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Localized Energy Associated with Bianchi-Type VI(A) Universe in *f(R)* Theory of Gravity

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Abstract

The energy momentum localization problem is one of the old, very interesting and unsolved puzzels in gravitational theories. Recently this significant problem has been extended to *f(R)*-gravity which is one of the famous modified theories gravity. In the present work, we consider generalized form of the Landau-Liftshitz energy-momentum relation in order to calculate energy distribution associated with the Bianchi VI(A) type space-time. Results were discussed numerically and specified by using of some well-known $f(R)$-gravity models given in literature.

Keywords: Bianchi-type VI(A) spacetime, energy localization, modified gravity

Kütle Çekimsel *f*(*R*) Kuramında Bianchi-Type VI(A) Evreni için Yerelleşmiş Enerji

Özet

Enerji momentum yerelleşme problemi oldukça eski ilginç ve halen çözüm bekleyen kütle-çekim kuramları bulmacasıdır. Son zamanlarda bu problem değiştirilmiş kütle-çekim kuramlarına genişletilmiştir. Sunulan bu çalışmada Bianchi VI(A) tipi uzay-zaman modeline eşlik eden enerji dağılımını hesaplamak için genelleştirilmiş Landau-Liftshitz enerji tanımı göz önünde bulunduruldu. Sonrasında, sonuçlar nümerik olarak analiz edildi. Ek olarak, literatürde iyi bilinen bazı $f(R)$-gravite modelleri için elde edilen sonuçlar özel durumlara indirgendi.

Anahtar Kelimeler: Bianchi-tipi VI(A) uzay zaman, enerji yerelleşmesi, modifiye kütle çekim kuramı

INTRODUCTION

Gravitational energy and momentum localization problem is one of the most popular puzzles in modern gravitation theories and it still remains unsolved. Einstein (1915) is known as the first scientist who worked on energy-momentum pseudo-tensors, later different energy momentum prescriptions such as Tolman (1934), Papapetrou (1948), Landau-Liftshitz (1951), Bergman and Thomson (1953) and Weinberg (1972), have followed his formulation. All of the energy-momentum descriptions except for the Møller (1958) formulation are restricted to make computations in cartesian coordinates. In 1990, Virbhadra (1990) and Rosen and his collaborators (1993) re-opened the energy-momentum localization problem and after those pioneering papers great numbers of work have been prepared by considering different energy momentum complexes and space-time models (Xulu, 2000; Vagenas, 2003; Aydoğdu et al., 2006; Salti and Aydogdu, 2006).

Recently, modified gravitation theories especially $f(R)$ gravity which extends the general theory of relativity have also been taken into account by many scientists to discuss the famous gravitational puzzle again (Capozziello, 2002 Carroll et al., 2004; Starobinsky, 2007; Sharif and Farasat, 2009; Hendi et al., 2014). Making use of the generalized Landau-Liftshitz prescription for the Schwarzschild-de Sitter universe, Multamäki et al. (2008) calculated energy distribution for some well-known $f(R)$ gravity models including constant curvature scalar. Later, Sharif and Farasat (2010), using generalized Landau-Liftshitz energy-momentum prescription, calculated the energy density of plane symmetric and cosmic string space-time models for some famous $f(R)$ gravity choices. Next, Amir and Naheed (2013) considered a spatially homogeneous rotating space-time solution of $f(R)$ gravity to obtain Landau-Liftshitz energy density. Moreover, using some well-known $f(R)$ theory suggestions, Salti et al. (2013)

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also discussed energy-momentum localization problem for Gödel-Type metrics. The above studies motivate to discuss energy-momentum problem for another background in *f*(*R*)-gravity and extend those works.

The *f*(*R*)-gravity is defined by modifying the Einstein-Hilbert action as given below

$$S = -\frac{1}{2\kappa}\int \sqrt{-g}f(R)d^4x + S_m \qquad (1)$$

here $\kappa = 8\pi G$, g represents the determinant of the metric tensor, *f*(*R*) denotes a general function of Ricci scalar and S_m is the matter part of action (Carrol et al., 2004). It is known that the Ricci curvature scalar is given by:

$$R = g^{\mu\nu}R_{\mu\nu}, \qquad (2)$$

where $R_{\mu\nu}$ is the Ricci tensor which is related with the Riemann tensor, i.e. $R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}$,

$$R^{\lambda}_{\mu\lambda\nu} = \partial_\nu \Gamma^{\lambda}_{\mu\nu} - \partial_\sigma \Gamma^{\lambda}_{\mu\nu} + \Gamma^{\eta}_{\mu\sigma}\Gamma^{\lambda}_{\eta\nu} - \Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\eta\sigma}, \qquad (3)$$

and $\Gamma^{\lambda}_{\mu\sigma}$ shows the Christoffel symbols:

$$\Gamma^{\lambda}_{\mu\sigma} = \frac{1}{2}g^{\lambda\beta}(\partial_\sigma g_{\mu\beta} + \partial_\mu g_{\sigma\beta} - \partial_\beta g_{\mu\sigma}). \qquad (4)$$

Varying Equation 1 with respect to the metric tensor yields the following field equation

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - [\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla_\alpha\nabla^\alpha]F(R) = \kappa T_{\mu\nu}. \qquad (5)$$

Here, it has been defined that $F(R) \equiv \frac{df(R)}{dR}$ and ∇_μ represents the covariant derivative. After construction for the vacuum case, i.e. $T = 0$, the corresponding field equation transforms to the following form

$$F(R)R - 2f(R) + 3\nabla_\alpha\nabla^\alpha F(R) = 0. \qquad (6)$$

It can be easily seen that for any constant curvature scalar Equation 6 becomes

$$F(R_0)R_0 - 2f(R_0) = 0, \qquad (7)$$

here we have used that $R = R_0 = constant$. In the non-vacuum case, the constant curvature scalar condition is described by

$$F(R_0)R_0 - 2f(R_0) = \kappa T. \qquad (8)$$

The paper is organized as follows. In the second section, we give a brief information about the Landau-Liftshitz distribution in *f*(*R*) gravity for the Bianchi-VI(A) type space-time. Next, in the third section, we calculate energy density Landau-Liftshitz definition for some specific *f*(*R*) models. Finally, we devote the last section to discussions.

GENERALIZED LANDAU-LIFTSHITZ PRESCRIPTION IN BIANCHI-TYPE VI(A) SPACETIME

The generalized Landau-Liftshitz formulation is given by (Multamäki et al., 2008)

$$\tau^{\mu\nu} = F(R_0)\tau^{\mu\nu}_{LL} + \frac{1}{6\kappa}[F(R_0)R_0 - f(R_0)]\frac{\partial}{\partial x^\gamma}(g^{\mu\nu}x^\gamma - g^{\mu\gamma}x^\nu), \qquad (9)$$

where $\tau^{\mu\nu}_{LL}$ denotes the original Landau-Lifshitz energy-momentum complex written in general relativity and it is defined by the following relation

$$\tau^{\mu\nu}_{LL} = (-g)(T^{\mu\nu} + t^{\mu\nu}_{LL}) \qquad (10)$$

with

$$t^{\mu\nu}_{LL} = \frac{1}{2\kappa}\Big[(2\Gamma^{\gamma}_{\alpha\beta}\Gamma^{\delta}_{\gamma\delta} - \Gamma^{\gamma}_{\alpha\delta}\Gamma^{\delta}_{\beta\gamma} - \Gamma^{\gamma}_{\alpha\gamma}\Gamma^{\delta}_{\beta\delta})(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}g^{\alpha\beta}) + g^{\mu\alpha}g^{\beta\gamma}(\Gamma^{\nu}_{\alpha\delta}\Gamma^{\delta}_{\beta\gamma} + \Gamma^{\nu}_{\beta\gamma}\Gamma^{\delta}_{\alpha\delta} - \Gamma^{\nu}_{\gamma\delta}\Gamma^{\delta}_{\alpha\beta} - \Gamma^{\nu}_{\alpha\beta}\Gamma^{\delta}_{\gamma\delta}) + g^{\nu\alpha}g^{\beta\gamma}(\Gamma^{\mu}_{\alpha\delta}\Gamma^{\delta}_{\beta\gamma} + \Gamma^{\mu}_{\beta\gamma}\Gamma^{\delta}_{\alpha\delta} - \Gamma^{\mu}_{\gamma\delta}\Gamma^{\delta}_{\alpha\beta} - \Gamma^{\mu}_{\alpha\beta}\Gamma^{\delta}_{\gamma\delta}) + g^{\alpha\beta}g^{\gamma\delta}(\Gamma^{\mu}_{\alpha\gamma}\Gamma^{\nu}_{\beta\delta} - \Gamma^{\mu}_{\alpha\beta}\Gamma^{\nu}_{\gamma\delta})\Big]. \qquad (11)$$

Considering 00-component of Equation 9 gives energy density associated with the sellected universe model and it can be written as given below (Multamäki et al., 2008).

$$\tau^{00} = F(R_0)\tau^{00}_{LL} + \frac{1}{6\kappa}[F(R_0)R_0 - f(R_0)](\frac{\partial}{\partial x^i}g^{00}x^i + 3g^{00}). \qquad (12)$$

In the canonical cartesian coordinates, the homogenous Bianchi-Type VI(A) spacetime is defined by the following line-element (Fagundes, 1992):

$$ds^2 = dt^2 - dx^2 - e^{2(A-1)x}dy^2 - e^{2(A+1)x}dz^2, \quad (13)$$

where A is a constant with $0 \leq A \leq 1$. Considering Equation 13 the metric tensor $g_{\mu\nu}$, its inverse form $g^{\mu\nu}$ and $\sqrt{-g}$ for the Bianchi-Type VI(A) model can be written, respectively, as:

$$g_{\mu\nu} = (1, -1, -e^{2(A-1)x}, -e^{2(A+1)x}), \quad (14)$$

$$g^{\mu\nu} = (1, -1, -e^{2(1-A)x}, -e^{-2(A+1)x}), \quad (15)$$

$$\sqrt{-g} = e^{2A} \ . \quad (16)$$

Next, using Equation 4 the nonvanishing component of Christoffel symbols are calculated as

$$\Gamma^1_{22} = (A-1)e^{2(A-1)x}, \quad \Gamma^1_{33} = -(A+1)e^{2(A+1)x}, \quad \Gamma^2_{12} = \Gamma^2_{21} = (A-1), \quad \Gamma^3_{13} = \Gamma^3_{31} = (A+1). \quad (17)$$

and using the above results in to the Equaion 3, the surviving components of Ricci tensor become

$$R_{11} = -2(A^2+1), \quad R_{22} = 2A(1-A)e^{2(A-1)x}, \quad R_{33} = -2A(1+A)e^{2(A+1)x}. \quad (18)$$

Additionally, the constant value of Ricci scalar in Equation 2 is

$$R = R_0 = 6A^2 + 2. \quad (19)$$

Making use of above calculations in to the Equation 11 , the non-vanishing components of $t^{\mu\nu}_{LL}$ are found as

$$t^{00}_{LL} = \frac{1}{\kappa}(1-5A^2), \quad t^{11}_{LL} = \frac{1}{\kappa}(1-A^2), \quad t^{22}_{LL} = \frac{1}{\kappa}\left[\frac{(1+A)^2}{e^{2(A-1)x}}\right], \quad t^{22}_{LL} = \frac{1}{\kappa}\left[\frac{(A-1)^2}{e^{2(A+1)x}}\right]. \quad (20)$$

Also, the non-zero components of $\tau^{\mu\nu}_{LL}$ in the Equations 10 are calculated as:

$$\tau^{00}_{LL} = -\frac{8A^2e^{4Ax}}{\kappa}, \quad \tau^{22}_{LL} = \frac{2}{\kappa}(A+1)^2e^{2(A+1)x}, \quad \tau^{33}_{LL} = \frac{2}{\kappa}(A-1)^2e^{2(A-1)x}. \quad (21)$$

Consequently, in the *f(R)*-gravity, one can easily write down Equation 21 in to the Equation 9, the generalized form of Landau-Liftshitz energy distribution as given below

$$\tau^{00} = -\frac{1}{2\kappa}\{[R_0F(R_0) - f(R_0)] - 16A^2e^{4Ax}F(R_0)\}, \quad (22)$$

and it is also calculated

$$\tau^{0i} = \frac{1}{6\kappa}[f(R_0) - R_0F(R_0)], \quad (i = 1,2,3),$$
$$\tau^{11} = \frac{2}{3\kappa}[f(R_0) - R_0F(R_0)],$$
$$\tau^{22} = \frac{e^{2(1-A)x}}{3\kappa}\{[2+(1-A)x]f(R_0) + [6(A+1)^2e^{4Ax} + (Ax - x - 2)R_0]F(R_0)\},$$
$$\tau^{33} = \frac{e^{-2(1+A)x}}{3\kappa}\{[2-(1+A)x]f(R_0) + [6(A-1)^2e^{4Ax} + (Ax + x - 2)R_0]F(R_0)\}. \quad (23)$$

ENERGY IN SPECIFIC *f(R)* MODELS

There are many suggested models in the $f(R)$ theory of gravity (Capozziello and Laurentis, 2011). In this section of study, we mainly consider five different well-known models to calculate energy momentum distribution associated with Bianchi-Tpe VI(A) spacetime exactly.

The first model (Starobinsky, 1980; Faulkner et al., 2007) is described in a polynomial form:

$$f_{1st}(R) = R + \xi R^2, \quad (24)$$

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where ξ denotes a positive real number.

The second model (Faulkner et al., 2007) is given by

$$f_{2nd}(R) = R - \frac{\varepsilon^4}{R}, \quad (25)$$

where ε is a constant parameter. This model is known also as the dark energy model of *f(R)*-gravity.

The next model (Nojiri and Odintsov, 2007) is defined as

$$f_{3th}(R) = R - pR^{-1} - qR^2, \quad (26)$$

with p and q are constant.

Another one is given by the following definition (Nojiri and Odintsov, 2004):

$$f_{4th}(R) = R - p\ln\left(\frac{|R|}{\sigma}\right) + (-1)^{n-1}qR^n. \quad (27)$$

Here n represents an integer and p, q, σ are constant parameters.

The final model is known as the chameleon model and it is given by (Nojiri and Odintsov, 2007).

$$f_{5th}(R) = R - (1-m)\lambda^2\left(\frac{R}{\lambda^2}\right)^m - 2\Lambda, \quad (28)$$

where Λ denotes the famous cosmological constant, m shows an integer and λ is a constant parameter.

For suitable choices of above constants, all of the *f(R)* models mentioned above can be reduced to the general relativity. Now, considering the above *f(R)* gravity models and Equation 22, one can obtain the following energy densities:

$$\tau_{1st}^{00} = \frac{1}{\kappa}\{2(3A^2+1)^2\xi - 8A^2e^{4A}\,[4\xi(3A^2+1)+1]\}, \quad (29)$$

$$\tau_{2nd}^{00} = \frac{1}{2\kappa(3A^2+1)^2}\{\varepsilon^4 + A^2[3\varepsilon^4 - 16e^{4A}\,(3A^2+1)^2 + 4\varepsilon^4e^{4Ax}]\}, \quad (30)$$

$$\tau_{3th}^{00} = \frac{1}{2\kappa(3A^2+1)^2}\{p(1+3A^2-4A^2e^{4Ax}) + 16Ae^{4Ax}(3A^2+1)^2[4q(3A^2+1)-1] - 4q(3A^2+1)^4\} \quad (31)$$

$$\tau_{4th}^{00} = \frac{1}{2\kappa}\left\{(-1)^n q(1-n)(6A^2+2)^n - p - 16A^2e^{4Ax}\left[1 - \frac{p+2nq(-1)^n(3A^2+1)^n}{2(3A^2+1) - p\ln\left(\frac{6A^2+2}{\sigma}\right)}\right]\right\}, \quad (32)$$

$$\tau_{5th}^{00} = \frac{1}{2\kappa}\left\{2\Lambda - 16A^2e^{4Ax} - \left(\frac{6A^2+2}{\lambda^2}\right)^m \frac{\lambda^2(m-1)[1-m+A^2(3-3m+8me^{4A}\,)]}{3A^2+1}\right\}.$$

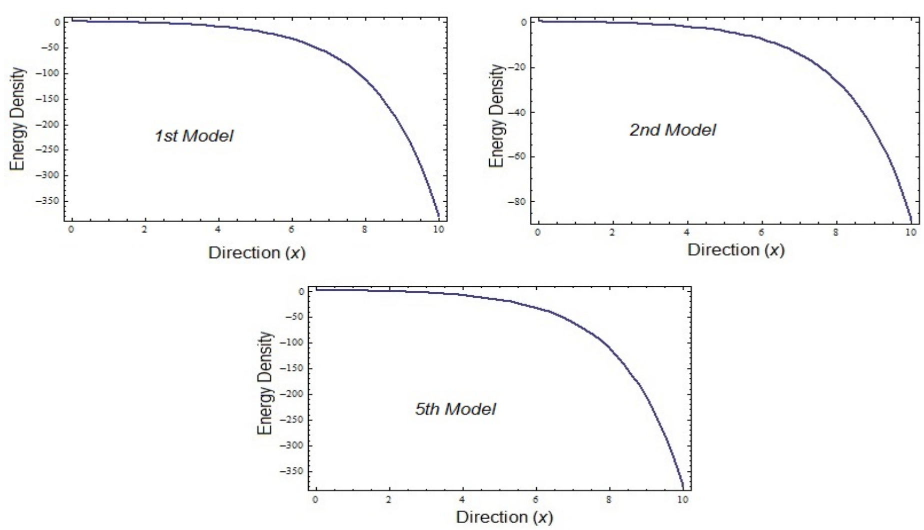

Figure 1. The energy density vs. x for the first, second and fifth $f(R)$ models, respectively. Auxiliary parameters are: $A = 0.5$, $\xi = \kappa = 1 = \varepsilon = \Lambda = \lambda = 1$ and $m = 2$

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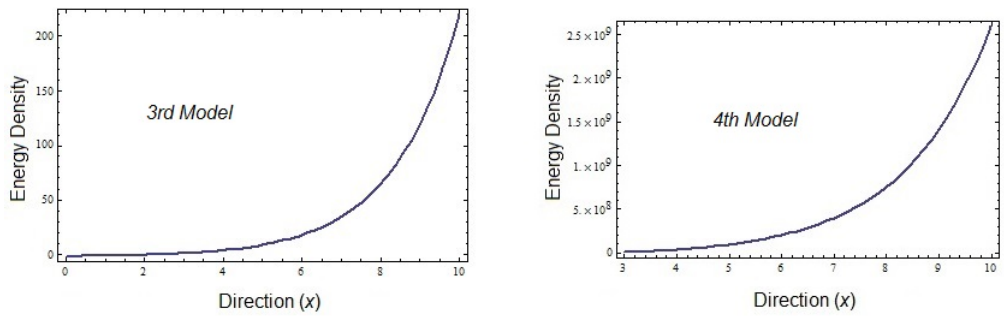

Figure 2. The energy density with respect to x coordinate for the third and fourth models of $f(R)$-gravity. Auxiliary parameters are: $A = 0.5$, $\kappa = p = q = \sigma = 1$ and $n = 20$

In the Figure 1, it is seen that energy densities have negative values and increase with x direction for the first, second and fifth $f(R)$ specific cases. It is obviously seen that from the Figure 2 that energy densities take positive values and increase exponentially.

CONCLUSION

Considering Bianchi-Type VI(A) spacetime representation and some popular models of $f(R)$ gravity including a constant Ricci curvature scalar, we have mainly evaluated the Landau-Liftshitz energy distribution. All of the corresponding calculations have been performed in cartesian coordinates. We have found, in $f(R)$ gravity, the energy distribution associated with the Bianchi-Type VI(A) model as given below:

$$\tau^{00} = -\frac{1}{2\kappa}\{[R_0 F(R_0) - f(R_0)] - 16A^2 e^{4A}\ F(R_0)\}. \tag{34}$$

Assuming $A_{min} = 0$, one can see that energy and momentum distributions transform into the following forms:

$$\tau^{00}_{1st(A=0)} = \frac{2\xi}{\kappa}, \tag{35}$$

$$\tau^{00}_{2nd(A=0)} = \frac{\varepsilon^4}{2\kappa}, \tag{36}$$

$$\tau^{00}_{3th(A=0)} = \frac{p-4q}{2\kappa}, \tag{37}$$

$$\tau^{00}_{4th(A=0)} = \frac{1}{2\kappa}[(-2)^n q(1-n) -p + p\ln\left(\frac{2}{\sigma}\right)], \tag{38}$$

$$\tau^{00}_{5th(A=0)} = \frac{1}{2\kappa}[2\Lambda +2^m(1-m)^2\lambda^2(1-m)]. \tag{39}$$

It is seen that each energy distributions are equal to a constant. Therefore, it can be generalized that

$$\tau^{00}_{All(A=0)} = constant \tag{40}$$

On the other hand, in case of $A = A_{max} = 1$, the energy momentum distributions for all models do not have constant values as expected. Moreover, when we take $f(R) = R_0$ in Equation 22 it can be concluded that

$$\tau^{00}_{GR} = -\frac{8}{\kappa}A^2 e^{4Ax}. \tag{41}$$

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