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INTUITIONISTIC MULTIPLICATIVE SET APPROACH FOR GREEN SUPPLIER SELECTION PROBLEM USING TODIM METHOD

ALI KÖSEOĞLU

0000-0002-2131-7141

ABSTRACT. Production and supply chain can be considered as the fundamentals of economic cycle for all countries. As a result of climate crisis in the world and the disasters it brought, the use of green products has become inevitable. With the restrictions imposed by some governments, suppliers have started to canalize their productions to environmental goods and the selection of most appropriate green supplier become a strategic decision. However, classical methods remain incapable to making these decisions due to the uncertain information contained in real life problems. Intuitionistic multiplicative information is a good choice dealing with these uncertainties in this kind of problems. Therefore, in this work, green supplier selection problem is discussed using TODIM method with intuitionistic multiplicative sets.

1. INTRODUCTION

As fuzzy set theory methods have evolved over time, the difficulties in real-life problems have increased significantly. So, the crisp numbers used so far have been insufficient for solving new kinds of problems. Fuzzy set theory, developed by Zadeh [1], is an excellent approach for processing information that includes both certainty and uncertainty. Thinking that there should be a non-membership function corresponding to the membership function, Atanassov [2] developed IFSs by generalizing FSs. These sets have been applied to many decision-making problems [3]–[10] and become useful mathematical tools to deal with problems that need uniform scaling or grading. Nevertheless, there are also numerous problems that require the use of unsymmetrical scaling and non-uniform distribution. The most typical example that needs such measurement is the grade systems of universities. Moreover, there is another example in economy called the principle of diminishing marginal utility which can also be considered as unsymmetrical situations [11]. To deal with these types of problems and make decisions based on them, 1-9 scales have been developed by Saaty [12], which are useful tools for assigning grades to variables in ways that best respond to decision needs. Based on this, Xia et. al [13] proposed a new approach called intuitionistic multiplicative sets, and these sets have been studied extensively so far [14]–[20]. The comparison of both uniform symmetric [0,1] and non-uniform unsymmetric $\left[\frac{1}{9}, 9\right]$ scales is shown in TABLE 1.

Multi-Criteria Decision Making (MCDM,) a decision-making technique studied by Churchman et al. [21] in the 1950s, has had an enormous impact on operations research over the

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last several decades. Its role has increased significantly over the past five decades and continues to grow as a result of their novel applications in a variety of fields [15], [22]–[25]. The TODIM method is a discrete MCDM method based on Prospect Theory [26] and proposed by Gomes and Lima [27] which has been widely used in recent years [28]–[30]. Unlike other MCDM methods, the TODIM method takes into account the limited rationality of decision makers. The ability of representing the decision maker behaviors is the strength of this method. Since the use of crisps numbers has some deficiencies in incomplete and uncertain information as classical sets, Krohling & De Souza [31] introduced the fuzzy TODIM method and Krohling et al. [32] proposed IF-TODIM method right after. However, like the IFSs, the IF-TODIM method is also insufficient in using non-uniform and unsymmetrical problem data. Therefore, in this work intuitionistic multiplicative TODIM method is proposed based on IMNs and it is applied to green supplier selection problem.

Multiplicative Scale	Fuzzy Scale	Explanation
1/9	0.1	Extremely undesirable
1/7	0.2	Very strongly undesirable
1/5	0.3	Strongly undesirable
1/3	0.4	Moderately undesirable
1	0.5	Equally desirable
3	0.6	Moderately desirable
5	0.7	Strongly desirable
7	0.8	Very strongly desirable
9	0.9	Extremely desirable
Other values	Other values	Intermediate values used to present compromise

TABLE 1. Comparison of both scales

2. PRELIMINARIES

In this section, we give essential information about IMS such as its aggregation operators and information measures.

Definition 2.1. [2] Let $X \neq \emptyset$ be a set, then the set

(2.1) $$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

is defined as IFS. The characteristic functions $\mu_A, \nu_A : \rightarrow [0,1]$ are called as the degree of membership and non-membership of the element $x \in X$. These functions hold the following property.

(2.2) $$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Definition 2.2. [11] Let $X \neq \emptyset$ be a set, then the set

(2.3) $$D = \{\langle x, \rho_D(x), \sigma_D(x), \rangle | x \in X\}$$

is defined as intuitionistic multiplicative set (IMS). Here, $\rho_D(x)$ is a membership function and $\sigma_D(x)$ is a non-membership function that satisfy the following conditions:

(2.4) $$\frac{1}{9} \leq \rho_D(x), \sigma_D(x) \leq 9, \qquad 0 < \rho_D(x)\sigma_D(x) \leq 1, \qquad \forall x \in X.$$

Using the pair $(\rho_D(x), \sigma_D(x))$, the number $\alpha = (\rho_\alpha, \sigma_\alpha)$ is called as intuitionistic multiplicative number (IMN). IMNs can be compared and ordered using a score function that is defined as $s(\alpha) = \rho_\alpha / \sigma_\alpha$. If the score function of two numbers is equal, then accuracy function is used to compare these numbers, which is defined as $h(\alpha) = \rho_\alpha$.

Definition 2.3. [11] Let $X = \{x_1, x_2, \dots, x_n\}$ be a set of n objects and $\alpha_{ij} = \left(\rho_{\alpha_{ij}}, \sigma_{\alpha_{ij}}\right)$ are IMNs where $i, j = 1,2, \dots n$. Then, the matrix $A = \left(\alpha_{ij}\right)_{n\times n}$ is called an intuitionistic multiplicative preference relation (IMPR). The elements of the number in this multiplicative relation satisfy the following conditions:

$$(2.5)\quad 0 < \rho_{\alpha_{ij}}\sigma_{\alpha_{ij}} \le 1, \qquad \rho_{\alpha_{ij}} = \sigma_{\alpha_{ji}}, \qquad \sigma_{\alpha_{ij}} = \rho_{\alpha_{ji}}, \qquad \rho_{\alpha_{ii}} = \sigma_{\alpha_{ii}} = 1, \qquad \frac{1}{9} < \rho_{\alpha_{ij}}, \sigma_{\alpha_{ij}} \le 9$$

Definition 2.4. [11] Let $\alpha_1 = (\rho_{\alpha_1}, \sigma_{\alpha_1})$, $\alpha_2 = (\rho_{\alpha_2}, \sigma_{\alpha_2})$ and $\alpha = (\rho_\alpha, \sigma_\alpha)$ be three IMNs. The basic operations are defined as:

$$(2.6)\quad \begin{array}{ll}
\text{i.} & \alpha^c = (\sigma_\alpha, \rho_\alpha) \\
\text{ii.} & \alpha_1 \wedge \alpha_2 = \left(\min\left(\rho_{\alpha_1}, \rho_{\alpha_2}\right), \max\left(\sigma_{\alpha_1}, \sigma_{\alpha_2}\right)\right) \\
\text{iii.} & \alpha_1 \vee \alpha_2 = \left(\max\left(\rho_{\alpha_1}, \rho_{\alpha_2}\right), \min\left(\sigma_{\alpha_1}, \sigma_{\alpha_2}\right)\right) \\
\text{iv.} & \alpha_1 \oplus \alpha_2 = \left(\frac{(1+2\rho_{\alpha_1})(1+2\rho_{\alpha_2})-1}{2}, \frac{2\sigma_{\alpha_1}\sigma_{\alpha_2}}{(2+\sigma_{\alpha_1})(2+\sigma_{\alpha_2})-\sigma_{\alpha_1}\sigma_{\alpha_2}}\right) \\
\text{v.} & \alpha_1 \otimes \alpha_2 = \left(\frac{2\rho_{\alpha_1}\rho_{\alpha_2}}{(2+\rho_{\alpha_1})(2+\rho_{\alpha_2})-\rho_{\alpha_1}\rho_{\alpha_2}}, \frac{(1+2\sigma_{\alpha_1})(1+2\sigma_{\alpha_2})-1}{2}\right) \\
\text{vi.} & \lambda\alpha = \left(\frac{(1+2\rho_\alpha)^\lambda-1}{2}, \frac{2\sigma_\alpha^\lambda}{(2+\sigma_\alpha)^\lambda-\sigma_\alpha^\lambda}\right) \\
\text{vii.} & \alpha^\lambda = \left(\frac{2\rho_\alpha^\lambda}{(2+\rho_\alpha)^\lambda-\rho_\alpha^\lambda}, \frac{(1+2\sigma_\alpha)^\lambda-1}{2}\right)
\end{array}$$

where $\lambda > 0$.

Definition 2.5. [11] Let a collection of IMNs is expressed as $\alpha_i (i = 1,2, \dots, n)$. Then, an intuitionistic multiplicative weighted averaging (IMWA) operator is a mapping $M^n \to M$, such that:

$$\text{IMWA}(\alpha_1, \alpha_2, \dots, \alpha_n) = \oplus_{i=1}^n (w_i\alpha_i)$$

which can be described as

$$(2.7)\qquad \text{IMWA}(\alpha_1, \alpha_2, \dots, \alpha_n) = \left(\frac{\prod_{i=1}^n\left(1+2\rho_{\alpha_i}\right)^{w_i}-1}{2}, \frac{2\prod_{i=1}^n \sigma_{\alpha_i}^{w_i}}{\prod_{i=1}^n\left(2+\sigma_{\alpha_i}\right)^{w_i} - \prod_{i=1}^n \sigma_{\alpha_i}^{w_i}}\right)$$

where $w = (w_1, w_2, \dots w_n)^\intercal$ is the weight vector of IMNs $\alpha_i (i = 1,2, \dots, n)$ with $w_i \in [0,1]$ and $\sum_{i=1}^n w_i = 1$

Definition 2.6. [11] Let a collection of IMNs is expressed as $\alpha_i (i = 1,2, \dots, n)$. Then, an intuitionistic multiplicative weighted geometric (IMWG) operator is a mapping $M^n \to M$, such that:

$$\text{IMWG}(\alpha_1, \alpha_2, \dots, \alpha_n) = \otimes_{i=1}^n {\alpha_i}^{w_i}$$

which can be expressed as

$$(2.8)\qquad \text{IMWG}(\alpha_1, \alpha_2, \dots, \alpha_n) = \left(\frac{2\prod_{i=1}^n \rho_{\alpha_i}^{w_i}}{\prod_{i=1}^n\left(2+\rho_{\alpha_i}\right)^{w_i} - \prod_{i=1}^n \rho_{\alpha_i}^{w_i}}, \frac{\prod_{i=1}^n\left(1+2\sigma_{\alpha_i}\right)^{w_i}-1}{2}\right)$$

where $w = (w_1, w_2, \dots w_n)$ is the weight vector of IMNs $\alpha_i (i = 1,2, \dots, n)$ with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$. Moreover, $\pi_D(x) = \frac{1}{\rho_D(x)\sigma_D(x)}$ is given as the hesitant information for the IMS.

Definition 2.7. [33] Let $X = \{x_1, x_2 \dots, x_n\}$ be a set that express the distance universe of discourse and d is mapping $d: \Psi(X) \times \Psi(X) \to [0,1]$ where $\Psi(X)$ is the set of all IMSs. If two IMSs A and B in X satisfy the following conditions:

(2.9)
(1) $0 \le d(A,B) \le 1$;
(2) $d(A,B) = 0 \Leftrightarrow A = B$;
(3) $d(A,B) = d(B,A)$;
(4) If $A \subseteq B \subseteq C$, then $d(A,C) \ge d(A,B)$ and $d(A,C) \ge d(B,C)$

then $d(A,B)$ is called a distance measure.

Definition 2.8. [33] Let A and B be two IMSs in X. Then the intuitionistic multiplicative normalized Manhattan distance between IMSs A and B is defined as

$$d_1(A,B) = \frac{1}{4n}\sum_{i=1}^{n}\left(\left|\log_9 \frac{\rho_A(x_i)}{\rho_B(x_i)}\right| + \left|\log_9 \frac{\sigma_A(x_i)}{\sigma_B(x_i)}\right|\right) \tag{2.10}$$

Definition 2.9. [33] Let a and b be two IMNs in X. Then the intuitionistic multiplicative normalized Manhattan distance between IMNs a and b is defined as

$$d_2(a,b) = \frac{1}{4}\left(\left|\log_9 \frac{\rho_a}{\rho_b}\right| + \left|\log_9 \frac{\sigma_a}{\sigma_b}\right|\right) \tag{2.11}$$

3. INTUITIONISTIC MULTIPLICATIVE TODIM METHOD

The proposed method is adapted from IF-TODIM which was proposed by Krohling et al. [32]. Let $A = \{A_1, A_2, \dots, A_m\}$ be a set of alternatives, $C = \{C_1, C_2, \dots, C_n\}$ be a set of criteria and $w = [w_1, w_2, \dots, w_n]$ be a weight vector with respect to criteria where $\sum_{j=1}^{n} w_j = 1$ and $w_j \ge 0$. Then, the steps of IMS-based TODIM method are given as follows:

Step 1. Construct decision making matrix $R = (r_{ij})_{m\times n} = (\langle \rho_{ij}, \sigma_{ij} \rangle)_{m\times n}$:

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} \langle \rho_{11}, \sigma_{11} \rangle & \langle \rho_{12}, \sigma_{12} \rangle & \cdots & \langle \rho_{1n}, \sigma_{1n} \rangle \\ \langle \rho_{21}, \sigma_{21} \rangle & \langle \rho_{22}, \sigma_{22} \rangle & \cdots & \langle \rho_{2n}, \sigma_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \rho_{m1}, \sigma_{m1} \rangle & \langle \rho_{m2}, \sigma_{m2} \rangle & \cdots & \langle \rho_{mn}, \sigma_{mn} \rangle \end{bmatrix} \end{array} \tag{3.1}$$

Step 2. Normalize the decision matrix R given by decision maker:
To normalize the decision matrix $R = (r_{ij})_{m\times n} = (\rho_{ij}, \sigma_{ij})$, the following formulation is operated:

$$\begin{cases} (r_{ij})_{m\times n} = (\rho_{ij}, \sigma_{ij}), & \text{for benefit type} \\ (r_{ij})_{m\times n} = (\sigma_{ij}, \rho_{ij}), & \text{for cost type} \end{cases} \tag{3.2}$$

Step 3. Determine the relative weight w_{cr} of criterion C_c with the equation $w_{cr} = \frac{w_c}{w_r}$ where $w_r = \max\{w_c \mid c = 1,2, \dots, n\}$.

Step 4. Calculate the dominance of each alternative A_i over each alternative A_j using the following expression:

(3.3)
$$\delta(A_i, A_j) = \sum_{c=1}^{n} \Phi_c(A_i, A_j)$$

where $\delta(A_1, A_2) = \Phi_1(A_1, A_2) + \Phi_2(A_1, A_2) + \cdots + \Phi_n(A_1, A_2)$ and $\delta(A_i, A_j)$ is an $n \times n$ matrix. The dominance score $\Phi_c(A_i, A_j)$ is calculated as:

(3.4)
$$\Phi_c(A_i, A_j) = \begin{cases} \sqrt{\dfrac{w_{cr}}{\sum_{c=1}^{m} w_{cr}} \cdot d(a_{ic}, a_{jc})}, & if\ a_{ic} > a_{jc} \\ 0, & if\ a_{ic} = a_{jc} \\ \dfrac{-1}{\theta}\sqrt{\dfrac{\sum_{c=1}^{m} w_{cr}}{w_{cr}} \cdot d(a_{jc}, a_{ic})}, & if a_{ic} < a_{jc} \end{cases}$$

where θ is the attenuation factor of losses. Qin et al. [34] suggested to take θ in the interval of [1,5]. Furthermore, they stated that the θ gets its optimal value in the [2.0, 3.0].

Step 5. Normalize the dominance matrix and calculate the global values of the alternative A_i according to the following expression:

(3.5)
$$\xi_i = \frac{\sum_{j=1}^{n} \delta(A_i, A_j) - \min_i\left(\sum_{j=1}^{n} \delta(A_i, A_j)\right)}{\max_i\left(\sum_{j=1}^{n} \delta(A_i, A_j)\right) - \min_i\left(\sum_{j=1}^{n} \delta(A_i, A_j)\right)}$$

Step 6. Rank the alternatives:
The ordering of the values ξ_i gives the rank for each alternative. The greatest value of ξ_i means the best alternative.

Numerical Example

Green supplier selection (GSS) is a significant MCDM problem for companies all over the world. It is vitally important in terms of companies' revenues and strategic competitiveness. As pollution approaches critical levels around the world, not only companies but also their supply partners are held responsible. As pollution around the world approaches critical levels, not only companies but also their supply partners are held responsible for this situation. Many major studies have been conducted on GSS in recent years [35]–[38] and it will still remain an open area of research as technology evolves. Therefore, an automobile manufacturing company in Turkey is selected for the GSS problem which is looking for a green supplier under five criteria regarding green competencies. As a result of the pre-evaluation, six supplier alternatives are determined, and five criteria are taken into account as:

C_1: Cost
C_2: Quality
C_3: Delivery
C_4: Technology capability
C_5: Environmental competency

The weights of these criteria are decided as w =[0.25 0.15 0.10 0.23 0.27]

Step 1. The evaluation values are given in TABLE 2:

	C_1	C_2	C_3	C_4	C_5
A_1	$\left(\frac{4}{9},1\right)$	$\left(\frac{1}{4},1\right)$	$\left(\frac{1}{9},\frac{8}{9}\right)$	$\left(\frac{5}{3},\frac{2}{5}\right)$	$\left(1,\frac{2}{3}\right)$
A_2	$\left(\frac{1}{3},\frac{6}{5}\right)$	$\left(\frac{1}{9},1\right)$	$\left(\frac{7}{9},\frac{6}{5}\right)$	$\left(\frac{5}{6},\frac{7}{6}\right)$	$\left(\frac{9}{2},\frac{1}{7}\right)$
A_3	$\left(\frac{1}{2},\frac{6}{7}\right)$	$\left(\frac{8}{5},\frac{1}{2}\right)$	$\left(\frac{2}{3},\frac{2}{3}\right)$	$\left(\frac{6}{7},\frac{2}{5}\right)$	$\left(\frac{4}{9},1\right)$
A_4	$\left(\frac{2}{5},\frac{1}{2}\right)$	$\left(\frac{1}{2},\frac{1}{6}\right)$	$\left(\frac{9}{5},\frac{3}{7}\right)$	$\left(\frac{1}{2},\frac{5}{8}\right)$	$\left(\frac{7}{6},\frac{1}{2}\right)$
A_5	$\left(\frac{7}{8},\frac{3}{5}\right)$	$\left(\frac{2}{9},1\right)$	$\left(1,\frac{1}{2}\right)$	$\left(\frac{1}{2},\frac{1}{9}\right)$	$\left(\frac{1}{9},8\right)$
A_6	$\left(\frac{7}{6},\frac{4}{5}\right)$	$\left(\frac{4}{5},\frac{1}{4}\right)$	$\left(\frac{5}{9},\frac{8}{5}\right)$	$\left(\frac{2}{3},\frac{4}{7}\right)$	$\left(\frac{5}{6},\frac{4}{7}\right)$

TABLE 2. Decision Matrix

Step 2. Normalize the decision matrix R given by decision maker using Eq. (3.2):
C_1 is cost type, so only the column of C_1 changes.

Step 3. Determine the relative weight w_{cr}:
Since w= [0.25 0.15 0.10 0.23 0.27], w_{cr} is calculated as:

$$w_{cr} = [0.9259 \quad 0.5556 \quad 0.3704 \quad 0.8519 \quad 1.0000]$$

Step 4. Calculate the dominance of each alternative A_i over each alternative A_j using Eq. (3.3):
Using Eq. (3.4), the dominance score values of $\Phi_c(A_i, A_j)$ for $\theta = 1$ are calculated and the matrix $\delta(A_i, A_j)$ is given in TABLE 3:

	$\delta(A_i, A_1)$	$\delta(A_i, A_2)$	$\delta(A_i, A_3)$	$\delta(A_i, A_4)$	$\delta(A_i, A_5)$	$\delta(A_i, A_6)$
$\delta(A_1, A_j)$	0	−2.5649	−3.0340	−1.5893	−3.8747	−2.4616
$\delta(A_2, A_j)$	−2.2799	0	−1.7686	1.1089	−3.2127	−2.0312
$\delta(A_3, A_j)$	−1.5067	−3.1799	0	−1.6361	−2.2838	−3.2779
$\delta(A_4, A_j)$	−3.6090	−5.9482	−2.8425	0	−3.5011	−4.2448
$\delta(A_5, A_j)$	−1.1032	−2.0872	−2.2945	−1.5494	0	−2.9248
$\delta(A_6, A_j)$	−2.3492	−2.4125	−0.6789	0.2269	−2.2164	0

TABLE 3. Dominance matrix

Step 5. Calculate the global value ξ_i:

$$\xi_i = [0.5207 \quad 0.9407 \quad 0.6497 \quad 0 \quad 0.8011 \quad 1.0000]$$

Step 6. Rank the alternatives:
By sorting the ξ_i from largest to smallest, we obtain the order of $A_6 > A_2 > A_5 > A_3 > A_1 > A_4$. This ranking shows that the alternative A_6 is the best option for choosing the green supplier.

Since θ is adaptable and Qin et al. [34] suggested to examine θ in [1,5] to analyze the outputs in decision making problems, we analyze the problem by taking $\theta \in [1,5]$. TABLE 4 gives the corresponding results:

	θ = 1		θ = 2		θ = 3		θ = 4		θ = 5	
	ξ_i	Order	ξ_i	Order	ξ_i	Order	ξ_i	Order	ξ_i	Order
A_1	0.5207	5	0.5710	5	0.6061	5	0.6229	4	0.6362	4
A_2	0.9407	2	0.9778	2	1.0000	1	1.0000	1	1.0000	1
A_3	0.6497	4	0.6424	4	0.6325	4	0.6139	5	0.5992	5
A_4	0	6	0	6	0	6	0	6	0	6
A_5	0.8011	3	0.8763	3	0.9287	3	0.9535	3	0.9729	2
A_6	1.0000	1	1.0000	1	0.9934	2	0.9711	2	0.9535	3

TABLE 4. Rankings with respect to θ

- The ranking of A_2 and A_6 changes between $\theta = 2$ and $\theta = 3$.
- The ranking of A_1 and A_3 changes between $\theta = 3$ and $\theta = 4$.
- The ranking of A_5 and A_6 changes between $\theta = 4$ and $\theta = 5$.

The replacement of alternatives orders can be noticed clearer in FIGURE 1. These outcomes confirm that the DMs' decisions can alter the orders of the alternatives.

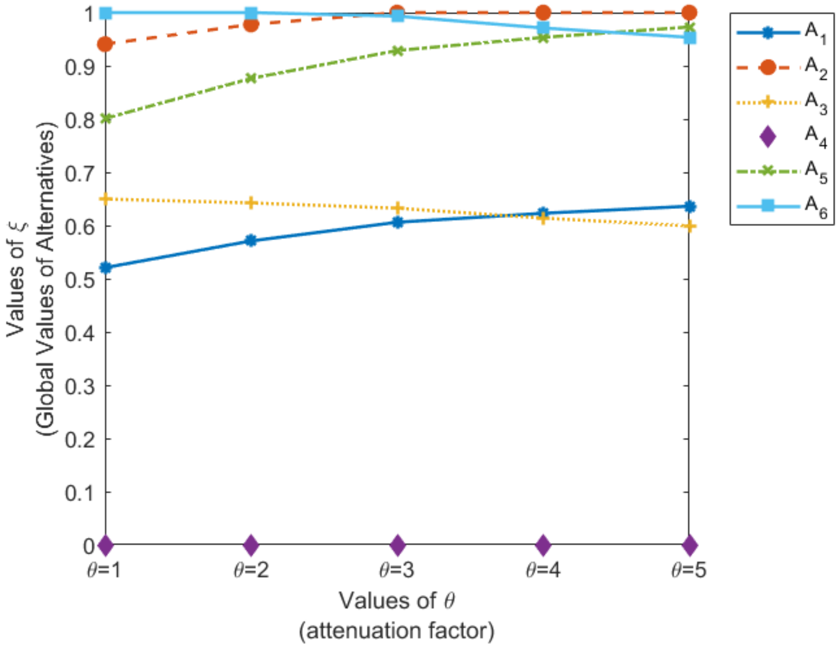

FIGURE 1. Ranking of Alternatives for Different θ's

Same example can be solved with the aggregation operators given in Eq. (2.7) or Eq. (2.8). We use the IMWA operator to aggregate the intuitionistic multiplicative values in decision matrix with the same weights as: $\alpha_i = \mathrm{IMWA}(A_1, A_2, \dots, A_5) = \bigoplus_{i=1}^{6} (w_i A_i) = (\rho_i, \sigma_i)$ where $i = 1,2,\dots,5$. and we obtain aggregated values as $\alpha_1 = (0.8448, 9.2474), \alpha_2 = (1.2932, 7.0640), \alpha_3 = (0.7942, 9.2966)$, $\alpha_4 = (0.7476, 6.5637)$, $\alpha_5 = (0.3892, 11.0401)$ and $\alpha_6 = (0.7504, 10.304)$. Using the score function for IMNs we obtain the final values of alternatives as $s(\alpha_1) = 0.0914$, $s(\alpha_2) = 0.1831$, $s(\alpha_3) = 0.0854$, $s(\alpha_4) = 0.1139$, $s(\alpha_5) = 0.0353$, $s(\alpha_6) = 0.0728$. Ranking them with descending order we obtain

$$A_2 > A_4 > A_1 > A_3 > A_6 > A_5$$

The ranking comparison between IM-TODIM and the IMWA operator is shown in TABLE 5.

Method	Att. Fac.	Ranking Order
IM-TODIM	$\theta = 1$	$A_6 > A_2 > A_5 > A_3 > A_1 > A_4$
	$\theta = 3$	$\boldsymbol{A_2} > \boldsymbol{A_6} > A_5 > A_3 > A_1 > A_4$
	$\theta = 4$	$A_2 > A_6 > A_5 > \boldsymbol{A_1} > \boldsymbol{A_3} > A_4$
	$\theta = 5$	$A_2 > \boldsymbol{A_5} > \boldsymbol{A_6} > A_1 > A_3 > A_4$
IMWA	-	$A_2 > A_4 > A_1 > A_3 > A_6 > A_5$

TABLE 5. Ranking comparison

4. CONCLUSION

This paper extends TODIM to IM-TODIM using intuitionistic multiplicative sets. The advantages of IM-TODIM are discussed in terms of grading scale of problem information. A numerical application is presented to demonstrate the advantages of the IM-TODIM method. It is seen that the ranking of the alternatives changes when the attenuation factor θ is adjusted. Moreover, IM-TODIM is compared with the IMWA aggregation operator. This comparison also shows the effectiveness of the IM-TODIM. Obviously, the IM-TODIM method produces more comprehensive results compared to aggregation operators.

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The Declaration of Research and Publication Ethics
The author(s) declared that they comply with the scientific, ethical, and citation rules of Journal of Universal Mathematics in all processes of the study and that they do not make any falsification on the data collected. Besides, the author(s) declared that Journal of Universal Mathematics and its editorial board have no responsibility for any ethical violations that may be encountered and this study has not been evaluated in any academic publication environment other than Journal of Universal Mathematics.

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RECEP TAYYIP ERDOGAN UNIVERSITY, FACULTY OF ARTS AND SCIENCES, DEPARTMENT OF MATHEMATICS, RIZE, TURKEY
E-mail address: ali.koseoglu@erdogan.edu.tr