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The Traveling Wave Solutions of Date–Jimbo–Kashiwara–Miwa Equation with Conformable Derivative Dependent on Time Parameter

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Abstract

In the paper, the traveling wave solutions of the conformable derivative Date–Jimbo–Kashiwara–Miwa equation were obtained by the modified exponential function method (MEFM). It has been seen that the wave solutions found are functions that have the feature of being periodic functions. The proper values for the parameters in the acquired wave solutions are then used to generate two contour and density graphs in three dimensions that simulate the solution functions.

Keywords: conformable date–jimbo–kashiwara–miwa equation, modified exponential function method, traveling wave solution.

Zaman Parametresine Bağlı Uyumlu Kesirli Mertebeden Date-Jimbo Kashiwara-Miwa Denkleminin İlerleyen Dalga Çözümleri

Öz

Bu makalede, uyumlu kesirli türevli Date–Jimbo–Kashiwara–Miwa denkleminin ilerleyen dalga çözümleri, değiştirilmiş üstel fonksiyon yöntemi (DÜFY) ile elde edilmiştir. Bulunan dalga çözümlerinin periyodik fonksiyon özelliği taşıyan fonksiyonlar olduğu görülmüştür. Elde edilen dalga çözümlerindeki parametreler için uygun değerler daha sonra, çözüm fonksiyonlarını simüle eden üç boyutlu iki tane kontur ve yoğunluk grafiklerini oluşturmak için kullanılmaktadır.

Anahtar Kelimeler: uyumlu date–jimbo–kashiwara–miwa denklemi, değiştirilmiş üstel fonksiyon metodu, ilerleyen dalga çözümü

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Introduction

In the last 20 years, nonlinear phenomena in applied mathematics and physics have played a crucial role in soliton theory, the calculation of analytical, numerical solutions, and especially the traveling wave solutions of nonlinear equations in mathematical physics. Nonlinear partial differential equations (NPDEs), often known as quasi-linear or nonlinear evolution equations, can be used to represent a wide variety of phenomena. This intricate mathematical equation, known as a space and time related, nonlinear evolution equation, or NLEE for short, has the potential to replicate almost every event that takes place in the natural world, particularly those that pertain to the fields of science and engineering. In order to provide an explanation for the natural occurrences that have taken place in a variety of scientific areas consisting of engineering, chemistry, biology, dynamics, plasma physics, electrodynamics, applied physics, and so on. It is necessary for us to search for specific answers to the NLEEs. There are many techniques to solve these kinds of problems that have been written about (Abdel-Gawad & Osman, 2013; Akturk et al., 2017; Baskonus & Bulut, 2015; Baskonus et al., 2017; Chen & Wang, 2005; Chen & Yan, 2005; Dubrovsky & Lisitsyn, 2002; Duran, 2020; Duran, 2021a; Duran, 2021b; Hossain & Akbar, 2017; Jafari et al., 2015; Jianming et al., 2011; Kubal & Aktürk, 2023Kumar & Pankaj, 2015; Lü, 2005; Malwe et al., 2016; Mohyud-Din & Noor, 2007; Salas & Gómez, 2010; Shen et al., 2013).

In this paper, we obtain the traveling wave solutions of the conformable Date-Jimbo-Kashiwara-Miwa equation (CDJKME) by the modified exponential function method (MEFM) (Baskonus et al., 2016; Xu, 2008).

The CDJKME is defined by (Guo & Lin, 2019; Ismael et al., 2021)

$$U_{xxxxy} + 4U_{xxy}U_x + 2U_{xxx}U_y + 6U_{xy}U_{xx} - \alpha U_{yyy} - 2\beta \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial^\theta U}{\partial t^\theta}\right)\right) = 0, \tag{1}$$

where α and β are non-zero constants and $0 < \theta \leq 1$, $U = U(x, y, t)$ is the wave-amplitude function, which describes long water waves. In case of $\theta = 1$, Eq. (1) reduces to Date-Jimbo-Kashiwara-Miwa equation.

This article is organized as follows: Basic definitions, theorems and the modified exponential function method are presented in material and method. In Section of results and discussion, we present the proposed method's application of the CDJKM equation. Finally, we present the section of conclusion.

Method

Basic definitions and theorems about fractional calculus in conformable sense are given.

Definition 1. Let a function $\boldsymbol{f}: [\mathbf{0}, \infty) \to \mathbb{R}$. The conformable fractional derivative (CFD) of $\boldsymbol{f}$ order $\boldsymbol{\theta}$ is given by (Abdeljawad, 2015; Gözütok & Gözütok, 2018; Khalil et al., 2014).

$$T_\theta[f(t)] = \lim_{\varepsilon \to 0} \frac{f\left(t + \varepsilon t^{1-\theta}\right) - f(t)}{\varepsilon}, \tag{2}$$

for all $t > 0, \theta \in (0, 1]$.

Theorem 1. Let $\boldsymbol{\theta} \in (\mathbf{0}, \mathbf{1}]$ and $\boldsymbol{f}, \boldsymbol{g}$ be $\boldsymbol{\theta}$ −differentiable at a point $\boldsymbol{x} > \mathbf{0}$. Thus, it is obtained as (Abdeljawad, 2015; Gözütok & Gözütok, 2018; Khalil et al., 2014).

$i. T_\theta(af + bg) = aT_\theta(f) + bT_\theta(g)$, for all $a, b \in \mathbb{R}$. (3)

$ii. T_\theta(t^p) = pt^{p-1}$, for all $p \in \mathbb{R}$. (4)

$iii. T_\theta(\lambda) = 0$, for all constant functions $f(t) = \lambda$. (5)

$iv. T_\theta(fg) = fT_\theta(g) + gT_\theta(f)$. (6)

$$v. T_\theta\left(\frac{f}{g}\right) = \frac{gT_\theta(f) - fT_\theta(g)}{g^2}. \tag{7}$$

If f is differentiable, then the derivative of the polynomial t is obtained as

$$T_\theta[f(t)] = t^{1-\theta}\frac{d}{dt}f(t). \tag{8}$$

Definition 2. Let $\boldsymbol{f}$ be an $\boldsymbol{n}$ −times differentiable at $\boldsymbol{t}$. Then, the CFD of $\boldsymbol{f}$ order $\boldsymbol{\theta}$ is defined by (Khalil et al., 2014).

$$T_\theta[f(t)] = \lim_{\varepsilon\to 0}\frac{f^{([\theta]-1)}\left(t+\varepsilon t^{([\theta]-\theta)}\right) - f^{([\theta]-1)}(t)}{\varepsilon}, \tag{9}$$

for all $t > 0, \theta \in (n, n+1]$, $[\theta]$ is the smallest integer greater than or equal to θ.

Theorem 2. Let $\boldsymbol{f}$ be an $\boldsymbol{n}$ −times differentiable at $\boldsymbol{t}$. So, there is the following equality (Khalil et al., 2014).

$$T_\theta[f(t)] = t^{[\theta]-\theta}f^{[\theta]}(t), \tag{10}$$

for all $t > 0, \theta \in (n, n+1]$.

The Modified Exponential Function Method

In the part, we are going to learn about MEFM. First, it will be helpful to go over some things that are already known about the MEFM.

To employ this method, understand about the NPDEs

$$P\left(U, T_\theta U, U_x, U_y, U_{xx}, U_{yy}, U_{tx}, \ldots\right) = 0, \tag{11}$$

where $U = U(x, y, t)$ is required function, P is a polynomial which has function of $u(x, y, t)$ and its partial derivatives according to x, y, t.

Step 1: Assume that the traveling wave transformation is as follows:

$$U(x, y, t) = U(\xi), \xi = kx + ry - c\frac{t^\theta}{\theta}, \tag{12}$$

where k and c are nonzero constants that is determined in the future. By substituting partial derivatives of equation (12) into equation (11), equation (11) is changed into a nonlinear ordinary differential equation (NODE) described by,

$$N(U, U', U'', U''', \ldots.) = 0, \tag{13}$$

where N is a polynomial depend on U.

Step 2: Assume that the traveling wave solution to equation (13) is written in below:

$$U(\xi) = \frac{\sum_{i=0}^{N} A_i[exp(-\Psi(\xi))]^i}{\sum_{j=0}^{M} B_j[exp(-\Psi(\xi))]^j} = \frac{A_0 + A_1 exp(-\Psi) + \cdots + A_N exp(N(-\Psi))}{B_0 + B_1 exp(-\Psi) + \cdots + B_M exp(M(-\Psi))}, \tag{14}$$

where A_i and $B_j, (0 \leq i \leq N,\ 0 \leq j \leq M)$ are constants which is specified in future, $A_N \neq 0$, $B_M \neq 0$ and $\Psi = \Psi(\xi)$ provides the ODE:

$$\Psi'(\xi) = exp(-\Psi(\xi)) + \mu exp(\Psi(\xi)) + \lambda. \tag{15}$$

Solving equation (15), then the five solution families are obtained (Naher & Abdullah, 2013).

Family 1: Let $\boldsymbol{\mu \neq 0}$, $\boldsymbol{\lambda^2 - 4\mu > 0}$,

$$\Psi(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \tag{16}$$

Family 2: Let $\boldsymbol{\mu \neq 0}$, $\boldsymbol{\lambda^2 - 4\mu < 0}$,

$$\Psi(\xi) = \ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu}\tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \tag{17}$$

Family 3: Let $\boldsymbol{\mu = 0}$, $\boldsymbol{\lambda \neq 0}$ and $\boldsymbol{\lambda^2 - 4\mu > 0}$,

$$\Psi(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1}\right). \tag{18}$$

Family 4: Let $\boldsymbol{\mu \neq 0}$, $\boldsymbol{\lambda \neq 0}$ and $\boldsymbol{\lambda^2 - 4\mu = 0}$,

$$\Psi(\xi) = \ln\left(-\frac{2\lambda(\xi + E) + 4}{\lambda^2(\xi + E)}\right). \tag{19}$$

Family 5: Let $\boldsymbol{\mu = 0}$, $\boldsymbol{\lambda = 0}$ and $\boldsymbol{\lambda^2 - 4\mu = 0}$,

$$\Psi(\xi) = \ln(\xi + E). \tag{20}$$

where $A_0, A_1, \ldots, A_N, B_0, B_1, \ldots, B_M, E, \lambda, \mu$ are constants and is determined in the future. Utilizing homogeneous balance principle between the highest nonlinear terms with the highest order derivatives of U in equation (14). It will be found a relationship between N and M.

Step 3: When Eq. (15) and the families solutions are put into Equation (14), we get a polynomial with $\boldsymbol{exp(\Psi(\xi))}$ terms. The algebraic equation system in terms of $\boldsymbol{A_0, A_1, \ldots, A_N, B_0, B_1, \ldots, B_M, E, \lambda, \mu}$ is found by putting the coefficients of the same power of exp ($\boldsymbol{\Psi}$ ($\boldsymbol{\xi}$)) to zero. Lastly, by plugging the values of the coefficients into equation (14), it provides the traveling wave solutions of equation (11).

Results and Discussion

In this part, solutions to the CDJKME will be found by using the MEFM. Handle the traveling wave transformation:

$$U(x, y, t) = U(\xi), \xi = kx + ry - c\frac{t^\theta}{\theta}. \tag{21}$$

Using the traveling wave transformation for Eq. (1), the following NODE is obtained:

$$k^4 rU''' + 3k^3 r(U')^2 - (\alpha r^3 - 2k^2 c\beta)U' = 0. \tag{22}$$

When $U' = V$ is applied to Eq. (22), we obtain the NODE

$$k^4 rV'' + 3k^3 rV^2 - (\alpha r^3 - 2k^2 c\beta)V = 0, \tag{23}$$

where $\mathrm{V} = U'$ and also both integral constants are zero. When we apply the balancing procedure to Eq. (23), it is obtained the relationship

$$n = m + 2.$$

Choosing $m = 1$, then we find $n = 3$. For m and n values, we obtain

$$U(\xi) = \frac{A_0 + A_1 e^{-\Psi} + A_2 e^{-2\Psi} + A_3 e^{-3\Psi}}{B_0 + B_1 e^{-\Psi}}. \tag{24}$$

The system of algebraic equations with $e^{-\Psi(\xi)}$ coefficients is generated by rearranging Eq. (24) according to the necessary term in Eq. (23).

Utilizing Mathematica, some appropriate coefficients acquired are as follows:

Case 1:

$A_0 = -2k\mu B_0,$

$A_1 = -2k(\lambda B_0 + \mu B_1),$

$A_2 = -2k(B_0 + \lambda B_1),$

$A_3 = -2kB_1,$

$c = \dfrac{r^3\alpha - k^4 r(\lambda^2 - 4\mu)}{2k^2\beta}.$

Using these coefficients in Equation (13), the solutions are obtained as:

Family 1: Let $\boldsymbol{\mu \neq 0, \lambda^2 - 4\mu > 0}$, we obtain solution of Eq. (1)

$$U_{1,1}(x,t) = \left(\frac{k.(\lambda^3 - 4\lambda\mu + 2\Gamma\mu sinh[\vartheta\Gamma])}{\lambda^2 - 2\mu + 2\mu cosh[\vartheta\Gamma]}\right), \tag{25}$$

where, $\xi = kx + ry - c\frac{t^\theta}{\theta}, \vartheta = EE + \xi\ , \ \Gamma = \sqrt{\lambda^2 - 4\mu}.$

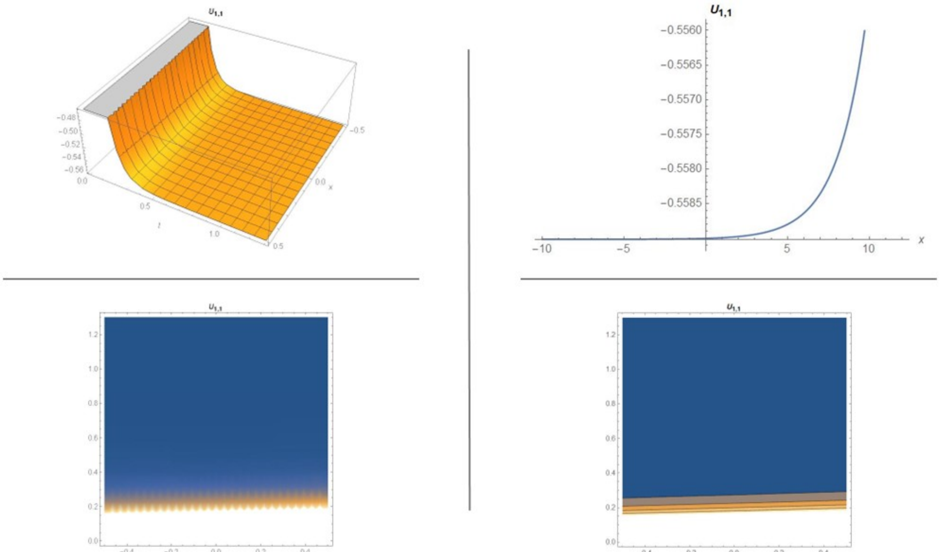

Figure 1. 2D, 3D, Density, Contour Graphs of Equation (24) at $\boldsymbol{\lambda = 3, \mu = 1, \theta = 0.5, y = 0.1, \beta = 1.3, \alpha = 0.543056, k = 0.25, r = 0.75, t = 0.1, c = 3.41466, A_1 = -2.13, A_2 = -1.59, B_1 = 0.66, EE = 0.82, A_3 = -0.33, B_0 = 1.2, A_0 = -0.6}$.

Family 2: Let $\mu \neq 0, \lambda^2 - 4\mu < 0$, the solution of Eq. (1) is found by

$$U_{1,2}(x,t) = \frac{\left((k(\lambda^2 - 4\mu - 2\mu\psi)sin[\vartheta]\psi)\right)}{(\lambda^2 - 2\mu + 2\mu cos[\vartheta]\psi)}. \tag{26}$$

where, $\xi = kx + ry - c\frac{t^\theta}{\theta}, \vartheta = EE + \xi\ , \psi = \sqrt{-\lambda^2 + 4\mu}.$

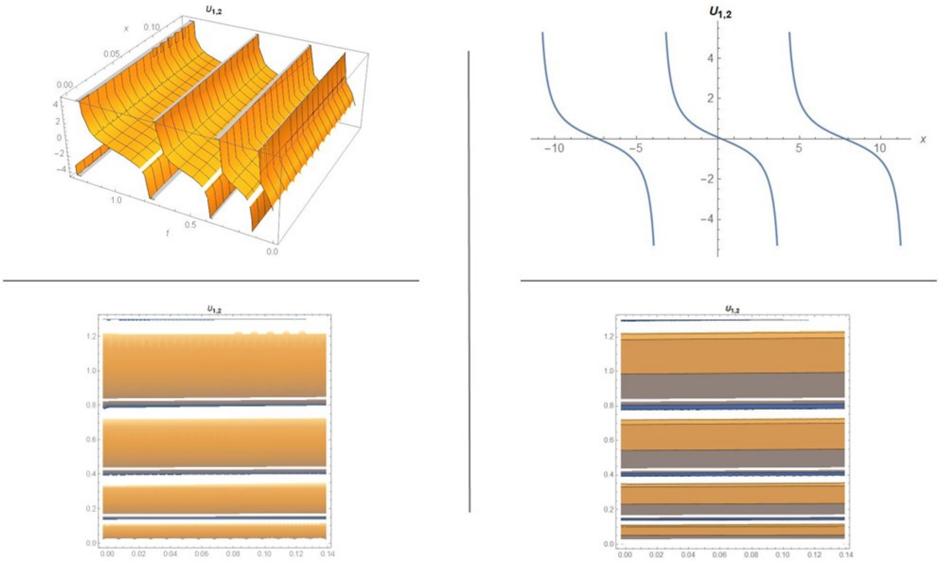

Figure 2. 2D, 3D, Density, Contour Graphs of Equation (25) at $\lambda = 1, \mu = 3, \theta = 0.5, y = 0.1, \beta = 1.3, \alpha = 1.35, t = 1, k = 0.25, r = 0.75, c = 3.70313, A_1 = -1.59, A_2 = -0.93, B_1 = 0.66, EE = 0.82, A_3 = -0.33, B_0 = 1.2, A_0 = -1.8$.

Family 3: Let $\mu = 0, \lambda \neq 0,\ \lambda^2 - 4\mu < 0$, the solution of Eq. (1) is obtained by

$$U_{1,3}(x,t) = \left(k\lambda \coth\left[\frac{1}{2}\vartheta\right]\lambda\right), \tag{27}$$

where, $\xi = kx + ry - c\frac{t^{\theta}}{\theta}, \vartheta = EE + \xi$.

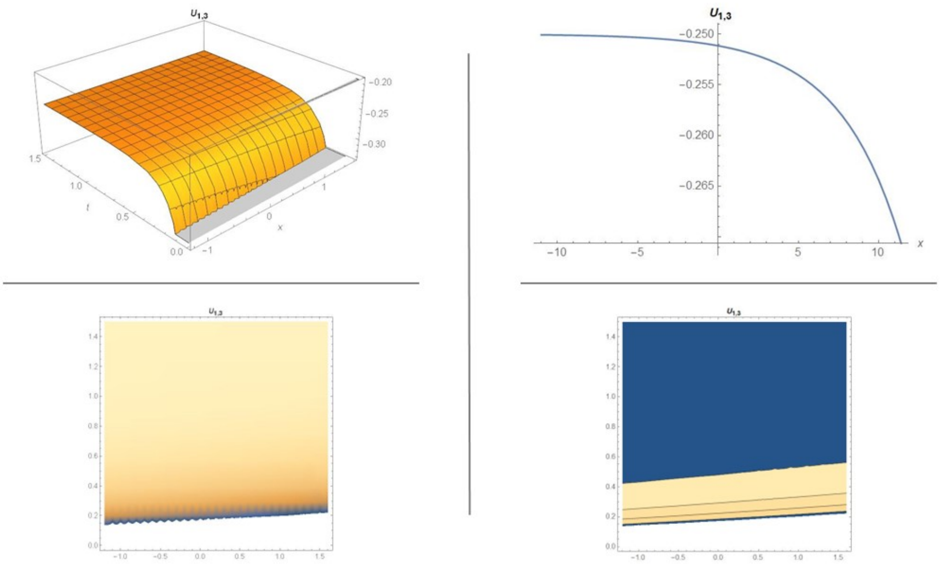

Figure 3. 2D, 3D, Density, Contour Graphs of Equation (26) at $\lambda = 1, \mu = 0, \theta = 0.5, y = 0.1, \beta = 1.3, \alpha = 1.35, k = 0.25, r = 0.75, c = 3.48678, t = 1, A_1 = -0.6, A_2 = -0.93, B_1 = 0.66, EE = 0.82, A_3 = -0.33, B_0 = 1.2, A_0 = 0$.

Family 4: Let $\mu \neq 0, \lambda \neq 0,\ \lambda^2 - 4\mu = 0$, the solution of Eq. (1) is acquired by

$$U_{1,4}(x,t) = \left(\frac{1}{2}k\left(\frac{4\lambda}{2+\vartheta\lambda} + +\lambda(2+\vartheta\lambda) - 4\,\xi\mu\right)\right), \tag{28}$$

where, $\xi = kx + ry - c\frac{t^{\theta}}{\theta}, \vartheta = EE + \xi$.

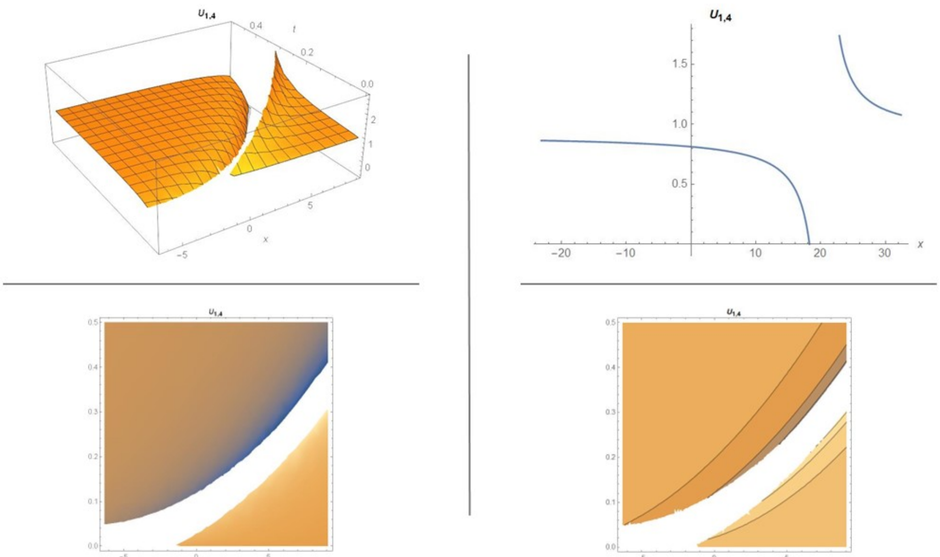

Figure 4. 2D, 3D, Density, Contour Graphs of Equation (27) at $\lambda = 2, \mu = 1, \theta = 0.5, y = 0.1, \beta = 1.3, \alpha = 1.35, k = 0.25, r = 0.75, c = 3.50481, A_1 = -1.53, A_2 = -1.26, B_1 = 0.66, EE = 0.82, A_3 = -0.33, B_0 = 1.2, A_0 = -0.60$.

Family 5: Let $\mu = 0, \lambda = 0, \lambda^2 - 4\mu = 0$, the solution of Eq. (1) is found by

$$U_{1,5}(x,t) = \frac{2k}{\vartheta}, \tag{29}$$

where, $\xi = kx + ry - c\frac{t^{\theta}}{\theta}, \vartheta = EE + \xi$.

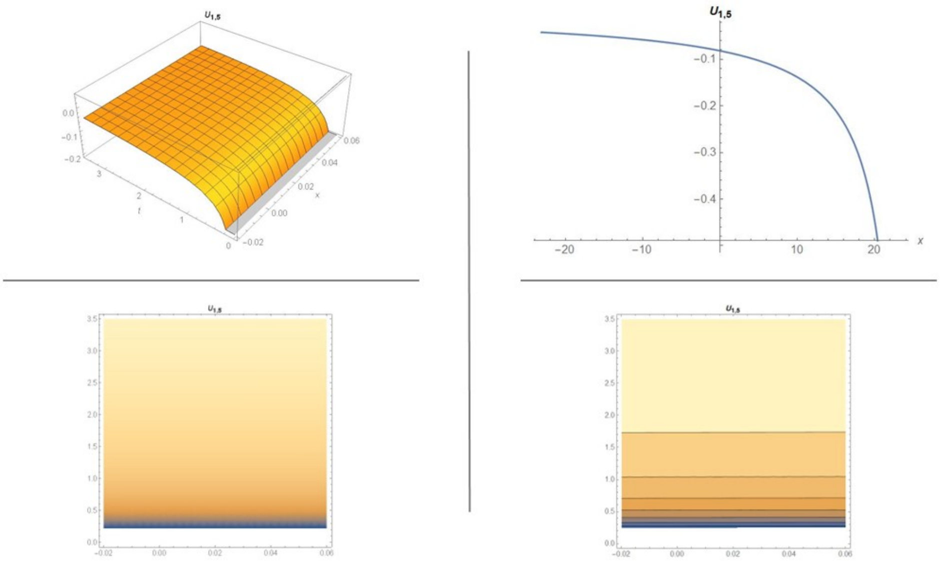

Figure 5. 2D, 3D, Density, Contour Graphs of Equation (28) at $\lambda = 0, \mu = 0, \theta = 0.5, y = 0.1, \beta = 1.3, \alpha = 1.35, k = 0.25, r = 0.75, c = 3.50481, A_1 = 0, A_2 = -0.6, B_1 = -1.1, EE = 0.82, A_3 = -0.33, B_0 = 1.2, A_0 = 0$.

Case-2:

$$A_0 = -\frac{1}{3}k(\lambda^2 + 2\mu)B_0,$$

$$A_1 = -\frac{1}{3}k(6\lambda B_0 + (\lambda^2 + 2\mu)B_1),$$

$$A_2 = -2k(B_0 + \lambda B_1),$$

$$A_3 = -2kB_1,$$

$$c = \frac{r^3\alpha + k^4 r(\lambda^2 - 4\mu)}{2k^2\beta}$$

When these coefficients are put into Equation (23), the following solutions are found:

Family 1: Let $\mu \neq 0, \lambda^2 - 4\mu > 0$, the solution of Eq. (1) is found by

$$U_{2,1}(x,t) = \left(\frac{1}{3}k(\lambda^2 - 4\mu)\left(-\vartheta + \frac{3\lambda}{\lambda^2 - 2\mu + 2\mu cosh[\vartheta\Phi]}\right) + \frac{2k\Phi\mu sinh[\vartheta\Phi]}{\lambda^2 - 2\mu + 2\mu cosh[\vartheta\Phi]}\right), \tag{30}$$

where, $\xi = kx + ry - c\frac{t^\theta}{\theta}, \vartheta = EE + \xi, \Phi = \sqrt{\lambda^2 - 4\mu}$.

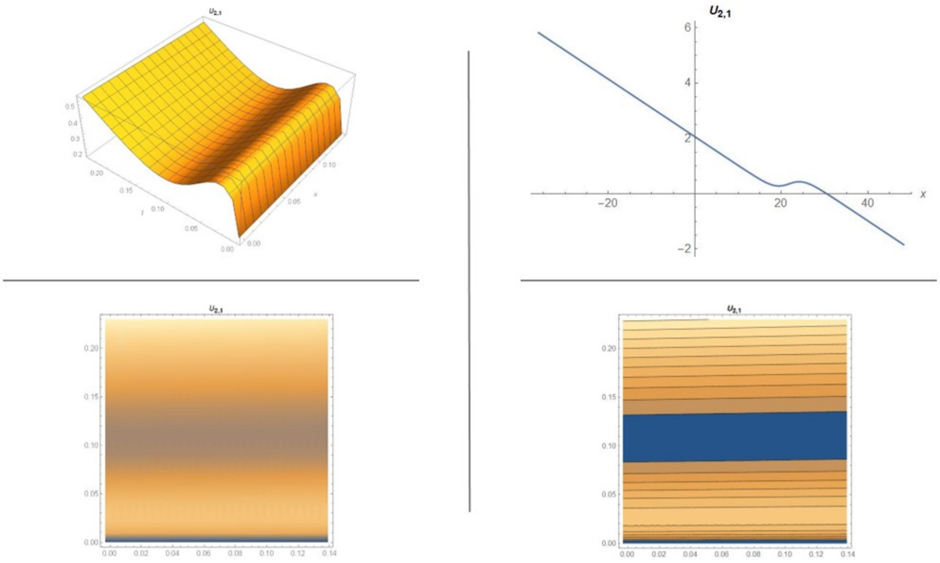

Figure 6. 2D, 3D, Density, Contour Graphs of Equation (29) at $\lambda = 3, \mu = 1, \theta = 0.5, y = 0.1, \beta = 1.3, \alpha = 1.35, k = 0.25, r = 0.75, c = 3.59495, A_1 = -2.405, A_2 = -1.59, B_1 = 0.66, EE = 0.82, A_3 = 1.6, B_0 = 1.2, A_0 = -1.1$.

Family 2: Let $\mu \neq 0, \lambda^2 - 4\mu < 0$, the solution of Eq. (1) is obtained by

$$U_{2,2}(x,t) = \left(\frac{1}{3}k(\lambda^2 - 4\mu)\left(-\vartheta + \frac{3\lambda}{\lambda^2 - 2\mu + 2\mu cosh[\vartheta\varsigma]}\right) + \frac{2k\varsigma\mu sinh[\vartheta\varsigma]}{\lambda^2 - 2\mu + 2\mu cosh[\vartheta\varsigma]}\right). \quad (31)$$

where, $\xi = kx + ry - c\frac{t^\theta}{\theta}, \vartheta = EE + \xi, \varsigma = \sqrt{-\lambda^2 + 4\mu}$.

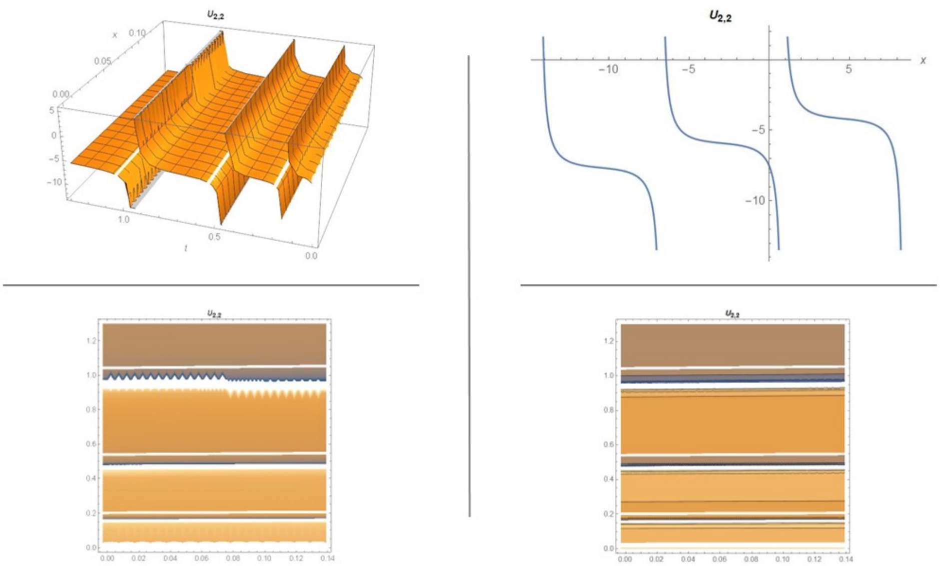

Figure 7. 2D, 3D, Density, Contour Graphs of Equation (30) at $\lambda = 1, \mu = 3, \theta = 0.5, y = 0.1, \beta = 1.3, \alpha = 1.35, k = 0.25, r = 0.75, c = 3.30649, A_1 = -0.985, A_2 = -0.93, B_1 = 0.66, EE = 0.82, A_3 = -0.33, B_0 = 1.2, A_0 = -0.7$.

Family 3: Let $\mu = 0, \lambda \neq 0,\ \lambda^2 - 4\mu < 0$, the solution of Eq. (1) is found by

$$U_{2,3}(x,t) = -\frac{1}{3}k\vartheta\lambda^2 + k\lambda\coth\left[\frac{1}{2}\vartheta\lambda\right]. \tag{32}$$

where, $\xi = kx + ry - c\frac{t^\theta}{\theta}, \vartheta = EE + \xi$.

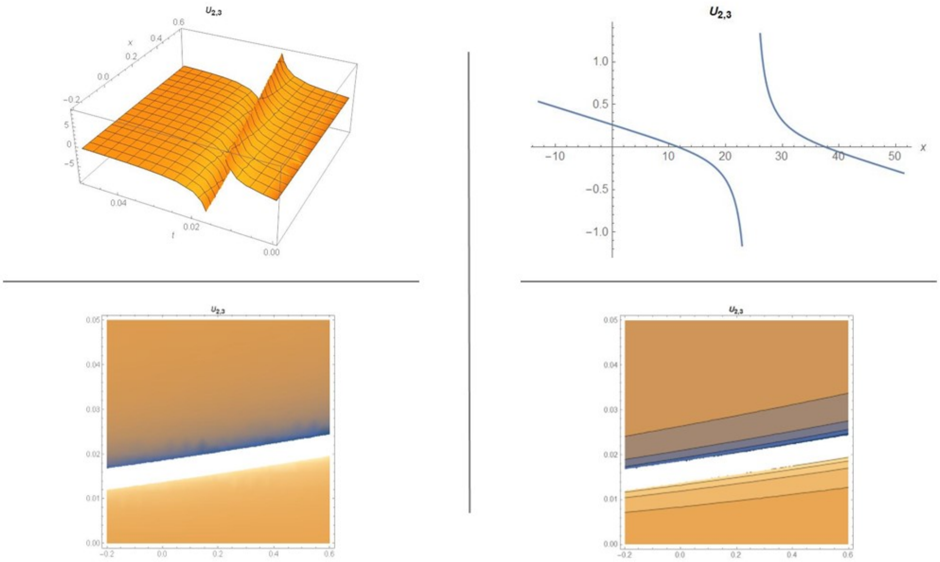

Figure 8. 2D, 3D, Density, Contour Graphs of Equation (31) at $\lambda = 1, \mu = 0, \theta = 0.5, y = 0.1, \beta = 1.3, \alpha = 1.35, k = 0.25, r = 0.75, c = 3.52284, A_1 = -0.655, A_2 = -0.93, B_1 = 0.66, EE = 0.82, A_3 = -0.33, B_0 = 1.2, A_0 = -0.1$.

Family 4: Let $\mu \neq 0, \lambda \neq 0,\ \lambda^2 - 4\mu = 0$, the solution of Eq. (1) is acquired by

$$U_{2,4}(x,t) = \left(\frac{1}{6}k\left(\frac{12\lambda}{2 + \vartheta\lambda} + \lambda(2 + \vartheta\lambda) - 4\xi\mu\right)\right). \tag{33}$$

where, $\xi = kx + ry - c\frac{t^\theta}{\theta}, \vartheta = EE + \xi$.

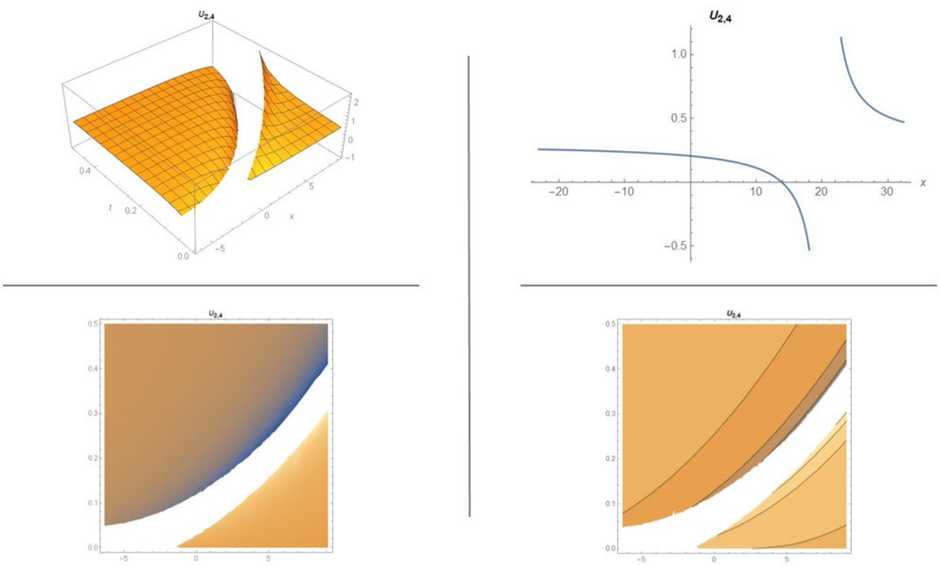

Figure 9. 2D, 3D, Density, Contour Graphs of Equation (32) at $\lambda = 2, \mu = 1, \theta = 0.5, y = 0.1, \beta = 1.3, \alpha = 1.35, k = 0.25, r = 0.75, c = 3.50481, A_1 = -1.53, A_2 = -1.26, B_1 = 0.66, EE = 0.82, A_3 = -0.33, B_0 = 1.2, A_0 = -0.60$.

Family 5: Let $\mu = 0, \lambda = 0, \lambda^2 - 4\mu = 0$, the solution of Eq. (1) is found by

$$U_{2,5}(x,t) = \frac{2k}{\vartheta}. \tag{34}$$

where, $\xi = kx + ry - c\frac{t^\theta}{\theta}, \vartheta = EE + \xi$.

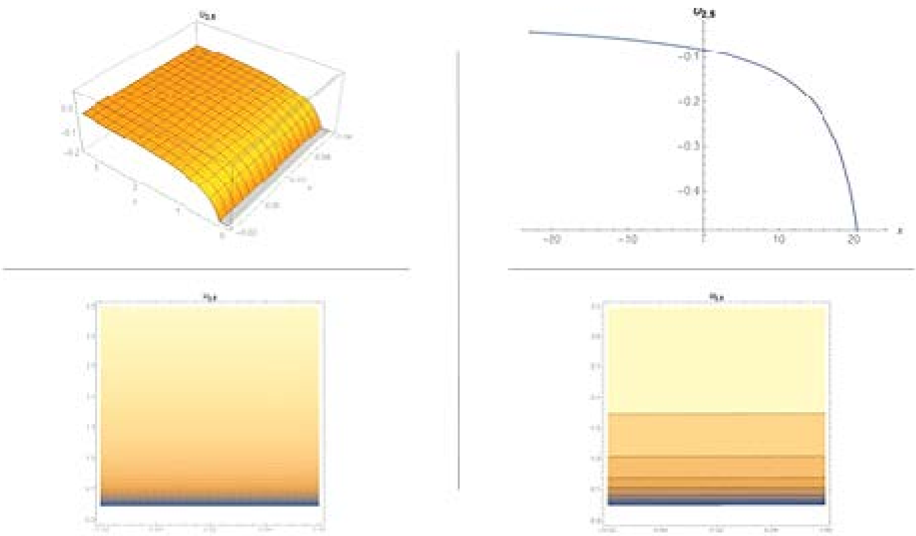

Figure 10: 2D, 3D, Density, Contour Graphs of Equation (33) at $\lambda = 0, \mu = 0, \Theta = 0.5, k = 0.25,$ $A_3 = -o.33, \alpha = 1.35, \beta = 1.3, B_0 = 1.2, A_0 = 0, c = 3.50481, A_1 = 0, A_2 = -0.6,$ $B_1 = 0.66, EE = 0.82, y = 0.1, r = 0.75$.

Conclusion

Lastly, we demonstrated how to acqiure the traveling wave solutions of the CDJKME via the MEFM. In the paper, we acquired traveling wave solutions of the CDJKME by utilizing MEFM. In Mathematica software, we obtain the traveling wave solutions of the CDJKME. The 2D,3D plots, density and contour surface graphs of the traveling wave solutions by choosing the proper parameters have been plotted in program. Based on our accomplishments, we can assert that our findings have made a substantial contribution to this field. The proposed method is highly efficient in obtaining the analytical solutions of such NPDEs. It is observed that the method is applicable to a wide range of difficult nonlinear models in applied mathematics, engineering, and physics, such as the Landau-Ginzburg-Higgs equation, Klein-Gordon equation and Duffing equation.

Author Contribution

All authors read and approved the final manuscript.

Ethics Statement

There are no ethical issues with the publication of this article.

Conflict of Interest

The authors declare that there is no conflict of interest.

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