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EFED, 2024; 15(1): 15-29 **Araştırma Makalesi**

12. Sınıf Öğrencilerinin Birim Çember Üzerinde Trigonometrik Fonksiyonları Anlamlandırmasının APOS Teorisi ile İncelenmesi[1]

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Öz: APOS teori, öğrenenin matematik kavramlarını yapılandırma ve inşa etme sürecinin nasıl gerçekleştiğinin tanımlandığı bir teoridir. Trigonometri konusu öğrencilerin güçlük yaşayabileceği ve kavram yanılgılarına sahip olabileceği bir konu olarak nitelendirilebilir. Aynı zamanda trigonometrik kavramlar cebir, geometri ve analiz konularında sıklıkla karşılaşılır. Ortaöğretim matematik dersi programında cebir ve geometri alt öğrenme alanlarının yoğun olması ve analiz konularının da olmasından dolayı öğrencilerin APOS teori ile trigonometrik kavramları yapılandırılmasının incelenmesi amaçlanmaktadır. Çalışma, nitel durum desenlerinden durum çalışması ile yapılandırılmıştır. Araştırmanın katılımcılarını farklı devlet liselerinin 12. sınıfında öğrenim görmekte olan 29 katılımcı oluşturmakta olup katılımcılar kolay örneklem yöntemiyle seçilmiştir. Katılımcılara ilk olarak Değerlendirme Testi (DT) uygulanmıştır. DT'nin sonuçlarına göre başarılı olan en yüksek 3 öğrenci ve en düşük 3 öğrenci ile görüşmeler yapılmıştır. Her öğrenciyle 3 görüşme yapılmış olup her görüşme 4 soru ve alt sorulardan oluşmaktadır. Görüşme verileri araştırmacılar tarafından hazırlanan rubriğe göre değerlendirilmiştir. Veriler içerik ve betimsel analiz yöntemleriyle analiz edilmiş olup APOS teorinin aşamaları olan eylem, süreç ve nesne temalarına uygun şekilde alt temalara ayrılmıştır. Bulgular öğrencilerin matematik kavramları anlamlandırmaları ile başarıları arasında bir ilişki olduğu göstermekte olup APOS teori aşamaları için farklı durumlar ortaya koyduğunu göstermektedir.

Anahtar kelimeler: APOS teori, Matematik eğitimi, Trigonometrik fonksiyonlar

12th Grade Students Interpretation of Trigonometric Functions on the Unit Circle with APOS Theory Investigation

Abstract: The APOS Theory is a theory that describes how the learner structures and constructs mathematical concepts. Trigonometry can be characterized as a subject in which students may have difficulties and misconceptions. At the same time, trigonometric concepts are frequently encountered in algebra, geometry and analysis. Since algebra and geometry sub-learning areas are intensive in the secondary mathematics curriculum and there are also analysis subjects, it is aimed to examine students' construction of trigonometric concepts with APOS theory. The study was structured as a case study, one of the qualitative case patterns. In this study, there are 29 participants studying in the 12th grade of different public high schools whom were selected by simple random sampling method. The participants were first applied the Evaluation Test (ET). Based on the results of the ET, three students with the highestsuccess and three students with the lowest success were interviewed. Each student was interviewed three times, and each interview consisted of four questions and sub-questions. Data from the interviews were evaluated according to the rubric prepared by the researchers. The data were analysed with content and descriptive analysis methods and were divided into sub-themes in accordance with the themes of action, process and object, which are the phases of the APOS Theory. Findings from the study have revealed that there is a relationship between students' making sense of mathematical concepts and their success and that they present different attitudes for the APOS Theory phases.

Key Words: APOS theory, Mathematics education, Trigonometric functions

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INTRODUCTION

It is the curriculum, specific course objectives and contents that determines what we need to learn and how much we will learn in mathematics. Although there are certain lines drawn for this determination, the fact that how the aforesaid matters will take place and how much of the learning outcomes can be achieved emerges as a situation that we, educators, should always investigate and question.

Thinking mathematically and doing mathematics is the flexible incorporation of concepts, ideas and relationships into everyday life problems and unusual mathematical problems (Velzen, 2016). Doing mathematics is something far beyond just applying formulas or performing operations, and producing a solution for a problem, applying this solution and then checking the result is a way of doing mathematics (Van de Walle et al., 2012). In addition, the authors state that mathematics is the science of patterns based on mathematical concepts and operations and that discovering this pattern can be defined as doing mathematics. According to Simon (2018), mathematical concepts are logical necessities established between mathematical relationsAnother point that is also emphasized is that proficiency in mathematics is based on conceptual understanding and operational understanding, and operational understanding is knowledge and rules about mathematical operations, while conceptual understanding is the views and connections of a subject matter (Van de Walle et al., 2012). In Scmittau's (2004) study, students were able to solve mathematical formulas that they thought were difficult by conceptualising them without relying on rules. Students' understanding of mathematical concepts enables them to understand mathematics subjects more easily (Velzen, 2016).

Dede and Argün (2004) emphasize that the making sense of mathematical concepts by students takes place as a result of a certain process and that it is important for students to interpret mathematical concepts correctly with the help of concrete models. The reason is that using an abstract language when making sense of mathematical concepts requires a longer process and also requires more effort by the educator. Mutual communication between teachers and students and the use of concrete materials by teachers enable students to understand mathematical concepts more clearly (Dominguez, 2021).

A few of the theories based on constructing and constructing mathematical knowledge are constructivist approach, sociocultural theory, heuristic rule theory and APOS theory. Constructivist approach is an approach that explains the process of constructing any knowledge that a student can acquire (Harlow, Cummings & Aberasturi, 2007); sociocultural theory defines the process of constructing knowledge with the help of the environment of human being as a social being (Hasan, 2005); heuristic rule theory is a theory that clarifies the question of whether students' answers to a mathematical problem are based on misconceptions or intuitions (Fischbein, 1987). Ed Dubinsky et al. Reinterpreted Jean Piaget's Constructivism Theory and introduced the APOS Theory, which investigates how an individual makes sense of mathematical concepts, constructs and uses mathematical knowledge (Oktaç & Çetin, 2016). The APOS Theory is explained with four main concepts, which are the mental structures of Action, Process, Object and Schema with the initials of the words ''APOS''. These mental structures are explained by the processes of reflective abstraction processes, which are encapsulation, internalization, generalization, coordination and reversibility. (Oktaç & Çetin, 2016). Dubinksy and his colleagues were able to define the general outlines of the APOS theory by organizing the mathematical concepts in detail by means of an analysis tool called genetic decomposition (Kabaca, 2006). Looking at the theoretical framework, Ed Dubinsky put forward a structure as the ACE teaching cycle. According to this structure, students construct mathematical knowledge through their activities in the computer environment, in-class discussions and students' homework (Asiala et.al., 1997). By using this theory and cycle, it has been observed that students can internalize mathematical knowledge better than traditional methods (Asiala et.al., 1999).

The phase ''Action'' is, in its most basic definition, is a phase in which the learner makes sense of a concept with the help of external tips (formulas, figures and models) and the guidance. Students are at a level where they can only apply a formula by looking at it. For example, using the equation of the unit circle shown to the students, students find the point written on the equation The phase ''Process'' is a phase in which the learner makes sense of mathematical concepts without external tips. As the learner in the phase ''Action'' thinks about the problem and repeats the problem, he/she internalizes it and proceeds to the phase ''Process''. With coordinating two different processes and reversing the process, the learner can also be in the phase ''Process''. The phase ''Object'' is a phase in which the learner is able to perceive mathematical knowledge, problem or concept as a whole. For example, students were able to solve the question easily without seeing the equation of the unit circle. This process is encapsulated and becomes an object. All of these phases take place in the schema in the learner's mind. As the learner repeats the action, he/she moves to the process stage as he/she thinks about it and concentrates on it. When the student realizes the applicability of the process, he/she performs encapsulation. Reflective abstraction is seen as the most difficult process. When a student in the process stage obtains a new process with more than one process, it is defined as a coordination process. A student in the action stage can also move to the process stage by performing a reverse operation. While this process is defined as reversibility, the generalization process is the student's ability to use their learning in different cases of mathematics in their own schema (Oktaç & Çetin, 2016).

Although defining and learning mathematical concepts are not the same, the APOS Theory explains how to teach and learn mathematical concepts (Öksüz, 2018). At the same time, since APOS theory is a theory that helps to see the difference between a student who memorizes mathematical concepts and a student who can associate mathematical concepts, the APOS Theory is also a theory that guides teachers in this way (Oktaç & Çetin, 2016; Salgado & Trigueros, 2014). The literature investigations show that studies conducted with the APOS Theory (Arnawa, et. al., 2007; Çekmez & Baki, 2019; Çetin, 2009; Deniz, 2014; Kılıçoğlu & Kaplan, 2019; Marsitin, 2017; Nagle, et. al., 2019; Şefik, 2017; Yorgancı, 2019) are very limited.

Çetin (2009) conducted a study on making sense of the concepts of limit with students studying in the mathematics department of a state university. As a result of semi-structured interviews with the students, it was concluded that the learning area created with APOS theory had a positive effect on students' understanding of mathematical concepts. In Yorgancı's (2019) study, it was observed that the academic success of pre-service elementary mathematics teachers at a state university and the students taking courses in APOS learning environment was higher than the students taking courses with the traditional teaching model and their attitudes towards the course were more positive. Kılıçoğlu and Kaplan (2019) conducted APOS-based learning with 7th grade students. In this way, it was observed that students were more active and more successful because they were engaged in activity-based learning. Similarly, in the study conducted by Çekmez and Baki (2019) with elementary mathematics teaching candidates, it was concluded that in the comprehension levels of the two groups studied with traditional methods and APOS theory, it was concluded that students had more advanced comprehension in the learning environment with APOS theory.

The subject ''trigonometry'', which is first introduced to the 11th grade students in the secondary education mathematics curriculum, is also included in the curriculum for the 12th grade students in the secondary education. Trigonometry can be characterised as a subject in which students may have difficulties and misconceptions (Malambo, 2021). At the same time, trigonometric concepts are frequently encountered in algebra, geometry and analysis subjects (Gürbüz et.al., 2011). Since algebra and geometry sub-learning areas are intensive and analysis subjects are also included in the secondary mathematics curriculum (Board of Education, 2018), it is aimed to examine students' construction of trigonometric concepts with APOS theory. Therefore, this study aims to investigate how the 12th grade students and high school graduates make sense of the concepts of trigonometry, which is a subject they are familiar with, with the APOS Theory.

With the outcome "explains trigonometric functions with the help of unit circle, one of the outcomes of the subject ''trigonometry'', it is thought that almost all the learning outcomes of trigonometry can be learned easily and will serve as the basis of the subject. For this reason, it will be important for students to work on the unit circle, and it is seen that investigating the subject ''trigonometry'' with the APOS Theory will be effective for learners and be guiding for teachers.

Based on these contexts, it is thought that it will be important to investigate the students making sense of the trigonometry-related concepts with the APOS Theory. Therefore, the statement ''How do 12th grade students make sense of trigonometric functions on the unit circle with the APOS Theory?'' is determined as the problem statement of this study. Accordingly, the sub-problems of the study are:

1. How do 12th grade students make sense of trigonometric functions on the unit circle in the phase ''Action''?
2. How do 12th grade students make sense of trigonometric functions on the unit circle in the phase ''Process''?
3. How do 12th grade students make sense of trigonometric functions on the unit circle in the phase ''Object''?

METHOD

The study is a case study, one of the qualitative study. Case study is, as defined by Yin (1994), an in-depth examination of the flow, space of events and all the developments experienced and all the variables that affect them. How and in what way the variables are affected is investigated holistically. The study's main case that will be described is the 12th grade students' making sense of trigonometric functions on the unit circle with the APOS Theory. Since all the cases experienced by the students during the application would be examined in depth and holistically, a holistic case study, which is one of the case study types, was chosen.

Participants

For the pilot study, the participants were 30 students graduated from different public high schools who had already taken the subject of trigonometric functions and 29 students who were studying in the 12th grade of different public high schools at the time the study was performed. The participants were determined using the easily accessible sampling method. According to the results of the Evaluation Test (ET) implemented, individual interviews were made with the first three students with highest scores and the last three students with lowest scores from the ET test, and the interviews were recorded using a voice recorder. The 6 selected students are of the same age and are 12th grade students. The successful students were the 3 highest students according to the average of the assessment test, while the unsuccessful students were the 3 lowest students according to the average. The aim was to compare how successful and unsuccessful students construct trigonometric concepts. In addition, the necessary permissions were obtained, and the participants were informed that both the pilot application and the study were on a voluntary basis.

Data Collection

The study data were collected using the Evaluation Test (ET) and interview form (IF) developed to determine the students' level of making sense of trigonometric functions on the unit circle.

(ET) consisted of 23 multiple-choice and 12 open-ended questions prepared by the researchers. Expert opinions were obtained from two mathematics educators and two mathematics teachers for the (ET) prepared. Necessary corrections were made in line with the expert opinions, and the pilot study was carried out accordingly. Based on the students' feedbacks obtained after the pilot study, the data were analysed with TAP, a data analysis program and the required adjustments were made, and thus, the (ET) was given its final form with a total of 30 questions, 20 of which were multiple choice and 10 were open-ended. ET consisted of 23 multiple-choice and 12 open-ended questions prepared by the researchers. Expert opinions were obtained from two mathematics educators and two mathematics teachers for the (ET) prepared. Necessary corrections were made in line with the expert opinions, and the pilot study was carried out accordingly. Based on the students' feedbacks obtained after the pilot study, the data were analysed with TAP (Test Analyzing Programme) a data analysis program and the required adjustments were made, and thus, the ET was given its final form with a total of 30 questions, 20 of which were multiple choice and 10 were open-ended. According to the TAP analysis, items with a discrimination index below 0.20 and items with a difficulty below or above 0.60 were removed. Corrections were made to items with a discrimination index of 0.20-0.30.

For the (ET) applied to the participants, an evaluation was made by scoring the correct answer as 1 point and the wrong answer as 0 point. The participants were ranked by the scores they received, and interviews were made with the first three and the last three participants based on their scores. Interviews were conducted with the 3 lowest and 3 highest students according to the average These interview forms were created in accordance with the phases ''Action'', ''Process'' and ''Object'' of the APOS Theory. Furthermore, expert opinions were obtained from two mathematics educators and two mathematics teachers for the interview forms. Each form was prepared consisting of four questions and of sub-questions of these questions. Interviews made with six students were recorded using a voice recorder. The interview forms were evaluated by measuring them with a four-grade rubric prepared by the researchers in accordance with the phases ''Action'', ''Process'' and ''Object'' of the APOS Theory. A pilot study was conducted with two participiant for the interview forms and also add changes after pilot study.

Data Analysis

The data obtained were analysed inductively in order to present a holistic picture of the situation by using content and descriptive analysis methods. In the descriptive analysis, the data were summarized and interpreted according to the previously determined themes, while in the content analysis, the data were analysed according to the themes that emerged after the data were collected. The students were analysed by giving them correct codes (S1, S2, S3…) from the students with the highest success to the students with the lowest success. The analysis of APOS phases was carried out by using the rubric which was based on the outcomes from trigonometry and prepared by the researchers. The rubric prepared is given in Table 1.

Table 1

Rubric

APOS Phases	0 point	1 point	2 points	3 points
Action	Cannot see or notice tips and formulas. (Cannot see unit circle equation, coordinate plane, sign table).	Sees and notices tips and formulas. Cannot use effectively. (Can see the unit circle equation, coordinate plane, sign table but cannot use them).	Sees and notices clues and formulas. Answers the question incorrectly. (See unit circle equation, coordinate plane, sign table but use them but answer incorrectly).	Sees and notices tips and formulas. Answers the question correctly. Explains his/her answer. (See the unit circle equation, coordinate plane, sign table, but use them and answer correctly and explain their answer).
Process	Cannot related without the phase ''Action''. (Cannot make relationships without equations and sign tables).	Interrelates and expresses concepts without the phase ''Action''. (Establishes and expresses relationships without equations and sign tables.)	Interrelates without the phase ''Action''. Expresses the concepts. Performs the operation of the problem. (Establishes, expresses and performs operations without equations and sign tables.)	Interrelates without the phase ''Action''. Expresses the concepts. Performs the operation of the problem. Explains the reason for his/her answer. (Establishes a relationship without equation and sign table, expresses, performs the operation and explains the answer).
Object	Cannot interpret the phase ''Process''. (Cannot interpret trigonometric concepts.)	Interprets the phase ''Process''. Cannot see and interrelate concepts holistically. (Interpret trigonometric concepts, cannot establish a relationship between concepts).	Interprets the phase ''Process''. Sees and interrelates concepts holistically. Cannot justify his/her answer**.** (Can see the relationship between trigonometric concepts but cannot express their reasoning).	Interprets the phase ''Process''. Sees and interrelates concepts holistically. Justifies his/her answer. (Relate, interpret and justify trigonometric concepts.)

The evaluation of the interviews with the students by the rubric is shown in Table 2.

Table 2

Evaluation Scores of Participants for APOS Phases

Students		1th Question	2th Question	3th Question	4th Question	Total Score
S1	A	3p	3p	3p	3p	12p
	P	3p	3p	3p	3p	12p
	O	3p	3p	3p	3p	12p
S2	A	3p	3p	3p	3p	12p
	P	3p	3p	3p	3p	12p
	O	3p	2p	3p	3p	11p
S3	A	3p	3p	3p	3p	12p
	P	3p	0p	3p	3p	9p
	O	1p	3p	3p	3p	10p
S4	A	1p	1p	2p	2p	6p
	P	0p	0p	0p	3p	3p
	O	0p	0p	3p	0p	3p
S5	A	1p	1p	1p	1p	4p
	P	0p	0p	0p	0p	0p
	O	0p	0p	0p	0p	0p
S6	A	3p	2p	2p	3p	10p
	P	0p	0p	0p	3p	3p
	O	0p	0p	1p	0p	1p

Ethics Committee Permission Information

This study was approved by Akdeniz University Institute of Education Sciences Ethics Committee on 07.11.2022 (Decision No: 391).

FINDINGS

Findings Of The Sub-Problem ''How Do 12th Grade Students Make Sense Of Trigonometric Functions On The Unit Circle According To The Phase Action?''

The sub-themes found as a result of the interviews with six students are given in Table 3. Accordingly, a frequency table was created with the aim to see the overall picture.

Table 3

Sub-Themes of Phase ''Action''

Theme	Sub-Theme	Participant	f
	The sum of the squares of the sine and cosine values of the same angles is 1.	S1, S2, S3, S6	4
	Sine is positive in the 1st and 2nd Sections and negative in the 3rd and 4th Sections Cosine is positive in the 1st and 4th Sections and negative in the 2nd and 3rd Sections Tangent and cotangent are positive in the 1st and 3rd Sections and negative in the 2nd and 4th Sections	S1, S2, S3, S6	4
	The unit circle equation can be derived with the Pythagorean Theorem.	S1, S2, S3	3
	In right triangle Sin=opposite/hypotenuse, Cos=adjacent/hypotenuse Tan=opposite/adjacent, Cot=adjacent/opposite.	S1, S2, S3	3
Action	The values of 30̂-45̂-60̂ angles can be easily found with the help of right triangle.	S1, S2, S3	3
	The sine-cosine and tangent-cotangent values of the angles completing each other by 90̂ are equal.	S1, S3	2
	Trigonometric functions take values from where they are discontinued.	S1, S2	2
	Values of trigonometric functions are less than 1, and some of them are 0.5.	S4, S5	2
	Trigonometric function values of angles are an ordered pair.	S4	1
	For sine and tangent, the larger the angle, the larger the value, and for cosine and cotangent, the larger the angle, the smaller the value.	S3	1
	Sine values of angles that complete each other by 360̂ are equal to each other.	S3	1
	I have noticed that the cosine is represented by the x-axis and the sine by the y-axis.	S6	1
	Sine and cosine values of angles are equal to the angles.	S6	1

According to the frequency table, students who can give the correct and desired answers according to the action stage and who can do the questions according to the given formula are mostly successful students. The table shows that unsuccessful students try to make inferences based on the given formula. It is seen that S1, one of the students with high success, has seen and noticed the tips and formulas, answered the question correctly and explained his/her answer. S2 and S3 stated that they saw the tips in both questions, but they still used them to make sure they remembered the formulas. The students were also able to see the rules and formulas of trigonometry on the unit circle and interrelated them. The three students saw and noticed the tips and formulas, answered the question correctly and explained their answers. Students' answers are as follows:

S1: *''The formula ''cos2x+sin2x=1'' can be noticed from the unit circle equation, and it can be also noticed when the Pythagorean Theorem is applied on the unit circle.''*

S2: *''Looking at the unit circle, you can see sine=opposite/hypotenuse, cosine=adjacent/hypotenuse, tangent=opposite/adjacent, cotangent=adjacent/opposite.''*

S3: *''Sine-cosine and tangent-cotangent values of angles completing each other by 90̂ are equal.''*

Figure 1

One of S3's Answers to the Action Phase

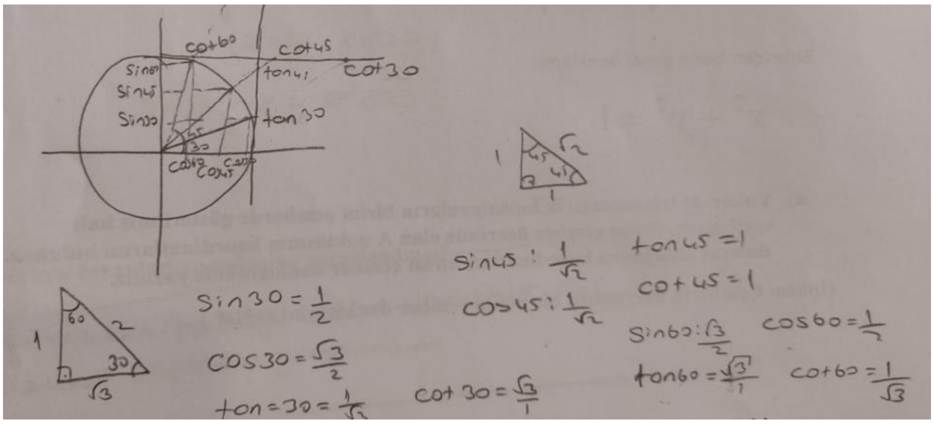
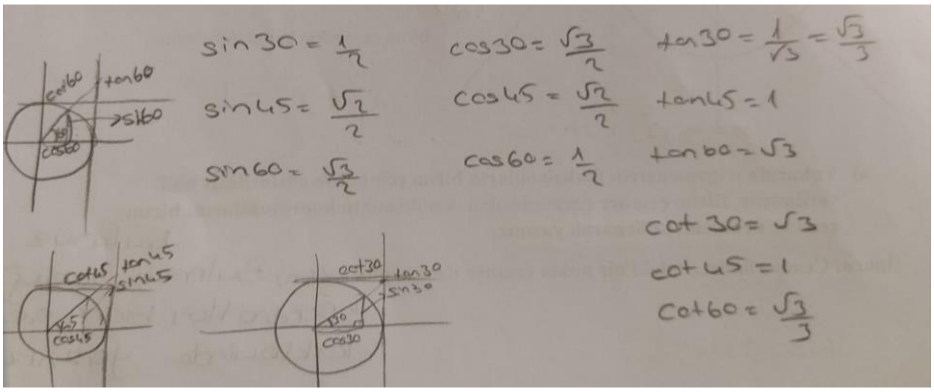


Figure 2

One of S2's Answers to the Action Phase



Although S5, one of the students with low success, noticed the formulas and tips, he/she could not use the formulas and tips effectively and could not give clear answers to the questions even if he/she was guided and helped by the interviewer. One of the students (S4) could not answer two of the four questions although he/she noticed the formulas and tips. He/she also noticed the formulas and tips about two questions, but gave wrong answers to them. Although S6 saw and noticed the tips and formulas in four questions, he/she answered two questions incorrectly. He/she answered the other two questions correctly and was able to explain his/her answer. Students' answers are as follows:

S5: *''Trigonometric values consist of ordered pairs.''*

S6: *''Trigonometric values are equal to angle''.*

Figure 3

One of S4's Answers to the Action Phase

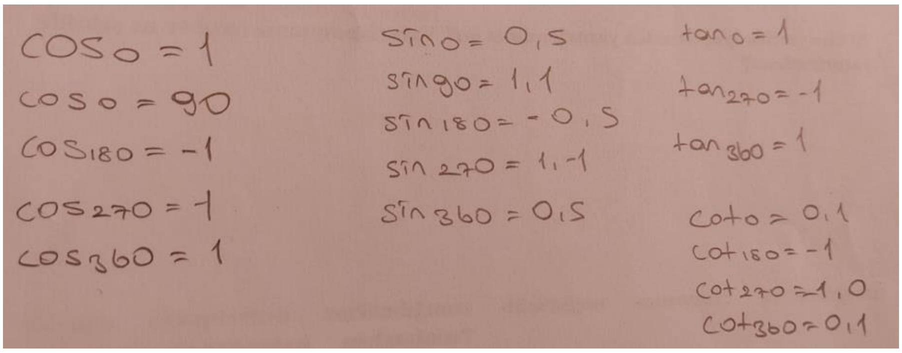
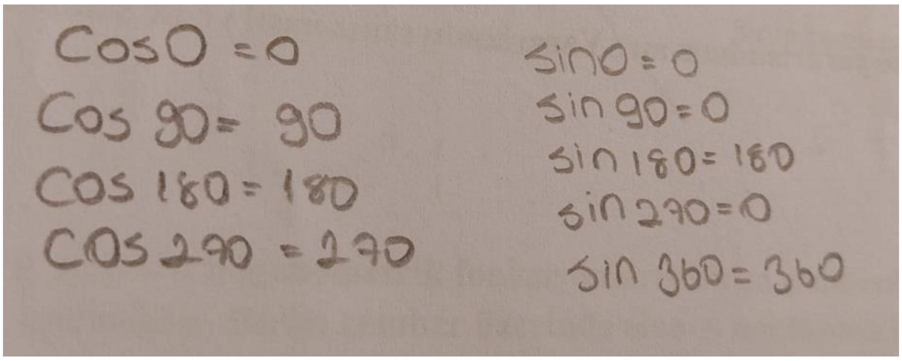


Figure 4

One of S6's Answers to the Action Phase



Findings of The Sub-Problem ''How Do 12th Grade Students Make Sense of Trigonometric Functions on the Unit Circle According to the Phase Process?'':

The sub-themes found as a result of the interviews with six students are given in Table 4. Accordingly, a frequency table was created with the aim to see the overall picture.

Table 4

Sub-Themes of the Phase ''Process''

Theme	Sub-Theme	Participant	f
Process	The larger the angle, the larger the sine value.	S1, S2, S3, S4, S6	5
	When finding the trigonometric values of the angles, when they are completed by 180° and 360°, the names for sine, cosine, tangent and cotangent do not change, but the signs change. When they are completed by 90° and 270°, sine and cosine and their tangents and cotangents change names.	S1, S2, S3	3
	The right triangle can be used when finding trigonometric function values of large angles.	S1, S2, S3	3
	Sine and cosine take -1 as the smallest value and 1 as the largest value.	S1, S2	2
	The sine values of the angles completing each other by 180° are equal.	S1, S2	2
	The largest and smallest values of the sine depend on the angle.	S3, S5	2
	The result of multiplication of tangent and cotangent values is 1.	S3	1
	For sine and tangent, the larger the angle, the larger the value, and for cosine and cotangent, the larger the angle, the smaller the value.	S1	1
	The sine axis is represented by the axis ''x'', while the cosine axis by the axis ''y''.	S6	1
	It can be at any point of the cotangent and tangent coordinate system.	S6	1
	Trigonometric function values of each angle are different.	S6	1
	The sine's largest value is 360 and smallest value is 0.	S6	1

S1 and S2, two of the students with high success, answered all the questions correctly. They justified their answers, did the correct operations and explained their answers clearly and in the desired way. The students (S1, S2) were also able to see the rules and formulas of trigonometry on the unit circle and interrelated in between trigonometric concepts on the unit circle. One student (S3) did not answer only one question. He/she could not interrelate to the question without the phase ''Action''. When asked to draw a unit circle in a question, he/she showed some trigonometric function values of the angles incorrectly on the circle. Nevertheless, he/she was able to interpret the questions. In the questions that the student answered correctly, he/she was able to justify his/her answers, do the correct operations and gave clear and desired answers. Accordingly, he was able to interrelate in between concepts. Students' answers are as follows:

> S1*: ''When angles greater than 90° are drawn on the unit circle, they are all equal to the values of the angles in Section 1, and some values change their names or signs.''*

S2: *''The largest and smallest values that sine can take 1 and -1, respectively, and the unit appears on the circle.''*

S3: *''The larger the angle value, the larger the sine value.''*

Figure 5

One of S3's Answers to the Phase ''Process''

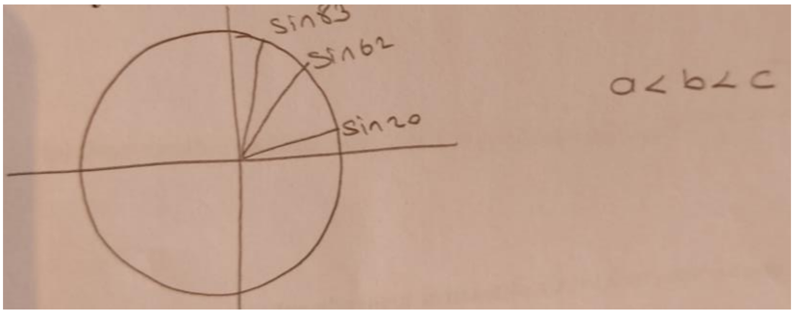
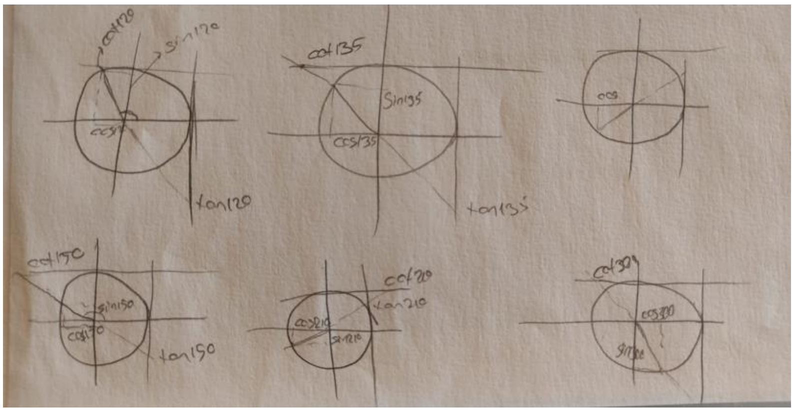


Figure 6

One of S2's Answers to the Process Phase



S4, one of the students with low success, could not interpret two of the questions. He/she misinterpreted one of the questions, answered one question correctly with a clear and desired explanation. He/she justified his/her answer by doing the operation for the question that the student had answered correctly. He/she stated that he/she did this by remembering the tips in the phase ''Action''. S5, one of the students with low success, could not answer and could not even interpret any questions. He/she could not interrelate without the phase ''Action''. Although S6 stated that he/she tried to do it by remembering the tips in the phase ''Action'', he/she could not answer them correctly. He/she also stated that he/she had better to have tips. He/she misinterpreted three of the questions and was able to answer one question by interpreting it correctly. In the question answered correctly, he/she explained the concepts by interrelating in between them without the phase ''Action''. He/she did the operation for the problem and explained the reason. Students' answers are as follows:

S4: *''The larger the angle value, the larger the sine value.''*

S6: *''The cosine axis is represented by the axis ''y'', while the sine axis by the axis ''x''.*

Figure 7

One of S6's Answers to the Process Phase

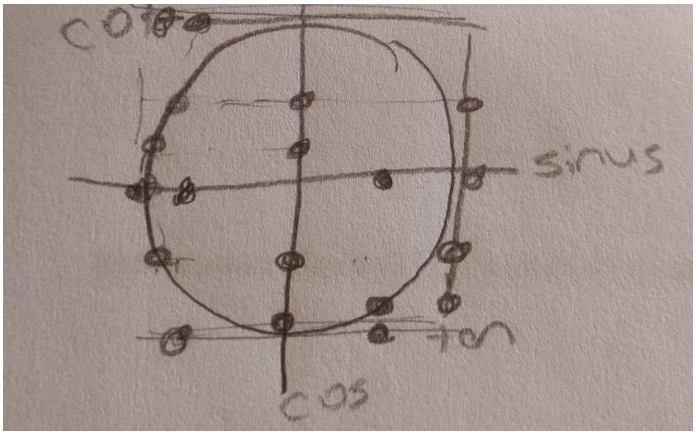
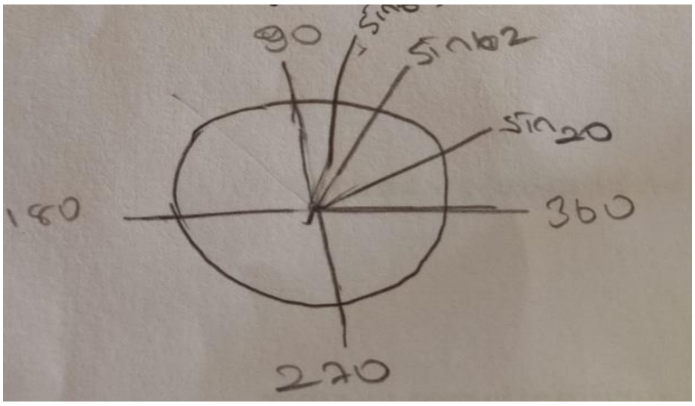


Figure 8

One of S6's Answers to the Process Phase



Findings Of The Sub-Problem ''How Do 12th Grade Students Make Sense Of Trigonometric Functions On The Unit Circle According To The Phase ''Object?''

The sub-themes found as a result of the interviews with six students are given in Table 5. Accordingly, a frequency table was created with the aim to see the overall picture.

Table 5

Sub-Themes of the ''Phase'' Object

Theme	Sub-Theme	Participant	f
	When a unit circle is drawn, the sine values in the 1st and 3rd Sections are the same, but with opposite signs.	S1, S2, S3, S4	4
	When the sine values in Section 1 and Section 3 are added on the unit circle, the result is always 0.	S1, S2, S3, S4	4
	Since the radius of the unit circle is 1, an isosceles triangle can appear inside the circle.	S1, S2, S3	3
Object	The sine values of the angles completing each other by 180̂ are equal	S1, S2, S3	3
	Although the cosine value is negative in Section 2, the length is indicated with a positive value.	S1, S2	2
	It is equal to the central angle facing the arc of the circle.	S2, S3	2
	The length of the arc of the circle is expressed in radians.	S1	1

S1, one of the three students with high success, answered all the questions correctly and explained his/her answers clearly and in the desired way. S1 was able to see trigonometric concepts holistically and also interpret the questions by interrelating in between the concepts and justify his/her interpretations. One of the students (S3) answered three of the four questions correctly, justified and explained his/her answers clearly and in the desired way. S3 could not see, and interrelate in between, the concepts holistically in only one question. S2 answered three of the four questions correctly, justified and explained his/her answers clearly and in the desired way. Although he/she was able to notice and interrelate in between the concepts holistically in a question, he failed to justify his/her answer. Students' answers are as follows:

S1: *''Since the sine has an opposite sign in the 1st and 3rd Sections, the sum of the sine values of the same angle will always be 0.''*

S3: *''The sine values of the angles completing each other by 180̊ are equal.''*

Figure 9

One of S1's Answers to the Object Phase

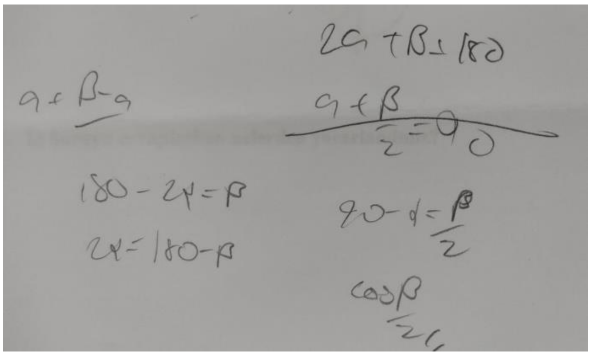
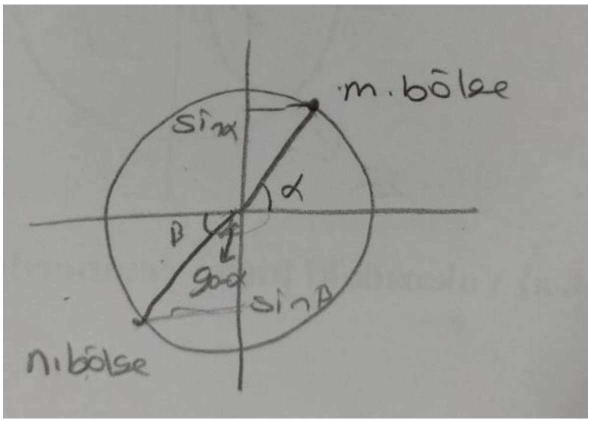


Figure 10

One of S3's Answers to the Object Phase



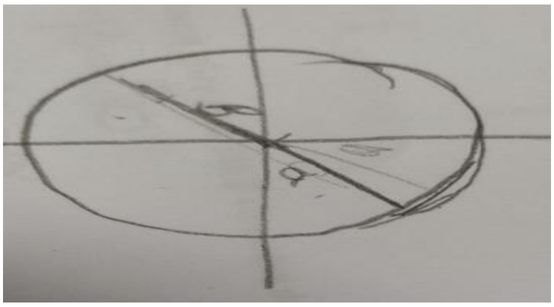
S5, one of the three students with low success, could not interpret the phase ''Process'' in the four questions asked. S4 could not give the desired answers to the questions since he/she was not able to interpret the phase ''Process'' in three of the four questions. In one question, he/she was able to interpret the phase ''Process'', see and interrelate in between the concepts holistically and justify his/her answer. S6 could not give the desired answers to the questions since he/she was not able to interpret the phase ''Process'' in three of the four questions. Although he/she was able to interpret the phase ''Process'' in one question, he could not see and interrelate in between the concepts holistically. Students' answers are as follows:

S4: *''The radii of the circle are equal to each other.''*

S5: *''The sine value is -1.''*

Figure 11

One of S6's Answers to the Phase ''Object''



CONCLUSION, DISCUSSION AND SUGGESTIONS

After the interview forms were analysed with the rubric, the student with the highest success according to the ET received the highest score, the student with the lowest academic achievement according to the ET did not rank at the bottom (Bonsilal, et.al., 2017). The student with the lowest academic achievement noticed the external clues in the action phase and tried to use them effectively. In the interviews prepared in accordance with the process and object stages, he received higher scores in the interview forms evaluated with the rubric because he tried to answer by remembering the extrinsic clues and formulas. Accordingly, it was concluded that extrinsic clues in the action stage can also be used in the process and object stages and that extrinsic clues are effective in making sense of mathematical concepts. When ranking according to academic achievement, the 5th of 6 students received the lowest score as a result of rubric analysis. The student could not use the external clues and formulas effectively in the interviews prepared in accordance with the action phase. Therefore, it was concluded that he could not answer the questions in the interviews appropriate for the process and object stages.

As a result of the interview in which trigonometric functions on the unit circle, which was prepared in accordance with the phase ''Action'', were explained, it was concluded that three students with high success answered all the questions correctly and explained their answers. Another result found was that the students did not use any external tips in the phase ''Action'' that they had prior knowledge in the questions they used them and that they wanted to be sure about their answers.

It was also found out that three students with low success did not use external tips effectively in any of the questions, although they noticed external tips and formulas in the phase ''Action''. Furthermore, although they used external tips and formulas in the solution of the questions, they could not answer all the questions correctly. It was concluded that the students were in the phase ''Action'' when making sense of the concepts. This result is similar to the result of the study conducted by Anam et al. (2019). This similarity was that the students were in the phase ''Action'' because they needed external tips. Moreover, the participants drew attention to the difficulties of the questions during the interviews. Similarly, Brijlall and Ndlovu's (2013) study conducted in Africa suggested that students were more successful in easy level questions and this was an indication that they were in the phase ''Action'', and this finding is similar to the finding of this study at the same phase.

It was concluded that the students with high success mostly answered all the questions correctly in the phase ''Process'' and were able to express the concepts without the phase ''Action'', that they explained the reasons for their answers by establishing correct and appropriate relationships. As a result of the interviews made with the students with low success, which were prepared in accordance with the phase ''Process'', it was concluded that the students could not give clear and correct answers to any of the questions, and accordingly, the students could not interrelate in between the concepts in the phase ''Process'' without external tips and formulas. Şefik's (2017) study is similar to this result. His study examines the understanding of the concepts of functions of two variables by pre-service teachers studying in the department of mathematics teaching. Accordingly, it was concluded that students who could not internalize the process stage did not use the external clues and formulas in the action stage effectively.

It was found out that the students with high success were able to interpret the process phase mainly in the phase ''Object'' and to see, and interrelate in between, the concepts holistically. In addition, it was found that they needed relationships in the phase ''Process'' because they did not answer the question correctly. In other words, it was discovered that the students with high success were able to make sense of the concepts of trigonometric functions on the unit circle at the end of the phase ''Object''. The study with a result similar to this result was carried out by Sumajji et al., (2020) in Indonesia. The study examined the reasoning situations in problem solving in high school students within the APOS theoretical framework, and it was seen that they were able to construct a mathematical concept at the end of the phase ''Object''. It is seen that the two results show parallelism.

Another result reached is that students with low success cannot interpret the phase ''Process'' in the phase ''Object''. It was also found out that they could not see the concepts of trigonometric functions on the unit circle in a holistic way and did not make sense of the concepts. Furthermore, another result found is that it is difficult for students with low success to switch to the phases ''Process'' and ''Object'' because their answers to the interview questions prepared in accordance with the phase ''Action'' were not correct and clear. This result is parallel to the result of the study of Batır (2022). The study conducted by Batır (2022) with high school graduates found out that the transition between the other phases was difficult for students who lacked the phase ''Action''.

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***Ethics approval**: This study was approved by Akdeniz University Institute of Education Sciences Ethics Committee on 07.11.2022*
***Conflict of interest**: The authors declare that they have no conflict of interest.*

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