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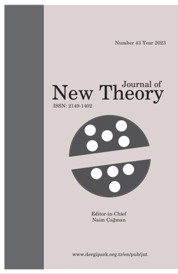

Magnetized Strange Quark Models in Lyra Theory

Kadir Özcan[1], **Can Aktaş**[2]

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Abstract — In this study, the behavior of magnetized strange quark matter (MSQM) distribution in Lyra theory was investigated for homogeneous anisotropic Bianchi III, locally rotationally symmetric (LRS) Bianchi I, and Kantowski-Sachs universe models. We have used the equations of state, anisotropy, and linearly varying deceleration parameters to obtain the exact solutions of field equations in Lyra theory. When switching from the anisotropic universe model to the isotropic universe model, the magnetic field was not observed in the LRS Bianchi I universe. Besides, the graphs of the dynamic quantities obtained for each universe model were analyzed in detail. Finally, we inquire whether further research should be conducted.

Keywords *Magnetized strange quark matter, Lyra theory, LRS Bianchi I, Kantowski-Sachs, Bianchi III*

Mathematics Subject Classification (2020) 83C05,83C15

1. Introduction

Recent experiments and observations show that the universe is accelerating and expanding [1,2]. What caused this accelerating expansion is still unknown. However, scientists have some assumptions about the causes of the accelerating expansion of the universe. The strongest of these assumptions can be counted as dark matter - dark energy. Einstein published General Relativity theory in 1916. General Relativity theory is one of the most important theories explaining the relationship between matter and space-time geometry. General Relativity Theory tries to explain the universe's structure on a large scale. However, General Relativity Theory falls short of explaining the universe's accelerating expansion. Edwin Hubble proved with observations that the universe is accelerating and expanding. After this proof, other theories that could be alternatives to General Relativity Theory were put forward. Among these alternative theories are Lyra, Brans-Dicke, $f(T)$, $f(G)$, and $f(R,T)$, etc. These alternative theories are reduced to the General Relative theory in special cases. Lyra theory, one of these alternative theories, was put forward in 1951 [3]. Lyra is a modified theory created by adding the term containing the scalar field to the left side of the field equations in theory.

There are many articles in the literature on both Lyra theory and magnetized strange quark matter dispersion. Some of these can be summarized as follows. Katore and Kapse [4] have investigated magnetized dark energy model behaviors in Lyra theory for axially symmetric space-time. Mishra et al. [5] have researched 5D Kaluza-Klein universe with magnetized anisotropic fluid matter distribution in Lyra theory. Katore and Hatkar [6] have studied magnetized anisotropic dark energy for Kaluza-Klein universe model in the context of Lyra manifold. Anisotropic dark energy and massive scalar

[1]kadirozcan09@hotmail.com; [2]canaktas@comu.edu.tr (Corresponding Author)
[1,2]Department of Mathematics, Faculty of Science, Çanakkale Onsekiz Mart University, Çanakkale, Türkiye

field for Bianchi VI_0 metric was investigated by Ram and Verma [7] within the framework of Lyra theory. The holographic Ricci dark energy universe model was analyzed by Das and Bharali [8] within the framework of Lyra theory in high-dimensional metric. The perfect fluid matter distribution was studied by Raushan et al. [9] within the framework of Lyra theory for a homogeneous isotropic Friedmann-Robertson-Walker (FRW) universe. Naidu et al. [10] have investigated massive scalar field and perfect fluid for Bianchi I universe model in Lyra manifold. Aktaş and Aygün [11] have researched magnetized strange quark matter (MSQM) distribution for FRW universe model in $f(R,T)$ theory. Aygün et al. [12] have investigated scalar field solutions both Lyra geometry and Riemannian geometry for Marder space-time. Kalkan and Aktaş [13] have studied MSQM for 5D Kaluza-Klein metric in $f(R,T)$ theory. The behavior of MSQM was investigated by Kalkan et al. [14] within the framework of $f(R,T)$ theory in inhomogeneous anisotropic space-time. In addition, the physical properties of MSQM for the Bianchi VI_0 metric $f(R,T)$ were examined in theory by Kalkan and Aktaş [15]. Güdekli et al. [16] have researched strange stars for Krori Barua space-time in $f(T,\tau)$ theory. Tsallis dark energy universe model was explored by Khan et al. [17] in the Saez–Ballester theory of gravity for the locally rotationally symmetric (LRS) Bianchi V metric. Can and Güdekli [18] have analyzed for conservative and non-conservative $f(R,T)$ models. Abebe et al. [19] have studied viscous fluid matter distribution for Bianchi V universe model. The role of the jerk parameter in $f(R,T)$ gravitation theory were analyzed by Tiwari et al. [20].

Our motivation in this study is to investigate the space-time geometry of magnetized strange quark matter in Lyra theory, one of the alternative gravitational theories, for Bianchi III, LRS Bianchi I, and Kantowski-Sachs metrics.

This article is organized as follows: In Section 2, the field equations in Lyra theory, the general form of Bianchi III, LRS Bianchi I, and Kantowski-Sachs metrics in spherical coordinates, and the energy-momentum tensor of MSQM are provided. In Section 3, solutions are obtained for each metric using the deceleration parameter, the anisotropy parameter, and the equation of state for the MSQM distribution. In Section 4, the solutions obtained for each metric are analyzed in detail both mathematically and physically, and their graphs are drawn. The final section discusses the need for further research.

2. Field Equations in Lyra Theory

The field equations in Lyra theory can be written as follows [3, 21]:

$$R_{ik} - \frac{1}{2}g_{ik}R + \frac{3}{2}\left(\phi_i\phi_k - \frac{1}{2}g_{ik}\phi_j\phi^j\right) = T_{ik} \tag{1}$$

Here, R_{ik} is Ricci tensor, R is Ricci scalar, g_{ik} is metric tensor, T_{ik} is energy momentum tensor, and ϕ_i is the displacement vector, defined by

$$\phi_i = (0,0,0,\beta(t)) = \delta_i^4 \cdot \beta(t) \tag{2}$$

where $i \in \{1,2,3,4\}$. The general form of homogeneous anisotropic Bianchi III, LRS Bianchi I, and Kantowski-Sachs metric in spherical coordinates (r,θ,Φ,t) is as follows:

$$ds^2 = -dt^2 + A^2dr^2 + B^2(d\theta^2 + K_l^2(\theta)d\Phi^2) \tag{3}$$

where the metric coefficients A and B are functions of t. Moreover, $K_l^2(\theta)$ is a function defined as follows [22]:

$$K_l^2(\theta) = \begin{cases} \sinh^2\theta, & \text{if } l=-1 \text{ Bianchi III model} \\ \theta^2, & \text{if } l=0 \text{ LRS Bianchi I model} \\ \sin^2\theta, & \text{if } l=1 \text{ Kantowski - Sachs model} \end{cases}$$

The energy-momentum tensor for magnetized quark matter distribution is

$$T_{ik} = (\rho + p)\, u_i u_k + p g_{ik} + (2u_i u_k + g_{ik})\, \frac{h^2}{2} - h_i h_k \tag{4}$$

where p, ρ, and h^2 denote pressure, energy density, and magnetic field, respectively [23,24]. Moreover, u_i and h_i denote the 4-velocity and magnetic field vector, respectively. Besides, h_i and u^i have the relations $h_i u^i = 0$ and $u_i u^i = -1$. Due to the condition $h_i u^i = 0$, the magnetic field is selected in the radial direction.

Kinematic quantities for the given metric; spatial volume (V), Hubble parameter (H), expansion scalar (θ), shear scalar (σ^2), deceleration parameter (q), and mean anisotropy parameter are defined as follows:

$$V = a^3 = AB^2 \tag{5}$$

$$H = \frac{\dot{a}}{a} = \frac{\dot{A}}{3A} + \frac{2\dot{B}}{3B} \tag{6}$$

$$\theta = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \tag{7}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{8}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{-3AB(\ddot{A}B + 2A\ddot{B}) + 2(A\dot{B} - \dot{A}B)^2}{(A\dot{B} + \dot{A}B)^2} \tag{9}$$

and

$$AP = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H} - 1\right)^2 = \frac{6\left(\dot{A}B - A\dot{B}\right)^2}{\left(2\dot{A}B + A\dot{B}\right)^2} \tag{10}$$

Here, the dot represents the derivative with respect to time and H_i is component of Hubble parameter such that $H_1 = \frac{\dot{A}}{A}$ and $H_2 = H_3 = \frac{\dot{B}}{B}$.

3. Magnetized Strange Quark Matter Solutions for Bianchi III, LRS Bianchi I, and Kantowski-Sachs Metrics

From Equations 1, 3, and 4, we obtain the field equations in Lyra theory as follows:

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}}{B} - \frac{K_l''}{K_l\, B^2} + \frac{3}{4}\beta^2 = -p + \frac{1}{2}h^2 \tag{11}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = -p - \frac{1}{2}h^2 \tag{12}$$

$$\frac{2\dot{A}\dot{B}}{B} + \frac{\dot{B}^2}{B} - \frac{K_l''}{K_l\, B^2} - \frac{3}{4}\beta^2 = \rho + \frac{1}{2}h^2 \tag{13}$$

where $K_l'' = \frac{d^2 K_l}{d\theta^2}$. As can be seen from Equations 11-13, we have three equations with six unknowns A, B, p, ρ, β^2, and h^2. We need three additional equations such as anisotropy parameter, deceleration parameter, and equation of state to solve the system of equations exactly.

Firstly, we can take the deceleration parameter as an additional equation. Deceleration parameter is known as one of the important parameters showing whether the universe is accelerating or not. In many studies, the deceleration parameter was taken as constant. However, in studies in recent years, the deceleration parameter is taken depending on time. One of the deceleration parameters taken depending on time, especially the one in linear form, has become prominent in recent years.

The deceleration parameter in linear form was proposed by Akarsu and Dereli [25]. The deceleration parameter in linear form is

$$q = -kt + m - 1 \tag{14}$$

where k and m are constants. From the solution of this equation, the metric potential A is obtained as follows:

$$A = c_1 \frac{e^{\left(\frac{\tanh^{-1}\left(\frac{kt-m}{\sqrt{m^2+6c_2k}}\right)}{\sqrt{m^2+6c_2k}}\right)^6}}{B^2} \tag{15}$$

such that c_1 and c_2 are integral constants. Without loss of generality, we can take $c_1 = 1$ and $c_2 = 0$. In this situation, we get the metric potential A as

$$A = \frac{\left(\frac{kt}{kt-2m}\right)^{\frac{3}{m}}}{B^2} \tag{16}$$

Secondly, we can use the anisotropy parameter as an additional equation. The anisotropy parameter is a parameter that gives information about the isotropy of the universe. It can take values between 0 and 1. If the anisotropy parameter is zero, then the universe is said to be isotropic. The anisotropy parameter is defined as follows:

$$\frac{\sigma}{\theta} = \xi \tag{17}$$

where ξ is constant and $0 \leq \xi \leq 1$. From Equations 7, 8, and 17, we get metric potential B

$$B = c_3 \left(\frac{t}{kt-2m}\right)^{\frac{\sqrt{3}\xi+1}{m}} \tag{18}$$

where c_3 is integral constant. From Equations 16 and 18, we have

$$A = \frac{(-1)^{\frac{3}{m}}}{c_3^2} \left(\frac{kt-2m}{t}\right)^{\frac{2\sqrt{3}\xi-1}{m}} \tag{19}$$

Finally, we can use the equation of state for strange quark matter as an additional equation. The equation of state for strange quark matter is defined as follows:

$$p = \frac{\rho - 4B_c}{3} \tag{20}$$

where B_c is a bag constant [26]. If Equations 18 and 19 are substituted in Equations 11-13, then the pressure

$$p = -\frac{2\left(\sqrt{3}\,\xi - 2\right)(kt-m+3)}{(kt-2m)^2\, t^2} - \frac{K_l''}{2c_3^2K_l}\left(\frac{kt-2m}{kt}\right)^{\frac{2+2\sqrt{3}\xi}{m}} - 2B_c \tag{21}$$

the energy density,

$$\rho = -\frac{6\left(\sqrt{3}\,\xi - 2\right)(kt-m+3)}{(kt-2m)^2\, t^2} - \frac{3K_l''}{2c_3^2K_l}\left(\frac{kt-2m}{kt}\right)^{\frac{2+2\sqrt{3}\xi}{m}} - 2B_c \tag{22}$$

the magnetic field,

$$h^2 = \frac{12\xi\,(kt-m+3)\sqrt{3}}{t^2\,(kt-2m)^2} - \frac{K_l''}{c_3^2K_l}\left(\frac{kt-2m}{kt}\right)^{\frac{2+2\sqrt{3}\xi}{m}} \tag{23}$$

and the displacement vector component β^2

$$\beta^2 = \frac{4(4kt - 9\xi^2 - 4m + 8)}{t^2(kt-2m)^2} - \frac{4K_l''}{3c_3^2K_l}\left(\frac{kt-2m}{kt}\right)^{\frac{2-3\xi}{m}} + \frac{8}{3}B_c \tag{24}$$

As can be seen from Equations 21-24, pressure, energy density, magnetic field, and displacement vector depend on $K_l(\theta)$. According to the states of $K_l(\theta)$, we obtain the solutions in Bianchi III, LRS Bianchi I, and Kantowski-Sachs universe models as follows:

i. If $K_l(\theta) = \sinh\theta$, then solutions are obtained in the Bianchi III universe model for magnetized strange quark matter distribution:

Pressure

$$p = -\frac{2\left(\sqrt{3}\,\xi - 2\right)(kt - m + 3)}{(kt-2m)^2\,t^2} - \frac{1}{2c_3^2}\left(\frac{kt-2m}{kt}\right)^{\frac{2+2\sqrt{3}\xi}{m}} - 2B_c \tag{25}$$

energy density

$$\rho = -\frac{6\left(\sqrt{3}\,\xi - 2\right)(kt - m + 3)}{(kt-2m)^2\,t^2} - \frac{3}{2c_3^2}\left(\frac{kt-2m}{kt}\right)^{\frac{2+2\sqrt{3}\xi}{m}} - 2B_c \tag{26}$$

magnetic field

$$h^2 = \frac{12\xi\,(kt - m + 3)\sqrt{3}}{t^2\,(kt-2m)^2} - \frac{1}{c_3^2}\left(\frac{kt-2m}{kt}\right)^{\frac{2+2\sqrt{3}\xi}{m}} \tag{27}$$

and displacement vector component

$$\beta^2 = \frac{4(4kt - 9\xi^2 - 4m + 8)}{t^2(kt-2m)^2} - \frac{4}{3c_3^2}\left(\frac{kt-2m}{kt}\right)^{\frac{2-3\xi}{m}} + \frac{8}{3}B_c \tag{28}$$

ii. If $K_l(\theta) = \theta$, then solutions are obtained in the LRS Bianchi I universe model for magnetized strange quark matter distribution:

Pressure

$$p = -\frac{2\left(\sqrt{3}\,\xi - 2\right)(kt - m + 3)}{(kt-2m)^2\,t^2} - 2B_c \tag{29}$$

energy density

$$\rho = -\frac{6\left(\sqrt{3}\,\xi - 2\right)(kt - m + 3)}{(kt-2m)^2\,t^2} - 2B_c \tag{30}$$

magnetic field

$$h^2 = \frac{12\xi\,(kt - m + 3)\sqrt{3}}{t^2\,(kt-2m)^2} \tag{31}$$

and displacement vector component

$$\beta^2 = \frac{4(4kt - 9\xi^2 - 4m + 8)}{t^2(kt-2m)^2} + \frac{8}{3}B_c \tag{32}$$

iii. If $K_l(\theta) = \sin\theta$, then solutions are obtained in the Kantowski-Sachs universe model for magnetized strange quark matter distribution:

Pressure

$$p = -\frac{2\left(\sqrt{3}\,\xi - 2\right)(kt - m + 3)}{(kt-2m)^2\,t^2} + \frac{1}{2c_3^2}\left(\frac{kt-2m}{kt}\right)^{\frac{2+2\sqrt{3}\xi}{m}} - 2B_c \tag{33}$$

energy density

$$\rho = -\frac{6\left(\sqrt{3}\,\xi - 2\right)(kt - m + 3)}{(kt-2m)^2\,t^2} + \frac{3}{2c_3^2}\left(\frac{kt-2m}{kt}\right)^{\frac{2+2\sqrt{3}\xi}{m}} - 2B_c \tag{34}$$

magnetic field

$$h^2 = \frac{12\xi\left(kt - m + 3\right)\sqrt{3}}{t^2\left(kt - 2m\right)^2} + \frac{1}{c_3^2}\left(\frac{kt - 2m}{kt}\right)^{\frac{2+2\sqrt{3}\xi}{m}} \tag{35}$$

and displacement vector component

$$\beta^2 = \frac{4(4kt - 9\xi^2 - 4m + 8)}{t^2(kt - 2m)^2} + \frac{4}{3c_3^2}\left(\frac{kt - 2m}{kt}\right)^{\frac{2-3\xi}{m}} + \frac{8}{3}B_c \tag{36}$$

4. Results and Discussions

From Equations 6-8, 10, 18, and 19, some of the kinematic quantities are obtained as follows:

Hubble parameter

$$H = \frac{2}{t(2m - kt)}$$

expansion scalar

$$\theta = \frac{6}{t(2m - kt)}$$

shear scalar

$$\sigma^2 = \frac{36\xi^2}{t^2\left(2m - kt\right)^2}$$

and mean anisotropy parameter

$$AP = 18\xi^2$$

As can be seen from Equations 29-36, there are singularities at points $t = 0$ and $t = \frac{2m}{k}$, for all three universe models (Bianchi III, LRS Bianchi I, and Kantowski-Sachs). In order to be valid these solutions, it must be $t \neq 0$ and $t \neq \frac{2m}{k}$. At these points, kinematic quantities have singularities. Moreover, c_3, k, and m must be non zero. For $t \to 0$, Hubble parameter, expansion scalar, and shear scalar approach infinity, while they approach zero, for $t \to \infty$. The metric potentials A and B increase with time. Figure of pressure and energy density are presented in Figures 1 and 2. As can be observed from Figures 1 and 2, pressure and energy density decrease with time.

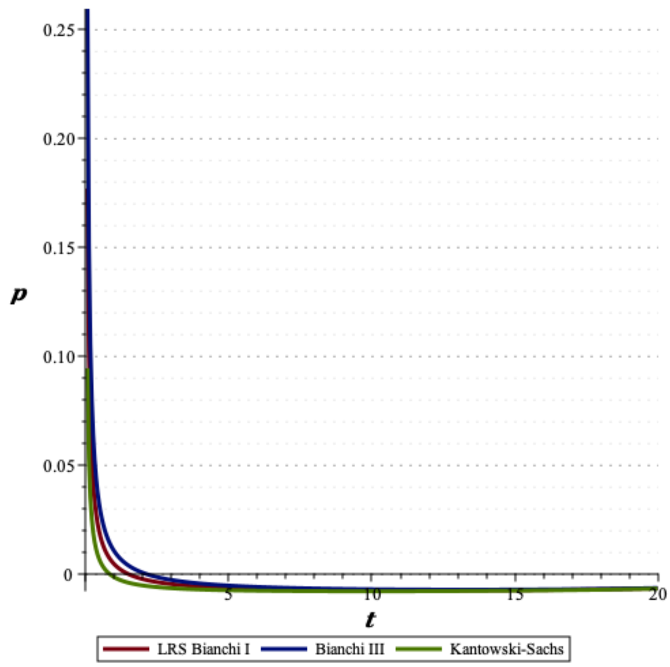

Figure 1. Pressure-time variation

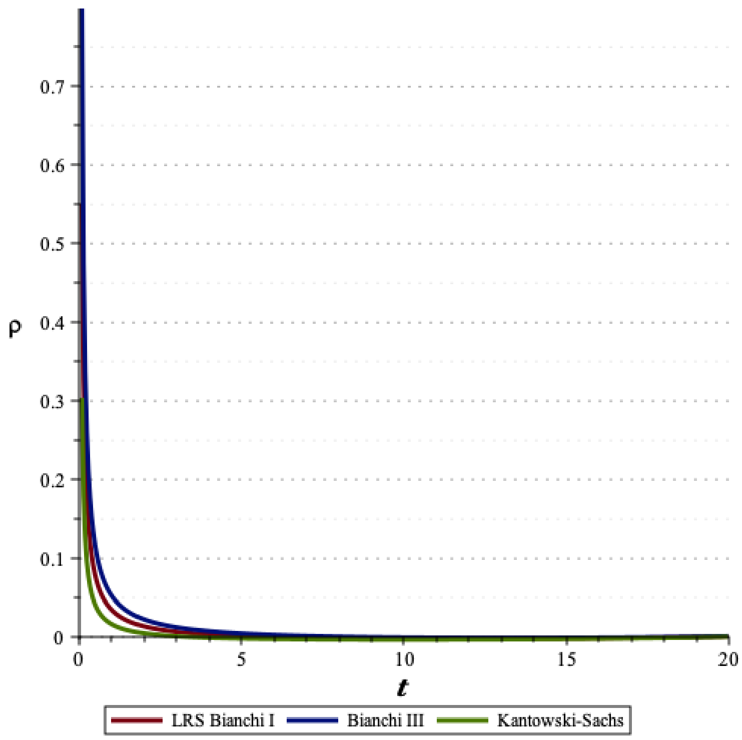

Figure 2. Energy density-time variation

The graphs of variation of magnetic field and displacement vector component with respect to time are provided in Figures 3 and 4. When Figures 3 and 4 are investigated, it is observed that the magnetic field and displacement vector component also decrease with time.

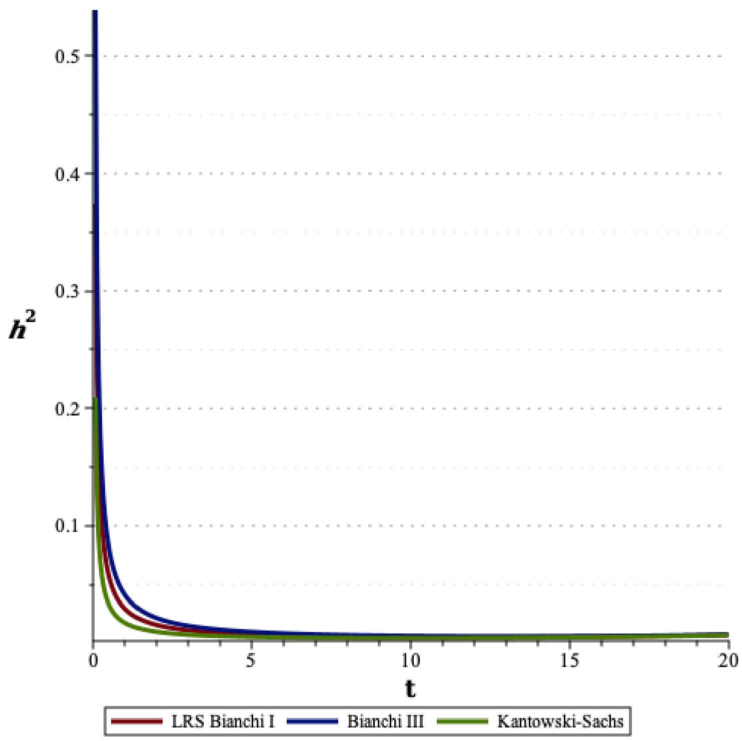

Figure 3. Magnetic field-time variation

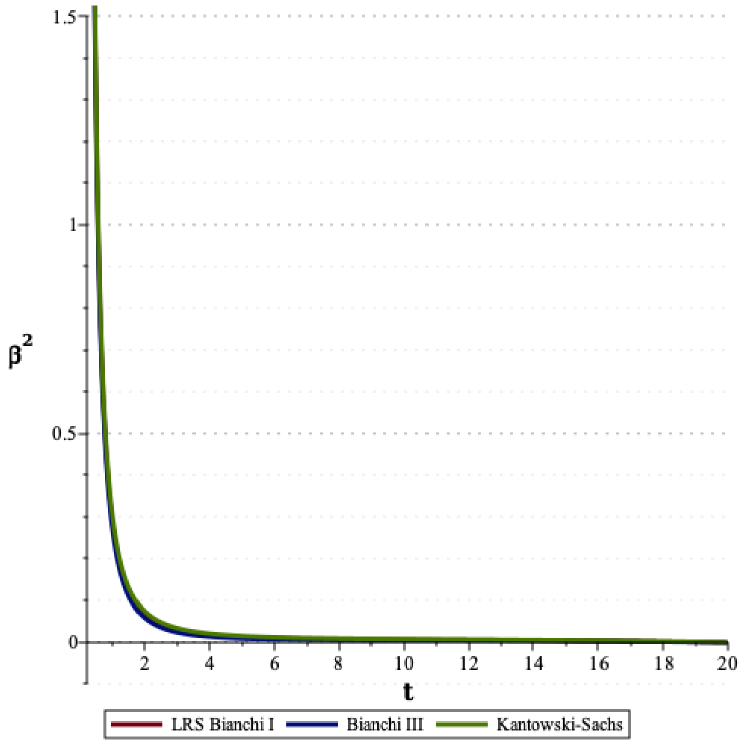

Figure 4. Displacement vector component-time variation

As can be observed from Equation 31 obtained in case of LRS Bianchi I metric, the magnetic field becomes zero when $\xi = 0$. $\xi = 0$ indicates that the universe model is isotropic. If we switch from the homogeneous anisotropic universe model to the homogeneous isotropic universe model, the magnetic field disappears. This shows that the source of the magnetic field may be the anisotropy of the universe. In other words, the anisotropy of the universe plays an important role in the formation of the magnetic field.

5. Conclusion

In this article, the behavior of MSQM distribution in homogeneous anisotropic Bianchi III, LRS Bianchi I, and Kantowski-Sachs metrics was investigated within the framework of Lyra manifolds. While investigating the solutions, the time-dependent linear deceleration parameter and anisotropy parameter were used. In future studies, investigating the space-time geometry of the MSQM distribution using other alternative gravity theories, such as $f(G)$, $f(Q)$, and $f(Q,T)$, or taking different deceleration parameters is worth studying.

Author Contributions

All the authors contributed equally to this work. This paper is derived from the first author's master's thesis supervised by the second author. They all read and approved the final version of the paper.

Conflicts of Interest

All the authors declare no conflict of interest.

References

[1] A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, R. P. Kirshner, B. Leibundgut, M. M. Phillips, D. Reiss, B. P.

Schmidt, R. A. Schommer, R. C. Smith, J. Spyromilio, C. Stubbs, N. B. Suntzeff, J. Tonry, *Observational Evidence From Supernovae for an Accelerating Universe and a Cosmological Constant*, The Astronomical Journal 116 (3) (1998) 1009–1038.

[2] D. N. Spergel, L. Verde, H. V. Peiris, E. Komatsu, M. Nolta, C. L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut, M. Limon, S. S. Meyer, L. Page, G. S. Tucker, J. L. Weiland, E. Wollack, E. L. Wright, *First-Year Wilkinson Microwave Anisotropy Probe (WMAP)* Observations: Determination of Cosmological Parameters*, The Astrophysical Journal Supplement Series 148 (1) (2003) 175–194.

[3] G. Lyra, *Über eine Modifikation der Riemannschen Geometrie*, Mathematische Zeitschrift 54 (1) (1951) 52–64.

[4] S. D. Katore, D. V. Kapse, *Dynamical Behavior of Coupled Magnetized Dark Energy in Lyra's Geometry*, Astrophysics 62 (3) (2019) 415–433.

[5] A. K. Mishra, U. K. Sharma, A. Pradhan, *A Comparative Study of Kaluza-Klein Model with Magnetic Field in Lyra Manifold and General Relativity*, New Astronomy 70 (2019) 27–35.

[6] S. D. Katore, S. P. Hatkar, *Kaluza Klein Universe with Magnetized Anisotropic Dark Energy in General Relativity and Lyra Manifold*, New Astronomy 34 (2015) 172–177.

[7] S. Ram, M. K. Verma, *Dynamics of Bianchi Type-VI_0 Anisotropic Dark Energy Cosmological Model with Massive Scalar Field in Lyra Manifold*, Indian Journal of Physics 96 (4) (2022) 1269–1275.

[8] K. Das, J. Bharali, *Higher-Dimensional Anisotropic Modified Holographic Ricci Dark Energy Cosmological Model in Lyra Manifold*, Astrophysics 64 (2) (2021) 258–275.

[9] R. Raushan, S. Angit, R. Chaubey, *Linear and Center Manifold Analysis of FRW Cosmological Model with Variable Equation of State in Lyra Geometry*, The European Physical Journal Plus 136 (4) (2021) Article Number 440 18 pages.

[10] R. L. Naidu, Y. Aditya, G. Ramesh, D. R. K. Reddy, *Axially Symmetric Bianchi Type-I Cosmological Model of the Universe in the Presence of Perfect Fluid and an Attractive Massive Scalar Field in Lyra Manifold*, Astrophysics and Space Science 365 (5) (2020) Article Number 91 5 pages.

[11] C. Aktaş, S. Aygün, *Magnetized Strange Quark Matter Solutions in $f(R,T)$ Gravity with Cosmological Constant*, Chinese Journal of Physics 55 (1) (2017) 71–78.

[12] S. Aygün, C. Aktaş, İ. Yılmaz, *Non-Existence of a Massive Scalar Field for the Marder Universe in Lyra and Riemannian Geometries*, Journal of Geometry and Physics 62 (1) (2012) 100–106.

[13] S. Kalkan, C. Aktaş, *Behavior of Magnetized Strange Quark Matter in 5D Cosmological Model*, Iranian Journal of Science and Technology, Transactions A: Science 46 (2022) 1505–1511.

[14] S. Kalkan, C. Aktaş, S. Aygün, *Does Magnetized Strange Quark Matter Exist in the Early Universe?*, Modern Physics Letters A 37 (39-40) (2022) 2250251.

[15] S. Kalkan, C. Aktaş, *Bianchi VI_0 Universe with Magnetized Strange Quark Matter in $f(R,T)$ Theory*, Journal of New Theory (37) (2021) 76–85.

[16] E. Güdekli, M. J. Kamran, M. Zubair, I. Ahmed, *Study of Anisotropic Strange Stars in Krori Barua Metric under f(T,$\mathcal{T}$) Gravity*, Chinese Journal of Physics 77 (2022) 592–604.

[17] R. A. A. Khan, R. K. Tiwari, J. Bharali, A. Bouali, G. D. A. Yıldız, E. Güdekli, *Hyperbolic Scenario of Accelerating Universe in Modified Gravity*, Symmetry 15 (6) (2023) 1238 18 pages.

[18] D. Can, E. Güdekli, *An Analysis for Conservative and Non-conservative f(R, T) Gravity Models*, Asian Journal of Research and Reviews in Physics 5 (1) (2021) 1–11.

[19] A. Abebe, A. H. A. Alfedeel, D. Sofuoğlu, E. I. Hassan, R. K. Tiwari, *Perturbations in Bianchi-V Spacetimes with Varying* Λ*, G and Viscous Fluids*, Universe 9 (2) (2023) 61 19 pages.

[20] R. K. Tiwari, S. Kumar, V. K. Dubey, D. Sofuoğlu, *Role of Constant Jerk Parameter in f(R,T) Gravity*, International Journal of Geometric Methods in Modern Physics 20 (3) (2023) 2350049 12 pages.

[21] D. K. Sen, K. A. Dunn, *A Scalar-Tensor Theory of Gravitation in a Modified Riemannian Manifold*, Journal of Mathematical Physics 12 (4) (1971) 578–586.

[22] T. Vinutha, V. U. M. Rao, B. Getaneh, M. Mengesha, *Dark Energy Cosmological Models with Cosmic String*, Astrophysics and Space Science 363 (9) (2018) 188 14 pages.

[23] J. D. Barrow, R. Maartens, C. G. Tsagas, *Cosmology with Inhomogeneous Magnetic Fields*, Physics Reports 449 (6) (2007) 131–171.

[24] C. G. Tsagas, J. D. Barrow, *A Gauge-Invariant Analysis of Magnetic Fields in General-Relativistic Cosmology*, Classical and Quantum Gravity 14 (9) (1997) 2539–2562.

[25] Ö. Akarsu, T. Dereli, *Cosmological Models with Linearly Varying Deceleration Parameter*, International Journal of Theoretical Physics 51 (2) (2012) 612–621.

[26] J. I. Kapusta, P. V. Landshoff, *Finite-Temperature Field Theory*, Journal of Physics G: Nuclear and Particle Physics 15 (3) (1989) 267–285.