International Journal of Educational Studies in Mathematics
IJESIM
EDUGARDEN

An Investigation of Sixth Grade Students' Skills of Solving and Posing Problems Which Require Using the Knowledge of Order of Operations*

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ABSTRACT

The aim of this research is to examine sixth grade students' skills of solving and posing problems which require using the knowledge of the order of operations to solve. The case study design, one of the qualitative research methods, was adopted in the research. The study group consisted of 44 sixth grade students attending a public secondary school in Eskisehir. Within the scope of the application, the forms including the questions to measure their problem solving and posing skills were directed to the study group. The data were analyzed by thematic analysis method. According to the results obtained, it was found that while most of the students were successful in solving problems which require the knowledge of the order of operations, they were not successful in posing such type of problems. In addition, it was found that students made mistakes in Turkish language grammar and expressions and in using the mathematical language. Students made mathematical terminology mistakes generally in subtraction and division operations. As a result, it is recommended to increase the problem posing and order of operations activities in mathematics lessons.

ARTICLE INFO

Article History:
Received: 22.07.2022
Received in revised form: 05.09..2022
Accepted: 05.09..2022
Available online: 10.09.2022
Article Type: Research paper
Keywords: mathematics education, sixth grade students, four basic operations, order of operations, problem solving, problem posing.

1. Introduction

Thanks to the developing information technologies in today's world, it is possible to reach any event, news or scientific information easily (Tulaev et al., 2020). For this reason, today, the need for people who can discover, organize and use information has increased (Rashidov, 2020). As a matter of fact, international exams such as the Program for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) (Organization for Economic Co-operation) and Development [OECD], 2004), as well as in our country in recent years, high school and university entrance exam questions have been prepared for these skills (Ekinci and Bal, 2019). Mathematics, which require intense reasoning use, is one of the main courses in which students acquire these skills (Özsoy, 2005). However, some students have difficulty in understanding mathematics lessons (Chinn, 2020), because of its specific symbols and language, abstract concepts, rules, calculation procedures (Barwell, 2020). For this reason, mathematics is often seen as a difficult and feared subject by many students (Andrade Medeiros and Muniz, 2022). This perception, which also exists among the public, is transmitted from generation to generation and among students in the school environment, causing some of them to be alienated from mathematics. Mathematics course does not deserve this

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DOI: https://doi.org/10.17278/ijesim.1146898
*This study is a part of the Master's Thesis named "An investigation of sixth grade students' problem solving and posing abilities for the order of operations" completed in Eskisehir Osmangazi University in 2020

widespread negative opinion (Russo and Minas, 2020). Mathematics is a vital basis of industrial and knowledge production where competition is experienced to the fullest, and it is not possible to be a strong individual, institution, company, state and nation without mathematics (Holt, 2021). In fact, mathematics is a course that prepares students for life and develops their creativity, estimation and reasoning skills (Zulnaidi et al., 2021), besides its perfection in its structure, artistic beauty, rich subjects and concepts, being the common language of sciences and being a tool used in all areas of life (Sullivan, 2020). In this respect, these features of mathematics should be explained well, be taught with appropriate teaching methods and techniques (Pietsch, 2020), and the formation of a negative opinion about mathematics in students should be prevented as much as possible (Andrade Medeiros and Muniz, 2022).

Problem solving, a teaching method that has been used in mathematics for a long time, is a skill that is frequently included and emphasized in the mathematics curriculum (Carroll and Isaacs, 2020). In the Mathematics Curriculum of the Ministry of National Education (MoNE, 2018), it is mentioned that students can easily express their own thoughts and reasoning in the problem solving process and that they can see the deficiencies or gaps in the mathematical reasoning of others. The MoNE's 2009 Mathematics Curriculum, problem solving is among the roles assigned to students. It is also mentioned that problem solving is a process, not a subject, that problem solving cannot be taught with rules and that it can be developed by providing environments where students can develop their creativity (Renatovna and Renatovna, 2021).

When the literature is examined, it is stated that in mathematics education, it is necessary to give importance to problem posing as well as problem solving in order to raise individuals who can not only do calculations, but also think mathematically and try to do mathematics (Cai and Hwang, 2020; English, 1997; Lowrie, 1999). Problem posing is a process that includes students' personal interpretations of concrete situations and forming them into meaningful mathematical problems (MoNE, 2018; NCTM, 2000). As it can be understood from the definitions of problem posing, student must make comments while posing a problem (Akben, 2020). It can be said that the student's interpretation helps her/him to work her/his mind to create different story situations and to develop her/his point of view (Li et al., 2020). In addition, problem posing activities in the educational environment motivate students to think flexibly and produce solutions to new problems, rather than doing the exercises offered by the teacher (Keşan et al., 2010; Prabawanto and Susilo, 2020). From this point of view, it can be said that problem posing is an important skill in acquiring problem solving skills (Chen and Cai, 2020).

According to Kilpatrick (1987), problem solving and posing activities should be included in mathematics lessons together in order to improve students' mathematical thinking skills. In the mathematics curriculum (MoNE, 2009), the expressions "solves and poses problems" are frequently encountered. The fact that these goals are often mentioned together in the curriculum is an indication that problem posing and solving skills are intertwined (Putra et al., 2020). Şengül-Akdemir and Türnüklü (2017) suggested in their research that both problem solving and posing skills should be developed together. Because the student's solving the problem may not be proof that he/she fully understands the problem (Putri et al., 2020; Voica et al., 2020). In order to understand the problem in a deeper way, it will be useful to establish problems that can be solved by the problem solving method (Korkmaz and Gür, 2006).

One of the most important skills that students should have in order to solve or pose a problem successfully is the four operations skills. Basic arithmetic skills including addition, subtraction, multiplication and division, which are expressed as four operations, form the skeleton of primary school mathematics (NTCM, 2000). For this reason, it can be said that it is not possible to do almost any operation without knowing the four operations skills. So, is having four operations skills enough to solve problems successfully? Of course, in problems involving four operations, the student may need to have the "order of operations" knowledge about which operation should be done first. This rule is known as the “order of operations” (shortened in this research as OO) in mathematics. Blando

et al. (1989) stated that the OO mistakes are among the most common arithmetic mistakes encountered in middle school students. These errors can cause the problem to be solved incorrectly, as well as cause the problem to be posed incorrectly (Amini et al., 2019). For example, if students does not know that multiplication will be done before addition, they can both solve the problem incorrectly and pose the related problem incorrectly. In this study, students' ability to solve operations that require the use of the OO knowledge and to pose problems for these operations were examined.

When the literature are examined, it is seen that the existing related researches are mostly based on problem posing (Deringöl, 2020; Dogan-Coskun, 2019). There are hardly any researches examining problem posing in the context of action priority. In these researches, problem posing was generally handled together with arithmetic operations, but the OO was not taken into account. This has revealed the necessity of the "Problem posing for the OO" research. For this purpose, the answers to the following questions were sought in this research. For the secondary school sixth grade students;

- What are their skills of solving problems requiring the knowledge of the OO?
- What are their skills of posing problems requiring the knowledge of the OO?

2. Methodology

In this part of the study, the research design, the study group, the data collection tools used, the methods used to analyze the data, and the reliability of the research are explained.

2.1. Research Design

In this study, the case study design, one of the qualitative research methods, was used in the collection, analysis and interpretation of data. According to Yin (2017), the case study is an up-to-date research method that is used to answer the questions of how and why in situations where the researcher's control is not over the variables.

2.2. Study Group

The study group of this research consists of 44 secondary school sixth grade students attending a public school in Eskisehir in the fall semester of the 2019-2020 academic year. The study group was selected by purposive sampling method, which is one of the non-random sampling methods. According to Patton (1987, cited in Yıldırım and Şimşek, 2011), purposive sampling allows for in-depth study of situations that are thought to have rich information. In addition, in accordance with the characteristics of the purposive sampling method, students of the sixth grade level were selected because of their readiness in the context of the OO pre-knowledge and the reason that the gains in problem posing and solving took place a lot at this level.

2.3. Data Collection Tools

Two data collection tools were used in the research: "Problem solving application form for OO" and "Problem posing application form for the OO". These application forms have been revised by taking expert opinion and then were administered to the students at one-week intervals.

2.3.1. Problem Solving Application Form for the OO

The first stage of the research was "Problem solving application form for the OO". The application form is presented in Appendix 1. For the application, four questions measuring OO skills including the binary operations "multiplication-addition", "multiplication-subtraction", "division-subtraction" and "division-addition" were created. These four questions measure four-operation skills. Routine problems are generally known as four operations problems, which are widely used in mathematics textbooks, and are also known as "word problems" in the literature (Ulu, 2011). Table 1 shows the operations and purposes of the questions in the problem solving application form.

Table 1. Questions in the problem solving application form for OO and their purposes

Questions	Operations Included	Purpose of the Question
2+7.5=?	Multiplication and addition	Knowing the superiority of multiplication over addition.
15-4.3=?	Multiplication and subtraction	Knowing the superiority of multiplication over subtraction.
8-4:2=?	Subtraction and division	Knowing the superiority of division over subtraction.
4+10:2=?	Addition and division	Knowing the superiority of division over addition.

2.3.2. Problem Solving Application Form for the OO

The second stage of the research was "Problem posing application form for the OO". The application form is presented in Appendix 2. The operations in this form are the same as the operations in the first stage of the research and aim to measure the students' problem posing skills regarding the OO.

2.4. Data Analysis

"Thematic analysis" method was used in the analysis of the data obtained in the study. Thematic analysis is the method used to identify, analyze and report the patterns (themes) in the data. It organizes and explains the dataset with minimal (rich) detail (Braun and Clarke, 2006). Thematic analysis not only goes further, but also makes sense of many dimensions of the research topic (Boyatzis, 1998). Thematic analysis consists of a series of individual or focus group interviews or an examination to find repetitive patterns of meaning in a dataset of various texts. In this research, all the data obtained from the applications were reviewed by two field experts, and as a result, codes and themes were created.

2.4.1. Problem posing form analysis for OO

The data form the "Problem posing form for the OO ", which is the second stage of the study were analyzed in three categories: "Students who take into account the OO, students who don't take into account the OO, and students who leave it blank". The problems posed by the students who took into account the OO rule, was examined in terms of "problem situation, suitability to the given operation, context situation and language/expression". The students were evaluated according to the analysis framework in Appendix 3A. The answers of the students who posed problems without taking into account the OO were classified in terms of "problem situation, context situation and language/expression". The students were evaluated according to the analysis framework in Appendix 3B. The category of "blank" is determined if a question is left unanswered or if there are words or sentences that are not related to the given operation in the answer section.

2.4.2. Problem posing form analysis for the operation

Table 2 shows what each of the categories in the problem posing form for the operation analysis framework represents.

Table 2. Questions in the problem solving application form for OO and their purposes

Problem Situation
-Problem Situation Exists: The problem has any question stem or statement. -Problem Situation Doesn't Exist: The problem has no question stem or statement.
Suitability for the Given Operation
-Suitable for the given operation: Problems posed by taking into account the OO. -Partially suitable for the given operation: The OO is taken into account but there are some unnecessary operations or numbers in the posed problem.
Context/Story
-Context/Story Exist: Problem having a story context related to real life. -Context/Story Doesn't Exist: Problem doesn't include story.
Language/Expression
-Language & Expression Mistakes: The problem posed contains an expression mistake or is not clearly expressed. -Punctuation Mistakes: The posed problem contains punctuation mistakes. -Terminology Mistakes: Incorrect or different use of the names of mathematical concepts in the posed problem.

3. Findings

The findings in this research are presented in two parts below, namely "Findings Obtained from the Problem Solving Form for the OO and Findings Obtained from the Problem Posing Form for the OO".

3.1. Findings Obtained from the Problem Solving Form for the OO

The analysis of the students' answers obtained from the four questions in the problem solving form is shown in Table 3.

Table 3. Findings obtained from the problem solving form

Problem Solving Application	The OO Taken into Account		The OO not Taken into Account		Left Blank
Questions	Correct Solution	Operation Mistake	Incorrect Solution	Other Solution	
1) 2+7.5=?	29	-	13	2	-
2) 15–4.3=?	28	-	14	2	-
3) 10–4:2=?	28	1	13	2	-
4) 6+9:3=?	25	3	13	3	-

According to the findings obtained at this stage, where the problem solving skills for the OO are measured, it was seen that more than half of the students could solve the problems by considering the operation priority. Looking at Table 3, it was seen that the most correct answers were given in the first question involving addition and multiplication, while the most incorrect answers were given in the second question involving subtraction and multiplication. It was determined that almost all of the wrong answers may be due to the fact that addition and subtraction is done before multiplication and division, and the tendency to perform operations from left to right on the OO. The student coded S6 made a wrong solution by first adding and then dividing in the fourth question. The solution of S6 is given in Figure 1.

Figure 1. A solution that didn't take into account the OO/wrong solution (S6)

It was seen that some students took the OO into account, but gave wrong answers because they made a mistake. In the fourth question, the student coded S37 applied the OO correctly by trying to do division and then subtraction, but he carelessly performed multiplication instead of division and found the result incorrect. The solution of the student coded S37 is given in Figure 2.

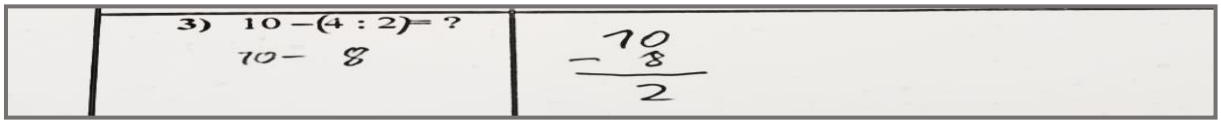

Figure 2. A solution that took into account OO / computation mistake (S37)

It is seen as in Figure 3 that the student coded S3, who didn't take into account the OO and whose answer included different solutions, is in the category of "Other Solution", solved the operation in two parts. S3 coded student added 2 and 7 and multiplied 7 and 5 and thought as if there were two separate operations.

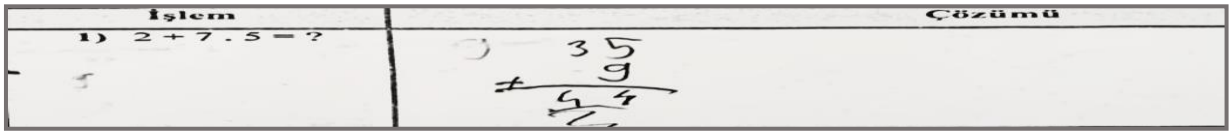

Figure 3. A solution that didn't take into account the OO / other solution (S3)

3.2. Findings Obtained from the Problem Posing Form for the OO

Findings obtained from the problem posing application which is the second stage of the research for the OO is given in Table 4.

Table 4. Findings obtained from the problem posing form

Problem Posing Application Task	Students Who Took into Account The OO	Students Who didn't Took into Account The OO	Students Not Answering(Blank)
1) 2+7.5=?	17	24	2
2)15–4.3=?	13	27	3
3)10–4:2=?	13	27	3
4) 6+9:3=?	13	23	7

According to the findings obtained from the application, the number of students who tried to pose a problem by not taking into account the OO in all tasks in the problem posing form was higher than the students who took the OO into account. It was observed that the OO was not taken into account when posing problems in the second and third questions, which mostly included subtraction/multiplication and subtraction/division by the students. It was determined that the first question, which includes the operations of addition and multiplication, is the question in which the OO is taken into account the most.

Considering the analysis framework of the problem posing form, the problems posed by the students who took into account the OO, were classified according to the status of "problem situation, suitability for the given operation, context situation and language/expression" as seen in Table 5.

Table 5. Findings of the problems posed by the students who didn't take the OO into account

Problem Posing Application	Problem Situation		Suitability for Given Operation		Context/ Story		Language & Expression		
Task	Yes	No	Suitable	Partially Suitable	Yes	No	Language & Expression Mistakes	Punctuation Mistakes	Terminology Mistakes
2+7.5=?	17	-	13	4	17	-	12	11	-
15–4.3=?	13	-	11	2	13	-	6	8	-
10–4:2=?	13	-	11	2	12	1	8	6	-
6+9:3=?	13	-	10	3	13	-	9	7	2

When the problems posed by the students who take into account the OO are analyzed according to the categories in Table 5, it was found that all of the expressions written by the students included a problem situation and context. Considering the suitability for the given operation, it was seen that the majority of the students, who took the OO into account, posed problems suitable for the given operation. The problem suitable for the given operation was posed mostly in the question involving addition and multiplication, and least in the question involving division and addition. "15–4.3=?" The answer of the student S42, who posed a suitable, that is, correct problem for the operation, is shown in Figure 4.

2) 15 – 4 . 3 = ?	Meltem, Kadir, Büşra and Nehir have a box of chocolates. Each of them have eaten three pieces of chocolate. There are 15 chocolates in the box. Find how many chocolates were left in the box.

Figure 4. A problem which took into account the OO / suitable for the given operation (S42)

Some of the students who pose problems by considering the operation priority took the operation priority into account, but included unnecessary data in the problem. Student coded S11 "2+7.5=?" In the problem he posed for the operation, he multiplied 7 by 5 and added 2, but at the beginning of the

problem "he had 7 balls." The problem he wrote was partially in the suitable category because he used unnecessary data. The problem of S11 is given in Figure 5.

1) 2 + 7 . 5 = ?	Mustafa had 7 toys. His grandfather bought 5 times more toys than his toys His mother bought 2 toys for Mustafa's birthday present. Do the order of operations showing how many toys Mustafa has?

Figure 5. A problem which took into account the OO / partially suitable for the given operation (S11)

Almost all of the problems of the students who posed problems by taking into account the OO include context, that is, a story. However, when the problems were examined in detail, it was seen that some of the students with context problems did not create a story for each of the data. For example, the student with the code S23 asked "15–4.3=?" Considering the problem he posed for the operation, there is no story related to the numbers 4 and 3 in the question. The expression "12 tickets" was used for the number 12, which is the product of these numbers (See Figure 6).

2) 15 – 4 . 3 = ?	Sıla buys 15 tickets to the amusement park with her brother. Her brother spends 12 tickets how many tickets are left for Sıla?

Figure 6. A problem which took into account the OO / partially suitable for the given operation (S23)

Likewise, the student with the code S8 asked "6+9:3=?" When we look at the problem he posed for the operation, it is seen that he did not create separate stories for the numbers 9 and 3 in the problem, but uses the operation directly and uses the expression "9:3 battery" (See Figure 7).

2) 15 – 4 . 3 = ?	Fatma and Aslı will experiment with battery. That's why Fatma brings 6 batteries, and Aslı brings 9:3 batteries, so how many batteries were brought for the experiment?

Figure 7. A problem which took into account the OO / suitable for the given operation (S8)

Language and expression mistakes were frequently encountered in the answers of the students who pose problems by considering the operation priority. It has been observed that these language and expression mistakes are generally such as not being able to clearly express what they want to ask in the problem, mis-expressing words, expression disorders and inconsistency between the data. Examples of some of these language and expression mistakes are given below. S31 coded student asked "10–4:2=?" The problem posed for the operation is shown in Figure 8. It was observed that while S31 was posing the problem, he started his story by sharing marbles and continuing with buying a car, writing an expression that contains inconsistency between the data, and made a language and expression mistake.

3) 10 – 4 : 2 = ?	Ayşe and Ali have 4 marbles. It is given equally to both. Ali will buy a toy car for 10 TL. How much should he save accordingly?

Figure 8. A problem which took into account the OO and with language and expression mistake (S31)

For example, the student with the code S41 said "6+9:3=?" Considering the OO for the operation, he posed suitable problem, but instead of asking the number of rabbits in a cage, he made a language and expression mistake by asking the total number of rabbits. The problem posed by the student coded S41 is given in Figure 9.

4) 6 + 9 : 3 = ?	They have put 9 rabbits in three cages. They have put 6 more rabbits on it. What is the total number of rabbits?

Figure 9. A problem which took into account the OO and with language and expression mistake (S41)

The answers of the students who posed problems without considering the OO in the research were also analyzed and classified in terms of "Existence of the problem situation, context/story and Language & Expression". The results obtained are listed in Table 6.

Table 6. Analysis of the problems posed by the students who didn't take into account the OO

Problem Posing Application	Problem Situation		Context/ Story		Language & Expression		
Task	Yes	No	Yes	No	Language & Expression Mistakes	Punctuation Mistakes	Terminology Mistakes
1) 2+7.5=?	23	1	19	5	18	20	2
2) 15–4.3=?	26	1	24	3	22	20	2
3) 10–4:2=?	26	1	23	4	23	20	-
4) 6+9:3=?	22	1	19	4	22	22	-

Considering the problems of the students who posed problems without taking into the OO, it is seen that most of the written statements have a problem situation and context. On the other hand, the majority of the problems contain language and expression mistakes. Below are examples of problems in which the OO is not taken into account.

S16's "15-4.3=?" The problem posed for the process is given in Figure 10. S16 did not take into account the OO by posing a problem for subtraction and then multiplication. In addition, he used the phrase "make it according to the OO" instead of the question statement.

4) 6 + 9 : 3 = ?	Ali has 15 TL, he spent 4 TL of it, his friend added 3 times the remaining money according to this how much money is there do it according to the order of operations

Figure 10. A problem which didn't took into account the OO / problem situation (S16)

The student with the code S24 said "2+7.5=?" While posing a problem for the operation, he posed a problem by not taking into account the OO. In addition, the answer of S24 does not include a problem situation (See Figure 11).

4) 6 + 9 : 3 = ?	Ali has 2 apples Ayşe has 12 apples. Ayşe gives Ali 7 of the apples. Uncle Ahmet gives Ali Ayşe 5 boxes of apples from the tree. 2+7.5= 45 whole apples

Figure 11. An answer which didn't took into account the OO / no problem situation (S24)

When the punctuation errors of all the answers obtained in the second stage of the research, "Posing a problem for the OO" stage are examined; it is seen that there are mistakes such as not separating the proper name with an apostrophe, starting with a lowercase letter after the ending sentence, not putting a question mark at the end of the question sentence, not writing the suffix "too" separately, not starting the proper name with a capital letter. It has been observed that terminology mistakes are mostly experienced when posing problems for operations involving multiplication and division. While expressing the division process, the students used mistaken expressions such as "Divisor, Divison". Expressions such as "two divisions, two parts, two halves, third half" were frequently encountered in the answers to questions involving division. It was observed that some students used the expression "half" while expressing division by three. While expressing the subtraction operation, it was seen that they used expressions such as "minus, less". It was observed that terminology mistakes are mostly experienced when posing problems for operations involving multiplication and division. When the problems of the students who took the OO into account and those who did not was compared; it was found that the students who took the OO into account in the areas of "Language & Expression, Punctuation, Terminology, Problem Situation and Context" were more successful.

4. Conclusion and Discussion

In this research, sixth grade middle school students' skills of solving and posing the problems requiring the knowledge of the OO were examined and results were obtained. The problem solving

application for the OO, which is the first stage of the research; was prepared to measure the skill of knowing the superiority of division and multiplication operations over addition and subtraction operations. Based on the fact that most of the students gave correct answers, it can be said that the students were successful in solving problems for the OO. Unlike this study, Yenilmez and Çoksöyler (2018) concluded in their research that students misunderstood a lot about the OO while solving operations.

The second stage of the research is the practice of posing a problem the OO. According to the findings obtained from the answers of the students who tried to pose a problem for the same operations in the first stage, it was seen that most of the students posed problems without taking into account the OO. It was found that the students mostly tried to pose problems in the questions involving subtraction/multiplication and subtraction/division without considering the OO. It was also found that the question in which the OO is taken into account the most is the question involving addition/multiplication operations. In the question involving multiplication and subtraction, multiplication was done before and after subtraction, and the wrong answer was reached as a result of the subtraction, since the subtraction and subtraction could not be written in the correct order. In their research, Yenilmez and Çoksöyler (2018) similarly concluded that although the students knew the OO, they reached erroneous results because they tried to do the operation with priority without considering the order of writing. In this research, as in the studies of Çetinkaya and Soybaş (2018), Dinç (2018) and Türnüklü et al. (2017) and Tabak (2019), it was concluded that the students' problem posing skills for the OO were weak. However, Ev Çimen and Yıldız (2018) found in their research that, unlike this research, students' problem posing skills were at a better level than expected.

In this research, it was observed that the students had more difficulties in the process of problem posing for OO than in problem solving. It was seen that students were significantly more successful in problem solving than in problem posing. In his research, Bunar (2011) concluded that, unlike this research, students were more successful in problem posing than problem solving. Gökkurt et al. (2015) concluded in their researches that students were not successful in both problem solving and problem posing. Cai and Hwang (2002) found that there was a strong relationship between Chinese students' problem solving and problem posing skills in their research comparing the problem solving and problem posing skills of Chinese and American students, while the same is not true for American students.

In this research, it was concluded that the students who were successful in taking into account the OO in problem posing activities had fewer punctuation mistakes, language/expression and terminology mistakes compared to those who did not. In Onkun-Özgür (2018) study, when the posed problems were examined in terms of grammatical mistakes, it was concluded that students with high academic success were able to pose problems with fewer mistakes.

In this research, it was concluded that students had more difficulties when posing problems involving subtraction and division compared to addition and multiplication. In Tertemiz's (2017) study, it was seen that students were more successful in posing problems for mathematical sentences containing addition and subtraction than for mathematical sentences containing multiplication and division operations. In this study, it was determined that the students had difficulties while posing the problem for the division operation. Similarly, from the studies conducted with pre-service teachers, Işık (2011) concluded that the candidates had difficulty in posing problems for dividing fractions, while Işık et al. (2012) found that the candidates had difficulty in posing problems for division with remainders.

4.1. Suggestions

In line with the findings and results obtained in this research, some recommendations can be made. This research was carried out with students at the sixth grade level. The same s research can be done at different grade levels and using different research methods. In this research, the superiority of multiplication and division operations over addition and subtraction operations was examined.

Different studies can be prepared and done with decimal numbers, negative integers, exponential and radical numbers that measure the OO. In addition, the students considered the OO issue as a separate issue from the four operations problems, and they thought that the OO were limited only to "the OO" subject. Therefore, teachers can prevent students from these misconceptions by giving more frequent place to problems that include the OO in different subjects. Finally, considering that students have difficulties in posing problems including subtraction and division, problem posing activities related to these operations can be given more place in mathematics lessons.

5. Declarations

Statement of Research and Publication Ethics Researchers have adhered to all ethical principles and rules in collecting, analyzing, and reporting data. The participant has given his voluntary consent to participate in the study and permission to use the data in a pseudonymized form in publications.

Declarations Conflict of interest. The authors declare no competing interests.

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Appendix 1. Problem solving form for OO (1st stage)

Dear Students,

Solve the following operations within the time given to you. The application consists of four questions.

Good luck.

Operation	Solution
1)2 + 7 . 5 = ?	
2) 15 – 4 . 3 =?	
3)10 – 4 : 2 = ?	
4)6 + 9 : 3 = ?	

Thank You…

Appendix 2. Problem posing form for OO (2nd stage)

Dear Students,

In the time given to you, pose a problem suitable for each of the operations below. The application consists of four problem posing tasks.

Good luck.

Operation	**Problem posed for the given operation**
1)2 + 7 . 5 = ?	
2) 15 – 4 . 3 =?	
3)10 – 4 : 2 = ?	
4)6 + 9 : 3 = ?	

Thank You…

Appendix 3A. Problem posing form evaluation framework for OO / students who take into account OO

	Problem Situation	Suitability to Given Operation	Context Situation	Language/Expression		
	Yes/No	Suitable/ Partially Suitable	Yes/No	Language and Expression Mistakes	Punctuation mistakes	Terminology Mistakes
Students Who Take into Account OO	(Student numbers are given for each category.)					
Total	(The total number of students is indicated.)					

Appendix 3B. Problem posing form evaluation framework for OO / students who don't take into account OO

	Problem Situation	Context Situation	Language/Expression		
	Yes/No	Yes/No	Language and Expression Mistakes	Punctuation Mistakes	Terminology Mistakes
Students Who Don't Take into Account OO	(Student numbers are given for each category.)				
Total	(The total number of students is indicated.)				

Thank You…