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On Vertex-Degree-Based Indices of Monogenic Semigroup Graphs

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Abstract — Albertson and the reduced Sombor indices are vertex-degree-based graph invariants that given in [7] and [20], defined as

$$Alb(G) = \sum_{uv \in E(G)} |d_u - d_v|, \qquad SO_{red}(G) = \sum_{uv \in E(G)} \sqrt{(d_u-1)^2 + (d_v-1)^2},$$

respectively. In this work we show that a calculation of Albertson and reduced Sombor index which are vertex-degree-based topological indices over monogenic semigroup graphs.

Subject Classification (2020): 05C12, 05C50, 15A18, 15A36, 15A42.

1. Introduction and Preliminaries

Let $G = (V, E)$ denotes a simple graph in which vertex and edge sets are indicated by $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$, respectively. Throught this paper the degree of a vertex $v \in V$ will be stipulated by d_v and if the u and v vertices are connected, this will be specified by uv. See [40] for detailed information about graph theory. In [15], the authors introduced a new type of graph relevant to monogenic semigroups, called monogenic semigroup graph. Initially, the authors identified a multiplicative monogenic semigroup that is finite and has one zero element as noted below

$$S_M = \left\{0, x, x^2, x^3, ..., x^n\right\}. \tag{1.1}$$

The researchers indicated the graph of a monogenic semigroup given in (1.1) with $\Gamma(S_M)$. The vertex set of $\Gamma(S_M)$ is $\left\{x, x^2, x^3, ..., x^n\right\}$ i.e. all elements in S_M except zero element and the necessary and the sufficent condition for any two different vertices, x^i and x^j in $\Gamma(S_M)$ to be linked to each other is that $i + j > n$. See [2–4] for detailed knowledge about a graph of a monogenic semigroup. Since graphs of monogenic semigroup are inspired by zero divisor graphs, we will briefly mention about zero divisor graphs. Zero divisor graphs were primarily studied on commutative rings [13]. After this study, studies were also carried out on commutative and non-commutative rings [10–12]. After the studies on zero-divisor graphs on rings, researchers worked on zero-divisor graphs over semigroups [17, 18]. Topological invariants, which have been

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studied for many years in the field of chemistry [39], have recently attracted the attention of mathematicians. Topological indices create structural properties of molecules and equip us with data for industrial science, applied physics, environmental science, and toxicology [19]. The vertex-degree based Sombor index initially presented by Gutman in [20]. It was first used in chemistry [8, 9, 14, 24, 25, 35]. Subsequently fascinated the interest of mathematicians. [16, 23, 26, 32–34]. Network science has exploited the dynamic effect of modeling complex systems in biology and social technology [38]. There are also studies on the Sombor index related to military use [21]. Since its beggining (less than a year after its publication), the Sombor index has also been of interest to mathematicians. In this paper we will compute the Albertson and reduced Sombor index of a monogenic semigroup graph. In [7] the Albertson index given as

$$Alb(G) = \sum_{uv\in E(G)} |d_u - d_v| \tag{1.2}$$

The reduced Sombor index which is a vertex-degree-based index introduced in [20], stated as given below

$$SO_{red}(G) = \sum_{uv\in E(G)} \sqrt{(d_u-1)^2+(d_v-1)^2} \tag{1.3}$$

Every vertex-degree-based topological index can be taken as a special case of a Sombor-type index [22]. Besides, for a real number r, we indicate by $\lfloor r \rfloor$ the greatest integer $\leq r$ and by $\lceil r \rceil$, the least integer $\geq r$. It is quite obvious that $r-1 < \lfloor r \rfloor \leq r$ and $r \leq \lceil r \rceil < r+1$. Furthermore, for a natural number n, we have

$$\left\lfloor \frac{n}{2} \right\rfloor = \begin{cases} \frac{n}{2} & if \ \text{n} \ is \ even \\ \frac{n-1}{2} & if \ \text{n} \ is \ odd \end{cases} \tag{1.4}$$

For further information about monogenic semigroup see [5, 6, 30, 31] and for the applications of some important topological indices in graph theory see [27–29]. In this study, we aim to calculate the Albertson and reduced Sombor index, which are vertex-degree-based indexes of a monogenic semigroup graph.

2. An Algorithm

In [2], the researchers developed an algorithm in relation to the neighborhood of vertices to facilitate the operations. We will consider this algorithm when calculating Albertson and reduced Sombor indices on monogenic semigroups graphs.

I_n: The vertex x^n is linked to every vertex x^{a_1} $(1 \leq a_1 \leq n-1)$ except itself.

I_{n-1}: The vertex x^{n-1} is linked to every vertex x^{a_2} $(2 \leq a_2 \leq n-2)$ except itself and the vertex x^n.

I_{n-2}: The vertex x^{n-2} is linked to every vertex x^{a_3} $(3 \leq a_3 \leq n-3)$ except itself and the vertices x^n and x^{n-1}.

Persisting the algorithm in this way and based on if the number n is odd or even, we get the result given below.

If n is even:

$I_{\frac{n}{2}+2}$: The vertex $x^{\frac{n}{2}+2}$ is linked not only to the vertices $x^{\frac{n}{2}-1}$, $x^{\frac{n}{2}}$ and $x^{\frac{n}{2}+1}$ also linked to the vertices x^n,

x^{n-1}, x^{n-2}, ..., $x^{\frac{n}{2}+3}$.

$I_{\frac{n}{2}+1}$: The vertex $x^{\frac{n}{2}+1}$ is linked not only to the single vertex $x^{\frac{n}{2}}$ also linked to the vertices x^n, x^{n-1}, x^{n-2}, ..., $x^{\frac{n}{2}+2}$.

If n is odd:

$I_{\frac{n+1}{2}}$: The vertex $x^{\frac{n+1}{2}+2}$ is linked not only to the vertices $x^{\frac{n+1}{2}-2}$, $x^{\frac{n+1}{2}-1}$, $x^{\frac{n+1}{2}}$ and $x^{\frac{n+1}{2}+1}$ also linked to the vertices x^n, x^{n-1}, x^{n-2}, ..., $x^{\frac{n+1}{2}+3}$.

$I_{\frac{n+1}{2}+1}$: The vertex $x^{\frac{n+1}{2}+1}$ is linked not only to the vertices $x^{\frac{n+1}{2}-1}$ and $x^{\frac{n+1}{2}}$ also linked to the vertices x^{n-1}, x^{n-2}, ..., $x^{\frac{n+1}{2}+2}$.

In Lemma 2.1, the degrees of vertices in $\Gamma(S_M)$ are indicated by $d_1, d_2, \ldots, d_n$. For more information about degree series see [1, 15] and citations in these papers. You see the proof of Lemma 2.1 in [15].

Lemma 2.1

$$d_1 = 1, \quad d_2 = 2, \quad \ldots \quad , d_{\lfloor \frac{n}{2} \rfloor} = \left\lfloor \frac{n}{2} \right\rfloor, \quad d_{\lfloor \frac{n}{2} \rfloor+1} = \left\lfloor \frac{n}{2} \right\rfloor, \quad d_{\lfloor \frac{n}{2} \rfloor+2} = \left\lfloor \frac{n}{2} \right\rfloor + 1, \quad \ldots \quad , d_n = n-1 \tag{2.1}$$

Remark 2.2 Note the repetitive terms given in Lemma 2.1 as follows,

$$d_{\lfloor \frac{n}{2} \rfloor} = \left\lfloor \frac{n}{2} \right\rfloor = d_{\lfloor \frac{n}{2} \rfloor+1}. \tag{2.2}$$

3. Calculating Albertson Index of $\Gamma(S_M)$

An accurate formula of Albertson index over a graph of a monogenic semigroup will be given in this part

Theorem 3.1. Let S_M denote a monogenic semigroup, as given in (1.1), the Albertson index of the monogenic semigroup graph $\Gamma(S_M)$ is stated as below

$$Alb(\Gamma(S_M)) = \begin{cases} \sum_{k=1}^{\frac{n}{2}-1} \sum_{i=k}^{n-k-1} |(n-k)-i| + \sum_{k=1}^{\frac{n}{2}} \left|(n-k) - \lfloor \frac{n}{2} \rfloor\right| & if \quad \text{n} \quad is \quad even \\ \sum_{k=1}^{\frac{n-1}{2}} \sum_{i=k}^{n-k-1} |(n-k)-i| + \sum_{k=1}^{\frac{n-1}{2}} \left|(n-k) - \lfloor \frac{n}{2} \rfloor\right| & if \quad \text{n} \quad is \quad odd \end{cases} \tag{3.1}$$

Proof.
Here our goal is to find a formula for $Alb(\Gamma(S_M))$ by utilising the algorithm given in Section 2. In addition, we will use (1.4), (2.1) equations and Remark 2.2 during our operations.
If n is odd:

$$\begin{aligned} [Alb]\,(\Gamma(S_M)) = {} & |d_n - d_1| + |d_n - d_2| + |d_n - d_3| + \ldots + |d_n - d_{n-2}| + |d_n - d_{n-1}| + \\ & + |d_{n-1} - d_2| + |d_{n-1} - d_3| + \ldots + |d_{n-1} - d_{n-2}| + \\ & + \ldots + \\ & + \left|d_{\frac{n+1}{2}+2} - d_{\frac{n+1}{2}-2}\right| + \left|d_{\frac{n+1}{2}+2} - d_{\frac{n+1}{2}-1}\right| + \left|d_{\frac{n+1}{2}+2} - d_{\frac{n+1}{2}}\right| + \left|d_{\frac{n+1}{2}+2} - d_{\frac{n+1}{2}+1}\right| \\ & + \left|d_{\frac{n+1}{2}+1} - d_{\frac{n+1}{2}-1}\right| + \left|d_{\frac{n+1}{2}+1} + d_{\frac{n+1}{2}}\right| \end{aligned} \tag{3.2}$$

Consequently, the Albertson index of $\Gamma(S_M)$ is written as given in the following

$$[Alb]\,(\Gamma(S_M) = \sum_{ij \in E(G)} \left|d_i - d_j\right| = [Alb]_n + [Alb]_{n-1} + \ldots + [Alb]_{\frac{n+1}{2}+2} + [Alb]_{\frac{n+1}{2}+1}. \tag{3.3}$$

Equality $\lfloor \frac{n}{2} \rfloor = \frac{n-1}{2}$ given in (1.4) is used while performing these operations (if n is odd).

$$\begin{aligned} [Alb]_n &= |(n-1)-1| + |(n-1)-2| + |(n-1)-3| + \ldots + \left|(n-1) - \left\lfloor \frac{n}{2} \right\rfloor\right| + \ldots + \\ &+ |(n-1)-(n-2)| + \left|(n-1) - \left\lfloor \frac{n}{2} \right\rfloor\right| \\ &= \sum_{i=1}^{n-2} |(n-1)-i| + \left|(n-1) - (\frac{n-1}{2})\right| \end{aligned} \tag{3.4}$$

In case analogous operations are applied to $[Alb]_{n-1}, \ldots [Alb]_{\frac{n+1}{2}+2}$ and $[Alb]_{\frac{n+1}{2}+1}$; we obtain

$$[Alb]_{n-1} = \sum_{i=2}^{n-3} |(n-2)-i| + \left|(n-2) - (\frac{n-1}{2})\right|, \tag{3.5}$$

$$[Alb]_{\frac{n+1}{2}+2} = \left|(\frac{n+3}{2}) - (\frac{n-3}{2})\right| + \left|(\frac{n+3}{2}) - (\frac{n-1}{2})\right| + \left|(\frac{n+3}{2}) - (\frac{n-1}{2})\right| + \left|(\frac{n+3}{2}) - (\frac{n+1}{2})\right| \tag{3.6}$$

and finally

$$[Alb]_{\frac{n+1}{2}+1} = \left|(\frac{n+1}{2}) - (\frac{n-1}{2})\right| + \left|(\frac{n+1}{2}) - (\frac{n-1}{2})\right|. \tag{3.7}$$

Hence

$$[Alb]_n + [Alb]_{n-1} + \ldots + [Alb]_{\frac{n+1}{2}+2} + [Alb]_{\frac{n+1}{2}+1} = \sum_{k=1}^{\frac{n-1}{2}} \sum_{i=k}^{n-k-1} |(n-k)-i| + \sum_{k=1}^{\frac{n-1}{2}} \left|(n-k) - (\frac{n-1}{2})\right| \tag{3.8}$$

If similar operations are performed in case n is even, the following sum is obtained

$$[Alb]_n + [Alb]_{n-1} + \ldots + [Alb]_{\frac{n}{2}+2} + [Alb]_{\frac{n}{2}+1} = \sum_{k=1}^{\frac{n}{2}-1} \sum_{i=k}^{n-k-1} |(n-k)-i| + \sum_{k=1}^{\frac{n}{2}} \left|(n-k) - (\frac{n}{2})\right|. \tag{3.9}$$

4. Calculating Reduced Sombor Index of $\Gamma(S_M)$

An accurate formula of replaced Sombor index over a graph of a monogenic semigroup will be given in this part.

Theorem 4.1. Let S_M denote a monogenic semigroup, as given in (1.1), the reduced Sombor index of the monogenic semigroup graph $\Gamma(S_M)$ is stated as below

$$SO_{red}(\Gamma(S_M)) = \begin{cases} \sum_{k=1}^{\frac{n}{2}-1} \sum_{i=k}^{n-k-1} \sqrt{(n-k-1)^2 + (i-1)^2} + \sum_{k=1}^{\frac{n}{2}} \sqrt{(n-k-1)^2 + \left(\frac{n}{2}-1\right)^2} & if \ \text{n} \ is \ even \\ \sum_{k=1}^{\frac{n-1}{2}} \sum_{i=k}^{n-k-1} \sqrt{(n-k-1)^2 + (i-1)^2} + \sum_{k=1}^{\frac{n-1}{2}} \sqrt{(n-k-1)^2 + \left(\frac{n-1}{2}-1\right)^2} & if \ \text{n} \ is \ odd \end{cases} \tag{4.1}$$

Proof.

Here our goal is to find an exact formula for $SO_{red}(\Gamma(S_M))$ by utilising the algorithm given in Section 2. Besides, we will utilise (1.4), (2.1) equations and Remark 2.2 during our operations. If n is odd:

$$
\begin{aligned}
[SO_{red}](\Gamma(S_M) &= \sqrt{(d_n-1)^2+(d_1-1)^2}+\sqrt{(d_n-1)^2+(d_2-1)^2}+\sqrt{(d_n-1)^2+(d_3-1)^2}+\ldots \\
&+\sqrt{(d_n-1)^2+(d_{n-2}-1)^2}+\sqrt{(d_n-1)^2+(d_{n-1}-1)^2}+\sqrt{(d_{n-1}-1)^2+(d_2-1)^2} \\
&+\sqrt{(d_{n-1}-1)^2+(d_3-1)^2}+\ldots+\sqrt{(d_{n-1}-1)^2+(d_{n-2}-1)^2}+\ldots+ \\
&+\sqrt{(d_{\frac{n+1}{2}+2}-1)^2+(d_{\frac{n+1}{2}-2}-1)^2}+\sqrt{(d_{\frac{n+1}{2}+2}-1)^2+(d_{\frac{n+1}{2}-1}-1)^2}+\sqrt{(d_{\frac{n+1}{2}+2}-1)^2+(d_{\frac{n+1}{2}}-1)^2} \\
&+\sqrt{(d_{\frac{n+1}{2}+2}-1)^2+(d_{\frac{n+1}{2}+1}-1)^2}+\sqrt{(d_{\frac{n+1}{2}+1}-1)^2+(d_{\frac{n+1}{2}-1}-1)^2}+\sqrt{(d_{\frac{n+1}{2}+1}-1)^2+(d_{\frac{n+1}{2}}-1)^2}
\end{aligned}
\tag{4.2}
$$

Consequently, the replaced Sombor index of $\Gamma(S_M)$ is written as stated below

$$
[SO_{red}](\Gamma(S_M) = \sum_{ij\in E(G)}\sqrt{(d_i-1)^2+(d_j-1)^2} = [SO_{red}]_n+[SO_{red}]_{n-1}+\ldots+[SO_{red}]_{\frac{n+1}{2}+2}+[SO_{red}]_{\frac{n+1}{2}+1}
\tag{4.3}
$$

$\lfloor \frac{n}{2} \rfloor = \frac{n-1}{2}$ equality which is given in (1.4) is utilised while performing these operations (if n is odd).

$$
\begin{aligned}
[SO_{red}]_n &= \sqrt{((n-1)-1)^2+(1-1)^2}+\sqrt{((n-1)-1)^2+(2-1)^2}+\sqrt{((n-1)-1)^2+(3-1)^2}+\ldots \\
&+\sqrt{((n-1)-1)^2+\left\lfloor\frac{n}{2}\right\rfloor^2}+\ldots+\sqrt{((n-1)-1)^2+((n-2)-1)^2}+\sqrt{((n-1)-1)^2+\left\lfloor\frac{n}{2}\right\rfloor^2} \\
&= \sum_{i=1}^{n-2}\sqrt{((n-1)-1)^2+(i-1)^2}+\sqrt{((n-1)-1)^2+(\frac{n-1}{2}-1)^2}
\end{aligned}
\tag{4.4}
$$

In case analogous operations are applied to $[SO_{red}]_{n-1}$, we have the following

$$
[SO_{red}]_{n-1} = \sum_{i=2}^{n-3}\sqrt{((n-2)-1)^2+(i-1)^2}+\sqrt{((n-2)-1)^2+(\frac{n-1}{2}-1)^2},
\tag{4.5}
$$

$$
\begin{aligned}
[SO_{red}]_{\frac{n+1}{2}+2} &= \sqrt{(\frac{n+3}{2}-1)^2+(\frac{n-3}{2}-1)^2} \\
&+\sqrt{(\frac{n+3}{2}-1)^2+(\frac{n-1}{2}-1)^2}+\sqrt{(\frac{n+3}{2}-1)^2+(\frac{n-1}{2}-1)^2}+\sqrt{(\frac{n+3}{2}-1)^2+(\frac{n+1}{2}-1)^2}
\end{aligned}
\tag{4.6}
$$

and finally

$$
[SO_{red}]_{\frac{n+1}{2}+1} = \sqrt{(\frac{n+1}{2}-1)^2+(\frac{n1}{2}-1)^2}+\sqrt{(\frac{n+1}{2}-1)^2+(\frac{n-1}{2}-1)^2}.
\tag{4.7}
$$

Hence

$$
\begin{aligned}
[SO_{red}]_n+[SO_{red}]_{n-1}+\ldots+[SO_{red}]_{\frac{n+1}{2}+2}+[SO_{red}]_{\frac{n+1}{2}+1} &= \sum_{k=1}^{\frac{n-1}{2}}\sum_{i=k}^{n-k-1}\sqrt{(n-k-1)^2+(i-1)^2} \\
&+\sum_{k=1}^{\frac{n-1}{2}}\sqrt{(n-k-1)^2+(\frac{n-1}{2}-1)^2}
\end{aligned}
\tag{4.8}
$$

If similar operations are performed in case n is even, the following sum is obtained

$$[SO_{red}]_n + [SO_{red}]_{n-1} + \ldots + [SO_{red}]_{\frac{n}{2}+2} + [SO_{red}]_{\frac{n}{2}+1} = \sum_{k=1}^{\frac{n}{2}-1} \sum_{i=k}^{n-k-1} \sqrt{(n-k-1)^2 + (i-1)^2} + \sum_{k=1}^{\frac{n}{2}} \sqrt{(n-k-1)^2 + (\frac{n}{2}-1)^2}. \tag{4.9}$$

The following examples reinforce Theorem 3.1 and Theorem 4.1.

Example 4.2. We will compute the Albertson index over a graph of a monogenic semigroup S_M^4. S_M^4 monogenic semigroup and the graph of S_M^4 is given below.

$$S_M^4 = \{x, x^2, x^3, x^4\} \cup \{0\}$$

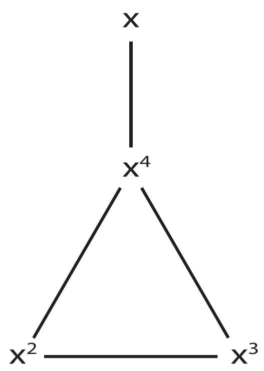

Figure 1. S_M^4 monogenic semigroup graph

$$Alb(\Gamma(S_M^4)) = \sum_{k=1} \sum_{i=1}^{2} |(4-k) - i| + \sum_{k=1}^{2} \left| (4-k) - \left(\frac{4}{2}\right) \right| \tag{4.10}$$
$$= |(4-1)-1| + |(4-1)-2| + |(4-1)-2| + |(4-2)-2| \quad = 2+1+1+0 = 4$$

The reduced Sombor index of $\Gamma(S_M^6)$ monogenic semigroup graph, is computed in the folowing example.

Example 4.3. We will compute the replaced Sombor index of $\Gamma(S_M^6)$ graph. S_M^6 monogenic semigroup and the graph of S_M^6 is given below:

$$S_M^6 = \{x, x^2, x^3, x^4, x^5, x^6\} \cup \{0\}.$$

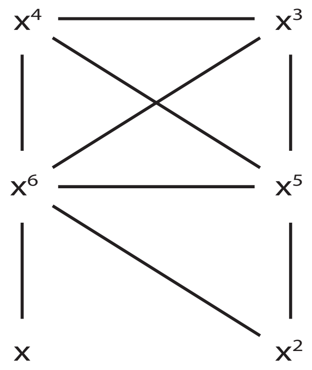

Figure 2. S_M^6 monogenic semigroup graph

$$\begin{aligned} SO_{red}(\Gamma(S_M^6) &= \sum_{k=1}^{2}\sum_{i=k}^{5-k}\sqrt{(6-k-1)^2+(i-1)^2}+\sum_{k=1}^{3}\sqrt{(6-k-1)^2+(3-1)^2} \\ &= \sqrt{4^2}+\sqrt{4^2+1^2}+\sqrt{4^2+2^2}+\sqrt{4^2+3^2}+\sqrt{3^2+1^2}+\sqrt{3^2+2^2} \\ &+\sqrt{4^2+2^2}+\sqrt{3^2+2^2}+\sqrt{2^2+2^2} \\ &= \sqrt{20}+\sqrt{13}+\sqrt{8} \end{aligned} \tag{4.11}$$

As can be clearly seen, with the help of the given main theorems, the monogenic semigroup graphs of Albertson and replaced Sombor indices are easily computed.

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