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RESEARCH ARTICLE

# Asset Allocation with Combined Models Based on Game-Theory Approach and Markov Chain Models

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**ABSTRACT**

The measurement of expected returns has a major impact on portfolio performance. While there are several methods used for estimating expected returns in existing studies, the mean-variance model most commonly used in portfolio theory utilizes the method of expected returns calculated from historical data. However, the problem with estimating expected returns is that estimating parameters based on historical data, such as the arithmetic mean, may not reflect the distributional characteristics of the return series and may not be an appropriate statistic for the population parameters. Therefore, using robust statistics or combined portfolio models can lead to better portfolios that minimize estimation error while maximizing expected returns. In this paper, we use game theory and Markov chain models to estimate expected asset returns and compare portfolios constructed based on these methods. The analysis results show that the portfolio constructed based on game theory yielded higher returns than the target index and mean-variance model, while the model based on Markov chains yielded portfolios with the lowest portfolio risk. In all out-of-sample investment periods, the game theory based portfolio produced better returns than the portfolios estimated in the study, except for the period from January 2022 to December 2022.

**Keywords:** Portfolio Theory, Game Theory, Markov Chains Model, BIST30

## Introduction

Asset allocation poses one of the greatest problems for investors in the process of constructing a portfolio. The investor who aims to reduce investment risk through diversification is interested in how many stocks should be included in a portfolio. Modern portfolio theory answers this question and aims to maximize the investor's expected return at a given level of risk. Markowitz's mean-variance model clearly articulates the relationship between risk and return. The model is built within a strong logical framework, taking into account the variance-covariance matrix between assets. Despite its advantages, the assumptions about return series have led to some criticisms of the model, the most important of which is the assumption of normality and the concern that the estimation of asset returns should be free from bias. However, return series are generally not normally distributed and have a fat-tailed distribution (Eugene Fama, 1965). Therefore, the use of the arithmetic mean in the case of non-normally distributed returns may lead to biased and incorrect results (Ibragimov, 2005). Under the normality assumption, the classical mean estimator is linear, unbiased, and the best (BLUE) estimator. If this assumption is not met, an estimator that better reflects the characteristics of the distribution should be used to create an efficient portfolio. Therefore, researchers have used a number of estimation methods to better predict the future. Using robust statistics for fat-tailed distributions can lead to the construction of more efficient portfolios. To avoid the estimation biases, Welsch and Zhou (2007) estimated asset returns using a weighted average for "fat-tailed" returns, giving more weight to observations around the mean and less weight to observations near the tail. DeMiguel and Nogales (2009) used robust M and R estimators to remove bias in the estimators. In addition, Hubert, Debruyne and Rousseeuw (2018), Reyna et al. (2005), Yang, Couillet, and McKay (2015) attempted to create portfolios that are superior to the mean-variance model using robust statistics with different properties. In addition to the statistics mentioned above, there are two other downside risk measures or robust risk measures widely used in financial studies: VaR (Value at Risk) and CVaR (Conditional Value at Risk). VaR and CVaR have been used to create efficient portfolios and eliminate the undesirable effect of biased estimators.

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It is widely accepted that a portfolio contains two different risks: systematic and unsystematic. The unsystematic risk can be eliminated by diversification. However, estimated returns have a major impact on portfolios and diversification. An incorrect estimate of returns may lead to an undiversified asset allocation. In this study, we search for a combined portfolio construction process to avoid or minimize estimation errors. We utilized the Markov chain model, which estimates stock returns based on transition probabilities of the system, and game theory to obtain a more accurate estimate of the returns used as parameters for portfolio construction while creating the portfolios. As with the Markov chain model, the game theory approach estimates returns based on probabilities. In game theory, probabilities are assigned using linear programming. In estimating the expected returns of the game theory and Markov chain models, the system was divided into three strategies according to the BIST30 returns: negative returns, neutral or zero returns, and positive returns. It was hoped that computing the weighted average of asset returns as a parameter estimator would yield more efficient portfolios than Markowitz's mean-variance model. Since they use the probabilities obtained for the three given states of the BIST 30 index, the parameters estimated by the combined method correspond to a robust statistical estimate. Thus, it does not require the assumption of normality.

**Literature Reviews**

The calculation of return has a crucial impact on portfolio risk. Unsystematic risk in a portfolio can be eliminated through diversification (Evans and Archer, 1968; Malkiel, 2002; Lhabitant, 2017; Koumou, 2020). However, return series are often not normally distributed and some stocks have infinite variance (Fama, 1965). Thus, although it is possible to reduce portfolio risk to some extent through diversification, it may be necessary to use other methods to eliminate such a risk. Diversification means benefiting from the appreciation of other assets while some assets lose value. However, the direction and strength of relationships among assets play an important role in gaining benefit from diversification (Campbell et al., 2001). In addition to the direction of the relationship, the measurement of asset returns also has a significant impact on the diversification of the models. The fact that returns are usually not normally distributed has led researchers to search for other solutions. Fabozzi et al. (2007) have shown that returns with a fat-tailed distribution significantly affect portfolio performance. It was concluded that other moments besides mean and variance are needed in portfolio optimization. Granito and Walsh (1978), Arditti and Levy (1977), Jobst and Zenios (2001), Chen and Zhou (2018), Gong et al. (2021) have used high moments in portfolio theory in different ways to avoid bias in the estimation of parameters included in the classical mean and variance model. These studies show that portfolios constructed with high moments may produce superior results when compared with the mean-variance model.

In the classical mean-variance model, the expected returns and risks are calculated based on past returns. However, the distributional properties of the return series have the effect of producing unbiased statistics of the parameters used in the model. Although the mean-variance model does not clearly express this, the model assumes that asset return series are normally distributed and will be normally distributed in the future. If the normality assumption is not satisfied, the calculated expected returns lose their reliability (Mandelbrot 1997; Bhansali, 2008; Sheikh and Qiao, 2009; Esch, 2010; Stoyanov et al., 2011; Eom, 2020). At this point, portfolio models that incorporate risk or uncertainty, as opposed to classical optimization, are necessary for asset allocation. Since the Markov chain model uses transition probabilities, it is one of the most important models used in financial studies (McQueen and Thorley, 1991; Özdemir and Demireli, 2014; Yenisu, 2020; Çam, 2021).

Besides the Markov chain model, many methods such as game theory are used to obtain unbiased estimators. Game theory is a method used in many fields from economics to international relations, from tourism to energy studies, and it provides superior results (Song and Zhang, 2013; Zhu-Gang, Wen-Jia, and Can, 2014; Tran and Thompson, 2015; Ruan et al., 2018; Norouzi, Fani, and Talebi, 2022). In the game theory, a well-defined payoff matrix can be used to determine which strategy should play with which probability. Since the probabilities of a game reflect the state of the system and the distributional properties, it can be applied to complex problems such as financial markets. To this end, many important studies have been conducted using game theory (Farias et al., 2006; Ding, 2006; Ferreira et al., 2009; Carfì and Musolino, 2012; Carfì, and Musolino, 2013; Tüfekçi and Avşarlıgil, 2016; Yavuz and Eren, 2016; Essid et al., 2018; Ibrahim et al., 2020; Evangelista, Saporito and Thamsten, 2022).

**Method**

In this paper, we combined three approaches to create more efficient portfolios: Game theory, Markov chains, and Markowitz's mean-variance model. Game theory and Markov chains are widely used models for many optimization problems. Here we compared the portfolios that combine game theory with the mean-variance model and the portfolio that combines the Markov chain model with the mean-variance model.

## Game Theory

Game theory is a simple tool for choosing the most efficient strategy against nature or other players, taking into account all their strategies. It is based on a payoff matrix that represents the outcome (reward) of players' choices in a game. A payoff matrix can be organized with two players: A row player with m strategies and a column player with n strategies.

**Table 1.** The Payoff Matrix of a Game Theory with Two Players

| Row Player's Strategy | Column Player's Strategy | | | |
|---|---|---|---|---|
| | Column 1 | Column 2 | $\cdots$ | Column n |
| Row 1 | $a_{11}$ | $a_{12}$ | $\cdots$ | $a_{1n}$ |
| Row 2 | $a_{21}$ | $a_{22}$ | $\cdots$ | $a_{2n}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | | $\vdots$ |
| Row $m$ | $a_{m1}$ | $a_{m2}$ | $\cdots$ | $a_{mn}$ |

In the table, each row represents the strategy of the row player, while each column defines the strategy of the column player, and $a_{ij}$ is the reward of the game if the row player chooses the i-th strategy and the column player chooses the j-th strategy. Linear programming is used to determine the game value and probability of each player's strategy. The expected return on assets can be determined using the probabilities resulting from linear programming. The basic linear programming formulation of the game can be stated as follows:

$$
\begin{gathered}
Z_{max} = v \\
v \leq a_{11}x_1 + a_{21}x_2 + \cdots + a_{m1}x_m \quad \text{(Column 1 constraint)} \\
v \leq a_{12}x_1 + a_{22}x_2 + \cdots + a_{m2}x_m \quad \text{(Column 2 constraint)} \\
\vdots \\
v \leq a_{1n}x_1 + a_{2n}x_2 + \cdots + a_{mn}x_m \quad \text{(Column n constraint)} \\
x_1 + x_2 + \cdots + x_m = 1 \\
x_i \geq 0 \quad (i = 1, 2, \ldots, m); \quad v\ urs
\end{gathered}
$$

The formulation shown above represents the maximum reward for a row player. But the mathematical formulation of a game theory can be organized for a column player:

$$
\begin{gathered}
Z_{min} = w \\
w \geq a_{11}y_1 + a_{12}y_2 + \cdots + a_{1n}y_n \quad \text{(Row 1 constraint)} \\
w \geq a_{21}y_1 + a_{22}y_2 + \cdots + a_{2n}y_n \quad \text{(Row 2 constraint)} \\
\vdots \\
w \geq a_{m1}y_1 + a_{m2}y_2 + \cdots + a_{mn}y_n \quad \text{(Row m constraint)} \\
y_1 + y_2 + \cdots + y_n = 1 \\
y_j \geq 0 (j = 1, 2, \ldots, n); w\ urs
\end{gathered}
$$

each player's mathematical formulation is a dual formulation of the other and the game value (solution value) would be equal for both. Once the probability of each strategy is calculated, the probabilities obtained from the linear programming solution can be used to determine the assets' expected returns based on game theory (Winston, 2004).

### Markov Chains Model

Markov chains express that a process observed for the current period depends on the state of one previous period (regime) and is independent of past states. In mathematical terms, let a process with N states be observed at time points T=0,1,2,3,...,t, and let $X_1, X_1, \ldots, X_t$ be the observed values for each time point. In this case:

$$P(X_t|X_{t-1}, X_{t-2}, \ldots, X_0) = P(X_t|X_{t-1})$$

The above mathematical formulation defines Markovian processes. Let $S_t$ be a randomly distributed variable that takes integer values and suppose the current value of $S_t$ depends only on period t-1. Now:

$$P\{s_t = j/s_{t-1} = i, s_{t-2} = k, \ldots..\} = Ps_t = j/s_{t-1} = i = P_{ij}$$

where $s_t$=j is the realization of the system in period t, $P_{ij}$ is the transition probability from state i to state j (Hamilton,1994). Such a process is defined as an N-state Markov process with probability matrix $\{P_{ij}\}_{i,j=1,2,3,\ldots,N}$ . The sum of the probabilities for the transition from state i to all other states is 1 and is represented as follows:

$$p_{i1} + p_{i2} + \ldots\ldots + p_{iN} = 1$$

A Markov chain model is said to be stationary (ergodic) if we assume that one eigenvalue of the P matrix is equal to one (unity) and the other eigenvalues lie within the unit circle. The ergodic probabilities for the Markov chain are represented as a $\pi$ vector of size (Nx1).

$$P.\pi = \pi$$

The solution of this equation will provide the ergodic probabilities of the system. Since the sum of the elements of the eigenvalue vector $\pi$ is equal to 1, the vector $\pi$ is normalized. Consequently, the ergodic probabilities represent the probabilities that the system would be in which regime in the long run, and the expected return of each asset from ergodic probabilities would yield the Markovian returns of the assets.

### Mean-Variance Model

The mean-variance model proposed by Markowitz (1952) is one of the most widely used models for determining efficient portfolios. The model takes into account both the expected return and the risk of a portfolio and tries to minimize the risk when maximizing the expected return. A model for portfolio optimization with general constraints is presented as follows:

$$Z_{max} = \mu' w - \lambda . w^1 \sum w$$
$$Aw \leq b$$
$$l \leq w \leq u$$

where μ is the coefficient vector of the objective function, ***w*** is the weight vector, ***A*** is the coefficients matrix of constraints, ***b*** is the vector of right-hand side coefficients, ∑ is the variance-covariance matrix,  is the risk aversion constant, and ***l*** and ***u*** are the lower and upper bound of the weights, respectively.

### Analysis and Results

We used the daily closing price of 29 stocks traded on BIST30 between January 02, 2015, and December 30, 2021. A stock was excluded in the analysis due to the lack of data and the aim was to create optimal portfolios from the remaining stocks. We grouped stock returns according to the three conditions of the BIST30 series: negative index returns, zero index returns, and positive index returns. When grouping daily returns, the returns that were close to but not equal to zero were accepted as neither positive nor negative. Since returns are rarely equal to zero, returns that were close to zero were treated as neutral returns or, in other words, zero returns. In the next step, the expected returns were calculated based on game theory and the Markov chain model using the following formula:

$$E(r_i) = P(R_m \leq 0).E(r_i|R_m \leq 0) + (R_m = 0).E(r_i|R_m = 0) + (R_m \geq 0).E(r_i|R_m \geq 0) or E(r_i) = \Sigma_{t=1}^{3} P(R_m|S_t).E(r_i|R_m = S_t) \quad (1)$$

Here, E($r_i$ ) is the expected return of the i-th stock. P($R_m$ |$S_t$ ) is the conditional probability of the market return based on three scenarios: negative returns, neutral or zero returns, and positive returns. $S_t$ represents the scenarios of the system in and can take three values: $S_t$=1 in which $R_m \leq 0$, $S_t$=2 in which $R_m$=0, and $S_t$=3 in which $R_m \geq 0$. The conditional probability of market returns was obtained from the solution of game theory and the Markov chain model. The game theory model was solved for the column player or, in other words, for the market. The solution of the game provides the conditional probability of market returns in the case of negative, zero and positive returns.

**Table 2.** Descriptive Statistics[1]

| The whole sample | | | | | | | $S_t = 1$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Variable | Obs | Mean | Std. Dev. | Min | Max | Jarque-Bera[1] | Obs | Mean | Std. Dev. | Min | Max |
| AKBNK | 1985 | 0.077 | 2.351 | -10.000 | 10.000 | 686.61*** | 65 | -0.124 | 1.189 | -4.405 | 4.502 |
| AKSA | 1985 | 0.210 | 2.516 | -19.802 | 9.979 | 1238.18*** | 65 | 0.451 | 1.911 | -2.948 | 8.471 |
| ARCLK | 1985 | 0.122 | 2.086 | -12.468 | 9.978 | 818.34*** | 65 | -0.067 | 1.358 | -3.150 | 4.581 |
| BIM | 1985 | 0.111 | 1.802 | -10.000 | 9.915 | 954.21*** | 65 | 0.059 | 1.474 | -1.995 | 7.995 |
| DOHOL | 1985 | 0.161 | 2.759 | -14.471 | 19.551 | 4265.37*** | 65 | 0.346 | 1.801 | -2.473 | 5.736 |
| EKGYO | 1985 | 0.084 | 2.386 | -12.615 | 12.037 | 999.46*** | 65 | -0.191 | 1.492 | -4.935 | 3.723 |
| EREGL | 1985 | 0.177 | 2.274 | -9.981 | 9.978 | 363.57*** | 65 | 0.338 | 1.845 | -3.857 | 7.112 |
| FROTO | 1985 | 0.181 | 2.430 | -10.929 | 10.005 | 733.74*** | 65 | 0.244 | 1.823 | -3.341 | 6.861 |
| GARAN | 1985 | 0.088 | 2.421 | -11.558 | 10.000 | 773.93*** | 65 | -0.043 | 1.087 | -2.795 | 3.033 |
| GUBRF | 1985 | 0.228 | 2.827 | -10.326 | 18.195 | 1550.79*** | 65 | 0.716 | 2.578 | -2.923 | 10.703 |
| ISCTR | 1985 | 0.103 | 2.361 | -13.225 | 9.991 | 891.99*** | 65 | -0.024 | 1.087 | -2.273 | 3.207 |
| KCHOL | 1985 | 0.117 | 2.026 | -9.968 | 9.436 | 680.32*** | 65 | -0.134 | 1.521 | -6.805 | 3.083 |
| KOZAA | 1985 | 0.231 | 3.632 | -20.000 | 20.000 | 2922.90*** | 65 | 0.525 | 4.404 | -10.366 | 19.101 |
| KOZAL | 1985 | 0.205 | 3.155 | -19.955 | 20.000 | 1863.08*** | 65 | 0.335 | 3.662 | -11.090 | 13.434 |
| KRDMD | 1985 | 0.146 | 2.744 | -12.271 | 11.157 | 202.61*** | 65 | 0.243 | 2.041 | -3.551 | 7.258 |
| PETKM | 1985 | 0.159 | 2.269 | -10.410 | 17.993 | 1988.82*** | 65 | 0.042 | 1.976 | -3.422 | 9.929 |
| PGSUS | 1985 | 0.165 | 2.961 | -11.294 | 14.837 | 458.94*** | 65 | -0.353 | 2.577 | -9.973 | 4.433 |
| SAHOL | 1985 | 0.100 | 2.031 | -9.970 | 9.972 | 692.72*** | 65 | -0.181 | 1.280 | -5.290 | 2.665 |
| SASA | 1985 | 0.393 | 3.355 | -16.886 | 20.000 | 1753.50*** | 65 | 0.384 | 3.236 | -9.958 | 15.220 |
| SISE | 1985 | 0.164 | 2.177 | -10.011 | 12.095 | 570.30*** | 65 | 0.231 | 1.418 | -3.394 | 4.190 |
| TAHVL | 1985 | 0.125 | 2.507 | -17.387 | 10.000 | 714.97*** | 65 | -0.313 | 2.058 | -6.405 | 4.090 |
| TCELL | 1985 | 0.087 | 2.014 | -10.000 | 9.955 | 804.78*** | 65 | 0.244 | 1.143 | -3.147 | 3.306 |
| THYAO | 1985 | 0.156 | 2.526 | -12.584 | 9.996 | 481.02*** | 65 | -0.092 | 1.350 | -3.611 | 2.619 |
| TKFEN | 1985 | 0.136 | 2.394 | -10.000 | 16.396 | 454.28*** | 65 | 0.193 | 2.256 | -6.808 | 6.229 |
| TOASO | 1985 | 0.161 | 2.312 | -10.000 | 10.020 | 404.69*** | 65 | 0.403 | 1.808 | -2.892 | 7.581 |
| TTKOM | 1985 | 0.081 | 2.307 | -17.269 | 9.980 | 1428.30*** | 65 | 0.163 | 1.458 | -3.178 | 4.883 |
| TUPRS | 1985 | 0.151 | 2.150 | -9.990 | 10.105 | 778.65*** | 65 | -0.026 | 1.761 | -2.981 | 7.039 |
| VESTL | 1985 | 0.161 | 3.022 | -17.498 | 20.836 | 4188.05*** | 65 | -0.084 | 2.382 | -6.489 | 5.485 |
| YKBNK | 1985 | 0.098 | 2.399 | -13.009 | 11.577 | 706.62*** | 65 | -0.152 | 1.599 | -4.035 | 6.518 |

(***) significance at 1%.

Table 1 shows the descriptive statistics of the stock returns used in the analysis. The descriptive statistics of the whole sample are presented on the left side of the table, while figures on the right side of the table were calculated for $S_t$=1 where market returns were negative. In a sense, these figures are the conditional expected values of the stocks and represent the rewards of the row player in the game when the nature or column player plays his first strategy (negative return).

The rewards of the row player (investor) in the case of the second and third strategy of the column player are summarized in table 2. The conditional expected returns shown on the left side of the table represent the rewards of the row player when $S_t$=2. The returns shown on the right side of the table are the rewards of the row player when $S_t$=3. The conditional probability of the game with respect to the column player's strategies was obtained by solving the linear programming.

1 The null hypothesis of Jarque-Bera test is that "the series is normally distributed."

**Table 3.** Descriptive Statistics (continued

| $S_t = 2$ | | | | | | $S_t = 3$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Variable** | **Obs** | **Mean** | **Std. Dev.** | **Min** | **Max** | **Obs** | **Mean** | **Std. Dev.** | **Min** | **Max** |
| AKBNK | 888 | -1.482 | 1.779 | -10.000 | 4.690 | 1032 | 1.432 | 1.973 | -9.962 | 10.000 |
| AKSA | 888 | -0.655 | 2.605 | -19.802 | 9.979 | 1032 | 0.939 | 2.223 | -9.667 | 9.966 |
| ARCLK | 888 | -0.847 | 1.928 | -12.468 | 7.950 | 1032 | 0.968 | 1.880 | -6.638 | 9.978 |
| BIM | 888 | -0.564 | 1.743 | -10.000 | 7.934 | 1032 | 0.695 | 1.663 | -7.466 | 9.915 |
| DOHOL | 888 | -0.774 | 2.820 | -11.750 | 19.551 | 1032 | 0.955 | 2.494 | -14.471 | 19.213 |
| EKGYO | 888 | -1.149 | 2.113 | -12.615 | 7.761 | 1032 | 1.163 | 2.120 | -8.299 | 12.037 |
| EREGL | 888 | -0.974 | 1.997 | -9.981 | 7.109 | 1032 | 1.158 | 2.050 | -4.257 | 9.978 |
| FROTO | 888 | -0.791 | 2.487 | -10.929 | 10.005 | 1032 | 1.013 | 2.082 | -5.174 | 9.994 |
| GARAN | 888 | -1.428 | 1.892 | -11.558 | 4.394 | 1032 | 1.400 | 2.102 | -9.946 | 10.000 |
| GUBRF | 888 | -0.892 | 2.666 | -10.326 | 18.195 | 1032 | 1.160 | 2.625 | -9.994 | 16.430 |
| ISCTR | 888 | -1.361 | 1.915 | -13.225 | 4.308 | 1032 | 1.370 | 2.012 | -10.000 | 9.991 |
| KCHOL | 888 | -1.085 | 1.726 | -9.968 | 4.163 | 1032 | 1.166 | 1.686 | -4.532 | 9.436 |
| KOZAA | 888 | -0.888 | 3.677 | -20.000 | 19.774 | 1032 | 1.175 | 3.252 | -10.390 | 20.000 |
| KOZAL | 888 | -0.749 | 3.185 | -19.955 | 20.000 | 1032 | 1.018 | 2.854 | -12.099 | 15.470 |
| KRDMD | 888 | -1.271 | 2.433 | -12.271 | 9.735 | 1032 | 1.360 | 2.436 | -6.574 | 11.157 |
| PETKM | 888 | -0.877 | 2.231 | -10.410 | 9.914 | 1032 | 1.059 | 1.911 | -7.593 | 17.993 |
| PGSUS | 888 | -1.160 | 2.720 | -11.294 | 10.000 | 1032 | 1.338 | 2.681 | -6.965 | 14.837 |
| SAHOL | 888 | -1.139 | 1.684 | -9.970 | 4.249 | 1032 | 1.184 | 1.702 | -4.336 | 9.972 |
| SASA | 888 | -0.605 | 3.321 | -14.880 | 19.203 | 1032 | 1.251 | 3.150 | -16.886 | 20.000 |
| SISE | 888 | -0.920 | 2.073 | -10.011 | 12.095 | 1032 | 1.092 | 1.850 | -5.597 | 9.970 |
| TAHVL | 888 | -0.844 | 2.444 | -17.387 | 9.959 | 1032 | 0.986 | 2.264 | -7.126 | 10.000 |
| TCELL | 888 | -0.832 | 1.910 | -10.000 | 9.320 | 1032 | 0.868 | 1.802 | -6.677 | 9.955 |
| THYAO | 888 | -1.229 | 2.246 | -12.584 | 5.790 | 1032 | 1.364 | 2.169 | -6.154 | 9.996 |
| TKFEN | 888 | -0.826 | 2.402 | -10.000 | 16.396 | 1032 | 0.960 | 2.067 | -7.592 | 9.992 |
| TOASO | 888 | -0.829 | 2.240 | -10.000 | 9.347 | 1032 | 0.997 | 2.054 | -6.662 | 10.020 |
| TTKOM | 888 | -1.014 | 2.261 | -17.269 | 9.980 | 1032 | 1.019 | 1.949 | -5.583 | 9.447 |
| TUPRS | 888 | -0.792 | 2.038 | -9.990 | 10.105 | 1032 | 0.974 | 1.919 | -5.971 | 9.979 |
| VESTL | 888 | -0.949 | 2.912 | -17.498 | 16.804 | 1032 | 1.131 | 2.814 | -13.889 | 20.836 |
| YKBNK | 888 | -1.403 | 1.932 | -13.009 | 7.528 | 1032 | 1.405 | 2.014 | -9.966 | 11.577 |

The full sample consists of 1985 daily returns. Of these, 65 were neutral or zero, 888 were negative, and 1032 were positive. For the full sample, the expected return on all stocks was positive, with SASA having the highest expected return at 0.393% and AKBNK having the lowest expected return at 0.077%. The standard deviation and range of returns were high compared to the subsample statistics. According to the Jarque-Bera normality test, the null hypothesis stating the normality of the series is rejected at 1% for all stocks. It can be concluded that none of the return series was normally distributed. For $S_t$=1, more than half of the stocks had positive expected returns. This means that a stock has almost a 50% chance of having a positive expected return when BIST30 has a neutral or zero return. When BIST30 had no positive or negative return, KOZAA had the highest positive return of 19.101% and KOZAL had the lowest negative return of -11.09% on a trading day. Conditional expected returns can be used to calculate the payoff matrix of the game theory.

**Table 4.** Payoff Matrix

| Stocks | $S_t = 1$ | $S_t = 2$ | $S_t = 3$ | Stocks | $S_t = 1$ | $S_t = 2$ | $S_t = 3$ |
|---|---|---|---|---|---|---|---|
| AKBNK | -0.124 | -1.482 | 1.432 | PETKM | 0.042 | -0.877 | 1.059 |
| AKSA | 0.451 | -0.655 | 0.939 | PGSUS | -0.353 | -1.160 | 1.338 |
| ARCLK | -0.067 | -0.847 | 0.968 | SAHOL | -0.181 | -1.139 | 1.184 |
| BIM | 0.059 | -0.564 | 0.695 | SASA | 0.384 | -0.605 | 1.251 |
| DOHOL | 0.346 | -0.774 | 0.955 | SISE | 0.231 | -0.920 | 1.092 |
| EKGYO | -0.191 | -1.149 | 1.163 | TAHVL | -0.313 | -0.844 | 0.986 |
| EREGL | 0.338 | -0.974 | 1.158 | TCELL | 0.244 | -0.832 | 0.868 |
| FROTO | 0.244 | -0.791 | 1.013 | THYAO | -0.092 | -1.229 | 1.364 |
| GARAN | -0.043 | -1.428 | 1.400 | TKFEN | 0.193 | -0.826 | 0.960 |
| GUBRF | 0.716 | -0.892 | 1.160 | TOASO | 0.403 | -0.829 | 0.997 |
| ISCTR | -0.024 | -1.361 | 1.370 | TTKOM | 0.163 | -1.014 | 1.019 |
| KCHOL | -0.134 | -1.085 | 1.166 | TUPRS | -0.026 | -0.792 | 0.974 |
| KOZAA | 0.525 | -0.888 | 1.175 | VESTL | -0.084 | -0.949 | 1.131 |
| KOZAL | 0.335 | -0.749 | 1.018 | YKBNK | -0.152 | -1.403 | 1.405 |
| KRDMD | 0.243 | -1.271 | 1.360 | | | | |
| | $P(S_t = 1)$ | $P(S_t = 2)$ | $P(S_t = 3)$ | | $P(S_t = 1)$ | $P(S_t = 2)$ | $P(S_t = 3)$ |

Table 3 represents the payoff matrix for the game with two players. The row player is an investor, and the column player is BIST30. In this game, BIST30, or nature, has three strategies: negative, neutral, and positive return, while the investor has 29 different strategies. The goal of the game is to maximize the investor's expected return. Fortunately, linear programming can be used to calculate the probabilities for each strategy that maximizes the expected value of the game. There are two ways of solving the problem: Solving the mathematical formulation for the row player or solving the mathematical formulation for the column player. Each formulation is a dual of the other and both would yield an equal game value. We solved the game for the column player, and the conditional probabilities are given below:

**Table 5.** The Probabilities of Column player's strategies

| | $P(S_t = 1)$ | $P(S_t = 2)$ | $P(S_t = 3)$ |
|---|---|---|---|
| Probabilities from Game Theory | 0.33 | 0.29 | 0.38 |
| Probabilities from Markov Chains | 0.46 | 0.05 | 0.49 |

According to the linear programming solution, the BIST30 or column player would earn negative, neutral, and positive returns with probabilities of 33%, 29%, and 38%, respectively. Using the probabilities of the column players, we calculated the expected returns of each stock. The stationary probabilities of the Markov chain model resulting from the transition matrix show that BIST30 had a 46% probability of being in the first strategy, a 5% probability of being in the second strategy, and a 49% probability of being in the third strategy. The obtained returns can be used to construct efficient portfolios.

**Table 6.** The Expected Returns Obtained from Game Theory and Markov Chains Models

| Stock | Expected Returns from Game Theory | Expected Returns from Markov Chains | Stock | Expected Returns from Game Theory | Expected Returns from Markov Chains |
|---|---|---|---|---|---|
| AKBNK | 0.010 | 0.019 | PETKM | 0.119 | 4.052 |
| AKSA | 0.267 | 4.171 | PGSUS | 0.016 | 3.862 |
| ARCLK | 0.063 | 2.661 | SAHOL | 0.015 | 4.837 |
| BIM | 0.091 | 1.623 | SASA | 0.382 | 3.680 |
| DOHOL | 0.203 | 1.478 | SISE | 0.173 | 3.242 |
| EKGYO | 0.000 | 2.998 | TAHVL | 0.000 | 3.002 |
| EREGL | 0.210 | 3.775 | TCELL | 0.121 | 2.552 |
| FROTO | 0.190 | 2.792 | THYAO | 0.078 | 3.687 |
| GARAN | 0.040 | 3.896 | TKFEN | 0.143 | 3.886 |
| GUBRF | 0.348 | 4.866 | TOASO | 0.217 | 2.636 |
| ISCTR | 0.056 | 4.458 | TTKOM | 0.094 | 2.985 |
| KCHOL | 0.039 | 4.940 | TUPRS | 0.096 | 2.905 |
| KOZAA | 0.300 | 3.878 | VESTL | 0.086 | 3.029 |
| KOZAL | 0.232 | 3.275 | YKBNK | 0.018 | 4.694 |
| KRDMD | 0.160 | 4.168 | | | |

Table 6 shows the expected returns calculated on the basis of game theory and the Markov chains model. In other words, the figures in Table 6 are the weighted average of the individual stocks. For example, the expected return of AKSA was calculated as (0.451x0.33)-(0.655x0.29)+(1.432x0.38). In Markowitz's mean-variance model, the returns were calculated as the equally weighted mean of the negative, neutral, and positive returns. Of course, the expected returns resulting from the different methods were not identical. Therefore, portfolios constructed based on a different measure of expected returns would yield different portfolios with different returns and risks. The Markov chain model was another method used in the study to determine expected returns. As in game theory, three different strategies were considered to estimate the expected returns. Here, we calculated the stationary probabilities for each strategy. These probabilities were used to calculate the expected returns of the assets that can be used to build efficient portfolios.

No matter how strong a model may be in theory, in practice it should produce a higher return than index returns and competing model portfolios. The strength of a model can be measured by the return achieved over a given term. An investor who invests in a proposed portfolio would want to beat the market at the end of the investment period. Otherwise, the proposed portfolio has no value in practice. To compare the combined models presented in this paper with the index and the mean-variance model, we planned four investment periods. The first period covered the last two months from September 2022 to December 2022, the second covered January 2022 to the end of the sample period, and the third began in April 2020, the start of the Covid 19 pandemic, and extended to the end of the sample period. The last investment period extended from January 2015 to the end of the sample period (December 2022). The rates of increase in the value of the portfolio over an actual investment period are shown in Table 7.

**Table 7.** Appreciation Rates of Proposed Models

| Investment Period | Markov Chain model | Game Theory Model | Mean-Variance model | BIST30 (Index) |
|---|---|---|---|---|
| September 2022 to December 2022 | 42.169% | 51.560% | 42.730% | 46.477% |
| January 2022 to December 2022 | 147.493% | 134.869% | 134.878% | 154.814% |
| January 2020 to December 2022 | 446.120% | 616.140% | 406.399% | 533.916% |
| January 2015 to December 2022 | 920.939% | 1472.345% | 1339.128% | 408.314% |

The figures in Table 7 show that an investment in the Markov chain model in January 2015 would have gained 920.94% in value at the end of the investment period. During this period, BIST30 only gained 408.31% in value. The combined model based on the Markov chain gained more than twice as much in value as the index. The increase in value of the portfolio proposed with the mean-variance model during this period was 1339.13%, while the investment in a portfolio created with the combined model based on the game theory achieved a return of almost 1500%. The Covid-19 pandemic had a negative impact on financial markets. During the pandemic most markets, both emerging and developed, suffered large losses. So, investing a portfolio in the financial market was riskier than ever.

An investment in a portfolio suggested by the mean variance model would yield a gain of 406.40% at the end of the investment period. From January 2020 to December 2022, the BIST30 gained 533.92% in value. Thus, the figures show that the classic mean-variance model did not work for exceptional periods such as the Covid-19 pandemic. The combined game theory-based model delivered a portfolio value gain of 616.14% from January 2020 to December 2022, while the Markov chain model delivered 446.12% over the same period. Among others, the game theory based portfolio beat all proposed portfolios including the target index. An investment in a portfolio combining mean-variance with the Markov chain model and game theory returned 147.49% and 134.87%, respectively, from the beginning of 2022 to December 2022. Over the same period, the mean-variance model returned 134.88%, while the index gained 154.91%. In all the investment periods, the portfolios based on game theory returned less than the other portfolios only during this period. If invested as planned over the last two months, a portfolio based on the Markov chain model and game theory would return 42.17% and 51.56%, respectively. BIST30 returned only 46.48% over the same period.

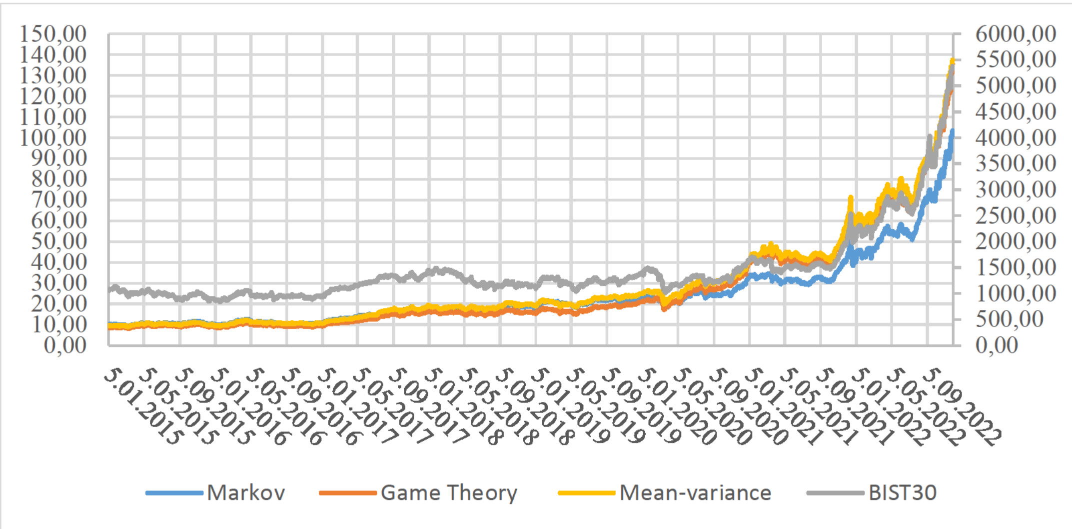


**Figure 1.** The Value of Created Portfolios and BIST30

Figure 1 shows the value of a portfolio based on the Markov chains model, game theory, the mean-variance model, and BIST30. The numbers on the left side of Figure 1 refer to the portfolios created, while the numbers on the right side refer to BIST30. According to the figure, at the beginning of the sample period, the constructed portfolios had a value of almost 10.00 Turkish liras. At the end of the period, the game theory portfolio increased to 137.00 Turkish Lira, the Markov chain portfolio increased to 103.37

Turkish Lira, and the mean-variance portfolio also increased to about 136.00 Turkish Lira. As is well known, the out-of-sample performance of portfolios is much more important to investors and researchers than the performance of portfolios within a sample period. Although the total sample period is an indicator of an efficient portfolio, the most important indicator is the out-of-sample performance. To compare the out-of-sample performance of portfolios in this context, we plotted the short-term line of portfolio values below.

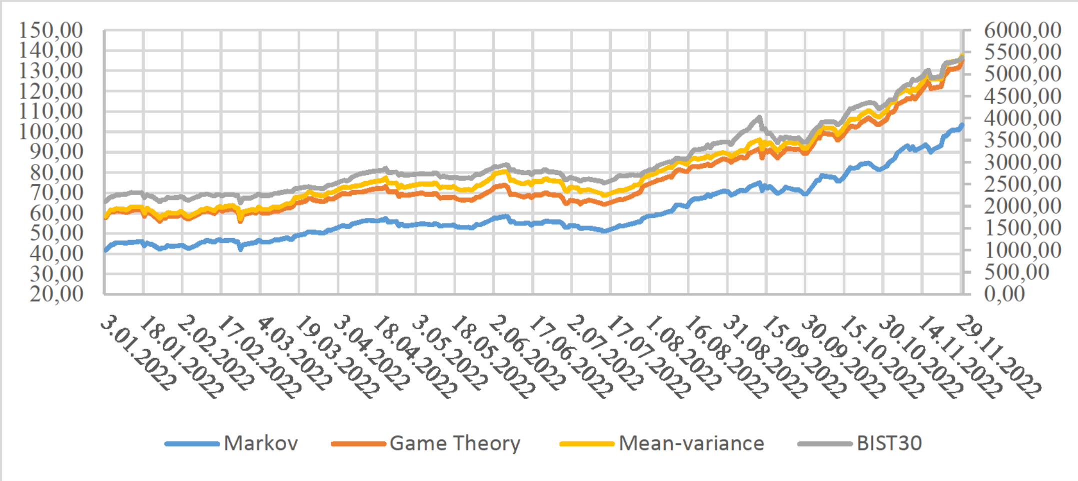


**Figure 2.** The Short-run Values of Created Portfolios and BIST30

As in Figure 1, the scale on the left side refers to created portfolios, while the scale on the right side refers to BIST30 in Figure 2. But here the increase in values in the invested portfolios is more evident. Parallel to the figures in Table 7, Figure 2 shows that the largest increase is in portfolios created based on game theory. Returns are important, but not the only criterion. The main objective is to minimize investment risk while maximizing the return on an investment. The portfolio risks of the mean-variance model, the portfolio based on game theory, and the Markov chain model were 1.91, 1.90, and 1.80, respectively. Thus, we can conclude that Markov chains construct portfolios with minimum risk but not maximum return.

## Conclusion

Modern portfolio theory aims to minimize portfolio risk while maximizing returns, taking into account the relationship between asset returns via the covariance matrix. However, the estimation of the parameters has a great impact on the created portfolios. In the existing literature, the analyzes of many studies, including the present work, show that the return series are mostly not normally distributed and that using the simple arithmetic mean of the assets would lead to biased parameter estimates and inefficient portfolios. As Markowitz (1991) says, "If we knew which stock gave the highest return, we would maximize our return by investing only in that stock, with no need for diversification. But none of us know the stock's distribution function for the investment period. So, we take some risk in investing and try to spread the risk through diversification." As Markowitz noted, we cannot know the distribution function of a stock. Using robust statistics that combine estimation methods would provide a robust estimate of the parameters used in the portfolio model. In this work, we used game theory and the Markov chain model to obtain unbiased estimators of the parameters and create efficient portfolios that beat the target index.

The results of the analysis suggest that the portfolio constructed on the basis of game theory is superior to the mean variance model and the model based on Markov chains. Over the long run, the portfolio combining game theory with the portfolio theory generated a return of almost 1500% in about eight years. With compound interest, this rate corresponds to an average annual growth of the portfolio of about 40%. Over the same period, the average annual growth of BIST30 was just over 19%, while it was 38% for the mean-variance model and 31% for the Markov chain model. In the short run, the portfolio which combines game theory with portfolio theory generated a return of almost 52% over a period of two months. This return represents an average monthly growth of the portfolio of about 2%. The returns of the other portfolios, including the target index, failed to beat this rate. BIST30, for example, had an average growth rate of only 1%. Overall, we conclude that the portfolio based on game theory is superior to the others. The Markov chain-based model is an alternative to the game theory-based portfolio and had the lowest portfolio risk among the others, including the game theory-based model. Since a combined model can reflect the distribution characteristics of parameters, an investor or a researcher using a combined model can create more efficient portfolios than the classical mean-variance model while avoiding investment risks.

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