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Research Article

Examples of the Use of Mathematics History in Mathematics Teaching: Ascalon Multiplication Method, Gelosia Multiplication Method and Napier's Rods Method

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Abstract – The aim of this study is to present the Ascalon multiplication method, Gelosia multiplication method and Napier's rods method, which take place in the history of mathematics, to sixth grade students through activities and to determine the effect of these methods on student achievement, and to get student opinions about this activity. The study group of the research consisted of 60 sixth grade students. A worksheet was used as a data collection tool and an interview form was used at the end of the activity to obtain students' opinions about the activity. According to the quantitative analysis results obtained from the research data, it was determined that the methods taught were effective in increasing student achievement. According to the results of the qualitative analysis, it was concluded that students generally did not encounter such activities much, that they learned the lesson by having fun with such activities, and that their interest in mathematics increased.

Key words: History of mathematics, Ascalon multiplication method, Gelosia multiplication method, Napier's rods method

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Introduction

The use of the history of mathematics in mathematics education is a long-standing and supported idea (Barwell, 1913; Fried, 2001; Groza, 1968). Radford and Santi (2022) argue that the history of mathematics in education is a necessity, not a choice, and that it is a central part of the process of understanding human nature in a fundamentally historical and cultural

way. In recent years, there has been an increase in domestic and international studies on the use of the history of mathematics in teaching environments and its integration into education (Baki & Bütüner, 2013; Bütüner, 2008; Furinghetti, 1997; Goodwin, 2007; İdikut, 2007; Özdemir & Göktepe, 2015; Siu, 2004; Swetz, 1994). Special issues on the history of mathematics are published in international journals (Jankvist, 2009). In these studies, it is also mentioned that integrating the history of mathematics into mathematics courses has many benefits. In addition, studies in this field, which contribute to research in both mathematics education and the history of mathematics, have also raised important theoretical issues related to mathematics by providing innovative teaching approaches (Chorlay et. al., 2022).

There are many reasons why educators are interested in the history of mathematics. In addition to this, there are many studies in literature on the use of the history of mathematics in mathematics lessons. Fauvel (1991) stated that there are fifteen reasons for being interested in the history of mathematics and that more reasons can be found in literature. Fried (2001) categorized these reasons under three main themes. The first one is that the history of mathematics helps mathematics to be a product of human activity, the second one is that the history of mathematics makes mathematics more interesting, more understandable and more accessible, and the third one is that the history of mathematics provides insight into mathematical concepts, problems and problem solving. Gulikers and Blom (2001) grouped the reasons why it is important to use the history of mathematics into three categories: conceptual discussions, multicultural discussions and motivational discussions. In discussions related to motivation, it has been stated by many studies that working on a problem taken from history will enable students to discover different solutions in addition to modern solutions and that they will have the opportunity to compare these solutions, which will increase their motivation and reduce their anxiety and fear towards mathematics and help them to develop a positive attitude toward mathematics (Fauvel, 1991; Liu, 2003; Swetz, 1997; Tzanakis & Arcavi, 2000). Therefore, including the history of mathematics in mathematics teaching can change students' perspectives on mathematics. By integrating the history of mathematics into mathematics lessons, students will realize that mathematics is a branch of science that can continuously improve itself, that mathematics has a cultural relationship, and how mathematics omit shapes and shapes our ideas (Baki, 2008).

Including the history of mathematics in mathematics lessons allows students to learn how some mathematical expressions and some calculation methods originated, and creates the opportunity to ask questions (Bidwell, 1993; Jankvist, 2009; Tzanakis & Thomaidis, 2000).

Jankvist (2009) stated that the history of mathematics is used in mathematics lessons in two different ways: as a tool and as a purpose. The use as a tool includes arguments about how students learn mathematics. For example, using the history of mathematics in mathematics lessons helps to maintain students' interest and excitement in the subject and is a motivating factor for students to learn and study mathematics (Jankvist, 2009). When used as an objective, the focus is on the developmental and evolutionary aspects of mathematics as a discipline. For example, an objective is to show students that mathematics exists and evolves over time and space and that many different cultures have been influential in shaping mathematics (Barabash and Guberman-Glebov, 2004; Høyrup, 2007; Tzanakis and Thomaidis, 2000;).

Bütüner (2008) taught the eighth grade equations subject by using examples from the history of mathematics. He solved algebraic problems with Ancient Egyptian, Babylonian, Ancient Chinese and Khwarezmi methods and made comparisons with today's solutions. Karakuş (2009) explained the Babylonian method, which is a different approach method for calculating square roots, in detail and gave examples. He provided students with a different experience from traditional square rooting methods in textbooks. Panasuk and Horton (2013) investigated the views of mathematics teachers on teaching by incorporating the history of mathematics, and the teachers stated that the history of mathematics attracted students' interest and mathematical concepts were learned by students with fun. Ersoy and Öksüz (2016) taught the topic of decimal fractions to fourth grade primary school students using the history of mathematics and investigated the effect of teaching on students' achievement, retention level and motivation. According to the results of the study, it was evident that the achievement, retention level and motivation to learn mathematics increased in students who learned decimal fractions using the history of mathematics. Altıntaş and Sidekli (2017) tried to teach multiplication to students studying in unified classes by using Napier's rods and concluded that students' achievement was positively affected. Başıbüyük and Soylu (2019) examined the change in student attitudes by using the history of mathematics in mathematics lessons and applied 15 activities related to the history of mathematics to students. It was concluded that mathematics history practices had a positive effect on students' attitudes towards mathematics course.

In our country, the introduction of the history of mathematics into the textbooks with the elementary mathematics curriculum in 2005 enabled students to make connections between the historical development of mathematics and the historical development of science

(Barry, 2000). In addition, within the framework of the general objectives of the elementary mathematics curriculum, it is emphasized that students should be able to recognize the role of mathematics in the development of historical and human thought and the importance of its use in different fields (Baki & Bütüner, 2013).

For this reason, it is also important to include the use of the history of mathematics in the curriculum. In the study, three multiplication methods, also included in the history of mathematics, were first introduced: " *Ascalon multiplication method, Gelosia multiplication method and Napier rods method*".

The Gelosia multiplication method is one of the most popular methods in 15th century algorithms that originated in India. It is believed that the Gelosia multiplication method was transferred westward by Arab traders. For this method it is enough to know multiplication and addition. With the emergence of new calculation techniques such as the Gelosia multiplication method in the early Renaissance, numerical calculation became easier and calculation became faster (Swetz, 1994). However, the rapid development of technology necessitated the search for more efficient tools to perform calculations. When John Napier from Scotland was doing research to facilitate numerical calculations, he found that the numerical entries in the Gelosia columns were almost multiples of the numbers at the beginning of these columns, and Napier transformed the calculation algorithm into a mechanical calculation device. This device, called "Napier's rods" or "Napier's rods", consists of an abacus, a board and a frame. Compared to the abacus, Napier's rods can perform square root operations in addition to multiplication and division.

The National Council of Teachers of Mathematics (NCTM) emphasized the necessity of integrating the history of mathematics into mathematics teaching, that mathematics is a success indicator for the history of humanity and that it is important and necessary to reveal the cultural influences that create this success. In addition, according to NCTM, using the history of mathematics in mathematics teaching will increase student motivation, provide a more positive approach towards mathematics, develop mathematical thinking with the solution of historical problems and guide history learning. From this point of view, it is thought that this study is important in terms of establishing inter-subject and interdisciplinary relationships in mathematics and providing a different perspective by enabling students to learn with fun. The fact that there are few studies in Turkey and abroad in which the history of mathematics is integrated into a teaching environment and activity applications are carried out

is one of the other factors that make the study important. The aim of the study is to determine the effect of the Ascalon multiplication method, Gelosia multiplication method and Napier's rods method, which take place in the history of mathematics, on student achievement by presenting them to sixth grade students through activities and to get student opinions about this activity.

Method

A phenomenological design was used for the qualitative part of this study. The purpose of the phenomenological design, one of the qualitative research methods, is to reveal the experiences, beliefs and meanings attributed to a phenomenon (Özmen and Karamustafaoğlu, 2019). Phenomenology is a careful and in-depth description of how participants experience phenomena (Creswell, 2013). The case considered here is the Ascalon multiplication method, Gelosia multiplication method and Napier's rods method, which were invented in history to facilitate numerical calculations. This study is a phenomenological study as it tries to determine students' views on the methods they used in history to facilitate numerical calculations, such as the Ascalon multiplication method, Gelosia multiplication method and Napier's rods method. In the research, a pilot study was conducted for both worksheets, pre-test, post-test and interview questions.

The study introduces the Ascalon multiplication method, Gelosia multiplication method and Napier's rods method, which were found in history to facilitate numerical calculations. The activity sheet (worksheet) prepared for these methods was created by the researchers and the process was carried out by the researchers. Each student answered the activity sheet individually. First of all, multiplication operations were done in normal ways. Then, Ascalon multiplication, Gelosia multiplication and Napier's rods methods were demonstrated with examples and students were asked to solve sample questions related to these methods. After these three methods, the students were asked to perform the normal multiplication operations on the activity sheet again.

The aim here is to compare the multiplication operations that the students first performed with normal methods with the final multiplication operations after different methods and to reveal their effectiveness on student achievement. Since the students' recall was taken into consideration, the numbers in the multiplication operations were chosen from different numbers. After the completion of the examples in the activity sheets, the opinions of fifteen students about this activity were taken.

Participants

The study group of the research consists of 60 sixth grade students studying in a public school in Istanbul. While selecting the student group, the fifth grade mathematics end-of-term averages were examined and the students of the first two grades whose averages were higher than the other grades constituted the sample. The study group was applied to sixth grade students since the activities were related to the subject of operations on natural numbers. Pre-test and post-test were applied to 60 students, and 15 students selected from among the volunteers were interviewed. Necessary permissions and ethics committee approval were obtained before the data collection process. Participation in the study was based on volunteerism.

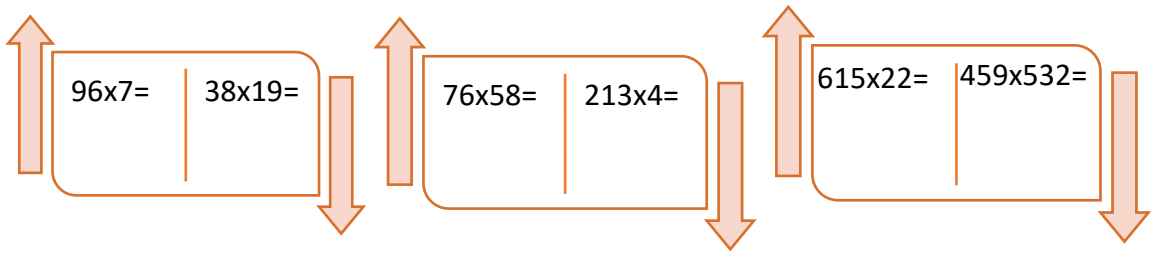
Data collection

The data collection tools of the research are worksheets, interviews, pre-test, post-test and students written documents. (The pre-test and post-test are given in the Appendix A and Appendix B).

Worksheets

Worksheets were prepared by the researchers. While creating the worksheets, Swetz's (1994) book "Learning activities from the History of Mathematics" was used. Multiplication methods in history were taught to sixth grade students by one of the researchers who completed her doctorate in mathematics education. Multiplication operations were given in the first part of the worksheet. In the following parts: Ascalon multiplication method, Gelosia multiplication method and Napier rods method were explained and examples were given to the students. Later, students were asked to give similar examples. In the last part of the study, multiplication operations consisting of different questions are included.

Ascalon Multiplication Method. Question 1 in the worksheet is given in Figure 1.



1. Find the result of the multiplication operations given above.

Figure 1 Worksheet question 1

Ascalon multiplication example is given in Figure 2.

Ascalon Multiplication

Example: To do the 628 x 4 operation with Ascalon multiplication, first the number 628 is decomposed into 600, 20 and 8 digits.

1.Then each of the solved numbers 600, 20 and 8 are multiplied by 4.

628 x 4 = 600 x 4 + 20 x 4 + 8 x 4

600 x 4 = 2400

20 x 4 = 80

8 x 4 = 32

Finally, the result is 2400 + 80 + 32 = 2512.

Figure 2 Ascalon multiplication example

Questions 2 and 3 on the worksheet are given in Figure 3.

2. Find the result of 84 x 7 using Ascalon multiplication.
3. Find the result of 567 x 9 using Ascalon multiplication.

Figure 3 Worksheet questions 2 and 3

Gelosia Multiplication Method. The Gelosia multiplication method is one of the most popular methods in 15th century algorithms that originated in India. For this method it is enough to know multiplication and addition.

An example of Gelosia multiplication method is given in Figure 4.

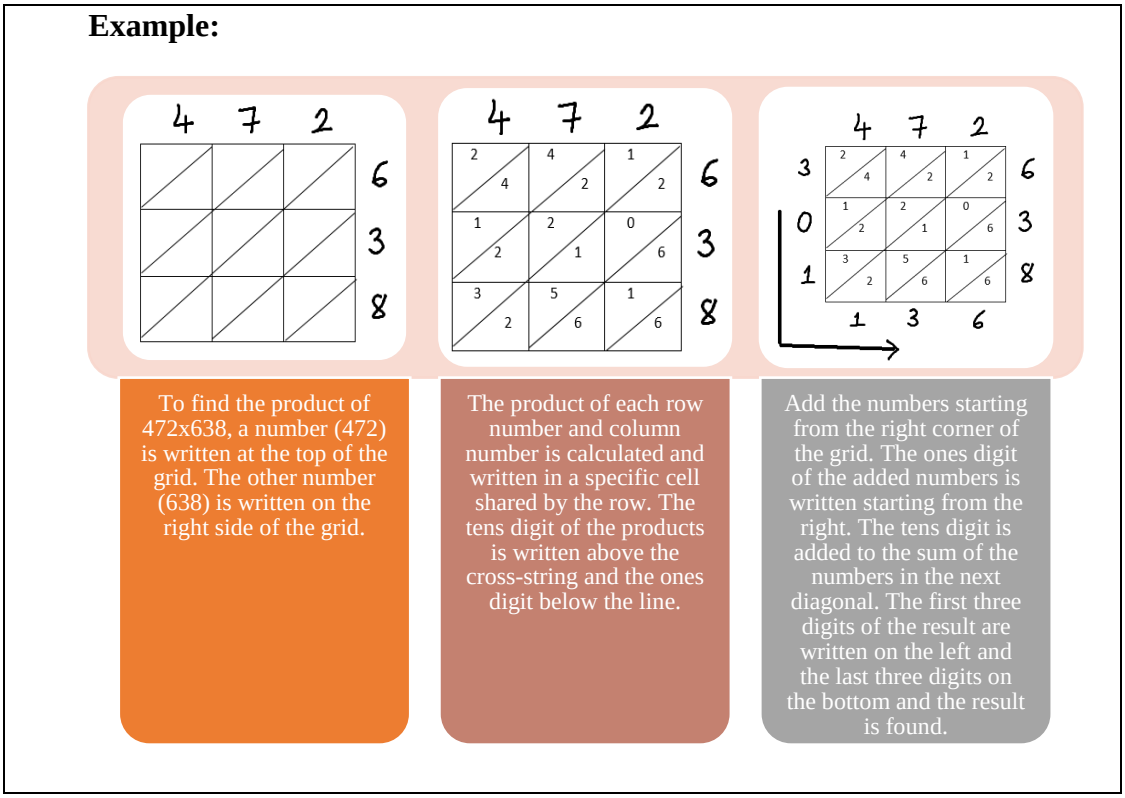


Figure 4 Gelosia multiplication method example

Questions 4 and 5 on the worksheet are given in Figure 5.

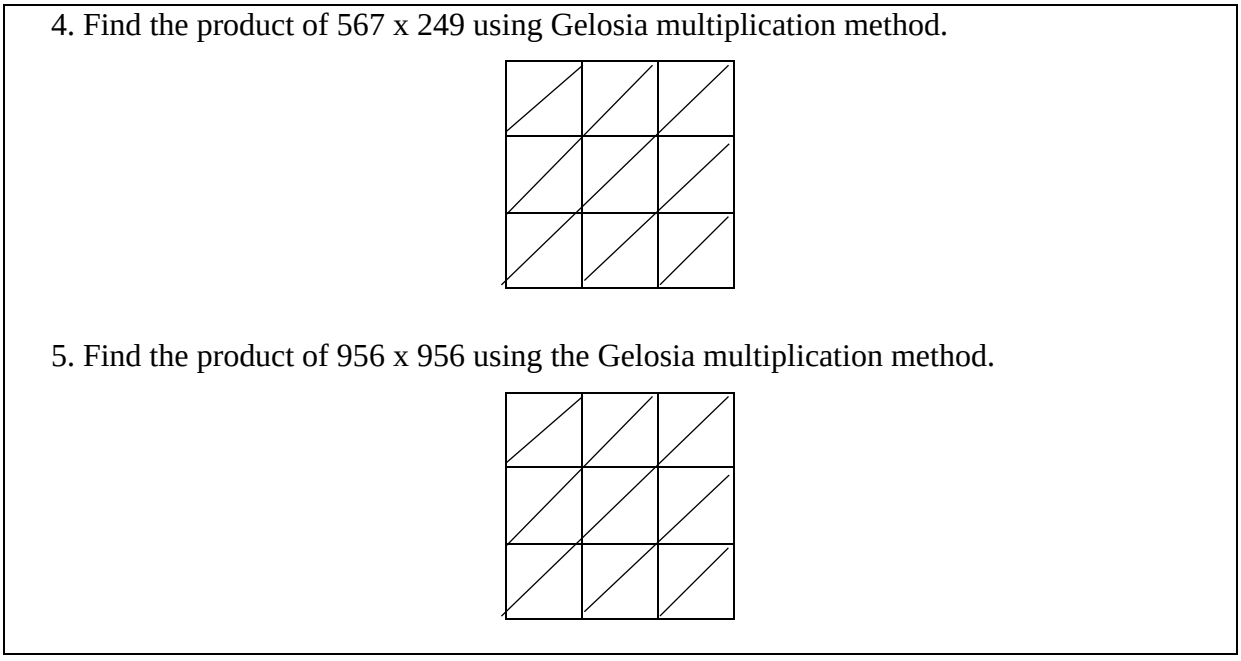


Figure 5 Worksheet questions 4 and 5

Napier's Rods Method. "Napier's rods" or "Napier's rods" is an abacus created by John Napier. Napier's rods can be used for multiplication, division and square rooting in practice (Figure 6).

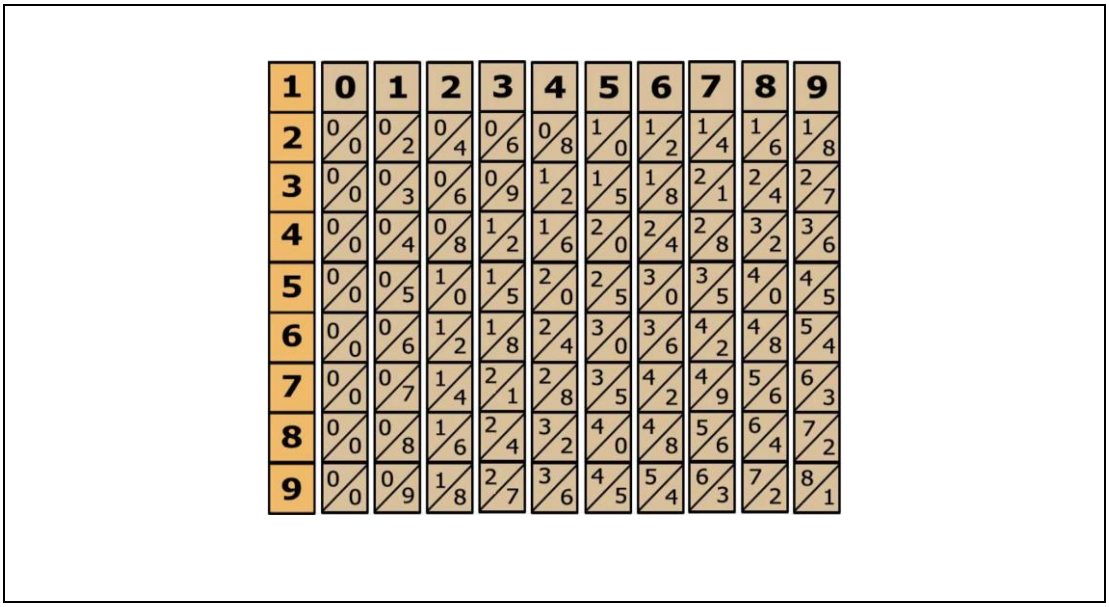
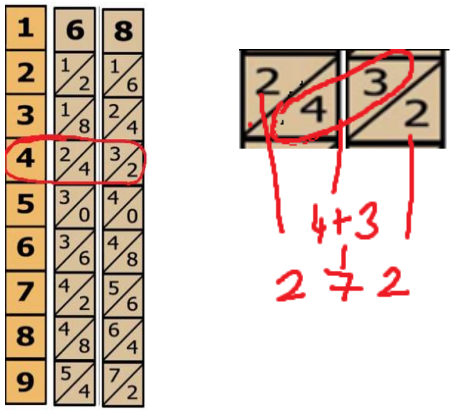


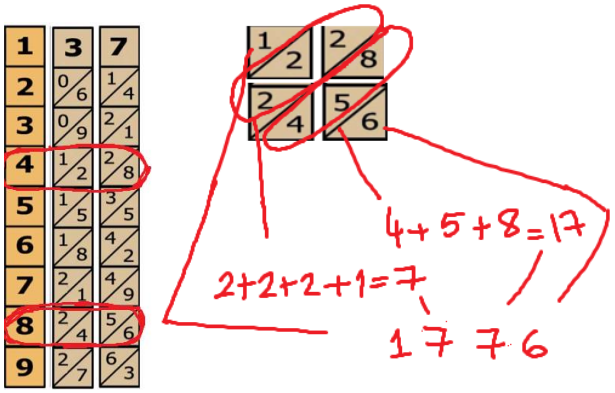
Figure 6 Napier’s rods abacus

An example of the Naiper’s rods method is given in Figure 7.

Example:



For the multiplication of 68x4, the boxes corresponding to the numbers 6 in column and 4 in row 8 in Figure 1 are taken.
The result of multiplication is found as shown on the side.



For the multiplication of 37x48, the boxes corresponding to the numbers 3 and 7 in the column and 4 and 8 in the row in Figure 1 are taken.
The result of multiplication is found as shown on the side.

Figure 7 Naiper's rods method example

Questions 6, 7 and 8 on the worksheet are given in Figure 8.

6. Find the product of 5 x 92 using Napier's rods method.
7. Find the product of 29 x 56 using Napier's rods method.
8. Find the result of the following multiplication operations.

Figure 8 Worksheet questions 6, 7 and 8

Question 9 on the worksheet are given in Figure 9.

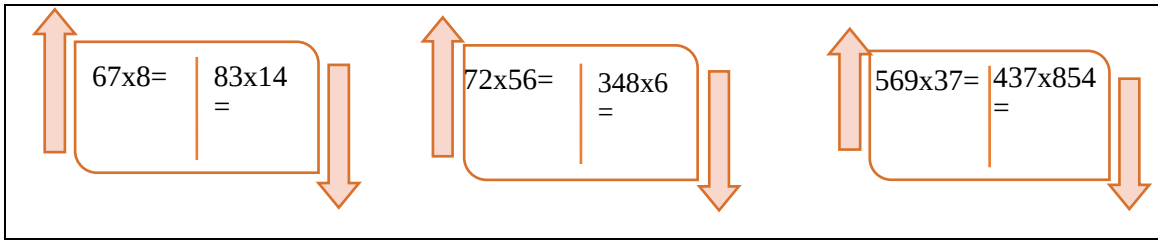


Figure 9 Worksheet question 9

Interview form

Five open-ended questions were prepared by the researchers in order to get the opinions of sixth grade students about the history of mathematics activity they did. 15 students, selected on a voluntary basis, were interviewed. A form containing interview questions was prepared and administered to the students at the end of the activity. The interview lasted around 30 minutes. To ensure the validity of the interview form, the opinions of three experts (two mathematics teachers with 15 years of experience and one mathematics teacher educator) were taken and the questions in the activity were finalized in line with their opinions. The questions in the interview form are given below:

1. Have you ever participated in an activity related to the history of mathematics during your mathematics class? If so, what kind of activity did you take part in?

2. Did you find the activity engaging? What are your opinions about including such activities in your lessons? What alternative activities could be considered?

3. Did you face any challenges while responding to the questions in the activity? If yes, what sorts of difficulties did you experience? Can you provide some examples?

4. Would you be interested in taking mathematics lessons that involve similar activities?

5. Can you establish connections between the topic you learned in the activity and its applicability in everyday life?

Data Analysis

The data obtained from the students' responses to pre-test, post-test, the activity sheet and the interview form were analyzed with qualitative and quantitative analysis methods.

For quantitative analysis, students' pre-test and post-test scores were analyzed and compared with t-test (Büyüköztürk, 2002). The pre-test and post-test scores consist of the first and last questions answered by the students on the activity sheet, which is the data collection tool. Students were given a total of 20 minutes to answer the first and last questions on the activity sheet, 10 minutes for the first question and 10 minutes for the last question. In the pre-test, the students applied the multiplication operations they knew, and in the post-test, after teaching different multiplication operations, most of the students applied the newly learned methods. While preparing the pre-test and post-test questions, the opinions of three

experts, two mathematics teachers with 15 years of experience and one mathematics educator, were taken. The questions were finalized in line with the expert opinions.

The answers given by the students in the activity sheets (worksheets) were evaluated according to the rubric prepared by the researchers. The answers given by the students to the activity were categorized as incorrectly answered (zero point), incompletely answered (one point) and correctly answered (two points), and the frequency, total score and student sample answers were given in tables (Table 2, Table 3, Table 4, Table 5, Table 6, Table 7). While scoring the student answers, zero points were given to students who answered incorrectly and left the answers blank, 1 point for each digit in Ascalon multiplication method, 1 point for each multiplication box in Gelosia multiplication method and 1 point for each digit in Napier's rods method.

Content analysis was performed for each interview question asked to the students. Content analysis brings similar data together under specific concepts and themes. It is an organized interpretation of such concepts and themes that help readers to understand better (Yıldırım & Şimşek, 2013). Content analysis is a systematic and renewable technique that summarizes a text's words into smaller content categories with respect to codes based on specific rules (Büyüköztürk, Çakmak, Akgün, Karadeniz, &Demirel, 2009). The findings obtained from the interviews with the students were first transcribed and each interview was recorded electronically as a separate document. Interview data were examined by researchers by content analysis. After the researchers were finished coding the data, the data were compared, and the inter-coder agreement (consistency) percentages were calculated according to the formula specified below. Frequency and percentage tables were created for each interview question. The purpose of content analysis is to organize similar data around certain concepts and themes (Yıldırım & Şimşek, 2013). In content analysis, the data obtained are coded, themes are established with inductive methods as a result of coding, and interpretations are made by organizing the themes. In content analysis, Miles and Huberman's (1994) consensus value was used when determining themes and organizing student responses. The reliability calculation determined by Miles and Huberman (1994) was used to determine the reliability of the study.

$$p = \text{Consensus Percentage} = \frac{N_a \; x \; 100}{N_t}$$

p: Percentage of consistency (compliance)

N_a: Number of students coded in the same way by two researchers

N_t: Total number of students coded by two researchers.

The data collected at the end of the interview were analyzed by coding. When the coding made by two researchers was examined, it was determined that there was a concordance rate of 0.90 (90%) between the coders. Şencan (2005) stated that a consistency percentage of over 80% indicates that coder reliability is achieved. After the determined codes and themes were processed into the program, the characteristics of the resulting data were presented in the findings using tables and graphs.

Findings and Discussions

In this section, findings regarding the pre-test and post-test scores, success scores, percentage and frequency tables of the scores obtained from the interview form regarding the answers of the 60 sixth grade students who participated in the research are included.

Firstly, the Kolmogorov-Smirnov normality test was performed to determine whether the data were normally distributed. Since p> .05, parametric tests were used. The results of the analyses of the students' pre-test and post-test scores are given in Table 1.

Table 1 Comparison of students' pre-test and post-test scores by t-test

Score	Groups	N	X	Ss	Sd	t	p
Achievement test	Pre-test	60	4.18	1.51	59	2.96	.004
	Post-test	60	4.71	1.23			

According to Table 1, it is seen that there is a significant difference between the students' pre-test scores obtained from the achievement test before the experimental procedure compared to the post-test scores obtained after the experimental procedure (p<.05). While the mean score of the students' answers to the achievement test before the experimental procedure was 4.18, the mean score of their answers to the achievement test after the experimental procedure was 4.71. From this point of view, it can be said that the method applied to the students had an effect on increasing their achievement.

The categories, frequencies, total scores and sample answers of the students' responses to the activity are given below (Table 2, Table 3, Table 4, Table 5, Table 5, Table 6, Table 7).

Table 2 Frequencies, total scores and examples of students' answers to question 2

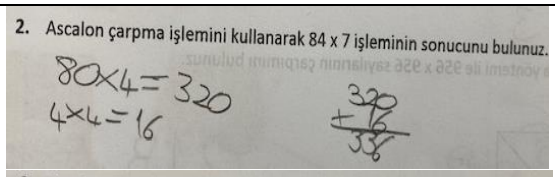
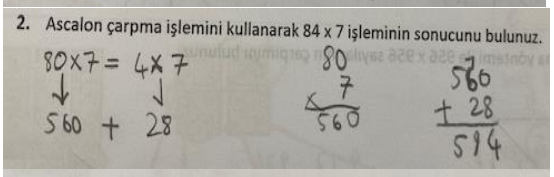
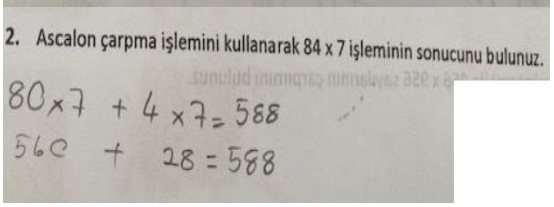
2. Find the result of 84 x 7 using Ascalon multiplication.	f	Total points	Student answers
Incorrect answer	3	0	
Who couldn't fully answer	5	9	
Correct answer	52	156	

Table 2 shows that 52 students answered the Ascalon multiplication correctly. When the student answers are analyzed, it is revealed that the students who could not answer the question correctly usually made mistakes in multiplication. Three students answered the question incorrectly.

Table 3 Frequencies, total scores and examples of students' answers to question 3

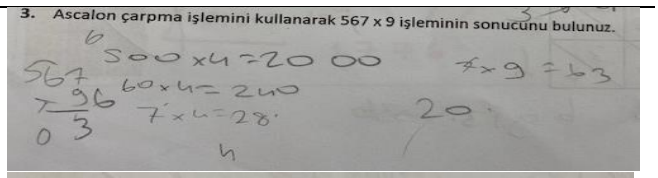
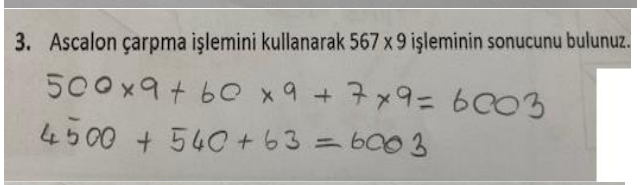
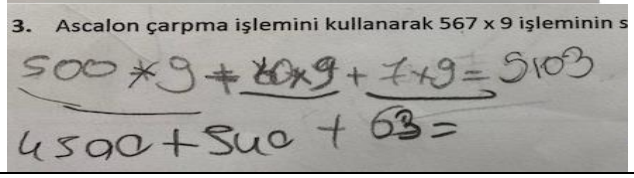
3. Find the result of 567 x 9 using Ascalon multiplication.	f	Total points	Student answers
Incorrect answer	4	0	
Who couldn't fully answer	10	25	
Correct answer	46	184	

Table 3 shows that 46 students answered the question correctly, 10 students could not find the correct answer and 4 students solved the question incorrectly. The students who could not find the correct answer made mistakes in multiplication and addition.

Table 4 Frequencies, total scores and examples of students' answers to question 4

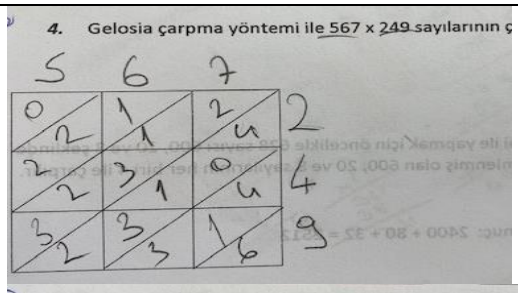
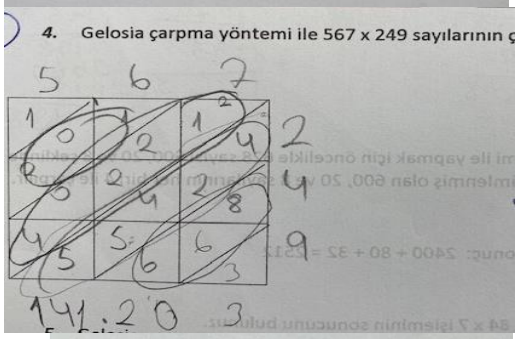
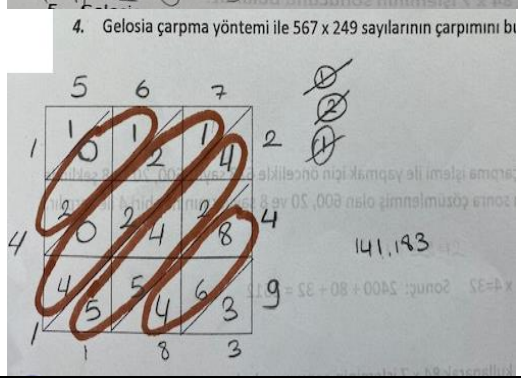
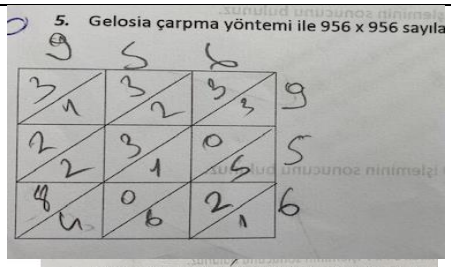
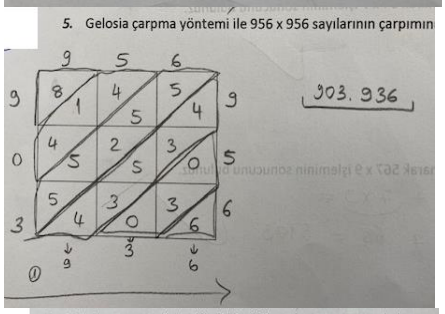
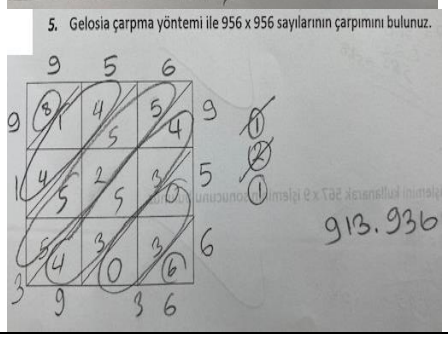
4. Find the product of 567 x 249 using Gelosia multiplication method.	f	Total points	Student answers
Incorrect answer	2	0	
Who couldn't fully answer	14	117	
Correct answer	44	440	

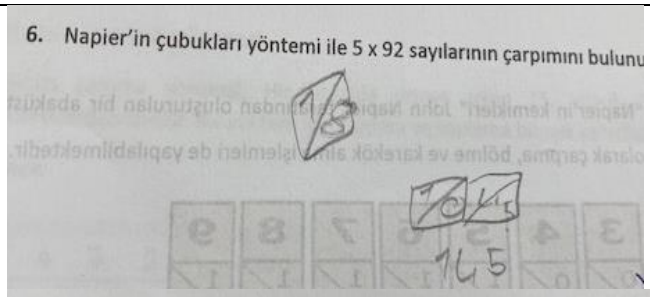
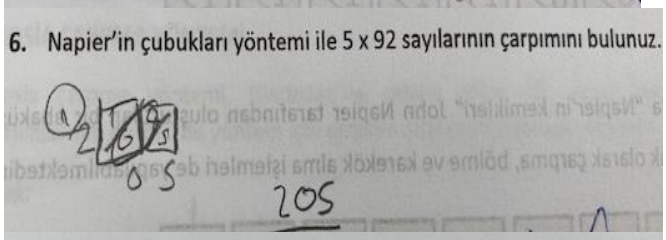
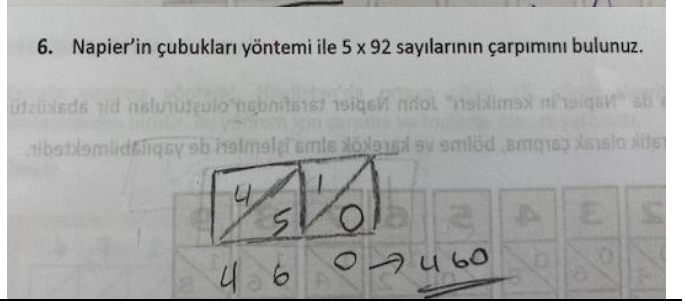
Table 4 shows that the number of students who answered the Gelosia multiplication method incorrectly and left it blank was 2. It can be inferred from this that the majority of the students understood the method. There were 44 students who answered correctly and 14 students who could not reach the correct result due to multiplication and addition errors.

Table 5 Frequencies, total scores and examples of students' answers to question 5

5. Find the product of 956 x 956 using the Gelosia multiplication method.	f	Total points	Student answers
Incorrect answer	5	0	
Who couldn't fully answer	15	107	
Correct answer	40	400	

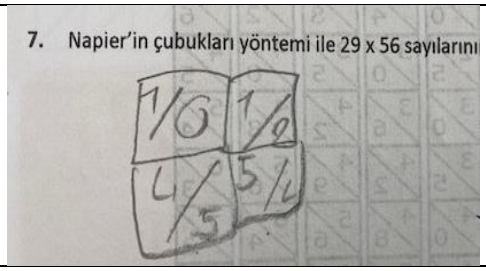
When the students' answers to question 5 are analyzed, it is seen that 40 students answered the question correctly, 15 students could not reach the correct answer due to errors arising from multiplication and addition, and five students left the question blank or answered incorrectly (Table 5).

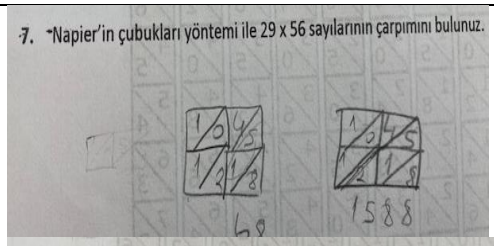
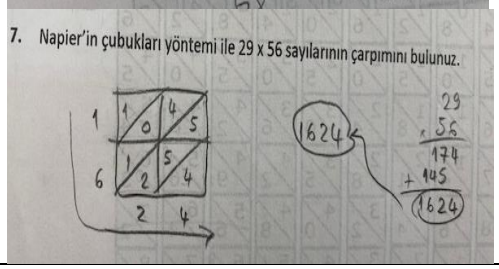
Table 6 Frequencies, total scores and examples of students' answers to question 6

6. Find the product of 5 x 92 using Napier's rods method.	f	Total points	Student answers
Incorrect answer	19	0	
Who couldn't fully answer	4	8	
Correct answer	37	111	

The frequency, total points and sample answers of the students' answers to the multiplication with Napier's rods method are given in Table 6. Nineteen students answered the question blankly or incorrectly because they did not understand the method; 37 students found the correct answer and 4 students could not reach the correct answer.

Table 7 Frequencies, total scores and examples of students' answers to question 7

7. Find the product of 29 x 56 using Napier's rods method.	f	Total points	Student answers
Incorrect answer	19	0	

Who couldn't fully answer	3	8	
Correct answer	38	190	

The students' answers to question 7 were similar to their answers to question 6 (Table 7). It is revealed that the number of students who answered correctly was 38, the number of students who could not find the exact answer was 3, and the number of students who answered incorrectly was 19. It is believed that the high number of errors in this method is due to the fact that the students could not find the boxes corresponding to the number values from the abacus.

The percentage and frequency values of the students' answers to the first question in the interview form are given in Table 8.

Table 8 Percentage and frequency values of the students' answers to the first question in the interview form

Have you ever participated in an activity related to the history of mathematics during your mathematics class?	f	%
Yes	2	13
No	13	87

Thirteen out of 15 students stated that they had not done any activities related to the history of mathematics before. Two of them stated that they had participated in activities related to the history of mathematics, but these activities took place outside the school. They stated that the activities were similar to the multiplication operations in the study and that they learned the old multiplication tables.

In the second question, students were asked whether the activity was interesting or not, what they thought about doing such activities in lessons and what different things could be done. Student responses are given in Table 9.

Table 9 Percentage and frequency values of students' answers to the second question in the interview form

Did you find the activity engaging? What are your opinions about including such activities in your lessons? What alternative activities could be considered?	f	%
Yes	14	93
No	1	7

The percentage of study participants who found the activity interesting was 93%. Some of the student responses about what they think about doing these activities are given below.

Ö1: "Yes, I think it is interesting. I think if activities like these are done in lessons, it will instill in us that mathematics is a fun lesson and help us learn new things."

Ö5: "Yes, it is good to see new methods. It is quite fun. Such activities help us to distract ourselves and help our lessons."

Ö7: "It was interesting, it is good to do such activities. I think such mathematics operations can be taught in mathematics lessons with easy and fun methods."

Ö12: "No, it did not interest me. It should be more illustrated and fun."

In the third question, students were asked whether they had any difficulties in answering the questions in the activity and if they had any difficulties, they were asked to exemplify what kind of difficulties they had. The frequency and percentage values of the students' answers are given in Table 10.

Table 10 Percentage and frequency values of students' answers to the third question in the interview form

Difficulties in answering the activity	f	%
I had no difficulties.	9	60
Multiplication with large numbers	6	40

Sixty percent of the students participating in the study stated that they did not have any difficulty in the activity. Some student responses regarding the difficulties experienced in answering the activity are given below.

Ö2: "I had difficulty in multiplying large numbers while doing normal multiplication operations. The newly taught multiplication operations were easier and more fun."

Ö3: "I had difficulty doing normal multiplication operations with large numbers."

Ö9: "I did not have much difficulty, but it is easier and more logical to use the method with Napier's rods when doing multiplication than other multiplication methods."

As the fourth question, students were asked whether they would like to teach mathematics lessons with such activities. The percentage and frequency values of the answers given by the students are given in Table 11.

Table 11 Percentage and frequency values of students' answers to the fourth question in the interview form

Would you be interested in instructing mathematics lessons that involve similar activities?	f	%
Yes	11	73
No	4	27

In general, 73% of the students who participated in the study stated that they wanted to teach mathematics lessons with such activities and that they enjoyed the activity. 4 students stated that they did not want to teach the lessons with such activities. Some of the answers of the students who want to teach mathematics lessons with such activities and those who do not want to teach mathematics lessons with such activities are given below.

Ö4: "I would like to do mathematics lessons with such activities because it is very nice and fun."

Ö10: "Yes, such activities are very fun. Thus, our interest in the lesson increases even more."

Ö11: "Yes, because we learn in a more memorable way."

Ö13: "Yes, because I had fun and learned at the same time."

Ö15: "No, it is very challenging and difficult to learn."

In the final question, students were asked to associate the subject learned in the activity with daily life. The answers given by the students are given in Table 12.

Table 12 Percentage and frequency values of students' answers to the fifth question in the interview form

Can you establish connections between the topic you learned in the activity and its applicability in everyday life?	f	%
I can make associations	4	27
I cannot make associations	11	73

The students participating in the study were generally (73%) unable to associate the subject learned in the activity with daily life. Some of the answers of the students who were able to make associations with daily life are given below.

Ö6: "I can use some of the methods I learned to make grocery shopping easier in daily life."

Ö8: "I had mathematical knowledge about multiplication operations done in the past, I got to know the past."

Ö14: "I can use these different calculation methods when doing difficult multiplication operations in daily life."

Conclusions and Suggestions

In this study, students were taught different methods in mathematics teaching with examples of activities from the history of mathematics. These methods were found to be effective in student achievement. There are many studies in literature that conclude that using different methods and techniques in mathematics teaching is effective in increasing students' academic achievement (Dereli, 2008; Erdağ, 2011; Çelik, 2013; Altıntaş & Sidekli, 2017).

In the study, activities were carried out with different methods supported by the history of mathematics and students were allowed to see the changing and developing aspect of mathematics. Considering that the history of mathematics has an important place in mathematics teaching, the activities enabled students to see the development of multiplication operations from ancient times to the present day. The achievement test averages of the students before the experimental procedure were found to be lower than the achievement test averages after the experimental procedure. From this point of view, it can be said that the methods taught are effective in increasing student achievement. Altıntaş and Sidekli (2017) found that the use of Napier rods in teaching multiplication positively affected students' academic achievement, which supports the results of the study. Yuriana and Suwardi (2019) examined the development of students' achievement in multiplication in mathematics lessons using Napier's rods teaching method and concluded that the teaching method increased students' achievement in mathematics. This result is in parallel with the findings of this study. Similarly, İdilkut (2007) concluded that conducting activities from the history of mathematics in lessons positively affected students' academic performance.

It was observed that the students generally answered the questions of Ascalon multiplication method and Gelosia multiplication method correctly, but the answers that they could not answer correctly were due to multiplication and addition errors. In the Napier's rods method, students had difficulty in finding the corresponding boxes on the abacus and therefore their incorrect answers were higher than their incorrect answers in other methods.

The answers to the interview questions revealed that the students had not encountered such activities in their mathematics lessons before. When the studies in the literature are examined, it is supported by this result that teachers continue the activities they have seen from their own teachers and do not include different activities (Furinghetti, 2000). In addition, Ceylan (2021) examined the use of the history of mathematics in secondary school mathematics textbooks and concluded that the history of mathematics is not sufficiently utilized in secondary school mathematics textbooks. It is important to include such activities in order to ensure students' active participation in the lesson and to help them have a positive attitude towards mathematics.

When asked whether they found the activity questions interesting or not, 93% of the students stated that they found them interesting, that they learned new things with such activities, that teaching mathematics with easy and fun methods increased their motivation and that they approached mathematics lessons more positively. Liu (2003) stated that students' developing positive attitudes towards mathematics is one of the purposes of including the history of mathematics in mathematics lessons. Similarly, Panasuk and Horton (2013) found that students had fun while engaging in such activities, which is in line with the result of the study.

In the opinions of the students participating in the study regarding the difficulties in answering the activity, it was determined that they had difficulty in multiplication with large numbers. Students stated that the methods taught were more fun and that they made calculations more easily than long multiplication operations.

Seventy-three percent (73%) of the students stated that they would like to teach mathematics lessons with such activities. Students stated that they experienced more memorable learning with such activities, that they learned by having fun and that their interest in mathematics lessons increased even more in this way. Özdemir and Yıldız (2015) examined student views on the use of the Babylonian counting system activity example in mathematics lessons and concluded that most of the students wanted to do an activity from the history of mathematics in their mathematics lessons. This result coincides with the study.

In his study with pre-service mathematics teachers, Clark (2012) stated that a relationship can be established between algebraic representation and geometric representation through the history of mathematics. Similarly, Baki and Bütüner (2013) made suggestions about the ways of using the history of mathematics in mathematics courses. These studies can be increased and such mathematics history activities that can attract students' interest can be included more in teaching environments.

Seventy-three percent (73%) of the students who participated in the study could not associate the subject learned in the activity with daily life. The students who were able to make an association were evaluated with the dimension of making calculations. This question was included because associating mathematics topics with daily life is considered very important. Because students who can make associations with daily life learn mathematical concepts in a more meaningful way and learn to perceive abstract mathematics as real by making it concrete.

In general, according to the results of the study, it is seen that Ascalon multiplication, Gelosia multiplication and Napier's rods methods are effective in increasing students' achievement and the use of such activities in teaching environments contributes to the development of positive attitudes towards mathematics courses. When the literature was examined, it was discovered that there were few studies in which the history of mathematics was used in lessons. Such studies can be increased, and in-service training can be given to pre-service teachers on how to use activities in lessons. Studies can be conducted with different study groups using different methods.

Compliance with Ethical Standards

Disclosure of potential conflicts of interest

The authors declare no conflict of interest. The research is a study involving human participants. Therefore, data were collected after the participants approved the written informed consent form.

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Research involving Human Participants and/or Animals

Prior to commencing the research, permission was obtained from the ethical committee of Biruni University under the number 2023/82-12, and it has been confirmed that there will be no ethical issues.

Matematik Öğretiminde Matematik Tarihi Kullanımına Örnekler: Ascalon Çarpma Yöntemi, Gelosia Çarpma Yöntemi ve Napier'in Çubukları Yöntemi

Özet:

Bu çalışmanın amacı, matematik tarihinde yer alan Ascalon çarpma yöntemini, Gelosia çarpma yöntemini ve Napier'in çubukları yöntemini etkinlikler yolu ile altıncı sınıf öğrencilerine sunmak ve bu yöntemlerin öğrenci başarısına etkisini belirleyerek bu etkinlik hakkında öğrenci görüşlerini almaktır. Araştırmanın çalışma grubunu 60 altıncı sınıf öğrencisi oluşturmuştur. Veri toplama aracı olarak çalışma yaprağı ve etkinlik bitiminde öğrencilerin etkinlik hakkındaki görüşlerini almak için görüşme formu kullanılmıştır. Öğrencilerin etkinlik kağıdına ve görüşme formuna verdiği cevaplar doğrultusunda elde edilen veriler nitel ve nicel analiz yöntemi ile analiz edilmiştir. Araştırma verilerinden elde edilen nicel analiz sonuçlarına göre kullanılan yöntemlerin öğrenci başarısını arttırmada etkili olduğu belirlenmiştir. Nitel analiz sonuçlarına göre ise genel olarak öğrencilerin bu tür etkinlikler ile fazla karşılaşmadığı, bu tür etkinlikler ile eğlenerek dersi öğrendikleri ve matematik dersine karşı ilgilerinin arttığı sonucuna ulaşılmıştır.

Anahtar kelimeler: Matematik tarihi, Ascalon çarpma yöntemi, Gelosia çarpma yöntemi, Napier'in çubukları yöntemi

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Appendix A

Pre Test
Do the following multiplication operations
96 x 7 = ?
38 x 19 = ?
76 x 58 = ?
213 x 4 = ?
615 x 22 = ?
459 x 532 = ?

Appendix B

Post Test
Do the following multiplication operations
67 x 8 = ?
83 x 14 = ?
72 x 56 = ?
348 x 6 = ?
569 x 37 = ?
437 x 854 = ?