

# The Control of A Non-Linear Chaotic System Using Genetic and Particle Swarm Based On Optimization Algorithms

Ercan Kose\*<sup>1</sup>, Aydin Muhurcu<sup>2</sup>

Accepted 20<sup>th</sup> December 2016

**Abstract:** In this study, the control of a non-linear system was realized by using a linear system control strategy. According to the strategy and by using the controller coefficients, system outputs were controlled for all reference points with the same coefficients via focused references. In the framework of this study, the Lorenz chaotic system as non-linear structure, and the discrete-time PI algorithm as the control algorithm has selected. The genetic algorithm and particle swarm optimization methods have used in the optimization process, and the success of both methods has been discussed among themselves. Closed-loop control system has run simultaneously under the Matlab / Simulink programmer. The results have discussed by using the ISE, IAE, ITAE error criteria, and improved dTISDSE purpose functions.

**Keywords:** Lorenz chaotic system, discrete-time PI controller, genetic algorithm (GA), particle swarm optimization (PSO).

## 1. Introduction

The use of the optimal parameters of the controllers affects the performances of the controllers [1]. Therefore, the optimal parameters of the controller can be determined by using many different optimization algorithms. The optimization process is an important determination factor for performance criteria like the obtained settling time and steady state error. In other words, the optimization process is a significant effect on steady-state and transient behaviours of the system. The Genetic algorithm and PSO technique used in the literature is still widely preferred for the optimization process of the controller [2].

In the 1970, John Holland invented genetic algorithm at Michigan University. Theory of evolution has used for optimization in genetic algorithms by Holland. Genetic algorithms create a solution set consisted of different solution instead of creating one solution for the problems. Thus, a lot of points can be regarded at the same time at search space and as a result, possibility of reaching holistic solutions has been raised [3-14]. In the 1995, Dr. Eberhart and Dr. Kennedy developed Particle swarm optimization (PSO). Particle swarm optimization (PSO) is a population based stochastic optimization technique, inspired by social behaviour of bird flocking or fish schooling [4]. Besides, the PSO unlike GA, it has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles.

Compared to the GA, the advantages of PSO are that PSO is easy to implement and there are few parameters to adjust. PSO has been successfully applied in many areas: function optimization, artificial neural network training, fuzzy system control, and other areas where GA can be applied [5].

Nedjah et al. developed parallel Implementations of the cooperative particle swarm optimization on many-core and multi-core architectures [6]. Mehdinejad et al. have been proposed for the solution of optimal reactive power dispatch of power systems

using hybrid particle swarm optimization and imperialist competitive algorithms [7]. Geng et al. improved to multi-objective operation optimization of ethylene cracking furnace based on an AMOPSO algorithm [8]. Zhang et al. proposed to particle swarm optimization algorithm based on ontology model to support cloud computing applications [9]. Wu et al. have practiced to the parallel particle swarm optimization on a graphics processing unit with application to trajectory optimization [10]. Jeyalakshmi and Subburaj have conducted to the particle swarm optimization-based fuzzy logic controller (PSO FLC) design for load frequency control in a two-area interconnected hydrothermal power system [11]. Saxena and Kumar have performed for reactive power control in a decentralized hybrid power system with STATCOM using GA, ANN and ANFIS methods [12]. Noshadi et al. have used for optimal PID-type fuzzy logic controller for a multi-input multi-output active magnetic bearing system [13]. Olszewski discusses the methodology for optimizing analysis of a complex pumping system with a set of parallel centrifugal pumps [14].

In this study, the optimal coefficients of the PI controller have been obtained by two different optimization algorithms. The chaotic system control has been performed by using the achieved coefficients and then the obtained results are compared with each other.

## 2. Optimization Methodology

Two different algorithms as GA and PSO was used for optimization. The studies related to genetic algorithm are described in detail in conducted a study earlier. In this study; genetic algorithm coefficients, flow diagram and the obtained results can be studied in detail [15].

### 2.1. PSO

The Particle Swarm Optimization (PSO) algorithm is an optimization technique based on swarm intelligence. The search process in the PSO algorithm likes genetic algorithm, and it is performed by the generation number. Each individual is called as the particles, and the swarm is occurred by the particles. Each individual is set own position toward the best position in the swarm via previously own experiences. The process continues until stop

<sup>1</sup>Mechatronic Engineering., Technology Faculty, Mersin University, Mersin, Turkey

<sup>2</sup>Electrical and Electronics Engineering, Faculty of Engineering, Sakarya University, Sakarya, Turkey

\* Corresponding Author: Email: ekose@mersin.edu.tr

criteria. The velocity of the each individual is a state performing randomly [16].

PSO algorithm has a basic two process such as location and velocity, Eq. (1) and Eq. (2).

$$v_i(k+1) = w * v_i(k) + c1 * rand1(p_i(k) - x_i(k)) + c2 * rand2(p_s(k) - x_i(k)) \quad (1)$$

$$x_i(k+1) = x_i(k) + w * v_i(k+1) \quad (2)$$

The equation parameters are given below.

$x_i$  : The swarm, and location of swarm elements,

$v_i$  : The swarm, and velocity of swarm elements,

$p_i$  : The best location of swarm elements,

$p_s$  : The best location of swarm,

$w$  : Swarm and inertia weight of swarm elements,  $0 < w \leq 1$ ,

$c_1, c_2$  : Scaling coefficients,  $0 < c_1, c_2$

$w, c_1$ , and  $c_2$  coefficients was defined like the Eq. (3) by Kennedy ve Eberhart.

$$\chi = \frac{2k}{\left| 2 - (\phi_1 + \phi_2) - \sqrt{(\phi_1 + \phi_2)^2 - 4(\phi_1 + \phi_2)} \right|}, 0 < k \leq 1 \quad (3)$$

$$w = \chi$$

$$c_1 = \chi \phi_1$$

$$c_2 = \chi \phi_2$$

The flow chart of the PSO optimization process of the Kp and Ki parameters of discrete-time PI based closed-loop control system are given in Figure 1. The parameters used in the flow diagram are given below.

$T_s$  : Sampling period,

$f(e)$  : Objective function,

$e_{sum}$  : The total error value.

The first operation performed in the flow diagram is to obtain the discrete time difference equations of the discrete-time PI-based closed-loop control system. Then, the number of swarm member is determined. The location, the speed, and error values for each particle in the swarm is kept constant. The position and velocity data format of the each swarm element for the PI controller is a vector format that is in the form of 2x1 size. For each particle in the swarm, the error value is calculated by using differential equations. The next step, the calculated the new error value is compared with the previous error value. If the error calculation gives the smaller value according to the comparison, then, the new location and speed value is calculated. This process for each of the swarm particles is repeated.

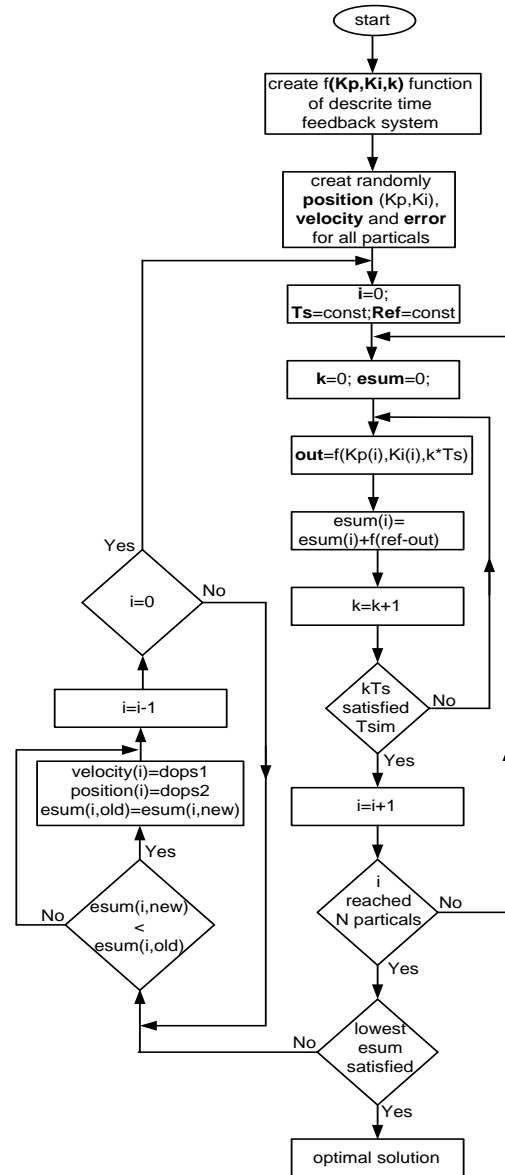


Figure 1. The flow chart of the PSO optimization algorithm

## 2.2. Cost Function

In this study, the cost function has used for two different processes. The first purpose of the using functions is the error minimization process which has been effective realized at the iterative GA and PSO optimization algorithm; The second purpose during the control process, the system controller success with optimized coefficients of the controller is to perform a more thorough detailed analysis.

The results of the ISE, IAE, and ITAE error criteria frequently used in the literature has been observed to be inadequate for the controller coefficient optimization. The objective function called as the discrete time integral sample based double square error (dTISDSE) is improved to obtain a more optimal results from the GA and PSO optimization. The equation is given below in Eq. (4).

$$dTISDSE(e) = \sum_{k=1}^n k * (e_k^2)^2 \quad (4)$$

## 2.3. Results of Optimizaion

The change of the cost function, and the parameter Ki of the PI controller during the GA optimization process is given in Figure 2 and Figure 3.

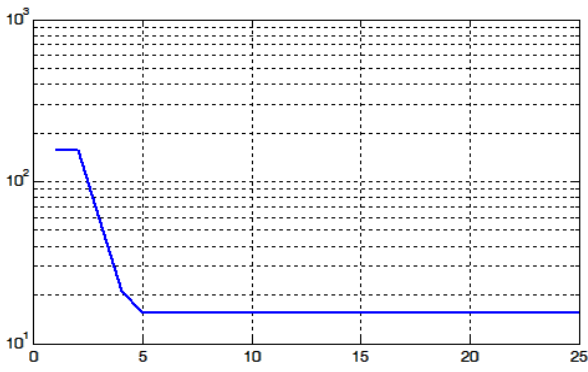


Figure 2. The logarithmic change of the cost function during the GA optimization process

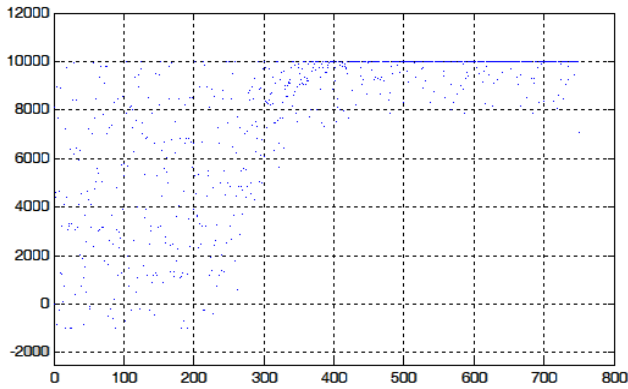


Figure 3. The Ki change in the GA optimization process

On the other hand, the change of the cost function, and the parameter Kp of the PI controller during the PSO optimization process is given in Figure 4 and Figure 5.

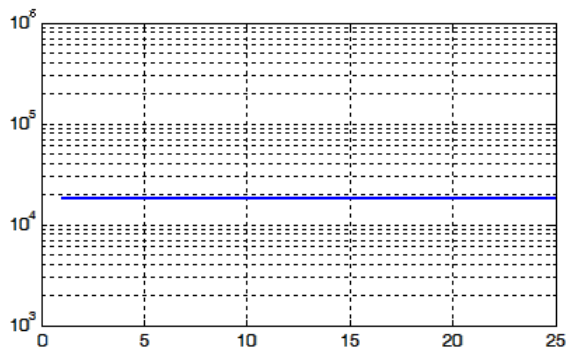


Figure 4. The logarithmic change of the cost function during the PSO optimization process

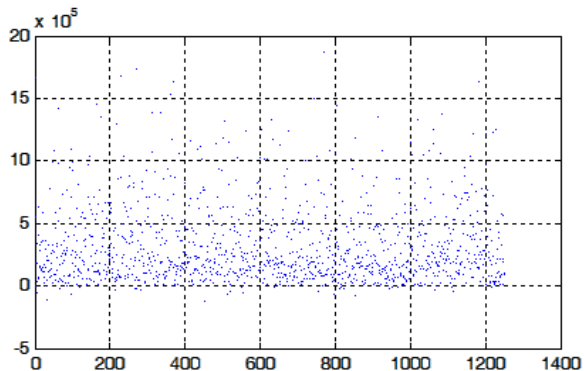


Figure 5. The Kp change in the PSO optimization process

### 3. Proposed Methodology

The mathematical model of the Lorenz chaotic system has given in Eq. (5) where  $x$ ,  $y$  and  $z$  are state variables;  $a$ ,  $b$  and  $c$  are positive constant parameters.

$$\left. \begin{aligned} \frac{\partial x}{\partial t} &= a(y - x) \\ \frac{\partial y}{\partial t} &= x(c - z) - y \\ \frac{\partial z}{\partial t} &= xy - bz \end{aligned} \right\} \quad (5)$$

The system has been added to the controller state variable ( $U_x(s)$ ) for controlling of the Lorenz chaotic system given in Eq. 5. Then the Laplace transform is applied to the system, as a result, Eq. 6 is obtained.

$$\left. \begin{aligned} X(s) &= \frac{1}{s} [aY(s) - aX(s) + U_x(s)] \\ Y(s) &= \frac{1}{s} [X(s)(c - Z(s)) - Y(s)] \\ Z(s) &= \frac{1}{s} [X(s)Y(s) - bZ(s)] \end{aligned} \right\} \quad (6)$$

Before the integral operators, the controller signal has been integrated with the closed-loop control system. Non-linear Lorenz chaotic system structure block diagram with the inclusion of the system and measurement noise is shown in Figure 6.

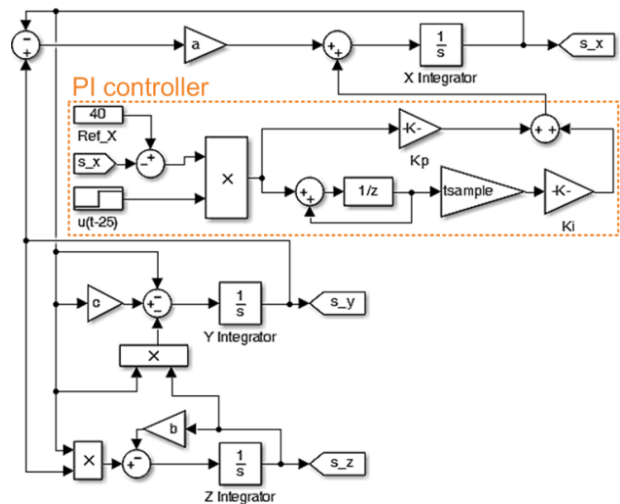
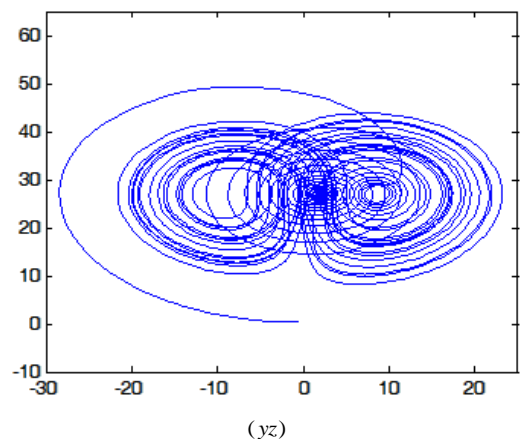
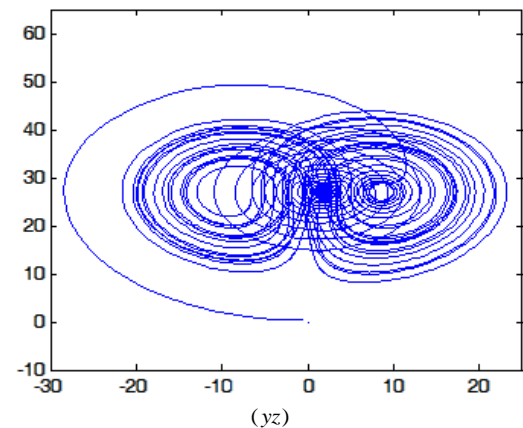
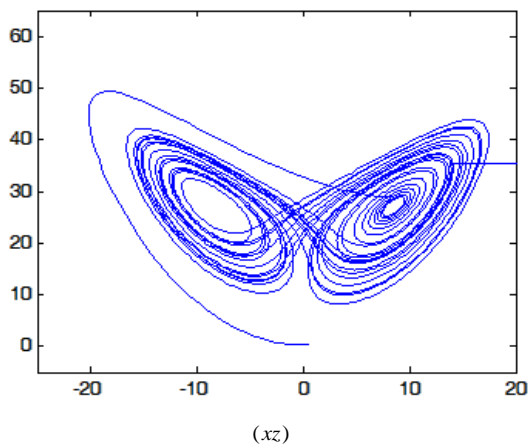
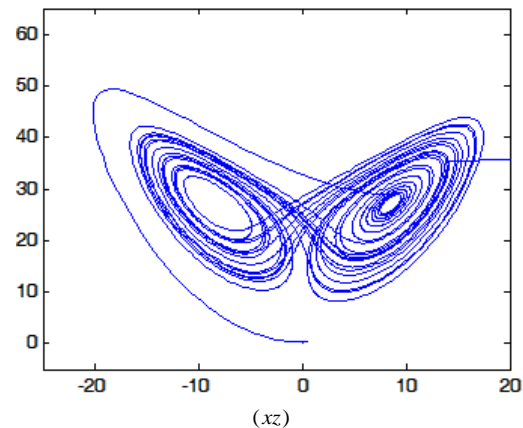
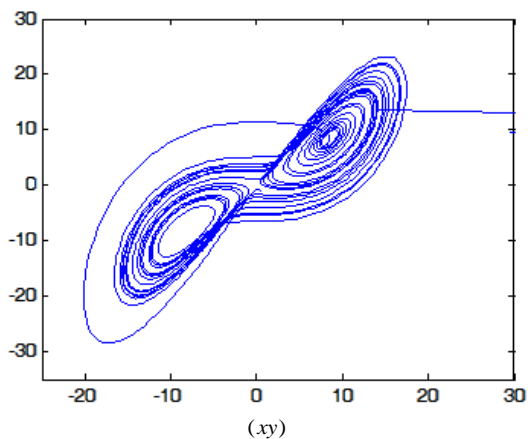


Figure 6. The Lorenz MATLAB/SIMULINK chaos control model for  $x$  state variable

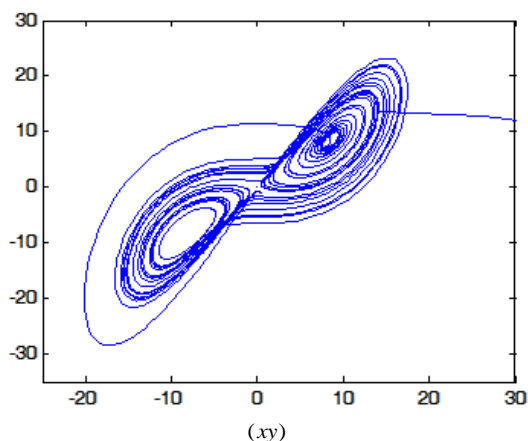
Equation 7,  $PI(s)$  has been converted to  $PI(z)$  block diagram.

$$s \approx \frac{1}{T_s} (1 - z^{-1}) \quad (7)$$

The 2D simulation results are shown in Fig. 7, and Fig. 8.



**Figure 7.** The  $xy$ ,  $xz$  and  $yz$  phase portraits of the Lorenz chaotic system base on GA optimization algorithm



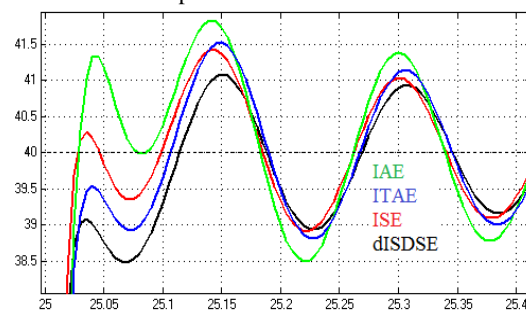
**Figure 8.** The  $xy$ ,  $xz$  and  $yz$  phase portraits of the Lorenz chaotic system base on PSO optimization algorithm

In order to assess of the  $K_p$  and  $K_i$  coefficients based on GA and PSO algorithms of the PI controller, 4 different objective functions given in the Table 1, were used. As shown Table 1, the GA, which is an iterative optimization algorithm, was found more optimal parameter values than the PSO. However, if maximum overshoot value will be close to zero in the control process, the PSO optimization algorithm should be used with simulation results given in Figure 11.

**Table 1.** The error performance criteria of controller

Controller Opt. with	IAE	ISE	ITEA	dISDSE
GA	0.2793	1.169	0.1358	4.27e-11
PSO	0.8594	3.102	0.4947	6.467e-9

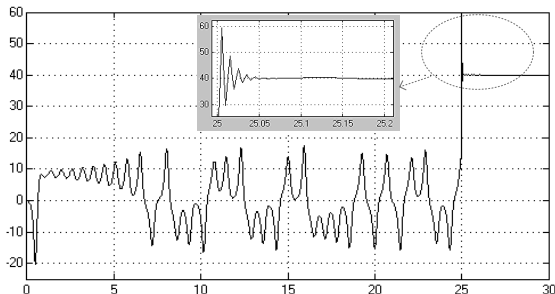
The performance success was given in Figure 9 for the four different objective functions of the PI algorithm based on PSO that activated control process at 25 seconds.



**Figure 9.** The performance success of the  $x$  state variable of the PI controller based on PSO optimization algorithm

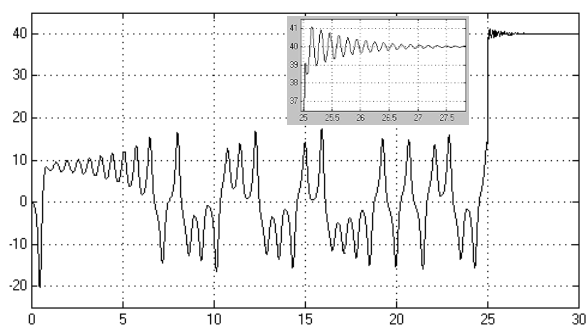
## 4. Results and Discussion

The lowest value of cost value was calculated 15:42 by the genetic algorithm. Besides,  $K_p$  and  $K_i$  controller coefficients were determined as 318.28 and 10000, respectively. The  $x$  state variable of the Lorenz nonlinear system and the control process of controller coefficient optimized by the GA was given in Figure 10. Here, the overshoot was % 50 and the settling time for %1 error band was 25 ms. In addition, the steady-state error was observed as zero.



**Figure 10.** The performance of the PI controller base on GA optimization algorithm for  $x$  state variable

The lowest value of the cost function was calculated as 1.8138e4 by the PSO algorithm. Besides,  $K_p$  and  $K_i$  controller coefficients were determined as 96.58 and 1112.7 respectively. The  $x$  state variable of the Lorenz nonlinear system and the control process of controller coefficient optimized by the PSO was given in Figure 11. Here, the overshoot was % 2.5 and the settling time for %1 error band was 250 ms. In addition, the steady-state error was observed as zero.



**Figure 11.** The performance of the PI controller base on PSO optimization algorithm for  $x$  state variable

## 5. Conclusion

The control process in a non-linear system was illustrated the behavior like a basic linear system control process by using discrete-time PI algorithm via the propose control strategy. During the control of the non-linear system, the control process was not performed around the required operating points. Only one time calculating of the PI coefficients were used for all of the operating points and the output dynamics of the control system also was saved. Besides, in the control of the non-linear system with classical control methods, the process swapping the system output to around of reference points and the run strategy of control mechanism was left. The performance of the proposed control strategy was evaluated with the synchronously simulations. The optimum values of the parameters were found than obtained values with the ISE, IEA and ITAE error functions by using the proposed objective function (dISDSE) for using optimization algorithms like

GA and PSO, and it was illustrated with the results of simulation.

## References

- [1] Puja Dash, Lalit Chandra Saikia, Nidul Sinha (2015). Automatic generation control of multi area thermal system using Bat algorithm optimized PD-PID cascade controller. *Electrical Power and Energy Systems*. Vol. 68. Pages. 364–372.
- [2] Marco Calvini, Mauro Carpita, Andrea Formentini (2015). PSO-Based Self-Commissioning of Electrical Motor Drives. *IEEE Transactions on Industrial Electronics*. Vol. 62. Pages. 768-776.
- [3] [https://tr.wikipedia.org/wiki/Genetik\\_algorithma](https://tr.wikipedia.org/wiki/Genetik_algorithma)
- [4] James Kennedy, Russell Eberhart (1995). Particle swarm optimization. *Proceedings of the Fourth IEEE International Conference on Neural Networks*. Pages. 1942-1948.
- [5] <http://www.swarmintelligence.org/tutorials.php>
- [6] Nadia Nedjah, Rogério de M. Calazan, Luiza de Macedo Mourelle, Chao Wang (2016). Parallel Implementations of the Cooperative Particle Swarm Optimization on Many-core and Multi-core Architectures. *Int J Parallel Prog*. Vol. 44. Pages. 1173–1199.
- [7] M. Mehdinejad, B.M. Ivatloo, R.D. Bonab, K. Zare (2016). Solution of optimal reactive power dispatch of power systems using hybrid particle swarm optimization and imperialist competitive algorithms. *Electrical Power and Energy Systems*. Vol. 83. Pages. 104–116.
- [8] Zhiqiang Geng, Zun Wang, Qunxiong Zhu, Yongming Han (2016). Multi-objective operation optimization of ethylene cracking furnace based on AMOPSO algorithm. *Chemical Engineering Science*. Vol. 153. Pages. 21–33.
- [9] Chijun Zhang, Yongjian Yang, Zhanwei Du, Chuang Ma (2016). Particle swarm optimization algorithm based on ontology model to support cloud computing applications. *Journal of Ambient Intelligence and Humanized Computing*. Vol. 7. Issue. 5. Pages. 633–638.
- [10] Q. Wu, F. Xiong, F. Wang & Y. Xiong (2016). Parallel particle swarm optimization on a graphics processing unit with application to trajectory optimization. *Engineering Optimization*. Vol. 48. Pages. 1679–1692.
- [11] V. Jeyalakshmi, P. Subburaj (2016). PSO-scaled fuzzy logic to load frequency control in hydrothermal power system. *Soft Computing*. Vol. 20. Pages. 2577–2594.
- [12] Nitin Kumar Saxena, Ashwani Kumar (2016). Reactive power control in decentralized hybrid power system with STATCOM using GA, ANN and ANFIS methods. *International Journal of Electrical Power & Energy Systems*. Vol. 83. Pages. 175–187.
- [13] A. Noshadi, J. Shi, Wee Sit Lee, P. Shi, A. Kalam (2016). Optimal PID-type fuzzy logic controller for a multi-input multi-output active magnetic bearing system. *Neural Computing and Applications*. Vol. 27. Issue. 7. Pages. 2031–2046.
- [14] Pawel Olszewski (2016). Genetic optimization and experimental verification of complex parallel pumping station with centrifugal pumps. *Applied Energy*. Vol. 178. Pages. 527–539.
- [15] Aydın Mühürçü, Ercan Köse (2016). Optimal Control of Output Signal of a Chaotic Oscillator Using Genetic Algorithm Based Discrete Time PI Controller. *International Journal of Innovative Research in Science, Engineering and Technology*. Vol. 5. Special Issue 12. Pages 187-195.
- [16] R.C. Eberhart, J. Kennedy (1995). Particle swarm optimization, in: *Proc. of IEEE Int. Conf. on Neural Network*, Perth, Australia. Pages 1942–1948.