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# Application of Kashuri Fundo Transform to Decay Problem

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**Abstract:** Recently, it has become quite common to investigate the solutions of problems that have an important place in scientific fields by using integral transforms. The most important reason for this is that this transform allows the simplest and least number of calculations to be made while reaching the solutions of the problems. In this study, we are looking for a solution to the decay problem, which has a very important place in fields such as economics, chemistry, zoology, biology and physics, by using the Kashuri Fundo transform, which is one of the integral transforms. In order to reveal the ease of use of this transform in reaching the solution, some numerical applications were examined. The results of these numerical applications reveal that the Kashuri Fundo transform is quite efficient in reaching the solution of the decay problem.

## Kashuri Fundo Dönüşümünün Bozunma Problemine Uygulanması

**Anahtar Kelimeler**
İntegral dönüşümü,
Kashuri Fundo dönüşümü,
Ters Kashuri Fundo dönüşümü,
Bozunma problemi

**Öz:** Son zamanlarda, bilimsel alanlarda önemli bir yere sahip olan problemlerin çözümlerinin integral dönüşümleri kullanılarak araştırılması oldukça yaygın hale gelmiştir. Bunun en önemli nedeni, bu dönüşümün problemlerin çözümüne ulaşırken en basit ve en az sayıda hesaplamanın yapılmasına olanak sağlamasıdır. Bu çalışmada ekonomi, kimya, zooloji, biyoloji ve fizik gibi alanlarda çok önemli bir yere sahip olan bozunma problemine integral dönüşümlerden biri olan Kashuri Fundo dönüşümü kullanılarak çözüm aranmaktadır. Çözüme ulaşmada bu dönüşümün kullanım kolaylığını ortaya koymak için bazı sayısal uygulamalar incelenmiştir. Bu sayısal uygulamaların sonuçları, Kashuri Fundo dönüşümünün bozunma probleminin çözümüne ulaşmada oldukça verimli olduğunu ortaya koymaktadır.

## 1. Introduction

Differential equations have a very important place in the mathematical modeling of problems in many different fields, especially in applied mathematics and engineering. Most problems in these areas are modeled via differential equations and made more understandable. One of these problems, the problem of decay, has a very important place in scientific fields. The decay problem [1-3] is mathematically defined as

$$\frac{dN(t)}{dt} = -kN(t) \qquad (1)$$

with initial condition as

$$N(0) = N_0 \qquad (2)$$

where , $N$ is the amount of the substance at $t$ and the amount of the substance at $t = 0$ is $N_0$.

The importance of the decay problem in the scientific field has led to the suggestion of various methods to solve these problems. Integral transforms are also included among these methods. These transforms can be considered as very useful methods, especially in terms of providing great convenience in solving initial value problems. The solutions of decay problem using various integral transformations in the literature also confirm this [4-14]. In this study, we investigate the solution of the decay problem by using Kashuri Fundo transform [15], which is one of the integral transformations and provides great convenience in solving differential equations.

Kashuri Fundo transform, like other integral transformations, transforms differential equations

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whose solutions seem complicated into algebraic equations, and provides ease of finding solutions with simpler operations. When the literature is examined, it is possible to come across studies showing that the Kashuri Fundo transform is a very useful method in searching for solutions to different mathematical models encountered in many fields [16-22]. At the same time, the success of finding approximate solutions to some models that seem complex or difficult to solve by blending them with other methods such as Adomian decomposition method and homotopy perturbation method in the literature has been demonstrated by studies [23-28].

## 2. Material and Method

### 2.1. Kashuri Fundo Transform

**Definition 1:** We consider functions in the set $F$ defined as

$$F=\left\{\begin{array}{l} f(t)\mid \exists M, k_1, k_2>0 \quad \text{such that} \\ \mid f(t)\mid \le Me^{\frac{|t|}{k_i^2}}, \\ \text{if} \quad t\in(-1)^i\times[0,\infty) \end{array}\right\} \tag{3}$$

For a function belonging to this set, $M$ must be finite number and $k_1, k_2$ may be finite numbers or infinite [15].

**Definition 2:** Kashuri Fundo transform denoted by the operator $K(.)$ is defined as

$$K[f(t)](v)=A(v)=\frac{1}{v}\int_0^\infty e^{\frac{-t}{v^2}} f(t)dt, \qquad t\ge 0, \quad -k_1<v<k_2 \tag{4}$$

Inverse Kashuri Fundo transform is denoted by $K^{-1}\left[A(v)\right]=f(t), \quad t\ge 0$ [15].

**Theorem 1 (Sufficient Conditions for Existence of Kashuri Fundo Transform):** If $f(t)$ is piecewise continuous on $\left[0,\infty\right)$ and of exponential order $\frac{1}{k^2}$, then $K[f(t)](v)$ exists for $|v|<k$ [15].

**Theorem 2 (Linearity Property):** Let $f(t)$ and $g(t)$ be functions whose Kashuri Fundo integral transforms exists and $c$ be a constant. Then,

$$K[(f\pm g)(t)]=K[f(t)]\pm K[g(t)] \tag{5}$$

$$K[(cf)(t)]=cK[f(t)] \tag{6}$$

hold [15].

**Theorem 3 (Kashuri Fundo Transform of Derivatives of the Function $f(t)$):** Let's assume that the Kashuri Fundo transform of $f(t)$ is $A(v)$. Then,

$$K[f'(t)]=\frac{A(v)}{v^2}-\frac{f(0)}{v} \tag{7}$$

$$K[f''(t)]=\frac{A(v)}{v^4}-\frac{f(0)}{v^3}-\frac{f'(0)}{v} \tag{8}$$

$$K[f^{(n)}(t)]=\frac{A(v)}{v^{2n}}-\sum_{k=0}^{n-1}\frac{f^{(k)}(0)}{v^{2(n-k)-1}} \tag{9}$$

are valid [15].

**Table 1**. Kashuri Fundo Transform of Some Special Functions [15,26].

| $f(t)$ | $K[f(t)]=A(v)$ |
|---|---|
| $1$ | $v$ |
| $t$ | $v^3$ |
| $t^n$ | $n!v^{2n+1}$ |
| $e^{at}$ | $\frac{v}{1-av^2}$ |
| $\sin(at)$ | $\frac{av^3}{1+a^2v^4}$ |
| $\cos(at)$ | $\frac{v}{1+a^2v^4}$ |
| $\sinh(at)$ | $\frac{av^3}{1-a^2v^4}$ |
| $\cosh(at)$ | $\frac{v}{1-a^2v^4}$ |
| $t^\alpha$ | $\Gamma(1+\alpha)v^{2\alpha+1}$ |
| $\sum_{k=0}^{n} a_k t^k$ | $\sum_{k=0}^{n} k!a_k v^{2k+1}$ |

## 3. Results

In this section, having been applied the Kashuri Fundo transform to the general form of the decay problem, sample applications to demonstrate the effectiveness of this transform will be provided.

### 3.1. Kashuri Fundo Transform for Decay Problem

If we apply the Kashuri Fundo transform to the equation (1), we have

$$K\left[\frac{dN(t)}{dt}\right]=K\left[-kN(t)\right] \tag{10}$$

in which the Kashuri Fundo transform of the first derivative on the left-hand side of the equation (10) exists. If we substitute the equivalent in equation (7) here, we find

$$\frac{A(v)}{v^2}-\frac{N(0)}{v}=-kA(v). \tag{11}$$

If we substitute the initial condition into the equation (11) and rearranging this equation, we get

$$A(v)=N_0\left(\frac{v}{1+kv^2}\right). \tag{12}$$

Having applied the inverse Kashuri Fundo transform bilaterally to the equation in (12) and then from table 1, we find

$$N(t)=N_0e^{-kt} \tag{13}$$

which is exactly coincides with the existing results obtained by other works [4-14].

### 3.2. Applications

**Application 1:** Any radioactive substance is known to decay in proportion to the amount available. Consider that $500\,mg$ of a radioactive substance is initially available and after five hours the radioactive substance has lost 25 percent of its available mass. Find the half-life of this radioactive substance.

We can express the mathematical model of the above-mentioned problem as follows:

$$\frac{dN(t)}{dt}=-kN(t) \tag{14}$$

where $N$ represents the amount of radioactive substance at $t$ and $k$ is the constant of proportionality. Assume that the initial amount at $t=0$ is $N_0$.

Having applied bilaterally the Kashuri Fundo transform to the equation in (14), we acquire

$$K\left[\frac{dN(t)}{dt}\right]=K\left[-kN(t)\right]. \tag{15}$$

Now, arranging equation (15) according to equation (7), we have

$$\frac{K\left[N(t)\right]}{v^2}-\frac{N(0)}{v}=-kK\left[N(t)\right]. \tag{16}$$

Substituting the condition $N(0)=N_0=500$ for $t=0$ in equation (16), we get

$$K\left[N(t)\right]-500v=-v^2kK\left[N(t)\right]. \tag{17}$$

Rearranging equation (17), we have

$$K\left[N(t)\right]=\frac{500v}{1+kv^2}. \tag{18}$$

Having applied bilaterally the inverse Kashuri Fundo transform to the equation in (18), we obtain

$$N(t)=500K^{-1}\left[\frac{v}{1+kv^2}\right] \tag{19}$$

$$N(t)=500e^{-kt}. \tag{20}$$

Since 25 percent of the available mass of this radioactive substance is lost at $t=5$, we have

$$N=500-125=375. \tag{21}$$

Using this in equation (20), we get

$$375=500e^{-5k} \tag{22}$$

$$e^{-5k}=0.75 \tag{23}$$

$$k=-\frac{1}{5}\ln 0.75=0.0575. \tag{24}$$

We are looking for $t$ when $N=\frac{N_0}{2}=250$, so from equation (20) we get,

$$250=500e^{-kt}. \tag{25}$$

Now, substituting the value of $k$ found in equation (24) into the equation (25), we obtain the required half-life of the radioactive substance.

$$250=500e^{-0.0575t} \tag{26}$$

$$e^{-0.0575t}=0.5 \tag{27}$$

$$t=-\frac{1}{0.0575}\ln 0.5 \tag{28}$$

$$t=12.05 \quad hours \tag{29}$$

which is exactly coincides with the result obtained by other method [8].

**Application 2:** Any radioactive matter is known to decay in proportion to the amount available. Find the half-life of the radioactive matter for the case where $100\ mg$ of radioactive matter is initially available and after six hours the radioactive matter has lost 30 percent of its available mass.

The mathematical model of the problem expressed here is the same as equation (14) in application 1. Therefore, the expression of the Kashuri Fundo transform of this model will be as in equation (16).

Substituting the condition $N(0)=N_0=100$ for $t=0$ in equation (16), we get

$$K\left[N(t)\right]-100v=-v^2kK\left[N(t)\right]. \tag{30}$$

Rearranging equation (30), we have

$$K\left[N(t)\right]=\frac{100v}{1+kv^2}. \tag{31}$$

Having applied the inverse Kashuri Fundo transform bilaterally to the equation in (31), we acquire

$$N(t)=100K^{-1}\left[\frac{v}{1+kv^2}\right] \tag{32}$$

$$N(t)=100e^{-kt}. \tag{33}$$

Now let's continue the solution using the information that the radioactive matter lost 30 percent of its available mass at $t=6$.

$$\begin{aligned} N &= 100-30 \\ &= 70 \end{aligned} \tag{34}$$

Using this in equation (33), we get

$$70=100e^{-6k} \tag{35}$$

$$e^{-6k}=0.7 \tag{36}$$

$$k=-\frac{1}{6}\ln 0.7=0.059. \tag{37}$$

We are looking for $t$ when $N=\frac{N_0}{2}=50$, so from equation (33) we get,

$$50=100e^{-kt}. \tag{38}$$

Now, substituting the value of $k$ found in equation (37) into the equation (38), we obtain the required half-life of the radioactive matter.

$$50=100e^{-0.059t} \tag{39}$$

$$e^{-0.059t}=0.5 \tag{40}$$

$$t=-\frac{1}{0.059}\ln 0.5 \tag{41}$$

$$t=11.75 \quad hours \tag{42}$$

which is fully agree with the existing result obtained by other method [11].

**Application 3:** It is known that the rate of degradation of a given substance in a given solution at any instant is proportional to the amount present in the solution at that moment. It is known that there are $27\ grams$ of substance in a solution initially and $8\ grams$ of this substance remain after three hours. How much substance remains in the solution after another one hour has passed?

We can express the mathematical model of the above-mentioned problem as follows:

$$\frac{dm(t)}{dt}=-pm(t) \tag{43}$$

where $m\ grams$ represents the amount of the substance left in the solution at any time $t$ and $p$ is the proportionality constant. Assume that the amount of the substance in the solution at $t=0$ is $m_0$.

Having applied bilaterally the Kashuri Fundo transform to the equation in (43), we obtain

$$K\left[\frac{dm(t)}{dt}\right]=K\left[-pm(t)\right]. \tag{44}$$

Now, arranging equation (44) according to equation (7), we have

$$\frac{K\left[m(t)\right]}{v^2}-\frac{m(0)}{v}=-pK\left[m(t)\right]. \tag{45}$$

Substituting the condition $m(0)=m_0=27$ for $t=0$ in equation (45), we get

$$K\left[m(t)\right]-27v=-v^2pK\left[m(t)\right]. \tag{46}$$

Rearranging equation (46), we have

$$K\left[m(t)\right]=\frac{27v}{1+pv^2}. \tag{47}$$

Having applied bilaterally the inverse Kashuri Fundo transform to the equation in (47), we acquire

$$m(t)=27K^{-1}\left[\frac{v}{1+pv^2}\right] \tag{48}$$

$$m(t)=27e^{-pt}. \tag{49}$$

Using the other given condition at $t=3$, i.e. $m(3)=8$, from equation (49) we have

$$8=27e^{-3p} \tag{50}$$

$$e^{-3p}=\frac{8}{27} \tag{51}$$

$$e^{-p}=\left(\frac{8}{27}\right)^{\frac{1}{3}}. \tag{52}$$

Now if we find $m(t)$ for $t=4$ using equation (49), we get

$$m(4)=27e^{-4p} \tag{53}$$

$$m(4)=27\left(\frac{8}{27}\right)^{\frac{4}{3}} \tag{54}$$

$$m(4)=\frac{16}{3}\quad grams \tag{55}$$

which is same as the result obtained by other method [12].

## 4. Discussion and Conclusion

In this study, we successfully applied the Kashuri Fundo transform to the decay problem. In order to demonstrate that the Kashuri Fundo transform is a very convenient method for solving differential equations, we reinforced its application to the decay problem with several numerical examples. By these applications, it has been revealed that decay problem can be solved without complex calculations using Kashuri Fundo transform. Due to the easy to use and effectiveness, many problems involving differential equations can be easily solved using this transform.

## Declaration of Ethical Code

*In this study, we undertake that all the rules required to be followed within the scope of the "Higher Education Institutions Scientific Research and Publication Ethics Directive" are complied with, and that none of the actions stated under the heading "Actions Against Scientific Research and Publication Ethics" are not carried out.*

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