European Journal of
Engineering and Applied Sciences

ISSN: 2651-3412 (Print) & 2667-8454 (Online)
Journal homepage: http://dergipark.gov.tr/EJEAS
Published by Çorlu Faculty of Engineering, Tekirdağ Namık Kemal University

European J. Eng. App. Sci. 5(2), 80-85, 2022

Research Article

A Microcontroller-based Liénard Oscillator

Ersoy Mevsim[1,a,*], **Reşat Mutlu**[2,b]

[1] **Computer Technology Department, Vocational College of Technical Sciences, Tekirdağ Namık Kemal University, Tekirdağ, Turkey, 59860**
[2] **Electronics and Communication Engineering Department, Çorlu Engineering Faculty, Tekirdağ Namık Kemal University, Tekirdağ ,Turkey, 59860**
[a]emevsim@nku.edu.tr, [b]rmutlu@nku.edu.tr

Received: 25.10.2022 *Accepted: 26.12.2022* *DOI: 10.55581/ejeas.1194452*

Abstract: Van der Pol Equation is a special case of Liénard equations. Both oscillators have significant historical importance. There are lots of different circuit topologies for Van der Pol and Liénard oscillators. Such oscillators can be made using vacuum tubes, diodes, etc. Some oscillators are made using microcontrollers, which are cheap and easy-to-use devices. They provide accurate adjustability of the frequency and magnitude of the waveforms. Arduino Nano Klon V3.0 microcontroller is a commonly used microcontroller. In this study, to the best of our knowledge, for the first time in literature, a Liénard Oscillator has been made with the Direct digital synthesis (DDS) method using the Arduino Nano Klon V3.0 microcontroller and two DACs. The experimental results of the oscillator are given. The circuit is able to produce the state variables of the oscillator, the effect of quantization can be seen on the waveforms, and it is shown that it performs well. The two variable outputs of the system let its phase portrait be examined easily. Also, using a microcontroller helps to design the oscillator in mere a few days.

Keywords: *Microcontroller-based circuit design, Liénard Equation, Liénard Oscillator, Van der Pol oscillator, Digital Circuit Synthesis, Limit Cycle, Circuit Dynamics.*

Mikrodenetleyici Tabanlı Bir Liénard Osilatörü

Öz. Van der Pol Denklemi, Liénard denklemlerinin özel bir halidir. Her iki osilatör de önemli bir tarihsel öneme sahiptir. Pek çok farklı Van der Pol ve Liénard osilatör devreleri mevcuttur. Bu tür osilatörler vakum tüpleri, diyotlar vb. kullanılarak yapılabilmektedir. Bazı osilatörler, ucuz ve kullanımı kolay cihazlar olan mikrodenetleyiciler kullanılarak yapılmaktadır. Mikrodenetleyiciler dalga biçimlerinin frekansının ve büyüklüğünün doğru şekilde ayarlanabilmesini sağlarlar. Arduino Nano Klon V3.0 mikrodenetleyici yaygın olarak kullanılan bir mikrodenetleyicidir. Bu çalışmada bildiğimiz kadarıyla literatürde ilk defa Arduino Nano Klon V3.0 mikrodenetleyici ve iki DAC denetleyici kullanılarak direct dijital sentezleme (DDS) yöntemi ile bir Liénard Osilatörü yapılmıştır. Çıkış akımını artırmak için bir tampon opamp kullanılmıştır. Osilatörün deneysel sonuçları verilmiştir. Devre, osilatörün durum değişkenlerini üretebilir, kuantalama hatasının dalga biçimleri üzerindeki etkisi görülebilmektedir ve bu devrenin iyi performans gösterdiği gösterilmiştir. Sistemin iki değişken çıkışı, faz portresinin kolayca incelenmesini sağlar. Ayrıca, bir mikrodenetleyici kullanmak, osilatörün sadece birkaç gün içinde tasarlanmasını sağlamaktadır.

Anahtar kelimeler: *Mikrodenetleyici tabanlı devre tasarımı, Liénard Denklemi, Liénard Osilatörü, Van der Pol osilatörü, Dijital Devre Sentezi, Limit Döngüsü, Devre Dinamiği.*

1. Introduction

The Van Der Pol Oscillator (VDPO), which has been reported in 1920 [1], made with a triode vacuum tube, is an ancient and still studied oscillator [2]. It is described by the Van der Pol equation [3, 4]. It does not have any analytical solutions [5]. That's why its approximate or numerical solutions are also still being studied [6]. Liénard's equations were suggested by Alfred-Marie Liénard in 1928 to model a

*** Corresponding author**
E-mail address: emevsim@nku.edu.tr (E. Mevsim)

set of oscillating circuits [7]. The Liénard equations define the VDPO as a special case [2]. Both VDPO and the Liénard Oscillator are historically important [2, 7]. Analog circuit implementations of Liénard and Van der Pol oscillators are commonly used for chaos studies [8-10]. A Liénard Oscillator or a VDPO can be made using various semiconductor circuit elements [1, 8- 12]. Liénard Oscillator circuit has numerous circuit variations. An optical resonance tunneling diode-based oscillator can also be modeled with the Liénard equation [12]. In [13], a VDPO is made with anti-parallel Schottky-diode strings. In [14], a VDPO is made with a nonlinear resistance circuit employing a Schottky-diode bridge fed JFET. An FPGA-based chaotic van der pol oscillator is made in [15] but FPGAs are still expensive devices. An alternative way to make Liénard and VDPO oscillators is to use the digital direct synthesis (DDS) method. Such a method is used to make classical signal generators successfully [16, 17]. The method is also used to make the signal generators requiring the solution of the time-dependent state-space equations numerically in [18-25]. There are already various types of microcontroller-based power supplies presented in the literature. In [18], a memristive chaotic circuit is made using a cheap microcontroller Arduino Mega 2560 R3. An ECG signal generator using experimental data is made in [19]. An H-R neuron is emulated with a microcontroller in [20]. The Lorentz system is emulated with an ARM microcontroller [21]. A hyper jerk system is emulated with a microcontroller [22]. Lotka-Volterra Equations are solved, and their variables are emulated with an ARM microcontroller in [23]. A synthetic ECG Generation is made with an ARM microcontroller in [24]. In [25], it is shown that an ARM microcontroller-based EEG signal generator, which makes use of the experimental data, can be implemented using both its DAC and PWM outputs. To the best of our knowledge, a microcontroller-based Liénard Oscillator has not been made yet. Such a circuit can be built with a cheap microcontroller such as PIC16F877A or an Arduino Nano Klon V3.0. In this study, it has been shown that a microcontroller-based Liénard Oscillator can be made using a cheap, rugged, easy-to-use microcontroller such as Arduino Nano Klon V3.0 and two DACs for the first time in the literature. In [18-25], more than one state variables are solved numerically and exported through either DACs or digital ports. In this study, Similarly, the discretized variables of the Liénard system are obtained and the equations are solved numerically using the Euler method with the microcontroller program. The solved state variables are sent out as binary numbers through the digital output ports and their time-dependent waveforms are obtained using two DACs. The simulations are done in Simulink™. Then, the circuit is assembled, and its experimental waveforms are acquired. The currents, voltages, and limit cycle of the Liénard Oscillator circuit are presented.

The paper is arranged as follows. In the second section, basic information on Liénard Equation and Equation System is given. Also, the discrete-time model of the system using the Euler equation is given. Some examples of analog Liénard and VDPO oscillators are given and the microcontroller-based Liénard oscillator circuit is introduced in the third section. In the fourth section, its flowchart is presented. In the fifth section, its Simulink simulation is given. In the sixth section, the experimental results of the circuit are given. The paper is finished with the conclusion section.

2. Liénard and Van Der Pol Equations

In this section, Liénard Equation and Liénard System Equations are given and briefly explained.

2.1. Liénard's Equation

The Liénard equation has been proposed by Alfred-Marie Liénard to model a set of oscillators called Liénard oscillators [7] or Liénard systems and it is given as

$$\frac{d^2x}{dt^2}+f(x)\frac{dx}{dt}+g(x)=0 \tag{1}$$

where f(x) is an even function and g(x) is an odd function.

It is a second-order differential equation and more about the definition of Liénard's equation can be found in [5]. Van der Pol equation is a subset of the Liénard equation and it is defined as

$$\frac{d^2V}{dt^2}-\frac{a}{C}(1-V^2)\frac{dV}{dt}+\frac{1}{LC}V=0 \tag{2}$$

2.2. Liénard System

Let's define

$$x_1 := x$$

$$x_2 := \frac{dx}{dt}+F(x) \tag{3}$$

where

$$F(x)=\int_0^x f(\xi)d\xi \tag{4}$$

With this transformation, the following system is defined as the Liénard System:

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\end{bmatrix}=h(x_1,x_2):=\begin{bmatrix}x_2-F(x_1)\\-g(x_1)\end{bmatrix} \tag{5}$$

Using the Liénard System for programming is more suitable to design the Liénard Oscillator.

2.3. Discretization of the Liénard System

In this study, it is decided to emulate a Van der Pol Oscillator since it is also a Liénard Oscillator. For a Van der Pol Oscillator,

$$f(x)=-(1-x^2) \tag{6}$$

Therefore,

$$F(x_1)=-\int_0^x(1-\xi^2)d\xi=-x_1+{x_1}^2/3 \tag{7}$$

and, then, the Liénard System is given as

$$\frac{dx_1}{dt}=x_2-F(x_1)=x_2+x_1-{x_1}^2/3 \tag{8}$$

$$\frac{dx_2}{dt} = -g(x_1) = -x_1 \quad (9)$$

The Liénard System is easier to discretize than the Liénard equation which would require finite differences. That's why the discrete time model of the Liénard System is derived using Euler method as the follows:

$$x_1[n+1] = \big(x_2[n] - F(x_1[n])\big).T_s + x_1[n]$$

$$(10)$$

$$x_1[n+1] = \big(x_2[n] - F([n])\big).T_s + x_1[n]$$

$$x_2[n+1] = -g(x_1[n]).T_s + x_2[n]$$

$$(11)$$

$$x_2[n+1] = -g([n]).T_s + x_2[n]$$

where $x_1[n]$ and $x_2[n]$ are the nth discrete time values of the state variables of the Liénard System and T_s is the sampling time.

3. Examples to Analog Van der Pol and Liénard Oscillator Circuits and the Proposed Design

In this section, first the generic Liénard Oscillator is summarized and then the Liénard Oscillator circuit proposed in this study is introduced.

3.1. A Few Examples to Analog Van der Pol and Liénard Oscillator Circuits

A Van der Pol Oscillator consists of an inductor, a capacitor and a nonlinear resistor. Since the Van der Pol Oscillator equation is a special case of the Liénard equation, it can be said that the Liénard Oscillator consists of an inductor, a capacitor, and a nonlinear resistor [2, 9, 11, 13, 14]. The general (generic) Liénard oscillator or Van der Pol oscillator circuit can be seen in Figure 1 [9]. Some examples to the Van der Pol and Liénard Oscillator circuits are shown in Figure 2.

As mentioned before, a nonlinear resistor must be used in the Liénard Oscillator. The microcontroller-based Liénard Oscillator made in this study does not need such a nonlinear resistor, whose nonlinear resistance is an even function of voltage and it has a simpler topology as given in the next section.

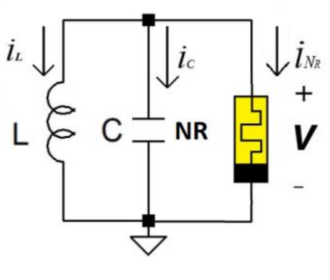


Fig. 1. The generic Liénard Oscillator or the generic Van der Pol Oscillator circuit [9].

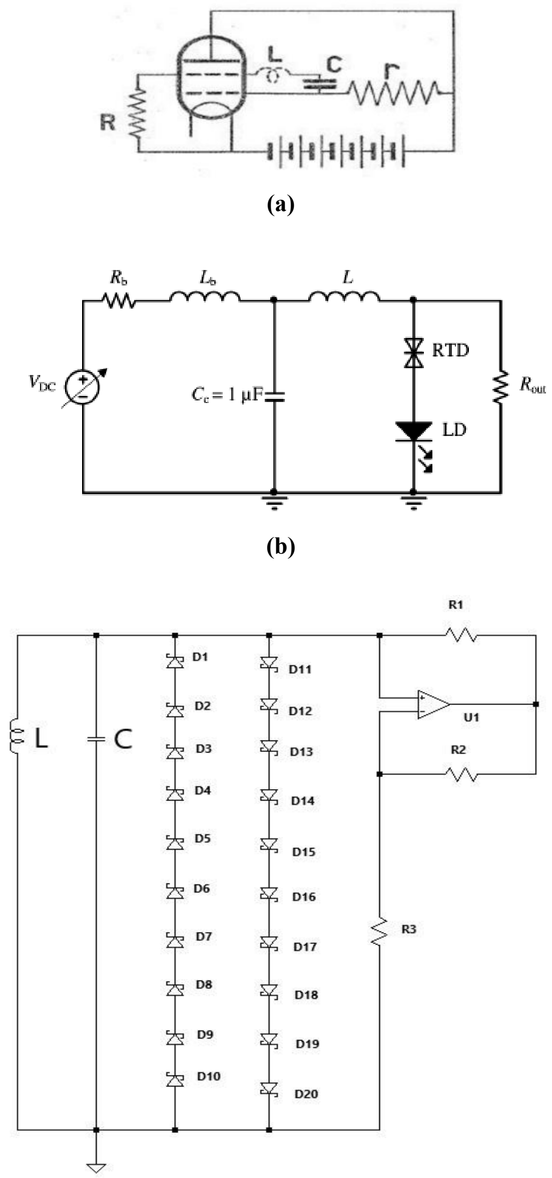


***Fig. 2.** **(a)** The VDPO made in 1926 [2], **(b)** A laser-based VDPO oscillator [11], and **(c)** An Anti-Parallel-Connected Schottky Diode String-Based Van der Pol Oscillator Circuit [13]*

3.2. Microcontroller-based Liénard Oscillator

The schematic of the microcontroller-based Liénard oscillator circuit is shown in Figure 3. It consists of the Arduino Nano Klon V3.0 microcontroller, and two DACs. The microcontroller solves the desired state variables of the Liénard equations x_1 and x_2 numerically using Euler method to obtain both $x_1[n]$ and $x_2[n]$ and sends them to the output using the DACs and operational amplifiers as shown in the Figure 3. At the output of the DACs connected to the microcontroller digital output ports for the time interval, $nT_s \leq t \leq (n+1)T_s$, the state-variables can be expressed as

$$x_1(t) = x_1[n][u(t-(n+1)T_s) - u(t-nT_s)], \quad (12)$$

$$x_2(t) = x_2[n][u(t-(n+1)T_s) - u(t-nT_s)], \quad (13)$$

where $u(t)$ is the unit step function.

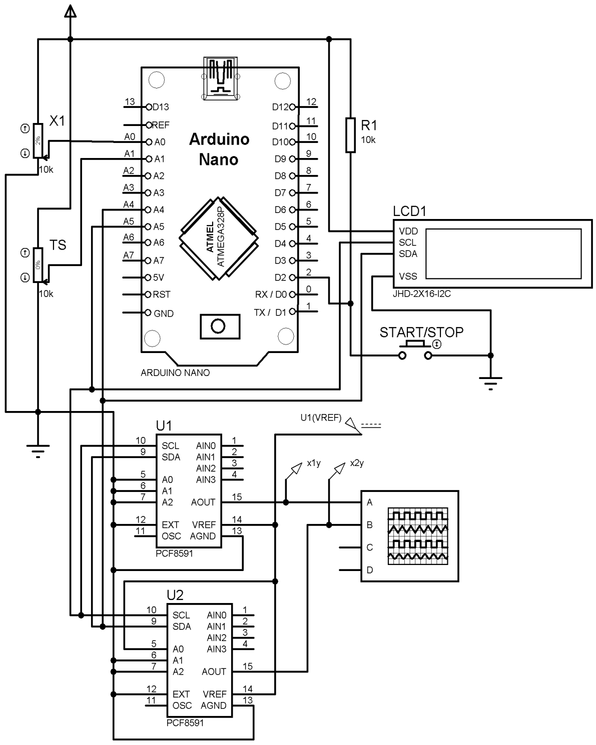


Fig. 3. The Circuit schematic.

4. The Microcontroller Program

The microcontroller is programmed in Arduino IDE 1.8.13. The algorithm of the microcontroller program is given as the flowchart shown in Figure 4.

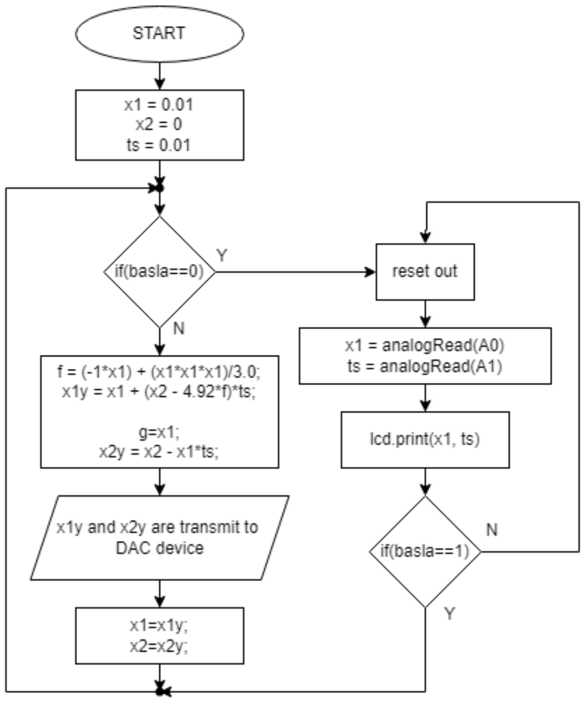


Fig. 4. Flowchart of the VDPO Oscillator Program.

5. The Oscillator Circuit's Simulation in Simulink

The circuit is simulated in Simulink. The Simulink block diagram of the oscillator is shown in Figures 5. x_1 and x_2 state variables obtained from the Simulink simulation is shown in Figure 6. The waveforms are in periodic steady state. Phase portrait of the oscillator is shown in Figure 7.

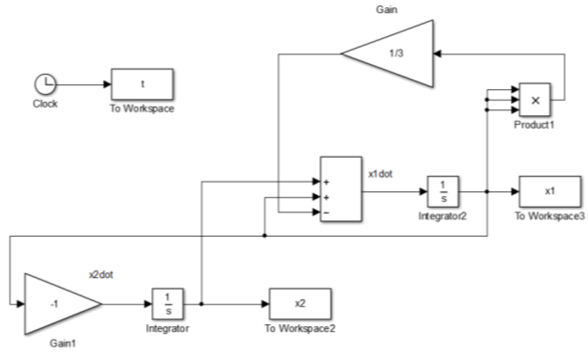
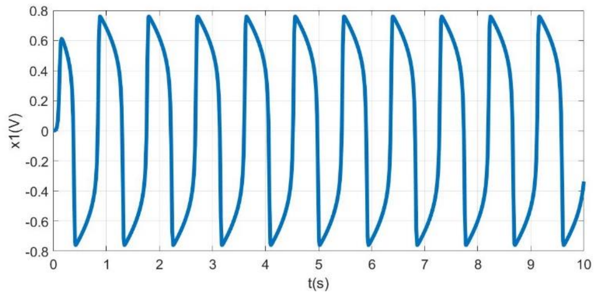
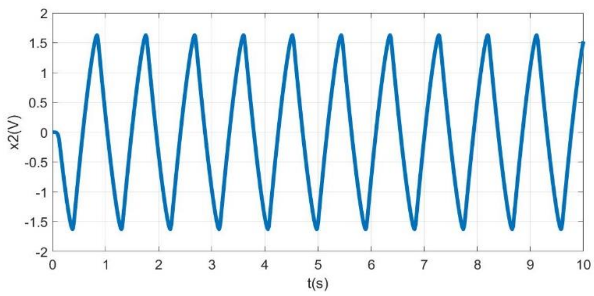


Fig. 5. Simulink block diagram of the Liénard Oscillator.



(a)



(b)

Fig. 6. Time domain waveforms obtained from the Simulink simulation: **(a)** x_1 and **(b)** x_2 versus time.

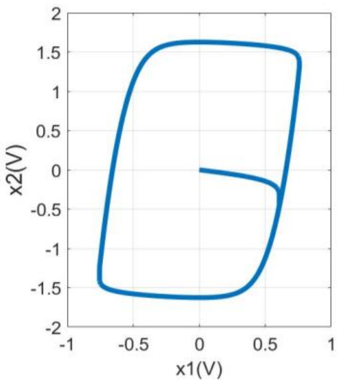


Fig. 7. Phase Portrait of the Oscillator for Van der Pol Equation: x_2 versus x_1.

6. Experimental Results of the Liénard Oscillator

In this section, the experimental results of the microcontroller-based Liénard Oscillator circuit are given.

The circuit whose photo is shown in Figure 8 has been assembled on a protoboard. The experimental waveforms are acquired by GW Instek GDS-1052-U 50 MHz digital oscilloscope. The experimental time domain waveforms are shown in Figure 9. The phase portrait of the oscillator is shown in Figure 10. As they can be seen from Figures 9 and 10, the experimental results resemble the simulated ones given in Figures 6 and 7. The microcontroller-based Liénard Oscillator is clearly able to produce both of the state variables and performs well. The effect of quantization on the waveforms can be seen in Figures 9 and 10. Increasing the DAC resolutions would result in decreasing the effect of quantization.

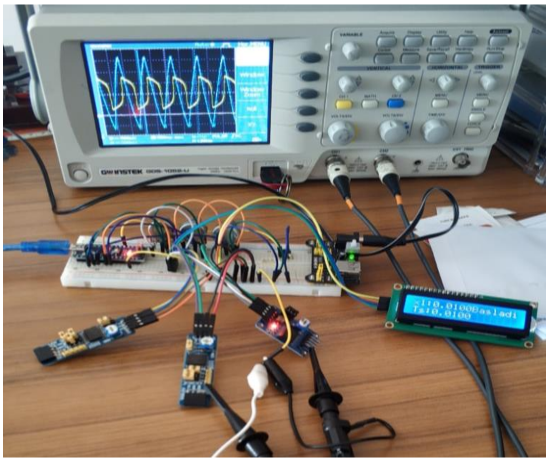
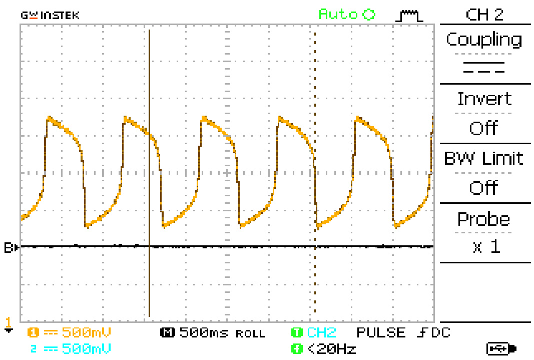
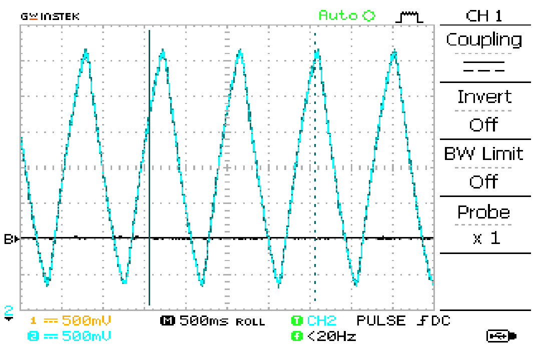


Fig. 8. Photograph of the implemented circuit.



(a)



(b)

Fig. 9. *The experimental state variable waveforms:* **(a)** $x_1(t)$ and **(b)** $x_2(t)$

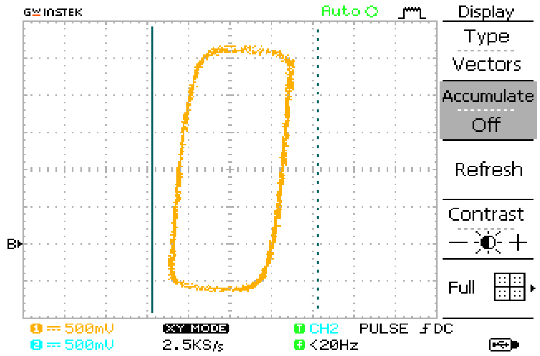


Fig. 10. *Experimental Phase Portrait of the Oscillator for Van der Pol Equation:* x_2 *versus* x_1.

7. Conclusions

In this study, a microcontroller-based Liénard Oscillator is made for the first time in the literature. In this work, a cheap, rugged, easy-to-use microcontroller Arduino Nano Klon V3.0 microcontroller is preferred for this purpose. Liénard System equations are solved with Euler method due to its simplicity. With its DACs, it can produce the state variable signals. The experimental waveforms and the phase portrait are acquired with an oscilloscope. The simulations have confirmed that the circuit operates as a Liénard Oscillator. By modifying its program for different Liénard oscillators, new types of oscillator waveforms can be obtained. The desired equations can be chosen by adding a keypad. By adding potentiometers to make the oscillator parameters adjustable, it can allow examination of the effect of the parameters on the output waveform. The effect of quantization on the waveforms can be reduced using DACs with a higher resolution.

Since the oscillator waveforms are produced in a hybrid way (DDS, direct digital synthesis) not in an analog way, the effect of using different numerical methods such as Euler method and different Runge-Kutta methods on the oscillator system can be inspected. Such an oscillator circuit can also be used in circuit laboratories for educational purposes. Perhaps, in biomedical engineering laboratories since it is an important oscillator circuit to model heart or breathing dynamics.

The coupling of the VDPO or Liénard Oscillators are a hot research area and using a microcontroller-based oscillator circuit for this purpose can make the examination of the coupling of such oscillators easier in the future.

Author Contribution

Formal analysis –Reşat Mutlu (RM); Investigation – RM; Experimental Performance – Ersoy Mevsim (EM); Data Collection RM– EM; Processing – EM; Literature review – RM; Writing – RM, EM; Review and editing – RM, EM;

Declaration of Competing Interest

The authors declared no conflicts of interest with respect to the research, authorship, and/or publication of this article.

References

[1] B. van der Pol, "A theory of the amplitude of free and forced triode vibrations", Radio Review, 1, pp. 701–710, 754–762, 1920.

[2] J. M. Ginoux, C. Letellier, "Van der Pol and the history of relaxation oscillations: Toward the emergence of a concept", Chaos: An Interdisciplinary Journal of Nonlinear Science, 22(2), 023120, 2012.

[3] B. Van der Pol, LXXXVIII. On "relaxation oscillations", The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 2(11), 978- 992, 1926.

[4] B. van der Pol, "The nonlinear theory of electric oscillations", Proc. IRE, 22, pp. 1051–1086, 1934.

[5] M. L. Cartwright, "I. Van der Pol's Equation for Relaxation Oscillations", In Contributions to the Theory of Nonlinear Oscillations (AM-29), Volume II, Princeton University Press., (pp. 1-18), 2016.

[6] T. Marios, "Theoretical and Numerical Study of the Van der Pol equation", Dissertation, 2006.

[7] A. Liénard, "Etude des oscillations entretenues," Revue générale de l'électricité, 23, pp. 901–912 and 946–954, 1928.

[8] J. Gleick, M. Berry, "Chaos-making a new science", Nature, 330, 293, 1987.

[9] S. Ahmad, "Study of Non-linear Oscillations Using Tunnel Diode", Doctoral dissertation, 1962.

[10] J. Brechtl, X. Xie, P. K. Liaw, "Investigation of chaos and memory effects in the Bonhoeffer-van der Pol oscillator with a non-ideal capacitor", Communications in Nonlinear Science and Numerical Simulation, 73, 195-216, 2019.

[11] T. J. Slight, B. Romeira, L. Wang, J. M. Figueiredo, E. Wasige, C. N. Ironside, "A Liénard oscillator resonant tunnelling diode-laser diode hybrid integrated circuit: model and experiment", IEEE journal of quantum electronics, 44(12), 1158-1163, 2008.

[12] T. J. Slight, B. Romeira, L. Wang, J. M. Figueiredo, E. Wasige, C. N. Ironside, "A Liénard oscillator resonant tunnelling diode-laser diode hybrid integrated circuit: model and experiment", IEEE journal of quantum electronics, 44(12), 1158-1163, 2008.

[13] Çakır, K., Mutlu, R., & Karakulak, E. Ters-Paralel Bağlı Schottky Diyot Dizisi Tabanlı Van der Pol Osilatörü Devresinin Modellenmesi ve LTspice ve Simulink Kullanarak Analizi. EMO Bilimsel Dergi, 11(21), 81-91.

[14] Çakır, K., Mutlu, R., Modeling and Analysis of Schottky diode bridge and JFET based Liénard oscillator circuit. DOI: 10.14744/sigma.2022.00082.

[15] M. Dursun, E. Kaşifoğlu, "Design and implementation of the FPGA-based chaotic van der pol oscillator", International Advanced Researches and Engineering Journal, 2(3), 309-314, 2018.

[16] Bilgin, S., Üser, Y., & Oktay, M. (2016). Low cost laboratory type signal generator using DDS method. *International Journal of Engineering and Applied Sciences*, *8*(4), 59-65.

[17] Abdullah, A. I., Mohammed, I. A., & AL-Helali, R. A. (2008). Microcontroller-Based Function Generator. *Al-Khwarizmi Engineering Journal, 4*(1).

[18] Yener, S. C., Barbaros, C., Mutlu, R., & Karakulak, E. (2017). Implementation of Microcontroller-Based Memristive Chaotic Circuit. *Acta Physica Polonica A, 132*(3), 1058-1061

[19] Yener, Suayb Cagri, and Resat Mutlu. "A microcontroller-based ECG signal generator design utilizing microcontroller PWM output and experimental ECG data." *2018 Electric Electronics, Computer Science, Biomedical Engineerings' Meeting (EBBT)*. IEEE, 2018.

[20] Yener, Ş. Ç., & Mutlu, R. (2019, November). A Microcontroller Implementation of Hindmarsh-Rose Neuron Model-Based Biological Central Pattern Generator. In *2019 1st International Informatics and Software Engineering Conference (UBMYK)* (pp. 1-4). IEEE.

[21] Yener, Ş. Ç., Mutlu. R., & Karakulak. E. (2020). Implementation of a Microcontroller-Based Chaotic Circuit of Lorenz Equations. *Balkan Journal of Electrical and Computer Engineering*, *8*(4), 355-360.

[22] Karthikeyan, R., Çiçek, S., Pham, V. T., Akgul, A., & Duraisamy, P. (2020). A class of unexcited hyperjerk systems with megastability and its analog and microcontroller-based embedded system design. *Physica Scripta*, *95*(5), 055214.

[23] Karakulak, E., Tan, R. K., & Mutlu, R. (2021). STM32F429 Discovery Board-Based Emulator for Lotka-Volterra Equations. *Journal of the Institute of Science and Technology*, *11*(3), 1887-1895.

[24] Usta, B. N., Tepeyurt, B., & Karakulak, E. (2021, October). Simple Synthetic ECG Generation via PWM Output of Microcontroller. In *2021 5th International Symposium on Multidisciplinary Studies and Innovative Technologies (ISMSIT)* (pp. 27-30). IEEE.

[25] Karakulak, E., ARM MCU-Based Experimental EEG Signal Generator Using Internal DAC and PWM Outputs. *Gazi University Journal of Science*, 35(3), 886-89