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Statistical Reliability Evaluation of a k-out-of-n:G System Subject to Competing Failure Processes

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k-out-of-n system,
Reliability analysis,
Gamma distribution.

Abstract: Many systems in real life can experience failure in two categories such as a soft failure process or a catastrophic failure process. Therefore, failure processes are competing since either failure can cause the system to fail. In this study, a reliability analysis of a k-out-of-n:G system which is exposed to a competing failure process is investigated based on different distribution assumptions for a shock magnitude, a damage size and a wear volume due to continuous degradation. Besides, unlike the researches in this field, in a more statistical frame, the reliability results are also examined in detail by considering normal distribution assumption. The reliability results are discussed based on a change in parameters, number of shocks and time. Graphical illustrations are also provided to observe the parameter effects explicitly on the reliability of the system.

Rakip Bozulma Süreçlerine Maruz Kalan bir n'den k çıkışlı:G Sistemine ait İstatistiksel Güvenilirlik Değerlendirmesi

Anahtar Kelimeler
Rakip bozulma süreçleri,
Şok modelleri,
n den k çıkışlı sistem,
Güvenilirlik analizi,
Gamma dağılımı.

Özet: Gerçek hayatta birçok sistem, yıkıcı olmayan bozulma veya yıkıcı bozulma süreçleri gibi bozulmaları iki kategoride deneyimleyebilirler. Her iki arıza da sistemin bozulmasına sebep olabileceğinden arıza süreçleri rekabet halindedir. Bu çalışmada, rakip bozulma süreçlerine konu olan şok büyüklüğü, hasar boyutu, sürekli bozulmaya sebep olan aşınma hacmi için farklı dağılım varsayımlarına dayalı n'den k çıkışlı:G sistem yapısının güvenilirlik analizine yer verilmektedir. Bunun yanında, bu alandaki çalışmalardan farklı olarak, güvenilirlik sonuçları normal dağılım varsayımı altında istatistiki açıdan daha ayrıntılı olarak incelenmiştir. Güvenilirlik sonuçları şok sayısı, zaman ve parametrelerdeki değişim dikkate alınarak tartışılmıştır. Sistem güvenilirliği üzerindeki parametre etkilerini açık bir şekilde gözlemlemek için sonuçlar ayrıca grafikler ile de sunulmuştur.

Nomenclature:

N(t) number of shocks arrived at t
For a soft failure process;
X(t) wear volume at t as a result of a continuous degradation
G(x;t) cumulative distribution function (cdf) of X(t)
$X_S(t)$ total wear volume at t
$F_X(x;t)$ cdf of $X_S(t)$
Y_j damage size contributing to soft failure
$f_Y(y)$ probability density function (pdf) of Y_j
$f_Y^{<k>}(y)$ probability density function (pdf) of the sum of k i.i.d Y_j variables
K(t) cumulative shock damage size at t.
H critical wear degradation failure threshold
For a catastrophic failure process;

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W size/magnitude of the jth shock load
$F_W\ (v)$ cdf of W_j
D threshold for catastrophic failure
Φ cdf of the standard normal random variable

1.Introduction

Many engineering systems can experience different shocks during their degradation process. These systems and their units therefore can fail due to those shocks along with their natural degdaration process. In this respect a competing failure process concept is arised. A soft failure or a catastrophic failure process can make the system fail in case of a competing failure process. The two failure processes have some threshold values and when they are exceeded the system or the units fail. In both of the failure processes, the failures are affected by the same random shock and this makes the processes dependent. Also, whenever one of the thresholds of each failure processes is exceeded, the system fails and that's why the failure processes are competing. The reliability research of systems under the influence of competing failure processes and shocks is worthy of interest especially in the field of reliability engineering. There has been plenty of studies regarding the shock models and competing failure processes in the literature. Gut [1] defined a cumulative shock model and proposed limit theorems for the failure and working times of a system. Eryilmaz and Kan [2] examined an extreme shock model in two stages. In the initial stage, the sizes of the shock are small in some degree, but in the following stage, the system experience more larger shocks due to an abrupt change in the environment. Then, the mean time to failure and reliability achievements were done considering an extreme shock model based on a random change point in that study. Wang et al. [3] obtained a function of reliability based on di§erent failure modes which were catastrophic, deterioration and failure due to shocks. They also classiffed the effect of shocks on the system in two categories. In one of them, they considered an increased failure rate after a shock arrived and in the other one they took into account a random change in the degradation after the occurence of the shock. An engineering application was given to support the model examined. Li and Pham [4] proposed a generalized multi-state model which considers shocks arriving at random and multi-competing failures. Two degradation processes were taken into account. In the model proposed, a reliability evaluation of a system was done by obtaining the system state probabilities considering both of the degradations. Peng, Feng and Coit [5] dealt with a reliability optimization problem of a surface-micromachined microengines ináuenced by wear degradation. Song et al. [6] considered the reliability analysis problem of a k-out-of-n structure which was exposed to multiple failure processes. Different from the previous studies failure times of the units are dependent. A soft failure or a catastrophic failure process can cause each unit to fail. Since the units fail due to the occurence of either of the failure processes, those failure processes are considered as competing. Pham et al. [7] developed a model for predicting the working probability of a k-out-ofn:G structure assuming that components were subjected to several stages of degradation and a catastrophic failure. State-dependent transition rates were used and also some functions for the reliability and MTTF were provided. Besides, another model was improved in Song et al. [8] in which s-dependent failure times of degradated components influenced by a shared shock were considered. The model was so useful for the reliability evaluation of many systems with several components that experience s-dependent competing failure processes. Rafiee et al. [9] also dealt with the working probability of a system under the influence of competing failure processes with one exception that is the changing failure rate. In complex systems, the system turns out to be capable of being affected from fatigue and deteriorates more quickly, in consequence of withstanding shocks. Therefore, four different shock patterns that could result with an increased degradation rates were considered within the study and the problem was discussed within this respect. Cha et al. [10] examined the dependency between the two competing processes where failures were categorized as the failures resulted from the accumulated wear and the catastrophic failure. An and Sun [11] classiffed the sock loads into two. According to their contribution, some shock loads especially which are beyond the threshold value cause the system's sudden failure. However, some of them which are between the certain level and shock threshold just cause sudden degradation increments. Under these circumstances a new reliability model was proposed under competing failures. Fan et al. [12] considered competing failures in which there was a dependency among the random shocks and the degradation process. The dependence effect was searched with reference to some classiffication of the shocks. The results were supported by a realistic application. Che et al. [13] took into account the reliability analysis problem based on mutually dependent degradation and shock processes in case of a competing failure process. The intensity of the shocks had a connection with the number of arrival shocks. Therefore, different from the previous studies, in this study Poisson process was not used to describe the arrival of the shocks. A multi-component system was considered by Shen et al. [14]. They took into account a reliability issue in which the deterioration behavior of a particular component can influence another component. A recursive method was proposed for the working probability evaluation problem of a series system. Moreover, a simulation was conducted to find the approximated failure times of a k-out-of-n system. Qiu and Cui [15] evaluated the reliability by considering two-stage dependent failures. The dependence of the stages were due to the shared non-homogenous Poisson modeled shock process. A reliability issue of a multiple dependent failure processes with a varied failure threshold was considered by Jiang et al. [16]. Different from the previous researches considering fixed threshold levels, within this study the threshold level of a catastrophic failure was affected by the shock process. In Wang et al. [17] a new age- and state-dependent competing risks model which took into account of random shocks was proposed. As a new contribution the overall degradation was affected by the current state and the degradation rates were accelerated by the shocks.

In almost all the the studies considering multiple dependent failure processes, besides the theoretical contributions, especially the applications were obviously conducted only under normal distribution assumptions. However, from the statistical point of view, some other distributions can be used to model the failure processes in nature. For instance; it is widely known that an electrical lamp's lifetime is exponentially distributed. In addition to the natural degradation process of the lamp, there can be some voltage increases that can affect the failure process of the lamp. Those increases result some wear outs for the lamp besides its natural deterioration. In addition, they can also lead the system to fail. Therefore, in the reliability evalution of the lamp, competing failure processes can be handled. This particular study, therefore, seeks to examine different distribution assumptions for the failure processes and their resulting effects on the working probability of a k-out-of-n system and its units. Besides, this study also aims to investigate the effects of the distribution parameters, the required and the total number of units in the system and the time change on the reliability and present the results within a more detailed frame. In section II, the indeed explanation of a competing failure process is given with reference to the literature and the model assumptions in this study are provided as well. In Section III, statistical reliability evaluation of a unit and a k-out-of-n system under normal distribution assumptions is done in detail by considering both soft failure and catastrophic failure processes, respectively. Section IV includes again a detailed statistical reliability evaluation problem of both a unit and a k-out-of-n system which is exposed to a competing failure process under different distribution assumptions than a normal distribution. Finally in section V, the major findings of the study are presented and suggestions are made for further studies.

2. Multiple Competing Failure Processes and Random Shocks

The two failure modes are generally used in the literature; one is a catastrophic failure in which units corrupted by some instantenous exterior shocks, the other one is a failure based on a degradation process in which units fail due to a physical deterioration. Therefore, it is important to examine probabilistic or stochastic models of systems exposed to competing failure processes lean on degradation and random shocks. Many researchers has been working on the problem of combining these two competing failure modes.

Shock models are also worth of interest in reliability researches and have great importance in the reliability evaluation of units or systems exposed to competing failure processes. Shock models in reliability analysis can be classified based on the time interval between two successive shocks or the damage size grew out of an single random shock and the system failure function. In shock modeling, the system is affected by shocks with random magnitudes and which occur at random times. Thus, shocks usually has a Poisson process. To supply mathematical formulations for the relaibility models of systems exposed to shocks many researchers provide many papers. Three classical random shock models are substantially used in the literature; cumulative, extreme and δ-shock models (Gong et al. [18], Lorvand et al. [19], Tuncel and Eryilmaz [20], Eryilmaz [21]). In this study, we consider a cumulative and an extreme type of a shock model. Besides, in real lifetime applications it is generally accepted that a graceful deterioration and a discrete random shock affect the system and makes it deteriorate. The failure processes are assumed to be independent in many of the studies regarding competing risks of degradation. However, in reality the degradation can increase with a sudden jump or with the increase in the degradation rate as a result of a shock. Therefore, due to this fact the processes is considered as dependent.

In the study of Song et al. [6], a working probability evaluation problem is discussed under two dependent failure processes. One of the failure processes is the soft failure process. It is resulted from both a lasting degradation and an additional suddenly terminating damage due to a shock process. The other one is the catastrophic failure which is resulted from an instantaneous stress from the same shock process. In real life applications of many systems and reliability studies, those two failure modes are handled together. Therefore, probabilistic and stochastic modeling of those competing and dependent failure processes which consists of both the random shocks and continuous degradation simultaneosuly is worth of notice. It is important to represent some assumptions regarding the model taken into account in Song et al. [6] in this part of the study as follows;

Assumptions of the Model:

1. Whenever Xs(t) is beyond a threshold value H, soft failure exists (according to a cumulative shock modeling).
2. Whenever W_j itself goes beyond the maximum strength D, catastrophic failure exists(according to an extreme shock modeling).
3. Poisson process explains the arrival of the random shocks.
4. The system is nonrepairable.
5. The system works when at least k units exist due to both soft failure and catastrophic failure processes.

2.1. Modeling catastrophic failures

The arrival of the shocks follows Poisson process with parameter λ. Let W_j be the size of the jth shock at time t_j , j = 1,2,.. ∞ and they arrive independently. Then, the probability of surviving the jth shock for a unit is;

$$P\,(W_j\ <\ D)\ =\ F_W\,(D), j\ =\ 1,2,\ldots,\infty \qquad (1)$$

If $W_i \sim N(\mu_W, \sigma_W)$, $i\ =\ 1,2,\ldots,\infty$, then the related probability turns out;

$$P_L\ =\ F_W(D)\ =\ \Phi\left(\frac{D-\mu_W}{\sigma_W}\right), \qquad (2)$$

2.2. Modeling soft failures

The total degradation of the system is;

$$X_S(t)\ =\ X(t)\ +\ K(t) \qquad (3)$$

where $X(t) = \phi + \beta t$ represents the degradation due to continual wear (ϕ; ; initial value, β ;degradation rate) and K(t) indicates the cumulative damage size till time t. K(t) is;

$$K(t)\ =\ \begin{cases} \sum_{j=1}^{N(t)} Y_j & \text{eğer } N(t) > 0 \text{ ise} \\ 0, & \text{eğer } N(t) = 0 \text{ ise} \end{cases} \qquad (4)$$

The probability of the total degradation being less than a certain value x can be obtained;

$$F_X(x,t) = \mathrm{P}(X_s(\mathrm{t}) < x)$$

$$\sum_{j=0}^{\infty} \mathrm{P}(\mathrm{X(t)} + K(t) < x | N(t) = j)\mathrm{P}(\mathrm{N(t)} = j) \qquad (5)$$

Let X(t) distribute with G(x;t) and Y_j distribute with f_Y(y), the distribution of Y_1+.....+Y_k is represented as $f_Y^{<k>}$(y). Thus, equation (5) transforms;

$$F_X(x,t)\ =\ \mathrm{G(x,t)exp(-\lambda t)}$$

$$+\sum_{j=1}^{\infty}\left(\int_0^x G(x-u,t) f_y^{<i>}\,(u)du\right)$$

$$\times\frac{exp(-\lambda\mathrm{t})(\lambda\mathrm{t})^j}{j!} \qquad (6)$$

If $Y_j \sim \mathrm{N}(\mu_y, \sigma_y^2)$) and are independent, and also there is a linear degradation with a constant φ and a β ~ $\mathrm{N}(\mu_\beta, \sigma_\beta^2)$ then, the probability that soft failure is not observed before t;

$$F_X(\mathrm{H,t})\ =\ \sum_{j=0}^{\infty}\Phi\left(\frac{H-(\mu_\beta t+\varphi+\mathrm{j}\mu_\mathrm{y})}{\sqrt{\sigma_\beta^2 t^2 + j\sigma_\mathrm{Y}^2}}\right)$$

$$\times\frac{exp(-\lambda\mathrm{t})(\lambda\mathrm{t})^j}{j!} \qquad (2)$$

(7)

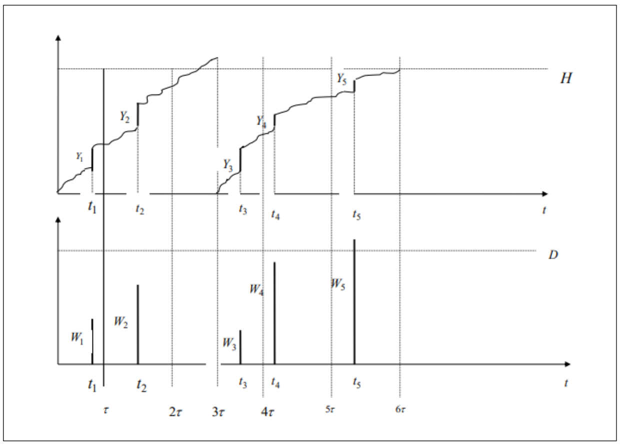

Figure 1. Two competing failure processes for a unit

2.3. Statistical reliability evaluation of a system

In a k-out-of-n system structure, the unit failure times are dependent because the units undergo the same shock effects. In such systems, the reliability is obtained as the probability of having at least k units surviving each of shock loads ($W_j < D$ for j = 1,2,...) and their total deterioration being less than the threshold ($X_s(t) < H$). For the reliability of the system we have S described as the set of system units and $\phi(t)$ as the set of the working units, $\phi(t) \subset S$) . In that case, it has been given as;

$$\phi(t) = \{index\ of\ components\ satisfying: W_1 < D, W_2 < D, \ldots, W_{N(t)} < D, X(t) + \sum_{j=1}^{N(t)} Y_j < H\} \quad (8)$$

$$\phi(t) = \{index\ of\ components\ satisfying: \bigcap_{j=1}^{N(t)} (W_j < D), X(t) + \sum_{j=1}^{N(t)} Y_j < H\} \quad (9)$$

Then, the working probability of the system is achieved by the probability of the intersection of the events that the unit degradations are below the failure thresholds. Therefore, the working probability;

$$R(t) = \mathrm{P}(|\Phi(t)| \geq k)$$

$$= \sum_{m=0}^{\infty} \mathrm{P}\{|\phi(t)| \geq k | N(t) = m\} \quad (10)$$

$$\times \mathrm{P}\{N(t) = m\} \quad (10)$$

Each unit is affected by the shocks arriving with a Poisson process. Then,

$$\mathrm{R(t)} = \sum_{m=0}^{\infty} \sum_{i=k}^{n} \binom{n}{i} P\{\bigcap_{j=1}^{N(t)} (W_j < D), X(t) + \sum_{j=1}^{N(t)} Y_j < H | N(t) = m\}^i \quad (10)$$

$$\{1 - P(\bigcap_{j=1}^{N(t)} (W_j < D), X(t) + \sum_{j=1}^{N(t)} Y_j < H | N(t) = m\}^{n-i} \times P\{N(t) = m\} \quad (11)$$

Based on the distributions of the related random variables, reliability can also be written as;

$$\mathrm{R(t)} = \sum_{i=k}^{n} \binom{n}{i} [P(X(t) < H)]^i \times [1 - P(X(t) < H)]^{(n-i)} \quad (10)$$

$$\times P(N(t) = 0) + \sum_{m=1}^{\infty} \sum_{i=k}^{n} \binom{n}{i} \{(P(W < D))^m \times \int_0^H G(H-u) f_Y^{<m>}(u)\, du\}^i \times \{1 - (P(W < D))^m \times \int_0^H G(H-u) f_Y^{<m>}(u)\, du\}^{n-i} \times \frac{e^{-\lambda t} (\lambda t)^m}{m!} \quad (12)$$

or as;

$$\mathrm{R(t)} = \sum_{i=k}^{n} \binom{n}{i} [P(X(t) < H)]^i \times [1 - P(X(t) < H)]^{(n-i)} e^{-\lambda t} \quad (10)$$

$$+ \sum_{m=1}^{\infty} \sum_{i=k}^{n} \binom{n}{i} \{F_W(D)^m \times \int_0^H G(H-u) f_Y^{<m>}(u)\, du\}^i \times \{1 - F_W(D)^m \times \int_0^H G(H-u) f_Y^{<m>}(u)\, du\}^{n-i} \times \frac{e^{-\lambda t} (\lambda t)^m}{m!} \quad (13)$$

3. Statistical Reliability Evaluation Under a Normal Distribution

In this part of the study, the statistical reliability evaluation of a unit and a system based on a competing failure process is examined by taking into account different distribution assumptions for wear volume, damage size and magnitude of a shock. First, the theoretical results presented in the literature regarding the success probability of a k-out-of-n structure under competing failure process is examined under a normal distribution. Second, under different distribution assumptions some theoretical achievements are provided with some numerical illustrations.

3.1. Statistical reliability evaluation of a unit

In this part of the study, under the theoretical achievements found in the literature regarding the reliability of a unit and a system under competing failure processes, the reliability results based on normal distribution assumption are discussed in detail by considering different parameter and k values or different number of units.

In Case of a Soft Failure:

In the the reliability evaluations under normal distribution assumption, we consider the following parameters given in Table 1(Song et al. [6]).

Table 1. Parameter values for both catastrophic and soft failure processes

Parameter	Values
H	0.00125 μm³
D	1.5 Gpa
ϕ	0
β	$\beta \sim N(\mu_\beta, \sigma_\beta^2)$, μ_β=8.4823*10^{-9} μm³, $\sigma_\beta = 6.0016 * 10^{-10}$ μm³
Y_j	$Y_j \sim N(\mu_Y, \sigma_Y^2)$, μ_Y=10^{-4}μm³, $\sigma_Y = 2 * 10^{-5}$ μm³
W_j	$W_j \sim N(\mu_W, \sigma_W^2)$, μ_W1.2 GPa $\sigma_W = 0.2$ GPa

When the unit is exposed to different number of shocks arrived, the reliability values of the unit are given in the following Table 2.

Table 2. Reliability of a Unit Based on Different Shock Numbers

	t=1.5		*t=5.0*		*t=6.5*	
	λ=0.9	λ=1.2	λ=0.9	λ=1.2	λ=0.9	λ=1.2
m=0	0.259	0.165	0.011	0.002	0.003	0.000
m=1	0.609	0.463	0.061	0.017	0.020	0.004
m=10	1.0	1.0	0.993	0.957	0.963	0.835
m=50	1.0	1.0	0.999	0.990	0.991	0.941

The working probability values decrease by the time increase. Also, as it is expected lower reliability values are observed with the large arrival rates of random shocks. For the changes in the parameters, the reliability results are also investigated and presented in the following Table 3-Table 4.

Table 3. Reliability of a Unit Based on the Change in μ_Y and σ_Y

	t=5.0		*t=10.0*	
μ_Y	λ=0.9	λ=1.2	λ=0.9	λ=1.2
10^{-4}	0.9989	0.9896	0.8705	0.5755
$3x10^{-4}$	0.5120	0.2709	0.0514	0.0070
$7x10^{-4}$	0.0611	0.0174	0.0012	0.00007
σ_Y				
$2x10^{-5}$	0.9989	0.9871	0.8705	0.5755
$5x10^{-5}$	0.9972	0.9822	0.8507	0.5743
$10x10^{-5}$	0.9876	0.9559	0.8024	0.5672

While searching the effect of the parameter μ_Y on the reliability, the results are obtained by assuming μ_β = 8.4823x10^(-9) ; σ_β = 6.0016x10^(-10) ; σ_Y = 2x10^(-5) and m=100 and for the effect of σ_Y, the results are obtained by assuming μ_β =8.4823x10^(-9), σ_β= 6.0016x10^(-10), μ_Y =10^(-10) and m=100 in Table 3. The reliability values decrease with an increase in μ_Y and in the arrival rate of shocks. Also, the reliability values decrase with an increase in σ_Y values. Besides, they also decrease with an ascent in the arrival rate of shocks.

Table 4. Reliability of a Unit Based on the Change in μ_β

	t=7.0		*t=12.0*	
μ_β	λ=0.9	λ=1.2	λ=0.9	λ=1.2
$8.4823x10^{-9}$	0.9852	0.9100	0.7069	0.3249
$10x10^{-9}$	0.9852	0.9100	0.7069	0.3249
$13x10^{-9}$	0.9852	0.9100	0.7069	0.3249

The working probability results are obtained by assuming μ_Y =10^(-4), σ_Y = 2x10^(-5), σ_β = 6.0016x10^(-10), and m=100 in Table 4. There is not an explicit effect on the reliability values of the parameter μ_β. However, by the time and the shock arrival rate increase, reliability decrease is explicitly observed. When the results in Table 5 are discussed, similar interpretations can be done for the reliability change of the unit based on the change in σ_β with the change in μ_β. Also, the reliability results are obtained by assuming μ_Y =10^(-4), σ_Y = 2x10^(-5), μ_β =8.4823x10^(-9) and m=100 in Table 5.

Table 5. Reliability of a Unit Based on the Change in σ_β

	t=5.0		*t=10.0*	
σ_β	λ=0.9	λ=1.2	λ=0.9	λ=1.2
$86.0016x10^{-10}$	0.9989	0.9896	0.8705	0.5755
$8x10^{-10}$	0.9989	0.9896	0.8705	0.5755
$10x10^{-10}$	0.9989	0.9896	0.8705	0.5755

In Case of a Catastrophic Failure:

When the reliability results of a unit are examined under a catastrophic failure, the effects of the parameters of the shock load's magnitude on the working probability are investigated and presented with the following figures.

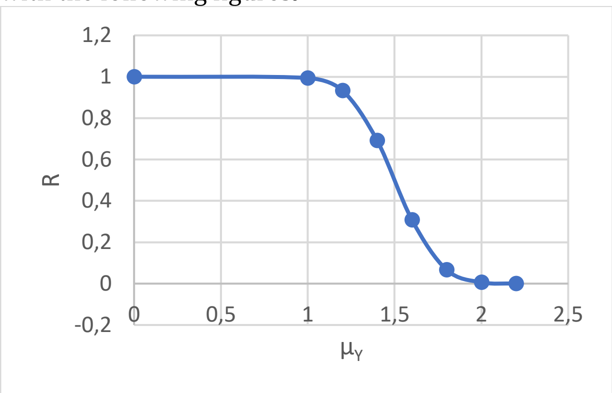

Figure 2. Reliability of a Unit Based on the Change in μ_Y

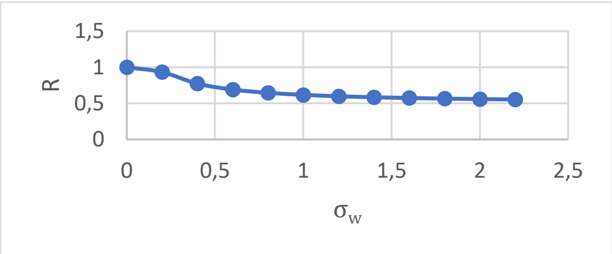

Figure 3. Reliability of a Componet Based on the Change in σ_W

3.2. Statistical reliability evaluation of a system

By the use of equation (13), the reliability function of a system is ;

$$R(t) = \sum_{i=k}^{n} \binom{n}{i} \left[\Phi\left(\frac{H-\mu_\beta t-\varphi}{\sigma_\beta t}\right)\right]^{i} \times \left[1-\Phi\left(\frac{H-\mu_\beta t-\varphi}{\sigma_\beta t}\right)\right]^{(n-i)} e^{-\lambda t} \quad (10)$$

$$+\sum_{m=1}^{\infty}\sum_{i=k}^{n}\binom{n}{i}\left(P_L{}^{m}\Phi\left(\frac{H-(\mu_\beta t+\varphi+m\mu_Y)}{\sqrt{\sigma_\beta{}^2t^2+m\sigma_Y{}^2}}\right)\right)^{i} \times\left(1-P_L{}^{m}\Phi\left(\frac{H-(\mu_\beta t+\varphi+m\mu_Y)}{\sqrt{\sigma_\beta{}^2t^2+m\sigma_Y{}^2}}\right)\right)^{n-i} \times\frac{e^{-\lambda t}(\lambda t)^m}{m!} \quad (14)$$

where P_L can be obtained by equation (2). The reliability results of a system are presented in the following Table 6 for different n and k values.

Table 6. Reliability Results for a k-out-of-n G system

n	k	Rs(0)	Rs(1.5)	Rs(5)	Rs(7)	Rs(10)
5	1	1.0	1.0	0.999	0.993	0.917
5	3	1.0	1.0	0.999	0.987	0.874
5	5	1.0	1.0	0.998	0.975	0.816
10	1	1.0	1.0	0.999	0.995	0.930
10	3	1.0	1.0	0.999	0.991	0.899
10	10	1.0	1.0	0.997	0.970	0.797

The reliability results are obtained by assuming μ_Y=10^(-4), σ_Y= 2x10^(-5), μ_β =8.4823x10^(-9), σ_β= 6.0016x10^(-10), μ_W =10^(-4)), σ_W=0.2, H=0.00125, D=1.5, λ =0.9 in Table 6. When the results are examined, it is clearly observed that there is a decrease in reliability with a time increase. When n is constant and k is increased, it results with a reliability decrease whereas when k is constant and n increases, it results with an increase in the reliability values. Therefore, when n increases in the system, the working probability values of system also increase. This result is also discussed in the Figure IV. In Figure IV, the black solid line indicates the working probability results of a 5-out-of-7:G system. The blue dashed line denotes the working probability results of a 5-out-of-15:G system and the red dashed line indicates the working probability results of a 5-out-of-30:G system. High reliability values are observed when n is large. Also, with the time change, all the reliability values are observed to decrease.

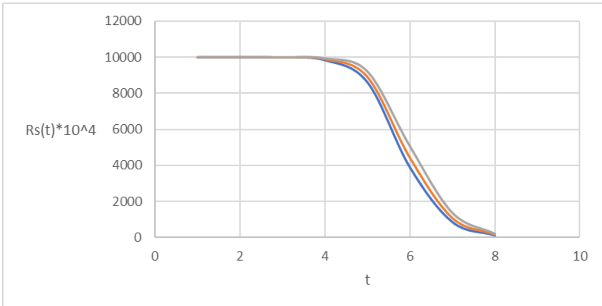

Figure 4. Reliability Results Based on Normal Distribution Assumption

4. Statistical Reliability Evaluation Under Exponential and Gamma Distributions

4.1. Statistical reliability evaluation of a unit

In the following part, we will provide reliability functions of a unit under competing failure processes considering an exponential distributed wear volume, a damage size and a shock load's magnitude whereas a gamma distributed sum of the damage sizes. First, we provide the reliability function of the unit under a soft failure process. Then, a reliability function of the unit under a catastrophic failure is given.

In Case of a Soft Failure:

Let the wear volume is; X(t) = Xt where X~ Exp(λ_1). Therefore, the distribution of X(t) is also exponential with the parameter $\frac{\lambda_1}{t}$. Let also $Y_j \sim$ Exp(λ_2). Besides, shocks arrive with a Poisson process. Then, the reliability function of the unit under a soft failure process is;

$$P(X_S(t) < x) = (1 - e^{-\frac{\lambda_1}{t}x})e^{-\lambda t} + \sum_{n=1}^{\infty} \int_0^x (1 - e^{-\frac{\lambda_1}{t}(x-u)}) \frac{\lambda_2^n u^{n-1}}{(n-1)!} du \times \frac{e^{-\lambda t}(\lambda t)^n}{n!} \quad (15)$$

In Case of a Catastrophic Failure:

Let $W_j \sim$ Exp(λ_S), then the reliability function of a unit under a catastrophic failure process is;

$$P(W_i < D) = 1 - e^{-\lambda_S D} \quad (16)$$

The reliability results of a unit is examined for different values of the parameters and discussed in the following Table 7-Table 9. In all the following tables H = 5 is assumed.

Table 7. Reliability of a Unit Based on the Change in λ

		t=0.5	*t=1.5*	t=5.0
$\lambda_1 = 1.5$, $\lambda_2 = 0.5$	λ=0.9	0.944	0.731	0.126
	λ=1.2	0.921	0.637	0.058
	λ=1.4	0.905	0.576	0.034

When the results in Table 7 are discussed, it is observed that the reliability values decrease by the increase in the parameter λ. This is an expected result actually because λ indicates the arrival rate of random shocks and an increased arrival rates result with more shocks to arrive and effect the unit. Therefore, with the increase in this parameter the reliability of the unit is expected to decrease. Also, another result which is also expected to be observed is the reliability decrease by the time increase in Table 7.

Table 8. Reliability of a Unit Based on the Change in λ_1

		t=0.5	*t=1.5*	t=5.0
$\lambda = 0.5$, $\lambda_2 = 0.6$	λ_1=0.5	0.963	0.676	0.165
	λ_1=0.7	0.974	0.768	0.216
	λ_1=1.5	0.981	0.888	0.362

Different from the previous interpretation in the reliability in Table 7, in Tables 8 and 9, with the increase in the parameters λ_1 and λ_2 , there is a decrease in the reliability of a unit. As the increase in λ_1 indicates a decrease in the wear volume and the increase in λ_2 denotes a decrease in the damage size, both cases result with an increase in the reliability values.

Table 9. Reliability of a Unit Based on the Change in λ_2

		t=0.5	*t=1.5*	t=5.0
$\lambda = 0.5$, $\lambda_1 = 0.7$	λ_2=0.5	0.964	0.740	0.190
	λ_2=0.9	0.990	0.822	0.282
	λ_2=1.7	0.998	0.872	0.382

Besides, Figure 5 shows the change on the reliability based on the change in the parameter λ_S . The reliability results are obtained by assuming D = 1.5. The reliability of a unit is observed to increase with an increase in the failure rate of random shocks in this figure. When the failure rate of random shocks increases, it is expected to have higher reliabilities of a unit.

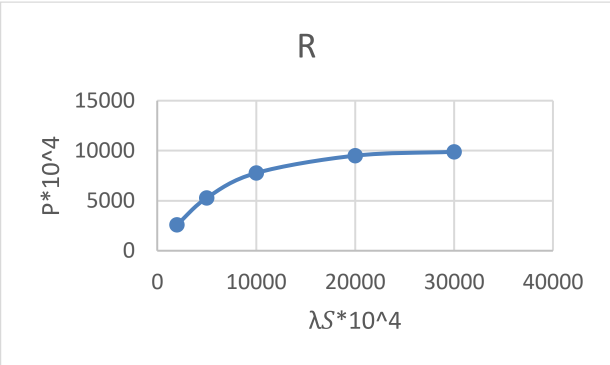

Figure 5. Reliability of a Unit Based on the Change in λ_S

4.2. Statistical reliability evaluation of a system

Considering an exponential distributed wear volume, damage size and shock magnitude and Gamma distributed damage size sum, we provide the reliability function of a k-out-of-n system as;

$$R(t) = \sum_{i=k}^{n} \binom{n}{i} \left[1 - e^{-\frac{\lambda_1}{t}H}\right]^i \times [1 - (1 - e^{-\frac{\lambda_1}{t}H})]^{n-i} \times e^{-\lambda t} + \sum_{m=1}^{\infty} \sum_{i=k}^{n} \binom{n}{i} [(1 - e^{-\lambda_S D})^m$$

$$\times \int_0^H (1-e^{-\frac{\lambda_1(H-u)}{t}}) \times \frac{\lambda_2^m u^{m-1} e^{-\lambda_2 u}}{(m-1)!} du]^i$$

$$\times\{1-(1-e^{-\lambda_s D})^m$$

$$\times (\int_0^H (1-e^{-\frac{\lambda_1(H-u)}{t}} \times \frac{\lambda_2^m u^{m-1} e^{-\lambda_2 u}}{(m-1)!} du)\}^{n-i}$$

$$\times \frac{e^{-\lambda t}(\lambda t)^m}{m!} \quad (17)$$

The reliability results are presented in the following Table 10 for various n and k.

Table 10. Reliability Results for a k-out-of-n system

n	k	Rs(0)	Rs(0.07)	Rs(0.15)	Rs(0.5)	Rs(1)
5	1	1.0	0.9920	0.9826	0.9352	0.8423
5	3	1.0	0.9694	0.9356	0.7986	0.5821
5	5	1.0	0.9656	0.9276	0.6035	0.1718
10	1	1.0	0.9980	0.9952	0.9753	0.9210
10	3	1.0	0.9826	0.9627	0.8731	0.7228
10	10	1.0	0.9656	0.9273	0.4673	0.0486

The reliability results are obtained by assuming $\lambda = 0.5, \lambda_1 = 0.3, \lambda_2 = 0.7, \lambda_s = 0.2$, H=5, D=1.5. When the results are discussed, the reliability decrease is explicitly observed with the time increase again for this system structure. When n increases, the working probability of the system also increases. Besides, with the increase in k, there is a decrease in the working probability of the system.

5. Conclusions

This study aims to examine the reliability of a k-out-of-n system and its units exposed to competing failure processes under different distribution assumptions. In previous studies, the reliability of the system is solely searched by considering normal distribution assumption. However, in real life the distributions of wear volume, damage size or the shock magnitudes can be different from a normal distribution. In this case, different parameters can have some effects on the reliabilities of the units and the system. Therefore, besides investigating the effect of the parameters of the normal distributed wear volume, damage size or shock magnitudes, it is worthy of consider to examine the effects of different distributions and their parameters on the working performance of the system under a competing failure mechanism. With this aim, first under normal distribution assumption the reliability results are discussed in detail with reference to the changes in the parameters, and by the use of reliability functions already achieved in the literature. Then, the reliability functions of a system is obtained under an exponential distributed wear volume, damage size and shock magnitude and Gamma distributed damage size sum. Moreover, the reliability results are examined based on the changes in the parameters of the related distributions. The effect of the time change on the working probability are also analysed along with different number of units and k values of a system.

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