{\beta}\ln(H_{q(i)c})-(H_{q(i-1)c})^{\beta}\ln(H_{q(i-1)c})\right]}{(H_{q(i)c})^{\beta}-(H_{q(i-1)c})^{\beta}}\right], \tag{9}$$

$$\frac{\partial\ln l_2}{\partial\vartheta}=\frac{m^*}{\vartheta}+\sum_{b=1}^{t_y}\left[\sum_{j=1}^{m}\ln(D_{q(j)b})+\sum_{j=1}^{m+1}\frac{B(j)\left[(D_{q(j)b})^{\vartheta}\ln(D_{q(j)b})-(D_{q(j-1)b})^{\vartheta}\ln(D_{q(j-1)b})\right]}{(D_{q(j)b})^{\vartheta}-(D_{q(j-1)b})^{\vartheta}}\right]. \tag{10}$$

The ML estimators of ϖ,β and ϑ are derived by solving numerically Equation (8), Equation (9) and Equation (10) using an iterative method, As a Result, $\mathbb{R}_{u,v}$ is obtained using Equation (4).

5. MSSR ESTIMATOR USING MRSS

In this section, the MSSR estimator is addressed when strengths and stress random variables are independent EEDs under MRSS. The first estimator is developed when the observed samples of stress and strengths are chosen from MRSS for odd set size (MRSSO), while the second estimator is formed when the observed samples of stress and strengths are taken from MRSS for even set size (MRSSE).

5.1. Estimator under MRSSO

Let $X=\{X_{i(g_n)c}, c=1,2,...,t_x;\ g_n=\left[(n+1)/2\right]^{th}, i=1,2,...,n\}$ and $Y=\{Y_{j(g_m)b}, b=1,2,...,t_y;\ g_m=\left[(m+1)/2\right]^{th}$, $j=1,2,...,m$, are MRSSO observed from EED (β,ϖ) and EED (ϑ,ϖ), with sample sizes nt_x and mt_y respectively, where n and m are the set sizes, t_x and t_y are the cycles numbers. The LF l_3 under the observed data is:

$$l_3=\prod_{c=1}^{t_x}\prod_{i=1}^{n}C_5\left[F(x_{i(g_n)c})\right]^{g_n-1}\left[1-F(x_{i(g_n)c})\right]^{g_n-1}f(x_{i(g_n)c})\prod_{b=1}^{t_y}\prod_{j=1}^{m}C_6\left[F(y_{j(g_m)b})\right]^{g_m-1}\left[1-F(y_{j(g_m)b})\right]^{g_m-1}f(y_{j(g_m)b}),$$

where, $C_\tau=\tau!\left[(g_\tau-1)!\right]^{-2}$, $\tau=n,m.$ Based on MRSSO, the LLF is

$$\begin{aligned}\ln l_3 \propto{}& n^*\left(\ln\varpi+\ln\beta\right)+\sum_{c=1}^{t_x}\sum_{i=1}^{n}\left\{(g_n-1)\ln\left[1-(\Lambda1_{i(g_n)c})^{\beta}\right]+(\beta g_n-1)(\ln\Lambda1_{i(g_n)c})-\varpi x_{i(g_n)c}\right\}\\&+m^*\left(\ln\varpi+\ln\vartheta\right)+\sum_{b=1}^{t_y}\sum_{j=1}^{m}\left[(\vartheta g_m-1)\ln\Lambda2_{j(g_m)b}-\varpi y_{j(g_m)b}+(g_m-1)\ln\left[1-(\Lambda2_{j(g_m)b})^{\vartheta}\right]\right].\end{aligned}$$

where, $\Lambda1_{i(g_n)c}=1-e^{-\varpi x_{i(g_n)c}}$, and $\Lambda2_{j(g_m)b}=1-e^{-\varpi y_{j(g_m)b}}$. The partial derivatives of l_3 under ϖ,β and ϑ are obtained as follows:

$$\begin{aligned}\frac{\partial\ln l_3}{\partial\varpi}={}&\frac{n^*+m^*}{\varpi}+\sum_{c=1}^{t_x}\sum_{i=1}^{n}\left[\left(\frac{x_{i(g_n)c}(\beta g_n-1)}{(e^{\varpi x_{i(g_n)c}}-1)}\right)-x_{i(g_n)c}-\left(\frac{\beta x_{i(g_n)c}(g_n-1)e^{-\varpi x_{i(g_n)c}}(\Lambda1_{i(g_n)c})^{\beta-1}}{1-(\Lambda1_{i(g_n)c})^{\beta}}\right)\right]\\&+\sum_{b=1}^{t_y}\sum_{j=1}^{m}\left[\left(\frac{(\vartheta g_m-1)y_{j(g_m)b}}{e^{\varpi y_{j(g_m)b}}-1}\right)-y_{j(g_m)b}-\left(\frac{\vartheta(g_m-1)(\Lambda2_{j(g_m)b})^{\vartheta-1}e^{-\varpi y_{j(g_m)b}}y_{j(g_m)b}}{1-(\Lambda2_{j(g_m)b})^{\vartheta}}\right)\right],\end{aligned}\tag{11}$$

$$\frac{\partial\ln l_3}{\partial\beta}=\frac{n^*}{\beta}+\sum_{c=1}^{t_x}\sum_{i=1}^{n}\left[g_n\ln(\Lambda1_{i(g_n)c})-\frac{(g_n-1)\ln(\Lambda1_{i(g_n)c})}{(\Lambda1_{i(g_n)c})^{-\beta}-1}\right],\tag{12}$$

$$\frac{\partial\ln l_3}{\partial\vartheta}=\frac{m^*}{\vartheta}+\sum_{b=1}^{t_y}\sum_{j=1}^{m}\left[g_m\ln(\Lambda2_{j(g_m)b})-\frac{(g_m-1)\ln(\Lambda2_{j(g_m)b})}{(\Lambda2_{j(g_m)b})^{-\vartheta}-1}\right].\tag{13}$$

Setting (11), (12), and (13) with zero to get the parameter estimators. Although such estimators produced cannot be stated in formal representation, they would be simply built using a numerical algorithm. Finally, the estimator of $\mathbb{R}_{u,v}$ is calculated using (4).

5.2. Estimator under MRSSE

Suppose that $X=\{X_{i(k_n)c}, k_n=(n/2)^{th}; c=1,2,...,t_x, i=1,2,...,n\}$ is the MRSSE observed from EED(β,ϖ) with $n^*=nt_x$, n is set size and t_x being the cycle's count. Also, let $Y=\{Y_{j(k_m)b}; k_m=(m/2)^{th}; j=1,2,...,m, b=1,2,...,t_y\}$ is the MRSSE collected from EED(ϑ,ϖ) with $m^*=mt_y$, m being the set size and t_y being the cycle's count. The LF l_4 under MRSSE is:

$$\begin{aligned}l_4={}&\prod_{c=1}^{t_x}\prod_{i=1}^{k_n}C_7 f(x_{i(k_n)c})\left[F(x_{i(k_n)c})\right]^{k_n-1}\left[1-F(x_{i(k_n)c}))\right]^{k_n}\prod_{c=1}^{t_x}\prod_{i=k_n+1}^{n}\left[F(x_{i(k_n+1)c})\right]^{k_n}f(x_{i(k_n+1)c})\left[1-F(x_{i(k_n+1)c})\right]^{k_n-1}\\&\times\prod_{b=1}^{t_y}\prod_{j=1}^{k_m}C_8\left[F(y_{j(k_m)b})\right]^{k_m-1}f(y_{j(k_m)b})\left[1-F(y_{j(k_m)b})\right]^{k_m}\prod_{b=1}^{t_y}\prod_{j=k_m+1}^{m}\left[F(y_{j(k_m+1)b})\right]^{k_m}f(y_{j(k_m+1)b})\left[1-F(y_{j(k_m+1)b})\right]^{k_m-1}\end{aligned}$$

where $C_7=n!\left[(k_n-1)!k_n!\right]^{-1}$ and $C_8=m!\left[(k_m-1)!k_m!\right]^{-1}$.

The LLF, viz MRSSE, is given by:

$$\begin{aligned}\ln l_4 \propto\; & n^*\left(\ln\varpi+\ln\beta\right)+\sum_{c=1}^{t_x}\sum_{i=1}^{k_n}\left\{k_n\ln\left\{1-(\Lambda 3_{i(k_n)c})^{\beta}\right\}-\varpi x_{i(k_n)c}+(\beta k_n-1)\left[\ln\Lambda 3_{i(k_n)c}\right]\right\}\\
&+\sum_{c=1}^{t_x}\sum_{i=k_n+1}^{n}\left[(\beta k_n+\beta-1)\ln\Lambda 3_{i(k_{n+1})c}+(k_n-1)\ln\left[1-(\Lambda 3_{i(k_{n+1})c})^{\beta}\right]-\varpi x_{i(k_n+1)c}\right]\\
&+m^*\left(\ln\varpi+\ln\vartheta\right)+\sum_{b=1}^{t_y}\sum_{j=1}^{k_m}\left\{k_m\ln\left[1-(\Lambda 4_{j(k_m)b})^{\vartheta}\right]-\varpi y_{j(k_m)b}+(\vartheta k_m-1)\ln\Lambda 4_{j(k_m)b}\right\}\\
&+\sum_{b=1}^{t_y}\sum_{j=k_m+1}^{m}\left[(\vartheta k_m+\vartheta-1)\ln\Lambda 4_{j(k_{m+1})b}-\varpi y_{j(k_m+1)b}+(k_m-1)\ln\left[1-(\Lambda 4_{j(k_{m+1})b})^{\vartheta}\right]\right]\end{aligned}$$

where $\Lambda 3_{i(k_n)c}=(1-e^{-\varpi x_{i(k_n)c}})$ and $\Lambda 4_{j(k_m)b}=(1-e^{-\varpi y_{j(k_m)b}})$. Differentiate l_4 with respect to ϖ,β and ϑ, then we have:

$$\begin{aligned}\frac{\partial\ln l_4}{\partial\varpi}=\;&\frac{m^*+n^*}{\varpi}+\sum_{c=1}^{t_x}\sum_{i=1}^{k_n}\left[(\beta k_n-1)x_{i(k_n)c}(e^{\varpi x_{i(k_n)c}}-1)^{-1}-\frac{\beta e^{-\varpi x_{i(k_n)c}}k_n x_{i(k_n)c}(\Lambda 3_{i(k_n)c})^{\beta-1}}{1-(\Lambda 3_{i(k_n)c})^{\beta}}-x_{i(k_n)c}\right]\\
&+\sum_{c=1}^{t_x}\sum_{i=k_n+1}^{n}\left[\frac{(\beta k_n+\beta-1)x_{i(k_n+1)c}}{e^{\varpi x_{i(k_n+1)c}}-1}-x_{i(k_n+1)c}-\frac{\beta(k_n-1)x_{i(k_n+1)c}(\Lambda 3_{i(k_n+1)c})^{\beta-1}e^{-\varpi x_{i(k_n+1)c}}}{\left[1-(\Lambda 3_{i(k_n+1)c})^{\beta}\right]}\right]\\
&+\sum_{b=1}^{t_y}\sum_{j=1}^{k_m}\left[\frac{(\vartheta k_m-1)y_{j(k_m)b}}{e^{\varpi y_{j(k_m)b}}-1}-\frac{\vartheta e^{-\varpi y_{j(k_m)b}}y_{j(k_m)b}(k_m)(\Lambda 4_{j(k_m)b})^{\vartheta-1}}{1-(\Lambda 4_{j(k_m)b})^{\vartheta}}-y_{j(k_m)b}\right]\\
&+\sum_{b=1}^{t_y}\sum_{j=k_m+1}^{m}\left[\frac{(\vartheta k_m+\vartheta-1)y_{j(k_m+1)b}}{(e^{\varpi y_{j(k_m+1)b}}-1)}-y_{j(k_m+1)b}-\frac{\vartheta(k_m-1)y_{j(k_m+1)b}(\Lambda 4_{j(k_{m+1})b})^{\vartheta-1}e^{-\varpi y_{j(k_m+1)b}}}{1-(\Lambda 4_{j(k_{m+1})b})^{\vartheta}}\right],\end{aligned}\tag{14}$$

$$\begin{aligned}\frac{\partial\ln l_4}{\partial\beta}=\;&\frac{n^*}{\beta}+\sum_{c=1}^{t_x}\sum_{i=1}^{k_n}\left[k_n\ln(\Lambda 3_{i(k_n)c})-\frac{(k_n)\ln(\Lambda 3_{i(k_n)c})}{\left\{(\Lambda 3_{i(k_n)c})^{-\beta}-1\right\}}\right]\\
&+\sum_{c=1}^{t_x}\sum_{i=k_n+1}^{n}\left[(k_n+1)\ln(\Lambda 3_{i(k_n+1)c})-\ln(\Lambda 3_{i(k_n+1)c})(k_n-1)\left\{(\Lambda 3_{i(k_n+1)c})^{-\beta}-1\right\}^{-1}\right],\end{aligned}\tag{15}$$

$$\begin{aligned}\frac{\partial\ln l_4}{\partial\vartheta}=\;&\frac{m^*}{\vartheta}+\sum_{b=1}^{t_y}\sum_{j=1}^{k_m}k_m\ln(\Lambda 4_{j(k_m)b})+\sum_{b=1}^{t_y}\sum_{j=k_m+1}^{m}(k_m+1)\ln(\Lambda 4_{j(k_m+1)b})\\
&-\sum_{b=1}^{t_y}\sum_{j=1}^{k_m}\frac{(k_m)\ln(\Lambda 4_{j(k_m)b})}{(\Lambda 4_{j(k_m)b})^{-\vartheta}-1}-\sum_{b=1}^{t_y}\sum_{j=k_m+1}^{m}\frac{(k_m-1)\ln(\Lambda 4_{j(k_m+1)b})}{(\Lambda 4_{j(k_m+1)b})^{-\vartheta}-1}.\end{aligned}\tag{16}$$

Equating Equations (14), (15), and (16) by zero yield the parameter estimators. Due to the fact that the offered estimators are not closed structures, they may easily be created by utilising a numerical tool. Accordingly, Equation (4) is used to produce the MSSR estimator.

6. NUMERICAL ILLUSTRATION AND DATA ANALYSIS

A simulation is undertaken as a study in this part to illustrate the behaviour of MSSR estimates produced via three sampling schemes, namely, RSS, MRSS and NRSS. Also, a real data analysis; for two data sets which depict the jute fibres' size, tensile traits, and coefficient of variations in fibre diameter (CVFD) at 10 mm as well as 20 mm lengths of gauge; is conducted to support the findings.

6.1. Simulation Study

We created the following simulated computation to explore and evaluate the behaviour of MSSR estimates produced via the indicated sampling pattern:

i. Take (n, m) = (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8), (9, 9), where n and m are set sizes.
ii. Take $t_x = t_y = t = 5$, where t_x and t_y are the number of cycles.
iii. The samples are (n^*, m^*) = (10, 10), (15, 15), (20, 20), (25, 25), (30, 30), (35, 35), (40, 40), (45, 45), where $n^* = nt_x = nt$, and $m^* = mt_y = mt$.

iv. The values of parameters at u=1 and v=3 are selected as: (β,ϑ) = (1.5, 1.5), (2, 1.5), (2, 1), (3.5, 1), and ϖ =1.4, therefore, the exact values of $\mathbb{R}_{1,3}$ = 0.75, 0.8, 0.85 and 0.91

v. The values of parameters at u=2 and v=4 are selected as: (β,ϑ) = (1.5, 1.5), (2, 1.5), (2, 1), (3.5, 1), and ϖ =1.4, therefore, the exact values of $\mathbb{R}_{2,4}$ = 0.6, 0.67, 0.76 and 0.85.

vi. Create 1000 samples from X~EED (β,ϖ) and Y~ EED (ϑ,ϖ) using the specified values of parameters.

vii. The validity of estimates is examined using mean squared error (MSER), absolute bias (AB), and relative effectiveness (RE).

viii. Regarding NRSS and MRSS, the RE of MSSR estimates of RSS, is defined as

$$RE_1 = MSE\,(\hat{\mathbb{R}}_{u,v}(X_{RSS} < Y_{RSS})) \,/\, MSE(\hat{\mathbb{R}}_{u,v}(X_{NRSS} < Y_{NRSS})), \qquad (u,v) = (1,3),(2,4)$$

$$RE_2 = MSE(\hat{\mathbb{R}}_{u,v}(X_{RSS} < Y_{RSS})) \,/\, MSE(\hat{\mathbb{R}}_{u,v}(X_{MRSS} < Y_{MRSS})), \qquad (u,v) = (1,3),(2,4).$$

Tables 1–4 show the values of the MSSR estimate's ABs, MSERs, and REs.

Table 1. *MSERs of* $\hat{\mathbb{R}}_{u,v}$ *and their efficiencies at* $\mathbb{R}_{1,3}$ *=0.75 and* $\mathbb{R}_{2,4} = 0.6$

t	(n,m)	RSS		NRSS		MRSS		RE_1	RE_2
		AB	MSER	AB	MSER	AB	MSER		
$\hat{\mathbb{R}}_{1,3}$									
5	(2,2)	0.0234	0.0053	0.01066	0.00498	0.0006	0.00515	1.06	1.03
	(3,3)	0.002	0.0038	0.00142	0.00243	0.00127	0.00268	1.56	1.42
	(4,4)	0.0028	0.0018	0.00072	0.00100	0.00055	0.00112	1.80	1.61
	(5,5)	0.0055	0.0013	0.00430	0.00059	0.00304	0.00114	2.20	1.14
	(6,6)	0.0038	0.0009	0.00059	0.00044	0.00059	0.00050	2.05	1.80
	(7,7)	0.0007	0.0005	0.00190	0.00034	0.00067	0.00045	1.47	1.11
	(8,8)	0.0001	0.0004	0.00188	0.00027	0.00054	0.00031	1.48	1.29
	(9,9)	0.0008	0.00039	0.00051	0.00021	0.00340	0.00035	1.86	1.11
$\hat{\mathbb{R}}_{2,4}$									
5	(2,2)	0.03	0.010	0.01190	0.00977	0.00078	0.00991	1.02	1.01
	(3,3)	0.0003	0.0075	0.00037	0.00498	0.00001	0.00546	1.51	1.37
	(4,4)	0.0053	0.0037	0.00173	0.00208	0.00003	0.00230	1.78	1.61
	(5,5)	0.007	0.0027	0.00578	0.00121	0.00359	0.00233	2.23	1.16
	(6,6)	0.0048	0.0019	0.00055	0.00090	0.00051	0.00103	2.11	1.84
	(7,7)	0.0015	0.0011	0.00250	0.00070	0.00128	0.00094	1.57	1.17
	(8,8)	0.0004	0.0009	0.00253	0.00056	0.00100	0.00065	1.61	1.38
	(9,9)	0.0015	0.0008	0.00088	0.00043	0.00467	0.00071	1.86	1.13

Table 2. *MSERs of* $\hat{\mathbb{R}}_{u,v}$ *and their efficiencies at* $\mathbb{R}_{1,3}$ =0.8 *and* $\mathbb{R}_{2,4} = 0.67$

t	(n,m)	RSS		NRSS		MRSS		RE_1	RE_2
		AB	MSER	AB	MSER	AB	MSER		
$\hat{\mathbb{R}}_{1,3}$									
5	(2,2)	0.0019	0.0045	0.0065	0.00404	0.0315	0.00445	1.11	1.01
	(3,3)	0.0001	0.0018	0.0027	0.00115	0.0119	0.00168	1.57	1.07
	(4,4)	0.0046	0.0013	0.0001	0.00071	0.0160	0.00100	1.83	1.30
	(5,5)	0.0027	0.0008	0.0016	0.00043	0.0005	0.00077	1.86	1.04
	(6,6)	0.0047	0.0006	0.0004	0.00034	0.0086	0.00045	1.76	1.33
	(7,7)	0.0026	0.0004	0.0024	0.00028	0.0035	0.00037	1.43	1.08
	(8,8)	0.0024	0.0003	0.0003	0.00019	0.0078	0.00026	1.68	1.23

t	(n,m)	RSS		NRSS		MRSS		RE_1	RE_2
		AB	MSER	AB	MSER	AB	MSER		
	(9,9)	0.0019	0.0003	0.0012	0.00014	0.0010	0.00022	2.14	1.36
$\hat{\mathbb{R}}_{2,4}$									
5	(2,2)	0.0059	0.0099	0.0124	0.00920	0.0444	0.00930	1.08	1.06
	(3,3)	0.0014	0.0041	0.0034	0.00255	0.0191	0.00388	1.61	1.06
	(4,4)	0.0061	0.0029	0.0007	0.00161	0.0235	0.00220	1.80	1.32
	(5,5)	0.0035	0.0020	0.0027	0.00098	0.0002	0.00176	2.04	1.14
	(6,6)	0.0075	0.0014	0.0008	0.00078	0.0127	0.00100	1.79	1.40
	(7,7)	0.0042	0.0010	0.0034	0.00063	0.0055	0.00086	1.59	1.16
	(8,8)	0.0038	0.0007	0.0006	0.00044	0.0116	0.00059	1.59	1.19
	(9,9)	0.0031	0.0006	0.0017	0.00033	0.0014	0.00050	1.82	1.20

***Table 3**. MSERs of* $\hat{\mathbb{R}}_{u,v}$ *and their efficiencies at* $\mathbb{R}_{1,3}$ =0.85 *and* $\mathbb{R}_{2,4} = 0.76$

t	(n,m)	RSS		NRSS		MRSS		RE_1	RE_2
		AB	MSER	AB	MSER	AB	MSER		
$\hat{\mathbb{R}}_{1,3}$									
5	(2,2)	0.0054	0.00279	0.0122	0.00225	0.0314	0.00272	1.24	1.03
	(3,3)	0.0019	0.00135	0.0006	0.00099	0.0019	0.00107	1.36	1.26
	(4,4)	0.0012	0.00075	0.0025	0.00050	0.0196	0.00065	1.50	1.15
	(5,5)	0.0013	0.00045	0.0014	0.00034	0.0005	0.00042	1.32	1.07
	(6,6)	0.0001	0.00036	0.0006	0.00023	0.0094	0.00031	1.57	1.16
	(7,7)	0.0018	0.00032	0.0011	0.00013	0.0020	0.00024	2.46	1.33
	(8,8)	0.0019	0.00026	0.00008	0.00009	0.0082	0.00015	2.89	1.73
	(9,9)	0.0012	0.00023	0.0007	0.00008	0.0010	0.00019	2.88	1.21
$\hat{\mathbb{R}}_{2,4}$									
5	(2,2)	0.0068	0.00662	0.0206	0.00564	0.0479	0.00637	1.17	1.04
	(3,3)	0.0021	0.00329	0.0017	0.00245	0.0038	0.00267	1.34	1.23
	(4,4)	0.0015	0.00187	0.0036	0.00125	0.0305	0.00159	1.50	1.18
	(5,5)	0.0024	0.00113	0.0025	0.00086	0.0005	0.00105	1.31	1.08
	(6,6)	0.0004	0.00091	0.0008	0.00058	0.0246	0.00084	1.57	1.08
	(7,7)	0.0030	0.00081	0.0016	0.00033	0.0034	0.0006	2.45	1.35
	(8,8)	0.0031	0.00066	0.00007	0.00024	0.0093	0.00046	2.75	1.43
	(9,9)	0.0018	0.00057	0.0012	0.00021	0.0017	0.00049	2.71	1.16

***Table 4.** MSERs of* $\hat{\mathbb{R}}_{u,v}$ *and their efficiencies at* $\mathbb{R}_{1,3}$ =0.91 *and* $\mathbb{R}_{2,4} = 0.85$

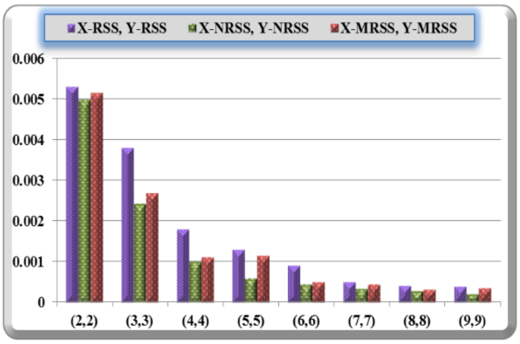
t	(n,m)	RSS		NRSS		MRSS		RE_1	RE_2
		AB	MSER	AB	MSER	AB	MSER		
$\hat{\mathbb{R}}_{1,3}$									
5	(2,2)	0.0006	0.00160	0.00930	0.00138	0.0094	0.00151	1.16	1.06
	(3,3)	0.0038	0.00065	0.00250	0.00051	0.0062	0.00061	1.27	1.07
	(4,4)	0.0006	0.00047	0.00130	0.00024	0.0072	0.00038	1.96	1.24
	(5,5)	0.0010	0.00024	0.00070	0.00015	0.0026	0.00021	1.60	1.14
	(6,6)	0.0008	0.00021	0.00010	0.00008	0.0063	0.00011	2.63	1.91
	(7,7)	0.0024	0.00017	0.00001	0.00006	0.0018	0.00014	2.83	1.21
	(8,8)	0.0009	0.00011	0.00010	0.00005	0.0058	0.00008	2.20	1.38
	(9,9)	0.0006	0.00008	0.00080	0.00003	0.0012	0.00007	2.67	1.14

t	(n,m)	RSS		NRSS		MRSS		RE_1	RE_2
		AB	MSER	AB	MSER	AB	MSER		
$\hat{\mathbb{R}}_{2,4}$									
5	(2,2)	0.0019	0.00406	0.0161	0.0038	0.0249	0.00401	1.07	1.01
	(3,3)	0.0067	0.0018	0.0039	0.00137	0.0106	0.00168	1.31	1.07
	(4,4)	0.0013	0.00126	0.0021	0.00065	0.0091	0.00081	1.94	1.56
	(5,5)	0.0016	0.00066	0.0011	0.0004	0.0044	0.00060	1.65	1.10
	(6,6)	0.0014	0.00057	0.0002	0.00023	0.0089	0.00050	2.48	1.14
	(7,7)	0.0041	0.00046	0.00006	0.00018	0.0031	0.00038	2.56	1.21
	(8,8)	0.0015	0.00032	0.00020	0.00015	0.0045	0.00021	2.13	1.52
	(9,9)	0.0010	0.00022	0.00130	0.00009	0.0021	0.00021	2.44	1.05

6.2. Numerical Results

The following are some of the results extracted from Tables 1–4 and Figures 1–10

- For certain sample size values (n^*, m^*) and real values of $\mathbb{R}_{u,v}$, we conclude that $\hat{\mathbb{R}}_{u,v}$ it is more effective to use NRSS strengths and stress random variables than the alternatives under RSS and MRSS (Figures 1, 2 and Tables 1–4).
- As the sample sizes (n^*, m^*) rise for different set sizes, the MSER of all reliability estimates reduces for various approaches, as seen in Figures 1 and 2.
- For each and every set size, the MSER of $\hat{\mathbb{R}}_{u,v}$ picks the least values using NRSS (Figures 1, 2).



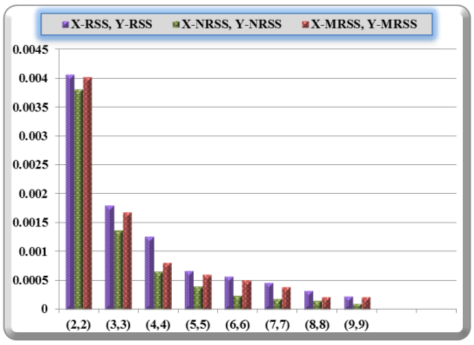


Figure 1. *MSER of* $\hat{\mathbb{R}}_{1,3}$ *at* $\mathbb{R}_{1,3}$ =0.75 ***Figure 2***. *MSER of* $\hat{\mathbb{R}}_{2,4}$ *at* $\mathbb{R}_{2,4}$ =0.85

- The MSER of $\hat{\mathbb{R}}_{u,v}$ declines as the value of $\mathbb{R}_{u,v}$ rises in all schemes (Figures 3, 4) and Tables 1–4).

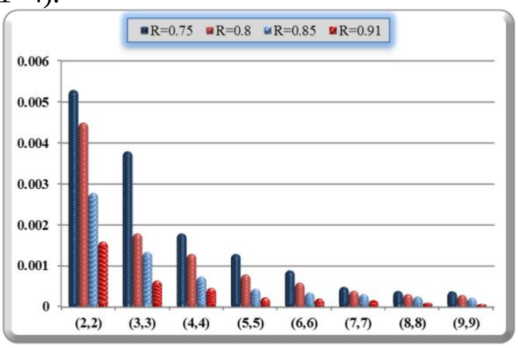


Figure 3. *MSER of* $\hat{\mathbb{R}}_{1,3}$ *at all true values of* $\mathbb{R}_{1,3}$ *based on RSS*

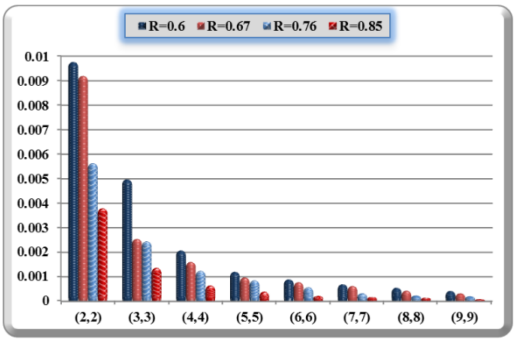


Figure 4. *MSER of* $\hat{\mathbb{R}}_{2,4}$ *at all true values of* $\mathbb{R}_{2,4}$ *based on NRSS*

- The MSER of $\hat{\mathbb{R}}_{1,3}$ takes the smallest value compared to $\hat{\mathbb{R}}_{2,4}$ for all values of (n, m) as well as sample sizes (n^*, m^*) (Figures 5, 6).

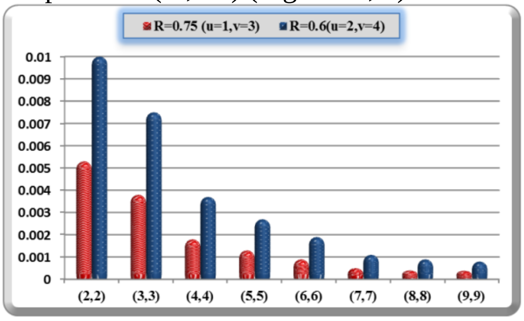


Figure 5. *MSER of $\hat{\mathbb{R}}_{1,3}$ and $\hat{\mathbb{R}}_{2,4}$ for all set sizes based on RSS*

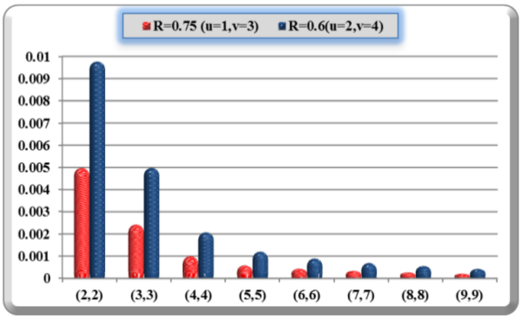


Figure 6. *MSER of $\hat{\mathbb{R}}_{1,3}$ and $\hat{\mathbb{R}}_{2,4}$ for all set sizes based on NRSS*

- The MSER of $\hat{\mathbb{R}}_{1,3}$ takes the smallest values compared to $\hat{\mathbb{R}}_{2,4}$ at all true values of $\mathbb{R}_{u,v}$ (Figures 7, 8).
- The MSER of $\hat{\mathbb{R}}_{1,3}$ is smaller than the MSER of $\hat{\mathbb{R}}_{2,4}$ for all exact values of $\mathbb{R}_{u,v}$ for all sample sizes at different set sizes.

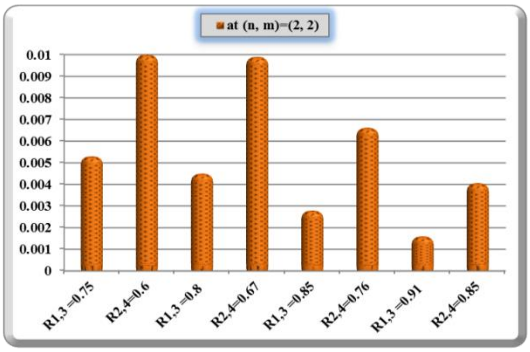


Figure 7. *MSER of $\hat{\mathbb{R}}_{1,3}$ and $\hat{\mathbb{R}}_{2,4}$ at set size* (2, 2) *based on RSS*

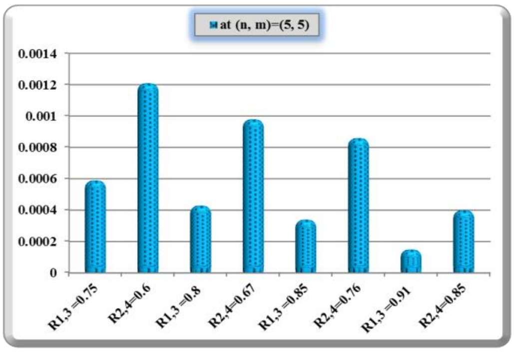


Figure 8. *MSER of $\hat{\mathbb{R}}_{1,3}$ and $\hat{\mathbb{R}}_{2,4}$ at set size* (5, 5) *based on NRSS*

- The MSSR estimates of $\mathbb{R}_{u,v}$ viz NRSS are preferred over the MSSR estimates of $\mathbb{R}_{u,v}$ based on RSS strengths and NRSS stress (see for example, Figures 9, 10 and Table 1).

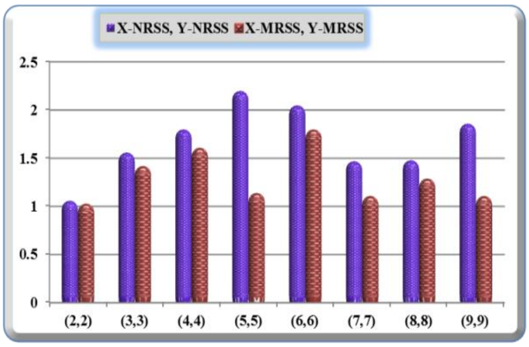


Figure 9. *RE of $\hat{\mathbb{R}}_{1,3}$ at $\mathbb{R}_{1,3}$ =0.75*

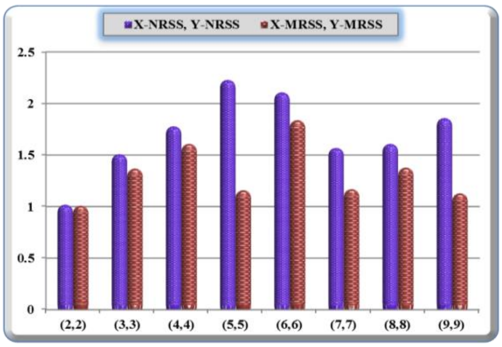


Figure 10. *RE of $\hat{\mathbb{R}}_{2,4}$ at $\mathbb{R}_{2,4}$ =0.6*

- In comparison to estimates made in light of RSS, the MSER of MSSR estimations under MRSS is lower.
- The equivalent RSS is less effective than the MSSR estimations under NRSS and MRSS.

6.3. Data Application

Here, we take into account two sets of data and demonstrate every aspect for illustration. The two data sets, which reflect CVFD of (5, 10, 15, and 20 mm) gauge lengths of jute fibres, were first published in [44], using the same data with sample sizes of $n^{*}=m^{*}=30$ (see Tables 5, 6).

Table 5 shows the MPa values for the first data set (10 mm) as follows:

693.73	704.66	323.83	778.17	123.06	637.66	383.43	151.48	108.94	50.16
671.49	183.16	257.44	727.23	291.27	101.15	376.42	163.40	141.38	700.74
262.90	353.24	422.11	43.93	590.48	212.13	303.90	506.60	530.55	177.25

Table 6. shows the MPa values for the second data set (20 mm) as follows:

71.46	419.02	284.64	585.57	456.60	113.85	187.85	688.16	662.66	45.58
578.62	756.70	594.29	166.49	99.72	707.36	765.14	187.13	145.96	350.70
547.44	116.99	375.81	581.60	119.86	48.01	200.16	36.75	244.53	83.55

Before continuing, some fundamental data analysis will be performed. The Kolmogorov-Smirnov (K-S) test and related P-value (P-V) are used to assess the performance of the adequate model. The K-S distance between the empirical and fitted distributions is 0.1012 for the first dataset with P-V = 0.918 and 0.14978 for the second dataset with P-V = 0.511, demonstrating strong fits. Figures 11 and 12 show the empirical survival function (ESF), PP plots and the estimated (PDF, CDF) for the data under consideration. the two sets of data may therefore be examined using the EED.

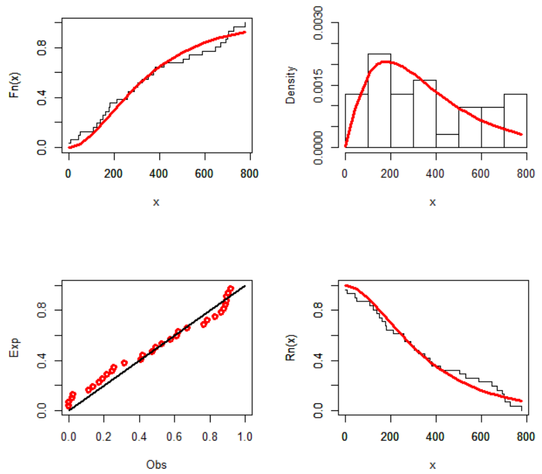


Figure 11. *The PDF, CDF, and ESF plots of the EED for Data 1*

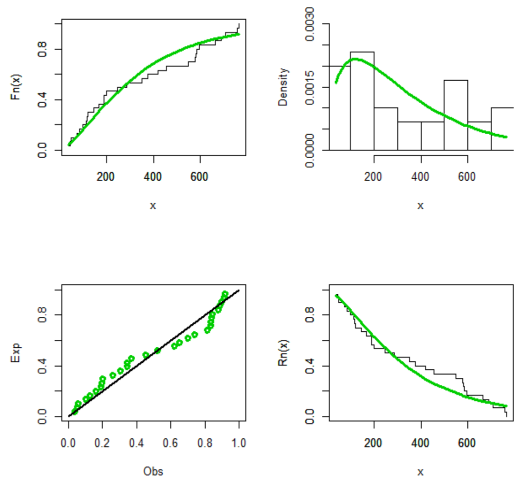


Figure 12. *The PDF, CDF, and ESF plots of the EED for Data 2*

Table 7. *MSSR estimate of the data sets*

(*n, m*)	RSS	NRSS	MRSS
$\mathbb{R}_{1,3}$			
(2,2)	0.701995	0.937863	0.773591
(3,3)	0.750732	0.945265	0.925312
(4,4)	0.846373	0.955738	0.939427
(5,5)	0.888889	0.969239	0.949332

(*n, m*)	RSS	NRSS	MRSS
(*n, m*)	RSS	NRSS	MRSS
$\mathbb{R}_{2,4}$			
(2,2)	0.532466	0.893466	0.634349
(3,3)	0.601054	0.905922	0.872494
(4,4)	0.744958	0.923656	0.896092
(5,5)	0.812698	0.946704	0.912793

Table 7 shows the MSSR estimates from the EED using a range of n^* and m^* of sample sizes values, with $t_x = t_y = 5$ cycles assuming the suggested sampling procedures. The NRSS, MRSS and RSS algorithms are generated from Data 1 and 2. The MSSR estimates via the three sampling strategies increase as n^* and m^* increase, for $n = m$, where $t_x = t_y = 5$ as shown in Figure 13. These findings demonstrate that, for large set sizes limits, the values of the estimated MSSR of $\mathbb{R}_{u,v}$ obtained using NRSS are higher than those obtained through RSS and MRSS, which led to the selection of NRSS samples. As a result, the findings in this subsection corroborate those in the one before it.

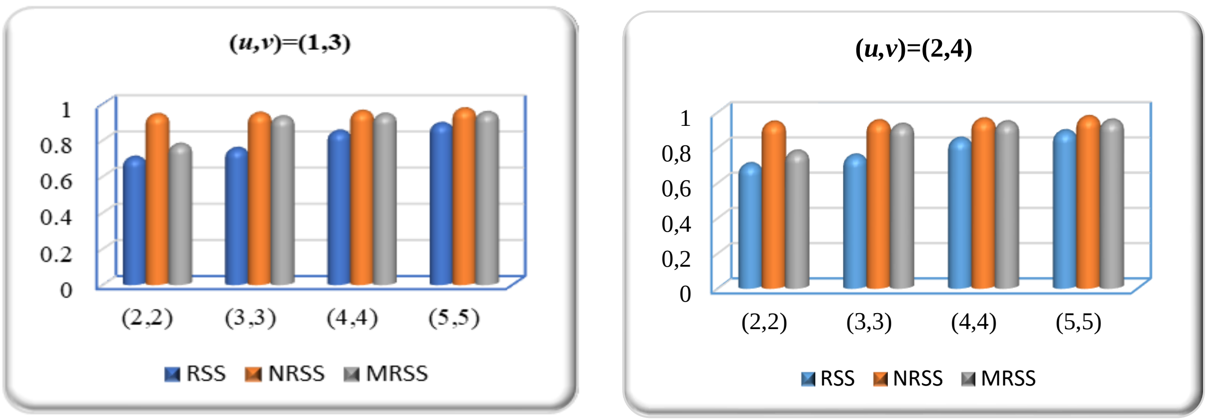


Figure 13. *The MSSR estimates for the three sampling strategies at various set sizes for* $\mathbb{R}_{1,3}$ *and* $\mathbb{R}_{2,4}$

7. CONCLUSION

The reliability estimation in MSSR, say $\mathbb{R}_{u,v}$, is covered in this paper, when stress and strengths random variables from the EED under the RSS, NRSS and MRSS designs. Under the MRSS scheme, we study the MSSR estimators in two different cases: odd and even set sizes. Despite the different sampling schemes, we execute a numerical analysis to assess the attitude of the different estimators. The study's findings show that, in all cases, the MSERs of MSSR estimates depending on NRSS data are lower compared to the equivalent in view of RSS and MRSS data. It's also worth noting that MSSR estimates based on MRSS are lower than those based on RSS. Hence, compared to the similar ones in RSS, the MSSR estimations under NRSS and MRSS are more effective. Also, real data applications illustrate these results. An extended study can be done when the observed data from stress and strengths random variables are distinct. In other words, the MSSR estimator is considered when the strengths data are derived from NRSS, and the stress measurements follow a simple odd/even set size MRSS pattern. The MSSR estimator is also taken into consideration when the strength data are produced from RSS and the stress data are presented as MRSS utilising an odd/even sizes of the sets. Future research will take into account the inference for MRSS reliability when the random variables for stress and strength have distinct distributions.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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