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# NEW HYBRID WINDOWS BASED ON COSH WINDOW AND THEIR PERFORMANCE ANALYSIS IN FIR DIGITAL FILTER DESIGN

**[1] Oğuzhan COŞKUN, [2,*] Kemal AVCI**

[1] *Emko Elektronik A.Ş., Bursa, TÜRKİYE*
[2] *Izmir Democracy University, Engineering Faculty, Electrical and Electronics Engineering Department, Izmir, TÜRKİYE*
[1] oguzhan.coskun@emkoelektronik.com.tr, [2] kemal.avci@idu.edu.tr

## *Highlights*

- Twenty hybrid window functions are proposed by hybridizing two-parameter Cosh window with various one-parameter windows in the existing literature
- Simulations in Matlab programming environment are performed to design FIR digital filter by using proposed window functions
- Comparative simulation results showed that nine proposed hybrid windows exhibit better filter characteristics compared to the Cosh window
- Among the proposed windows, Cosh-Hamming and Cosh-Von Hann hybrid windows outperformed other well-known two-parameter windows in the literature in designing FIR filters

***Corresponding Author:** Kemal AVCI, kemal.avci@idu.edu.tr

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[1] oguzhan.coskun@emkoelektronik.com.tr, [2] kemal.avci@idu.edu.tr

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**ABSTRACT:** FIR filters find extensive use in various applications such as audio processing, image processing, communications, and control systems. Improving the design methods for FIR filters can lead to better performance in these critical areas. This study introduces novel hybrid window functions, or abbreviated as "windows", to design FIR filters. These newly proposed windows are created by hybridizing two-parameter Cosh window with various one-parameter windows in the existing literature. The performance of these hybrid windows in designing filters is thoroughly examined. Simulation results, conducted in Matlab programming environment, demonstrate that nine proposed hybrid windows exhibit better filter characteristics compared to the Cosh window. Furthermore, for the filter lengths of N = 51 and 101, the filters designed using the proposed windows are compared with those designed using two-parameter windows from the literature, focusing on achieving minimum stopband attenuation for a fixed transition width. The findings reveal that the Cosh-Hamming and Cosh-Von Hann hybrid windows outperform other well-known windows in producing filtering.

***Keywords:*** *Cosh Window, Digital Filter Design, FIR Filters, Hybrid Windows, Window Functions*

## 1. INTRODUCTION

A digital filter is used to perform filtering operation on digital signals, such as audio, images, or other time-dependent data, to achieve specific desired effects. Software and hardware realization, non-critical circuit element tolerance, high accuracy, small physical size, high reliability, insignificance of the effect of environmental electrical noises, more efficient cost and being the only option in non-real time systems can be listed as the advantages of digital filters compared to analog filters [1].

Digital filters can be categorized into finite impulse response (FIR) and infinite impulse response (IIR) based on the length of their impulse response. Both FIR and IIR filters come with their own advantages and disadvantages. For example, FIR filters are always stable. This means they won't exhibit unstable behavior, even for certain types of filter designs. IIR filters, on the other hand, can become unstable if not designed properly. FIR filters can achieve a linear phase response, meaning that the input signal experiences a uniform delay across all its frequency components. This is important in applications like audio processing where phase distortion can cause audible artifacts.

Among the FIR filter design techniques, the most superior designs are achieved through optimization methods, and ongoing research is dedicated to advancing this methodology [2-5]. Nonetheless, optimization-driven approaches demand substantial computational resources, rendering them impractical for real-time applications. Conversely, the windowing-based Fourier series method stands out as the most direct method for designing FIR filters, entailing considerably fewer computational requirements compared to alternative methods. The incorporation of a window function in Fourier series serves to truncate and smooth out the ideal filter [6].

A window is a discrete time-domain function that is non-zero only in a certain time interval as in Figure 1 [7]. Windows are classified as either fixed or adjustable. Windows that depend on a single parameter are called fixed, and those with two or more are called adjustable. Practical applications often

***Corresponding Author:** Kemal AVCI, kemal.avci@idu.edu.tr

favor the use of adjustable windows because of their flexibility. Among the window functions, the rectangular window is the most simple one and it performs a straightforward cut without any smoothing effect. Other well-known fixed windows include Von Hann, Hamming, and Blackman [1].

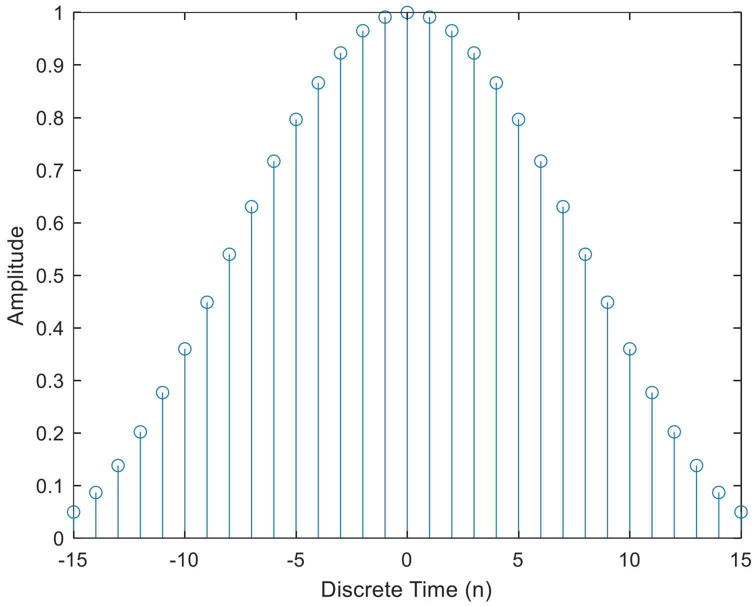



**Figure 1.** A discrete time domain window for N = 31

As for the adjustable windows, there are well known two [8-10] and three parameter [11-13] windows. The Dolph-Chebyshev window, introduced by Dolph in 1946, yields the narrowest main lobe width for a given maximum sidelobe level, outperforming all other windows in literature. Kaiser, in 1966, introduced the Kaiser window, which is among the most known adjustable windows in the field. Subsequently, Kaiser published the Kaiser window related filter design equations in 1974. This window exhibits superior performance in FIR filter design applications when compared to many two parameter windows in literature. In 1989, Saramaki proposed a two-parameter window, known as Saramaki window [9]. The findings of this study demonstrated that filters designed using the Saramaki window exhibited better stopband attenuation compared to those designed with the Kaiser window. However, it relies on iterative expressions as in the Dolph-Chebyshev window in the time domain, which could be considered a drawback. Various two-parameter adjustable windows were suggested in the literature, including the Poisson, Cauchy, Gaussian, and Parzen-Cos6 ($\pi$t) window families. However, these alternatives do not surpass the performance of the Kaiser window in filter design. Bergen and Antoniou [11] conducted an in-depth study on the use of three-parameter Ultraspherical windows in non-recursive digital filter design. Their research demonstrated that the Ultraspherical window family allows for the design of filters with lower degrees compared to those designed using the Saramaki, Kaiser, and Dolph-Chebyshev windows. The Ultraspherical family of windows has one important feature that is special cases of Saramaki and Dolph-Chebyshev windows. In recent years, studies have been continuing to increase the effectiveness of window functions in different application areas [14-23].

The motivation behind proposing new window functions for digital FIR filter design in literature lies in overcoming limitations associated with existing window functions to achieve improved filter characteristics, making them more suitable for diverse signal processing applications. In order to enhance filter design, it is essential for a window function to effectively minimize undesirable Gibbs oscillations and optimize filtering performance metrics, thereby improving the overall frequency response of the filter. By definition, Gibbs oscillations refer to the phenomenon where oscillations or ripples occur in the vicinity of a sharp transition in a function. Gibbs oscillations are inherent in Fourier Series based FIR filter design method due to the need to truncate the infinite ideal impulse response of the filter derived from the desired frequency response which is identified by specifying passbands, stopbands, and transition bands, as well as any other relevant parameters based on the application requirements. In the frequency domain, truncating process corresponds to convolving with a sinc function, leading to overshoot and ringing (i.e., Gibbs oscillations) in the frequency response [1]. Minimizing these oscillations lead to an improvement in filter design. Various performance metrics such as minimum stopband attenuation, maximum passband attenuation, and transition width are used to evaluate and compare different filters. Stopband and passband attenuations are related with the ripples in stopband and passband, respectively. The transition width of a filter refers to the rate at which the filter response changes from the passband to the stopband. Proposing new window functions allows for the optimization of these performance metrics, tailoring the filter design to achieve the best compromise based on the specific application requirements. For example, different window functions can provide different transition width characteristics, therefore developing new window functions allows for the exploration of designs that provide sharper or smoother transitions.

Since undesired Gibbs oscillations inherently occur in the windowing-based filter design method, it is important to minimize them to obtain a better filter design by proposing new window functions. In this study, it is aimed to propose new adjustable hybrid windows based on Cosh window [10], which will allow to design better window method-based filters in terms of transition width and minimum stop band attenuation parameters.

The article is organized as follows: The second section provides an overview of fixed and adjustable windows used in this study and introduces the method for designing FIR filters using windows, along with the proposed window functions. In the third section, the performance of filters designed with the proposed windows is analyzed through a comparison with digital filters designed using adjustable windows in the literature. The last section summarizes the results obtained in this study.

## 2. MATERIAL AND METHODS

### 2.1. Fixed Windows in The Literature

Window functions that have only window length as their argument, that is, that depend on only one parameter, are called fixed window functions. Fixed window functions proposed in the literature are given below for the range $|n| \leq (N-1)/2$ in alphabetical order.

The Bartlett window is defined by Eq. (1) [24].

$$w(n) = \begin{cases} \dfrac{n + \frac{N-1}{2}}{\frac{N-1}{2}} & -\dfrac{N-1}{2} \leq n \leq 0 \\ 2 - \dfrac{n + \frac{N-1}{2}}{\frac{N-1}{2}} & 0 \leq n \leq \dfrac{N-1}{2} \end{cases} \tag{1}$$

The Bartlett-Hann window is a modified version of the Bartlett and Von Hann windows. It is defined by Eq.(2) [25].

$$w(n) = 0.62 - 0.48 \left| \frac{n + \frac{N-1}{2}}{N-1} - 0.5 \right| + 0.38 \cos\left(2\pi\left(\frac{n + \frac{N-1}{2}}{N-1} - 0.5\right)\right) \tag{2}$$

Blackman window is defined as in Eq. (3) [26].

$$w(n) = 0.42 + 0.5cos\ 2\pi n/(N-1) + 0.08cos\ 4\pi n/(N-1) \tag{3}$$

Blackman-Harris Window is defined by Eq. (4) [26].

$$w(n) = 0.35875 - 0.48829\cos\left(\frac{2\pi\left(n+\frac{N-1}{2}\right)}{N-1}\right) + 0.14128cos\left(\frac{4\pi\left(n+\frac{N-1}{2}\right)}{N-1}\right) - 0.01168cos\left(\frac{6\pi\left(n+\frac{N-1}{2}\right)}{N-1}\right) \tag{4}$$

Bohman window is defined as in Eq. (5) [26].

$$w(n) = \left(1 - \frac{|n|}{(N-1)/2}\right)cos\left(\frac{\pi n}{(N-1)/2}\right) + \frac{1}{\pi}\frac{sin(\pi|n|)}{(N-1)/2} \tag{5}$$

Cos(x) window is defined as in Eq. (6) [8].

$$w(n) = cos\left(\frac{\pi n}{N-1}\right) \tag{6}$$

Cos³(x) window is defined as in Eq. (7) [8].

$$w(n) = 0.75cos\left(\frac{\pi n}{N-1}\right) + 0.25cos\left(\frac{3\pi n}{N-1}\right) \tag{7}$$

Cos⁴(x) window is defined as in Eq. (8) [8].

$$w(n) = 0.375 + 0.5cos\left(\frac{2\pi n}{N-1}\right) + 0.125cos\left(\frac{4\pi n}{N-1}\right) \tag{8}$$

Flat-Top window is defined as in Eq. (9) [27].

$$w(n) = 0.21557895 - 0.41663158\cos\left(\frac{2\pi\left(n+\frac{N-1}{2}\right)}{N-1}\right) + 0.277263158cos\left(\frac{4\pi\left(n+\frac{N-1}{2}\right)}{N-1}\right) - 0.083578947cos\left(\frac{6\pi\left(n+\frac{N-1}{2}\right)}{N-1}\right) + 0.006947368cos\left(\frac{8\pi\left(n+\frac{N-1}{2}\right)}{N-1}\right) \tag{9}$$

Hamming window is defined as in Eq. (10) [25].

$$w(n) = 0.54 + 0.46cos\frac{2\pi n}{N-1} \tag{10}$$

Lanczos window is defined as in Eq. (11) [28].

$$w(n) = sinc\left(\frac{2n}{N-1}\right) \tag{11}$$

The Nuttal Window is formed by modifying the coefficients of the Blackman-Harris window equation. It is given in Eq. (12) [29].

$$w(n) = 0.3635819 - 0.4891775\cos\left(\frac{2\pi\left(n+\frac{N-1}{2}\right)}{N-1}\right) + 0.1365995cos\left(\frac{4\pi\left(n+\frac{N-1}{2}\right)}{N-1}\right) - 0.106411cos\left(\frac{6\pi\left(n+\frac{N-1}{2}\right)}{N-1}\right) \quad (12)$$

The Optimized Blackman Window is created by changing the coefficients in the Blackman window equation. It is given in Eq. (13) [8].

$$w(n) = 0.412 + 0.5cos\frac{2\pi n}{N-1} + 0.088cos\frac{4\pi n}{N-1} \quad (13)$$

Parabolic window is defined as in Eq. (14) [8].

$$\mathrm{w(n)} = 1 - \left(\frac{2\mathrm{n}}{\mathrm{N}-1}\right)^2 \quad (14)$$

The Parzen window is defined as in Eq. (15) [30].

$$w(n) = \begin{cases} 1 - 6\left[\frac{n}{(N)/2}\right]^2 + 6\left[\frac{n}{(N)/2}\right]^3 & 0 \le |n| \le \frac{N-1}{4} \\ 2\left[1 - \frac{|n|}{N/2}\right]^3 & \frac{N-1}{4} \le |n| \le \frac{N-1}{2} \end{cases} \quad (15)$$

Rectangular window is defined as in Eq. (16) [1].

$$w(n) = 1 \quad (16)$$

The Riemann window is defined as in Eq. (17) [8].

$$w(n) = \frac{sin\left(\frac{2\pi n}{N-1}\right)}{\frac{2\pi n}{N-1}} \quad (17)$$

Triangular window is defined as in Eq. (18) [31].

$$w(n) = 1 - \frac{|2n|}{N} \quad (18)$$

Von-Hann window is defined as in Eq. (19) [21].

$$w(n) = 0.5 + 0.5cos\frac{2\pi n}{N-1} \quad (19)$$

Welch window is defined as in Eq. (20) [32].

$$w(n) = 1 - \left(\frac{n - \frac{N-1}{2}}{\frac{N-1}{2}}\right)^2 \quad (20)$$

## 2.2. Adjustable Windows in The Literature

Window functions that depend on more than one parameter are called adjustable window functions. There are various adjustable window functions proposed in the literature. Some well-known adjustable windows used in this study are given below for the range $|n| \leq (N-1)/2$ in alphabetical order.

Cosh window is defined as in Eq. (21) [10].

$$w(n) = \frac{\cosh\left(\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right)}{\cosh(\alpha)} \tag{21}$$

Cosh-Hamming window is defined as in Eq. (22) [33].

$$w(n) = \frac{\cosh\left(\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right)}{2\cosh(\alpha)} + 0.27 - 0.23\cos 2\pi\left(\frac{n}{N-1} + 0.5\right) \tag{22}$$

Dolph Chebyshev window is defined as in Eq. (23) [1].

$$w(n) = \frac{1}{N}\left[\frac{1}{r} + 2\sum_{i=1}^{\frac{N-1}{2}} T_{N-1}\left(x_0 cos\left(\frac{i\pi}{N}\right)\right) cos\left(\frac{2ni\pi}{N}\right)\right] \tag{23}$$

where

$$r = 10^{-R/20}\ ,\ x_0 = cosh\left(\frac{1}{N-1} cosh^{-1}\left(\frac{1}{r}\right)\right),\ T_k(x) = \begin{cases} cos\left(kcos^{-1}(x)\right) & |x| \leq 1 \\ \mathrm{Cosh}\left(cosh^{-1}(x)\right) & |x| \geq 1 \end{cases} \tag{24}$$

Exponential window is defined as in Eq. (25) [34].

$$w(n) = \frac{exp\left(\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right)}{exp(\alpha)} \tag{25}$$

Exponential-Hamming window is defined as in Eq. (26) [35].

$$w(n) = \frac{exp\left(\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right)}{2exp(\alpha)} + 0.27 - 0.23cos2\pi\left(\frac{n}{N-1} + 0.5\right) \tag{26}$$

Gaussian window is defined as in Eq. (27) [8].

$$w(n) = e^{-\frac{1}{2}\left[\alpha\frac{2n}{N-1}\right]^2} \tag{27}$$

Hann-Poisson window is defined as in Eq. (28) [8].

$$w(n) = 0.5\left[1 + cos\left(\frac{2\pi n}{N-1}\right)\right] e^{\alpha\frac{2|n|}{N-1}} \tag{28}$$

Kaiser window is defined as in Eq. (29) [1].

$$w(n) = \frac{I_0\left(\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right)}{I_0(\alpha)} \tag{29}$$

where

$$I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2 \tag{30}$$

Kaiser-Hamming window is defined as in Eq. (31) [7].

$$\mathrm{w}(n) = \frac{I_0\left(\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right)}{2I_0(\alpha)} + 0.27 - 0.23cos2\pi\left(\frac{n}{N-1}+0.5\right) \tag{31}$$

Poisson window is defined as in Eq. (32) [8].

$$w(n) = e^{\alpha\frac{2|n|}{N-1}} \tag{32}$$

Saramaki window is defined as in Eq. (33) [9].

$$w(n) = \widehat{w}(n)/\widehat{w}(0) \quad |n| \le \frac{N-1}{2} \tag{33}$$

where

$$\widehat{w}(n) = v_0(n) + 2\sum_{k=1}^{(N-1)/2} v_k(n), \quad \mathrm{v}_0(\mathrm{n}) = \begin{cases}1 & \mathrm{n}=0\\ 0 & \text{other}\end{cases}, \quad \mathrm{v}_1(\mathrm{n}) = \begin{cases}\gamma-1 & \mathrm{n}=0\\ \gamma/2 & |\mathrm{n}|=1\\ 0 & \text{other}\end{cases} \tag{34}$$

$$\mathrm{v}_\mathrm{k}(\mathrm{n}) = \begin{cases}2(\gamma-1)\mathrm{v}_{\mathrm{k}-1}(\mathrm{n}) - \mathrm{v}_{\mathrm{k}-2}(\mathrm{n}) + \gamma[\mathrm{v}_{\mathrm{k}-1}(\mathrm{n}-1) - \mathrm{v}_{\mathrm{k}-2}(\mathrm{n}-1)] & |\mathrm{n}| \le \mathrm{k}\\ 0 & \text{other}\end{cases} \tag{35}$$

### 2.3. FIR Filter Design Using Window Functions

The Fourier series method is one of the FIR filter design methods. The purpose of using a window in this method is to cut and smooth the ideal filter having the infinite impulse response. The impulse response of the noncausal filter obtained using the window is found from Eq. (36) [1,7].

$$\mathrm{h}_{\mathrm{nc}}(\mathrm{n}) = \mathrm{w}(\mathrm{n})\mathrm{h}_{\mathrm{id}}(\mathrm{n}) \tag{36}$$

where $h_{id}(nT)$ is the infinite time impulse response of the ideal filter that cannot be realized. For a low pass filter, this impulse response can be found from Eq. (37) [1,7].

$$h_{id}(n) = \begin{cases}\frac{w_{ct}}{\pi} & for\ n=0\\ \frac{sinw_{ct}n}{n\pi} & for\ n\neq 0\end{cases} \tag{37}$$

The undesired oscillations in the passband and stopband regions of filters designed using windows are equal. Therefore, when finding the simulation results, it is sufficient to calculate the attenuation parameter (Generally $A_s$) in only one of the two regions.

The amplitude spectrum and spectral parameters of a low-pass filter are given in Figure 2.

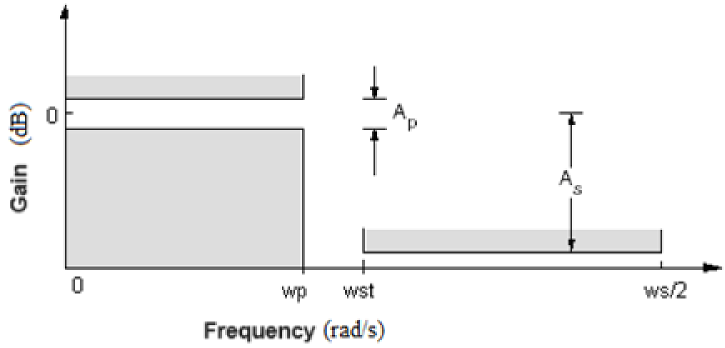



**Figure 2.** Amplitude spectrum and spectral parameters of a low-pass filter

The specified parameters in Figure 2 are: pass-band frequency ($\omega_p$), stop-band frequency ($\omega_{st}$), sampling frequency ($\omega_s$), maximum pass-band attenuation ($A_p$), minimum stop-band attenuation ($A_s$).

Apart from these, there are also cut-off frequency ($\omega_c$) and transition width ($\Delta\omega$) parameters defined by Eq. (38), respectively.

$$\omega_c = (\omega_{st} + \omega_p)/2 \text{ and } \Delta\omega = \omega_{st} - \omega_p \tag{38}$$

### 2.4. Proposed Hybrid Windows

When the fixed windows given in Section 2.1 are combined with the Cosh windows in equal weight, the hybrid windows given in Table 1 are obtained [36].

**Table 1.** Proposed hybrid windows.

| Window | Equation | Window | Equation |
|---|---|---|---|
| w1 | 0.5(Cosh + Bartlett) | w11 | 0.5(Cosh + Lanczos) |
| w2 | 0.5(Cosh + Bartlett-Hann) | w12 | 0.5(Cosh + Nuttal) |
| w3 | 0.5(Cosh + Blackman) | w13 | 0.5(Cosh + Optimized |
| w4 | 0.5(Cosh + Blackman-Harris) | w14 | 0.5(Cosh + Parabolic) |
| w5 | 0.5(Cosh + Bohman) | w15 | 0.5(Cosh + Parzen) |
| w6 | 0.5(Cosh + Cos(x)) | w16 | 0.5(Cosh + Rectangular) |
| w7 | 0.5(Cosh + $Cos^3$(x)) | w17 | 0.5(Cosh + Riemann) |
| w8 | 0.5(Cosh + $Cos^4$(x)) | w18 | 0.5(Cosh + Triangular) |
| w9 | 0.5(Cosh + Flat-Top) | w19 | 0.5(Cosh + Von-Hann) |
| w10 | 0.5(Cosh + Hamming) | w20 | 0.5(Cosh + Welch) |

In Figure 3, time domain amplitude characteristics of 20 hybrid windows plotted for N = 51 and $\alpha$ = 3 are given.

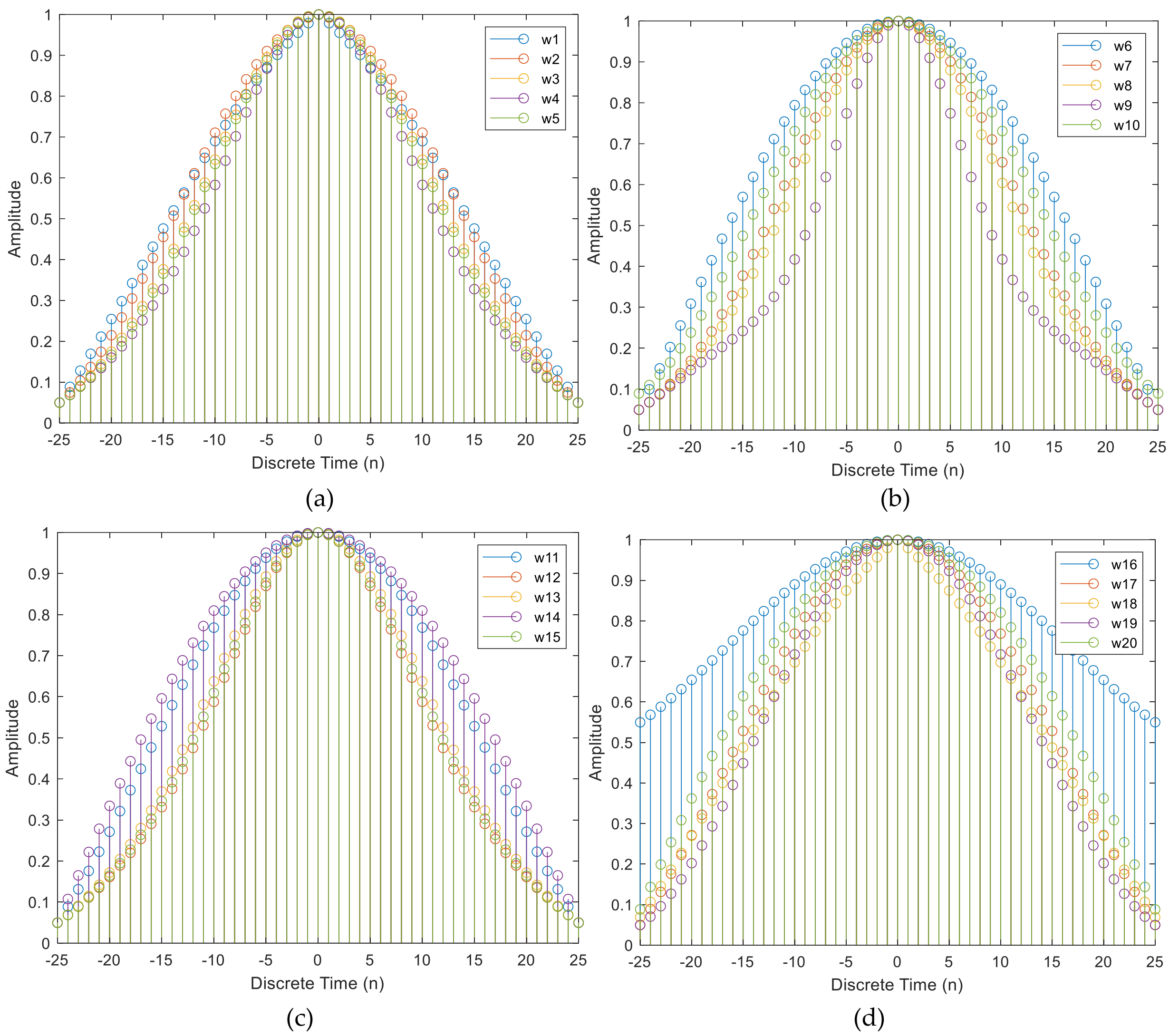


Figure 3. For N = 51 and $\alpha = 3$, time domain plots of a) w1-w5 b) w6-w10 c) w11-w15 d) w16-w20

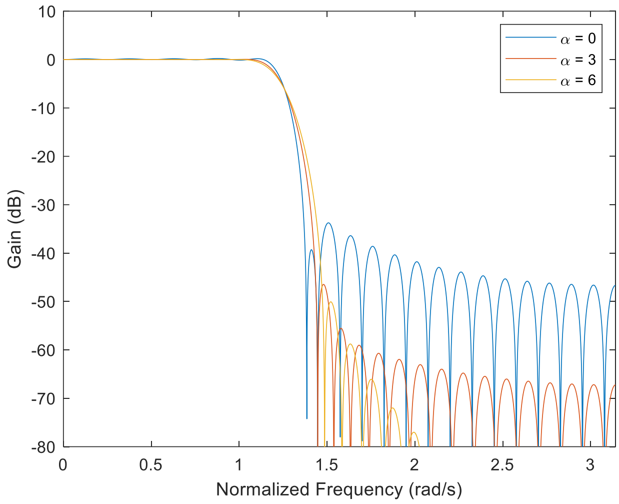
## 3. RESULTS AND DISCUSSION

In this section, first, the effect of window parameters $\alpha$ and N on the filter characteristics will be observed through the Cosh-Von Hann window, which is one of the hybrid windows. Then, the minimum stopband attenuation and transition width characteristics of 20 hybrid windows obtained by many filter design results will be achieved and the filter performances will be observed by comparing them with the filters designed with Cosh window. Then, the numerical filter design performances of the proposed hybrid windows and two parameter adjustable windows in the literature will be compared.

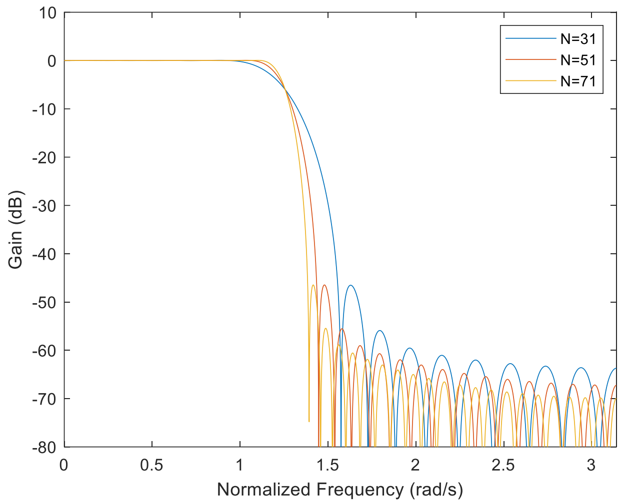
### 3.1. Effect of Proposed Hybrid Windows Parameters on Designed Filter Characteristics

In this section, performance analyzes in terms of minimum stop band attenuation and transition width parameters of digital filters designed with equal filter length - different alpha values and equal alpha value - different filter lengths for the new hybrid window obtained by hybridizing Cosh window and Von-Hann window are given. Figure 4a shows the effect of $\alpha$ on the filter characteristic at a fixed filter length. It is observed that when $\alpha$ value increases, $\Delta\omega$ increases (gets worse) and $A_s$ increases (gets better). Figure 4b illustrates the impact of the filter length on the filter characteristic at a constant $\alpha$

value. It is seen that as N increases, $\Delta\omega$ decreases (gets better) and $A_s$ remains almost the same. The numerical data of the filters designed in Figure 4 are presented in Table 2. The effects of the window parameters on the filter spectral parameters are also seen in this table explicitly.




(a)




(b)

**Figure 4.** Amplitude spectra of filters designed with a Cosh-Von Hann hybrid window a) for different values of $\alpha$ at a fixed filter length and b) for different filter lengths at a fixed value of $\alpha$

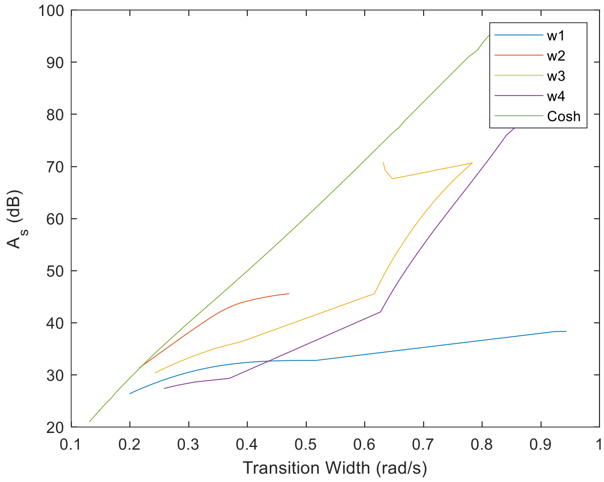
**Table 2.** Transition width and minimum stopband attenuation values for filters designed with Cosh-Von Hann window at different lengths and $\alpha$ values

| | N = 31 | | N = 51 | | N = 71 | |
|---|---|---|---|---|---|---|
| $\alpha$ | $\Delta\omega$ (rad/s) | $A_s$ (dB) | $\Delta\omega$ (rad/s) | $A_s$ (dB) | $\Delta\omega$ (rad/s) | $A_s$ (dB) |
| 0 | 0.368 | 33.77 | 0.221 | 33.74 | 0.158 | 33.70 |
| 3 | 0.610 | 46.52 | 0.365 | 46.22 | 0.261 | 46.46 |
| 6 | 0.737 | 50.06 | 0.443 | 50.06 | 0.316 | 50.06 |

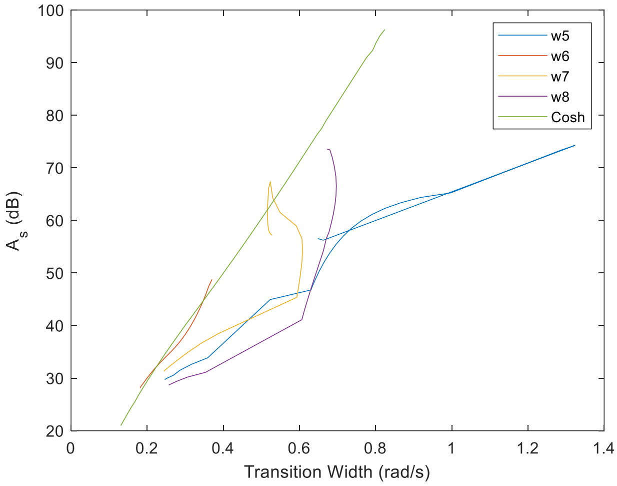
### 3.2. Comparison of Filters Designed with Proposed Hybrid Windows and Cosh Window

In this section, the comparisons of filters designed for 20 hybrid windows obtained by hybridization of Cosh window and fixed windows with filters designed with Cosh window in terms of minimum stopband attenuation and transition width parameters for a filter length of N = 51 are presented.

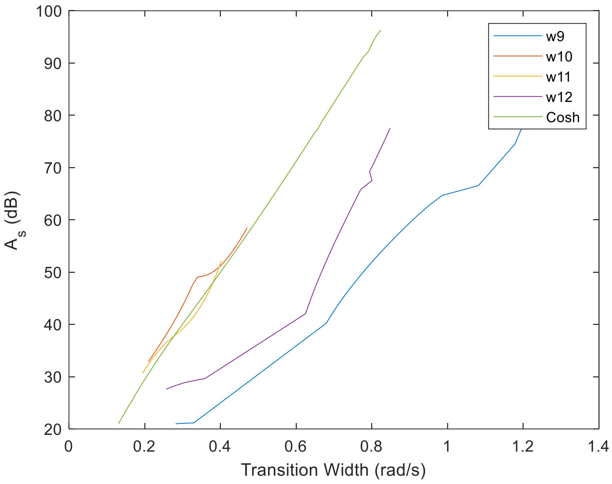
In Figure 5a, it is seen that none of the filters designed with Cosh-based hybrid windows exhibited better filter spectral characteristics than Cosh-based filters. From Figure 5b, it is observed that the filters designed with hybrid windows based on w6 and w7 can exhibit better filter spectral characteristics than Cosh based filters at certain intervals. As for Figure 5c, it is seen that filters designed with hybrid windows based on w10 and w11 can exhibit better filter spectral characteristics than Cosh based filters at certain intervals. In Figure 5d, it is seen that filters designed with hybrid windows based on w14 and w16 can exhibit better filter spectral characteristics than Cosh based filters at certain intervals. And, from Figure 5e, it is observed that the filters designed with hybrid windows based on w17, w19 and w20 can exhibit better filter spectral characteristics than Cosh based filters at certain intervals. As a result, since the filter performance of w1, w2, w3, w4, w5, w8, w9, w12, w13, w15 and w18 hybrid windows is always worse than the Cosh window, the hybridization in these windows was not significant for filter design applications.



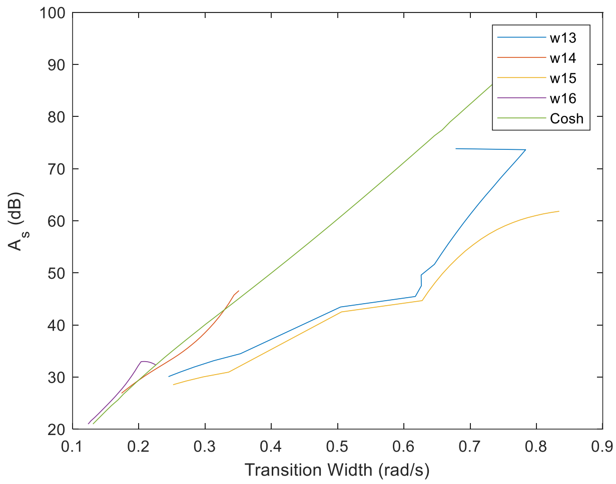

(a)



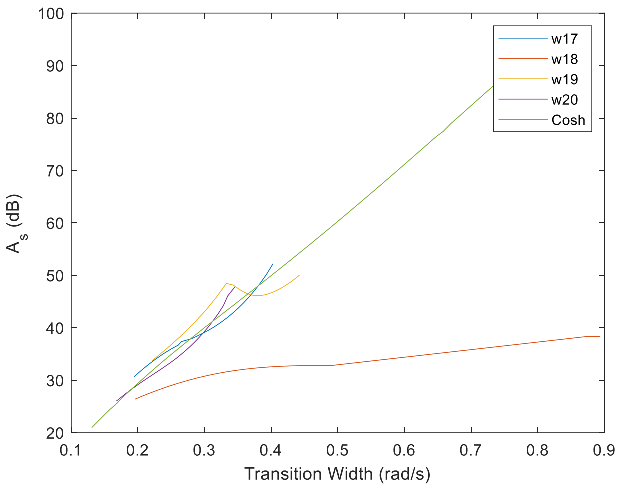

(b)




(c)




(d)




(e)

**Figure 5.** For window length N = 51 a) w1-w4, b) w5-w8, c) w9-w12, d) w13-w16 and e) w17-w20 hybrid windows with minimum stop band attenuation of filters designed with Cosh window-pass band width characteristics.

### 3.3. Comparison of Filters Designed Using Proposed Hybrid Windows with Filters Designed Using Other Two-Parameter Windows in The Literature

As a result of the filter spectral analysis performed in the previous section, it was seen that filters designed with hybrid windows based on w6, w7, w10, w11, w14, w16, w17, w19 and w20 can exhibit better filter spectral characteristics than Cosh based filters at certain intervals. Therefore, in this section, the performance of these 9 hybrid windows proposed in the filter design with other two-parameter windows proposed in the literature is examined. For a fair comparison, the minimum stopband attenuation values of the designed filters are compared while the filter lengths and transition widths are selected to be the same. Fixed window functions such as Hamming window are not included in the comparison, because they cannot provide the filter characteristics specified in the comparison, because they can perform only one filter characteristic for a fixed filter length.

Table 3 presents the filter spectral numerical values related to the digital filters designed by using nine mentioned proposed windows and well known two parameter adjustable windows (Kaiser-Hamming, Dolph-Chebyshev, Exponential-Hamming, Saramaki, Kaiser, Cosh, Exponential, Gaussian, Hann-Poisson and Poisson windows) in literature for the filter lengths N = 51 and 101. The amplitude spectrums of the designed filters for a filter length N = 51 are shown in Figure 6. The results show that the best filter design performance is exhibited by filters designed with Cosh-Hamming and Cosh-Von Hann windows, respectively, while the worst performance is exhibited by filters designed with Poisson and Cosh-Nuttal windows. In addition, from the values given in Table 3, it is seen that when the filter length is increased, the minimum stop band attenuation remains almost the same, while the transition bandwidth is reduced by half to a better value.

**Table 3.** Comparison of the digital filters designed with hybrid windows and other two-parameter windows in terms of minimum stop band attenuation at different filter length values

| **Window** | **N = 51** | | **N = 101** | |
|---|---|---|---|---|
| | **$\Delta\omega$ (rad/s)** | **$A_s$ (dB)** | **$\Delta\omega$ (rad/s)** | **$A_s$ (dB)** |
| Cosh-Hamming (w10) | $0.1065\pi$ | 48.92 | $0.0533\pi$ | 48.75 |
| Cosh-Von Hann (w19) | $0.1065\pi$ | 48.86 | $0.0533\pi$ | 48.71 |
| Kaiser-Hamming [7] | $0.1065\pi$ | 48.16 | $0.0533\pi$ | 48.01 |
| Dolp-Chebyshev [1] | $0.1065\pi$ | 48.10 | $0.0533\pi$ | 47.99 |
| Exponential-Hamming [35] | $0.1065\pi$ | 47.10 | $0.0533\pi$ | 47.07 |
| Saramaki [9] | $0.1065\pi$ | 46.23 | $0.0533\pi$ | 46.07 |
| Kaiser [1] | $0.1065\pi$ | 45.89 | $0.0533\pi$ | 45.86 |
| Cosh-Welch (w20) | $0.1065\pi$ | 45.68 | $0.0533\pi$ | 44.84 |
| Gaussian [8] | $0.1065\pi$ | 44.15 | $0.0533\pi$ | 44.02 |
| Cosh-Parabolic (w14) | $0.1065\pi$ | 43.91 | $0.0533\pi$ | 44.01 |
| Cosh [10] | $0.1065\pi$ | 43.91 | $0.0533\pi$ | 43.88 |
| Exponential [34] | $0.1065\pi$ | 43.47 | $0.0533\pi$ | 43.51 |
| Cosh-Cosx (w6) | $0.1065\pi$ | 42.68 | $0.0533\pi$ | 42.71 |
| Cosh-Lancsoz (w11) | $0.1065\pi$ | 42.02 | $0.0533\pi$ | 42.04 |
| Cosh-Riemann (w17) | $0.1065\pi$ | 42.02 | $0.0533\pi$ | 42.04 |
| Cosh-Bartlett Hann (w2) | $0.1065\pi$ | 40.78 | $0.0533\pi$ | 40.74 |
| Cosh-$Cos^3x$ (w7) | $0.1065\pi$ | 36.30 | $0.0533\pi$ | 36.24 |
| Cosh-Blackman (w3) | $0.1065\pi$ | 34.70 | $0.0533\pi$ | 34.66 |
| Cosh-Opt. Blackman (w13) | $0.1065\pi$ | 33.90 | $0.0533\pi$ | 33.86 |
| Hann-Poisson [8] | $0.1065\pi$ | 33.46 | $0.0533\pi$ | 33.70 |
| Cosh-Bohman (w5) | $0.1065\pi$ | 33.21 | $0.0533\pi$ | 33.18 |
| Cosh-Triangular (w18) | $0.1065\pi$ | 31.65 | $0.0533\pi$ | 31.47 |
| Cosh-Nuttal (w12) | $0.1065\pi$ | 29.36 | $0.0533\pi$ | 29.36 |
| Poisson [8] | $0.1065\pi$ | 22.11 | $0.0533\pi$ | 22.15 |

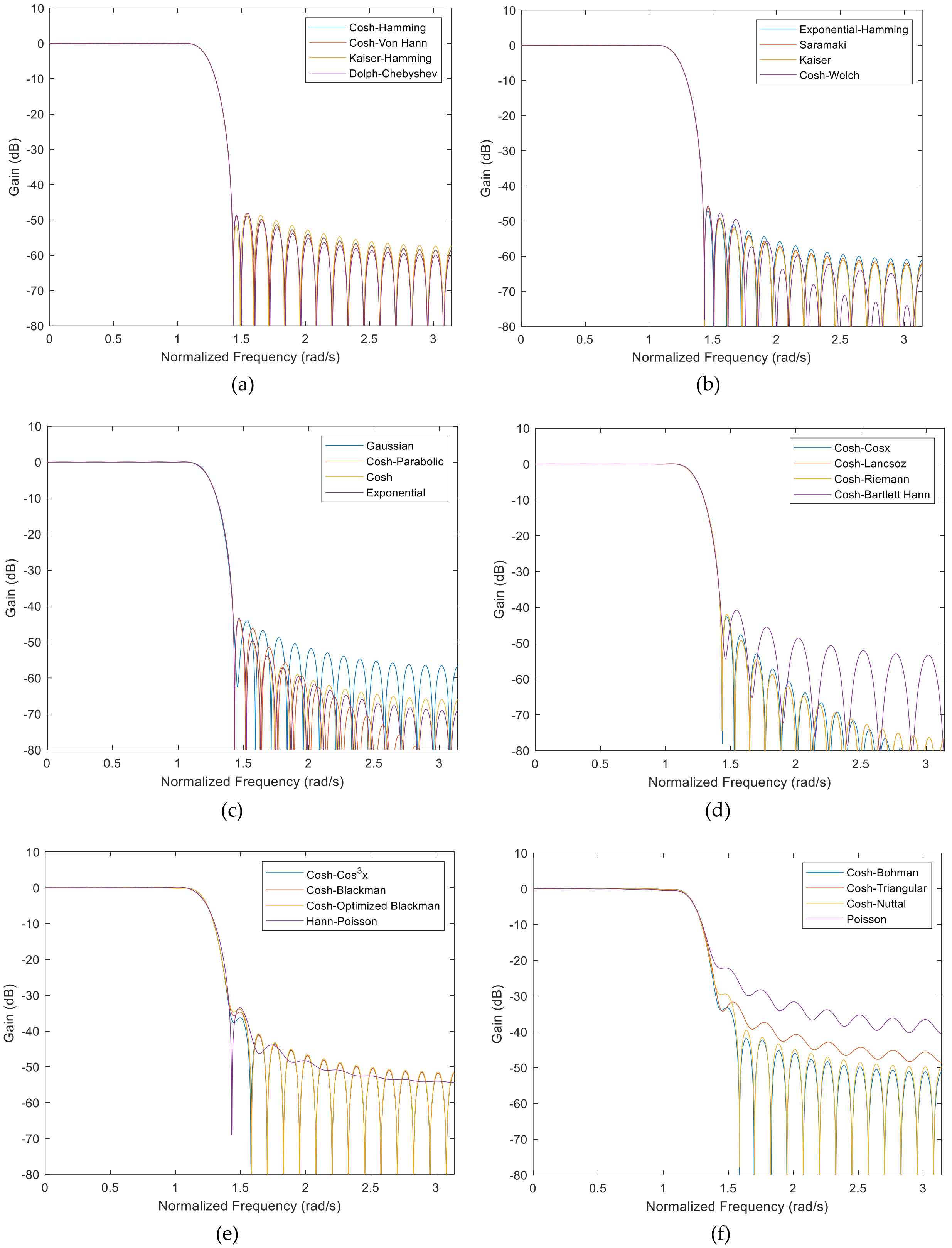



**Figure 6.** Amplitude spectrums of digital filters designed with the proposed windows and the adjustable windows in the literature for filter length of N = 51

## 4. CONCLUSIONS

In this study, new hybrid windows are proposed by weighting the fixed windows in the literature with equal coefficients with the two-parameter adjustable window, namely Cosh window, and their performances in filter design are examined, as extended version of our study [36]. First, the minimum stop band attenuation and transition width characteristics of 20 hybrid windows obtained by many filter design results are achieved. The filter performances are observed by comparing them with the filters designed with the Cosh window. The results show that filters designed with only nine hybrid windows (Cosh-Cosx, Cosh-Cos$^3$x, Cosh-Hamming, Cosh-Lancsoz, Cosh-Parabolic, Cosh-Rectangular, Cosh-Riemann, Cosh-Von Hann, and Cosh-Welch) can exhibit better filter spectral characteristics than Cosh window-based filters at certain intervals. Meanwhile, among hybrid windows, the filtering performances of Cosh-Bartlett, Cosh-Bartlett Hann, Cosh-Blackman, Cosh-Blackman Harris, Cosh-Bohman, Cosh-Cos$^4$(x), Cosh-Flat Top, Cosh-Nuttal, Cosh- Optimized Blackman, Cosh-Parzen, Cosh-Triangular are always worse than the Cosh window, therefore the hybridization in these windows is not significant for filter design applications. Finally, the performances of the nine hybrid windows in filter design are examined in comparison with other two-parameter windows in the literature. As a result of the filter spectral analysis for N = 51 and N = 101, it is observed that the filters designed based on Cosh-Hamming and Cosh-Von Hann windows can provide better results than filters designed with other windows. The proposed windows can also be used in applications such as communication [37-39], image processing [40, 41] and biomedical processing [42, 43] instead of the Kaiser window and other well-known windows.

### Declaration of Ethical Standards

As the authors of this study, we declare that all ethical standards have been complied with.

### Credit Authorship Contribution Statement

Author contribution rates are equal in this study.

### Declaration of Competing Interest

The authors declare that there is no conflict of interest.

### Funding / Acknowledgements

No funding is available for this research.

### Data Availability

This study does not contain any dataset.

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