Int. J. Pure Appl. Sci. 9(2);333-343 (2023)

ijpas@munzur.edu.tr
ISSN: 2149-0910

Research article/Araştırma makalesi
DOI: 10.29132/ijpas.1369826

Scattering State Solutions of Vector Bosons Interacting with Sun Potential

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Received date: 02.10.2023, Accepted date: 14.11.2023

Abstract

For vector bosons with spin-1, scattering state solutions have been attained by considering the Duffin-Kemmer-Petiau equation with the Sun interaction field. Based on the obtained solution, relations for phase shift and scattering amplitude have been derived. Furthermore, the bound state energy eigenvalue relation has been derived by taking the scattering amplitude to infinity. The results obtained through the Mathematica software program are presented graphically and numerically. In addition, the effects of the variables in the interaction function on the obtained results are discussed.

Keywords: Scattering states, bound states, Duffin-Kemmer-Petiau equation, sun potential

Sun Potansiyel ile Etkileşen Vektör Bozonlarının Saçılma Durumu Çözümleri

Öz

Spini-1 olan vektör bozonlar için Sun potansiyeli varlığında Duffin-Kemmer-Petiau denklemi ele alınarak saçılma durumu çözümleri elde edilmiştir. Elde edilen çözümler kullanılarak faz kayması ve saçılma genliği için bağıntılar türetilmiştir. Ayrıca saçılma genliğini sonsuza götürerek bağlı durum enerji özdeğerleri denklemi elde edilmiştir. Mathematica yazılım programı aracılığıyla elde edilen sonuçlar grafiksel ve nümerik olarak verilmiştir. Bunlara ek olarak etkileşme fonksiyonunda yer alan değişkenlerin elde edilen sonuçlara olan etkileri tartışılmıştır.

Anahtar Kelimeler: Saçılma durumları, bağlı durumlar, Duffin-Kemmer-Petiau denklemi, sun potansiyeli

INTRODUCTION

The Duffin-Kemmer-Pettiau (DKP) equation defines scalar (spin 0) and vector (spin 1) bosons on the same basis and is a first-order relativistic equation (Kemmer, 1939; Duffin, 1938; Petiau, 1936). This equation is of great importance of these various applications in nuclear physics, particle physics, quantum chromo dynamics (QCD) and cosmology. For instance, in QCD theory it can be used to investigated deuteron-nucleus elastic scattering (Kozack, Clark, Hama, Mishra, Mercer and Ray, 1989; Gribov, 1999). This equation is a Dirac-type equation (by replacing the algebra of the gamma matrices with beta matrices) and can be expressed by different matrices that follow different commutative rules and represented by five and ten component representations that work for spin-0 and spin-1 bosons, respectively. Under a vector potential, the scalar boson representation of the DKP equation has the same mathematical structure as the Klein–Gordon (KG) equation, and the vector boson representation has the same mathematical structure as the Proca equation. However, the DKP equation is more comprehensive than the KG and Proca equations due to its more complex structure (Nedjadi and Barrett, 1993; Nedjadi and Barrett, 1994; Nedjadi and Barrett, 1994)

Scattering and bound state solutions to the wave equation are of great importance in quantum mechanics because the wave functions obtained from these solutions contain all the information needed to describe the entire quantum system. Therefore, there are many studies using different methods on physical potentials related to the relativistic and the non-relativistic particle equations (Taş and Havare, 2017; Taş, Aydoğdu, and Salti, 2017; Taş and Havare, 2018; Taş, Aydoğdu and Saltı, 2018; Yanar, Taş, Saltı and Aydoğdu, 2020; Edet, Amadi, Okorie, Taş, Ikot and Rampho, 2020; Okorie, Taş, Ikot, Osobonye and Rampho, 2021). In recent years, many studies have been conducted to

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ISSN: 2149-0910

Research article/Araştırma makalesi
DOI: 10.29132/ijpas.1369826

consider different interaction types for various representation of the DKP equation (Taş, 2021; Hassanabadi, Forouhandeh, Rahimov, Zarrinkamar and Yazarloo, 2012; Hamzavi and Ikhdair, 2013; Zarrinkamar, Rajabi, Yazarloo and Hassanabadi, 2013; Bahar, 2013; Bahar, and Yasuk, 2013; Bahar and Yasuk, 2014; Onate, Ojonubah, Adeoti, Eweh and Ugboja, 2014; Ikot, Molaee, Maghsoodi, Zarrinkamar, Obong and Hassanabadi, 2015; Zarrinkamar, Panahi and Rezaei, 2016; Oluwadare and Oyewumi, 2017; Oluwadare and Oyewumi, 2018). However, when the existing literature is examined, it is seen that most of the research is on the spin-0 representation of the DKP equation. This is mainly due to the mathematical resemblance of the DKP equation with the KG equation under a vector potential. Since the form of this equation for vector bosons has a more complex structure and is much more difficult to solve. For this reason, it has not been discussed extensively (Hassanabadi, Yazarloo, Zarrinkamar and Rajabi, 2011). The first goal of this study is to obtain the scattering state solutions of vector bosons interacting with the Sun potential field, which have been previously described in the literature and are successful in describing many diatomic structures, and to obtain the phase shift equation, scattering amplitude and bound state energy eigenvalues, respectively, through these solutions. Its other goal is to investigate the effect of the parameters in the interaction field on the physical quantities obtained.

This paper is planned as follows: first, the properties of the DKP equation will be given in the material method section. In the result and discussion section, scattering state solutions of the DKP equation in the (1+3) dimension will be obtained for vector bosons in the presence of the Sun potential. Phase shift and scattering amplitude relations will be derived by using scattering state solutions in the same section. Additionally, the singular points of the scattering amplitude will be discussed and through this feature, the bound state energy eigenvalues relation will be attained. Finally, in the conclusion section, phase shift and bound state energy eigenvalues for different quantum states will be calculated numerically and expressed in tables using the Mathematica software program. Besides, the influence of the variables in the interaction field on the physical quantities obtained will be presented graphically.

MATERIAL AND METHODS

The DKP equation for a $\mathcal{U}$ interaction with m_0 field is given in the following form $(\hbar = c = 1)$ (Kemmer, 1939; Duffin, 1938; Petiau, 1936).

$$(i\beta^{\mu}\delta_{\mu} - m_0 - \mathcal{U})\Psi = 0 \tag{1}$$

β^{μ} are DKP matrices of 5×5 for spin-0 and 10×10 for spin-1. These matirces satisfy the following commutation relation:

$$\beta^{\mu}\beta^{\nu}\beta^{\lambda} + \beta^{\lambda}\beta^{\nu}\beta^{\mu} = g^{\mu\nu}\beta^{\lambda} + g^{\lambda\nu}\beta^{\mu} \tag{2}$$

β^{μ} matrices for spin-1 are given as

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^T & 0 & I & 0 \\ \bar{0}^T & I & 0 & 0 \\ \bar{0}^T & 0 & 0 & 0 \end{pmatrix},$$

$$\beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^T & 0 & 0 & -iS_i \\ -e_i^T & 0 & 0 & 0 \\ \bar{0}^T & -iS_i & 0 & 0 \end{pmatrix} \tag{3}$$

Here S_i, I, and 0 are spin-1, identity and zero matrices, respectively. The matrices $\bar{0}$ and e_i are defined as follows:

$$e_1 = (1,\quad 0,\quad 0),\quad e_2 = (0,\quad 1,\quad 0),$$
$$e_3 = (0,\quad 0,\quad 1),\ \bar{0} = (0,\quad 0,\quad 0) \tag{4}$$

The general form of the interaction potential in Eq. (1) is given as follows:

$$\mathcal{U} = S(r) + PS_{\mu}(r) + \beta^{\mu}V_{\mu}(r) + \beta^{\mu}PV_{P\mu}(r) \tag{5}$$

This expression takes the following form under rotational invariance and parity conservation for an elastic scattering

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$$\mathcal{U} = S(r) + PS_{\mu}(r) + \beta^0 V(r) + \beta^0 PV_P(r) \tag{6}$$

Each term in this equation has Lorentz character. Under rotational invariance and parity conservation, the two Lorentz vectors β^{μ} and $P\beta^{\mu}$ can be written, so that the projection operator is $P = (\beta^{\mu}\beta_{\mu} - 2) = diag(1,1,1,1,0,0,0,0,0,0)$. Thus, there are four admissible representation of interplay potential expressed as follows:

$$\mathcal{U} = PS(r) + \beta^0 PV(r) \tag{7}$$

$$\mathcal{U} = S(r) + \beta^0 V(r) \tag{8}$$

$$\mathcal{U} = PS(r) + \beta^0 V(r) \tag{9}$$

$$\mathcal{U} = S(r) + \beta^0 PV(r) \tag{10}$$

These states are concerted for the study of different physical systems. For instance, Eq. (8) is connected with the investigation of deuteron-nucleus scattering (Kozack, Clark, Hama, Mishra, Mercer, and Ray, 1989). In this study, Eq. (7) will be used as the interaction potential. (Molaee, Ghominejad, Hassanabadi and Zarrinkamar, 2012; Bahar, and Yasuk, 2014). The DKP equation is written as follows in the presence of the interaction potential defined in Eq. (7):

$$[i\beta^{\mu}\delta_{\mu} - m_0 - \beta^0 PV(r)]\Psi = 0 \tag{11}$$

Here Ψ is a ten-component spinor describing the dynamics of spin-1 particles. To get time-independent solutions, the solution of Eq. (11) is suggested as follows:

$$\Psi_{n,l}^T(x,y,z,t) = e^{(-iE_{n,l}\,t)}\psi_{n,l}(x,y,z) \tag{12}$$

For spin-1 representation, ten-component wave function is written as:

$$\psi_{n,l}^T(x,y,z) = \left(\varphi_{n,l}^{(1)}, \varphi_{n,l}^{(2)}, \varphi_{n,l}^{(3)}, \varphi_{n,l}^{(4)}, \varphi_{n,l}^{(5)}, \varphi_{n,l}^{(6)}, \varphi_{n,l}^{(7)}, \varphi_{n,l}^{(8)}, \varphi_{n,l}^{(9)}, \varphi_{n,l}^{(10)}\right)^T \tag{13}$$

Writing the wave functions expressed in Equation (14) as follows:

$$\varphi_{n,l}^{(1)} = i\phi_{n,l}\,, \qquad \vec{F} = \left(\varphi_{n,l}^{(2)}, \varphi_{n,l}^{(3)}, \varphi_{n,l}^{(4)}\right),$$

$$\vec{G} = \left(\varphi_{n,l}^{(5)}, \varphi_{n,l}^{(6)}, \varphi_{n,l}^{(7)}\right), \;\; \vec{H}\left(\varphi_{n,l}^{(8)}, \varphi_{n,l}^{(9)}, \varphi_{n,l}^{(10)}\right), \tag{14}$$

and by substituting these functions in Eq. (11) and performing the necessary intermediate operations, the following ten coupled equations are obtained.

$$i\left(\frac{\partial F_{n,l}^{(2)}}{\partial x} - \frac{\partial F_{n,l}^{(1)}}{\partial y}\right) = m_0 H_{n,l}^{(3)}, \tag{15}$$

$$i\left(\frac{\partial F_{n,l}^{(1)}}{\partial z} - \frac{\partial F_{n,l}^{(3)}}{\partial x}\right) = m_0 H_{n,l}^{(2)}, \tag{16}$$

$$i\left(\frac{\partial F_{n,l}^{(3)}}{\partial y} - \frac{\partial F_{n,l}^{(2)}}{\partial z}\right) = m_0 H_{n,l}^{(1)}, \tag{17}$$

$$\left(\frac{\partial G_{n,l}^{(3)}}{\partial z} + \frac{\partial G_{n,l}^{(2)}}{\partial y} + \frac{\partial G_{n,l}^{(1)}}{\partial x}\right) = m_0 \phi_{n,l}, \tag{18}$$

$$E_{n,l}G_{n,l}^{(1)} + i\left(\frac{\partial H_{n,l}^{(3)}}{\partial z} - \frac{\partial H_{n,l}^{(2)}}{\partial y}\right) = m_0 F_{n,l}^{(1)}, \tag{19}$$

$$E_{n,l}G_{n,l}^{(2)} + i\left(\frac{\partial H_{n,l}^{(1)}}{\partial z} - \frac{\partial H_{n,l}^{(3)}}{\partial x}\right) = m_0 F_{n,l}^{(2)}, \tag{20}$$

$$E_{n,l}G_{n,l}^{(3)} + i\left(\frac{\partial H_{n,l}^{(2)}}{\partial x} - \frac{\partial H_{n,l}^{(1)}}{\partial y}\right) = m_0 F_{n,l}^{(3)}, \tag{21}$$

$$[E_{n,l} - V(r)]F_{n,l}^{(1)} + \frac{\partial \varphi}{\partial x} = m_0 G_{n,l}^{(1)}, \tag{22}$$

$$[E_{n,l} - V(r)]F_{n,l}^{(2)} + \frac{\partial \varphi}{\partial y} = m_0 G_{n,l}^{(2)}, \tag{23}$$

$$[E_{n,l} - V(r)]F_{n,l}^{(3)} + \frac{\partial \varphi}{\partial z} = m_0 G_{n,l}^{(3)}. \tag{24}$$

Combining the above ten equations, we get

$$i\vec{\nabla} \times \vec{F} = m_0 \vec{H} \tag{25}$$

$$\vec{\nabla}.\vec{G} = m_0 \phi_{n,l}, \tag{26}$$

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ISSN: 2149-0910

Research article/Araştırma makalesi
DOI: 10.29132/ijpas.1369826

$$E_{n,l}\vec{G}+i\vec{\nabla}\times\vec{H}=m_0\vec{F}, \quad (27)$$

$$\left[E_{n,l}-V(r)\right]\vec{F}+\vec{\nabla}\phi_{n,l}=m_0\vec{G}, \quad (28)$$

and thus, using the above equations, we arrive at the following expression

$$\{E_{n,l}[E_{n,l}-V(r)]-m_0^2\}\vec{F}$$
$$-\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right)+\vec{\nabla}\left(\vec{\nabla}.\vec{F}\right)=0 \quad (29)$$

If the following identity for the term $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right)$ is used in this expression as

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right)=\vec{\nabla}\left(\vec{\nabla}.\vec{F}\right)-\nabla^2\vec{F}=$$
$$\vec{\nabla}\left(\vec{\nabla}.\vec{F}\right)-\left(\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}-\frac{L^2}{r^2}\right)\vec{F} \quad (30)$$

we get the following differential equation

$$\left\{\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}+E_{n,l}^2-E_{n,l}V(r)-m_0^2-\frac{l(l+1)}{r^2}\right\}\vec{F}=0 \quad (31)$$

This equation formally has the same structure as the Proca equation (Castro and De Castro, 2014). In order to remove the first derivative in this expression, the wave functions recommended as

$$\vec{F}=r^{-1}\vec{\mathrm{R}}$$

$$\begin{pmatrix}\varphi_{n,l}^{(2)}\\ \varphi_{n,l}^{(3)}\\ \varphi_{n,l}^{(4)}\end{pmatrix}=r^{-1}\begin{pmatrix}\mathrm{R}_{n,l}^{(2)}\\ \mathrm{R}_{n,l}^{(3)}\\ \mathrm{R}_{n,l}^{(4)}\end{pmatrix}$$

$$\left(\mathrm{R}_{n,l}^{(2)}=\mathrm{R}_{n,l}^{(3)}=\mathrm{R}_{n,l}^{(4)}=\mathrm{R}_{n,l}\right) \quad (32)$$

and if Eq. (31) is rearranged, we gain

$$\left\{\frac{d^2}{dr^2}+E_{n,l}^2-E_{n,l}V(r)-m_0^2-\frac{l(l+1)}{r^2}\right\}\mathrm{R}_{n,l}(r)=0 \quad (33)$$

RESULTS AND DISCUSSION

The Sun interaction field describing diatomic molecules is defined as follows (Sun, 1999):

$$V_{Sun}(r)=\frac{D_e e^{-2\left(\frac{\alpha}{r_e}\right)r}(e^{\alpha}-\lambda)^2}{\left(1-\lambda e^{-\left(\frac{\alpha}{r_e}\right)r}\right)^2}-\frac{2D_e e^{-\left(\frac{\alpha}{r_e}\right)r}(e^{\alpha}-\lambda)}{1-\lambda e^{-\left(\frac{\alpha}{r_e}\right)r}} \quad (34)$$

where D_e indicates the dissociation energy, r_e indicates the equilibrium bond length, α and λ are two dimensionless variables. This function can be reduced to the Tietz function with appropriate selection of parameters (Jia, Wang, He and Sun, 2000; Liang, Tang and Jia, 2013). It will be sufficient to use Eq. (33) to find the scattering state solutions of spin-1 vector bosons interacting with this field function.

By substituting Eq. (34) into Eq. (33), the following expression is obtained

$$\left\{\frac{d^2}{dr^2}+E_{n,l}^2-E_{n,l}\left(\frac{D_e e^{-2\left(\frac{\alpha}{r_e}\right)r}(e^{\alpha}-\lambda)^2}{\left(1-\lambda e^{-\left(\frac{\alpha}{r_e}\right)r}\right)^2}-\frac{2D_e e^{-\left(\frac{\alpha}{r_e}\right)r}(e^{\alpha}-\lambda)}{1-\lambda e^{-\left(\frac{\alpha}{r_e}\right)r}}\right)-m_0^2-\frac{l(l+1)}{r^2}\right\}\mathrm{R}_{n,l}=0 \quad (34)$$

For the term $1/r^2$ in this expression, a Pekeris-Type approach is applied as (Pekeris, 1934)

$$\frac{1}{r^2}\approx\frac{1}{r_e^2}\left(\mathfrak{D}_0+\frac{\mathfrak{D}_1 e^{-\left(\frac{\alpha}{r_e}\right)r}}{1-\lambda e^{-\left(\frac{\alpha}{r_e}\right)r}}+\frac{\mathfrak{D}_2 e^{-2\left(\frac{\alpha}{r_e}\right)r}}{\left(1-\lambda e^{-\left(\frac{\alpha}{r_e}\right)r}\right)^2}\right) \quad (36)$$

If this equation is expanded to the Taylor series, the $\mathfrak{D}_0$, $\mathfrak{D}_1$ and $\mathfrak{D}_2$ coefficients in the expression are calculated as follows:

$$\mathfrak{D}_0=1+\frac{1}{\alpha^2}(3-3\alpha-6\lambda e^{-\alpha}+3\lambda^2e^{-2\alpha}+2\lambda\alpha e^{-\alpha}+\lambda^2\alpha e^{-2\alpha})$$

$$\mathfrak{D}_1=\frac{2}{\alpha^2}(9\lambda-3\lambda\alpha-3e^{-\alpha}+2\alpha e^{-\alpha}-9\lambda^2e^{-\alpha}+3\lambda^3e^{-2\alpha}+\lambda^3\alpha e^{-2\alpha})$$

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ISSN: 2149-0910

Research article/Araştırma makalesi
DOI: 10.29132/ijpas.1369826

$$\mathfrak{D}_2 = \frac{1}{\alpha^2}(18\lambda^2 - 12\lambda e^{\alpha} + 3e^{2\alpha} + 2\lambda\alpha e^{\alpha} - \alpha e^{2\alpha} - 12\lambda^3 e^{-\alpha} + 3\lambda^4 e^{-2\alpha} - 2\lambda^3\alpha e^{-\alpha} + \lambda^4\alpha e^{-2\alpha}) \quad (37)$$

The suitability of this approach can be seen in Figure 2. By substituting Eq. (36) into Eq. (35) and defining a new variable in the form $\eta = 1 - \left(1 - \lambda\, e^{-\left(\frac{\alpha}{r_e}\right)r}\right)^{-1}$, the following expression is obtained

$$\eta(1-\eta)\frac{d^2R_{nl}(\eta)}{d\eta^2} + (1-2\eta)\frac{dR_{nl}(\eta)}{d\eta} + \frac{1}{\eta(1-\eta)}[-\gamma_1 + \gamma_2\eta - \gamma_3\eta^2]R_{nl}(\eta) = 0 \quad (38)$$

where

$$\gamma_1 = \frac{(m_0^2 - E_{n,l}^2)r_e^2}{\alpha^2} + \frac{[(e^{\alpha}-\lambda)^2 + 2(e^{\alpha}-\lambda)D_e\lambda]E_{n,l}r_e^2}{\alpha^2\lambda^2} + \frac{l(l+1)(\mathfrak{D}_0\lambda^2 - \mathfrak{D}_1\lambda + \mathfrak{D}_2)}{\alpha^2\lambda^2},$$

$$\gamma_2 = \frac{2[(e^{\alpha}-\lambda)^2 + (e^{\alpha}-\lambda)D_e\lambda]E_{n,l}r_e^2}{\alpha^2\lambda^2} + \frac{l(l+1)(2\mathfrak{D}_2 - \mathfrak{D}_1\lambda)}{\alpha^2\lambda^2},$$

$$\gamma_3 = \frac{(e^{\alpha}-\lambda)^2 D_e E_{n,l} r_e^2}{\alpha^2\lambda^2} + \frac{l(l+1)\mathfrak{D}_2}{\alpha^2\lambda^2}, \quad (39)$$

In order to remove the singularity at the points $\eta = 0$ and $\eta = 1$ in Eq. (39), the wave function is proposed again as $R_{nl}(\eta) = \eta^{\beta}(1-\eta)^{-\frac{ikr_e}{\alpha}}\mathcal{F}_{nl}(\eta)$ and when the necessary operations are carried out, we reach

$$\eta(1-\eta)\frac{d^2\mathcal{F}_{nl}}{d\eta^2} + [\varrho_3 - (\varrho_1 + \varrho_2 + 1)]\frac{d\mathcal{F}_{nl}}{d\eta} - \varrho_1.\varrho_2\mathcal{F}_{nl} = 0 \quad (40)$$

Where

$$\beta = \sqrt{\gamma_1}, \qquad k = \frac{\alpha}{r_e}\sqrt{\gamma_2 - \gamma_3 - \gamma_1},$$

$$\sigma = \pm\frac{1}{2}\sqrt{1+4\gamma_3}, \quad \varrho_1 = \beta - \frac{ikr_e}{\alpha} + \sigma + \frac{1}{2},$$

$$\varrho_2 = \beta - \frac{ikr_e}{\alpha} - \sigma + \frac{1}{2}, \quad \varrho_3 = 1 + 2\beta \quad (41)$$

Using the property of the Gauss Hypergeometric equation (Flügge, 1999), the solution for Eq. (40) becomes:

$$\mathcal{F}_{nl}(\eta) = A_1\, {}_2F_1(\varrho_1, \varrho_2, \varrho_3, \eta) + A_2\eta^{1-\varrho_3}\, {}_2F_1(\varrho_1 - \varrho_3 + 1, \varrho_2 - \varrho_3 + 1, 2 - \varrho_3, \eta). \quad (42)$$

Thus, we get:

$$R_{nl}(\eta) = A_1\eta^{\beta}(1-\eta)^{-\frac{ikr_e}{\alpha}} \times {}_2F_1(\varrho_1, \varrho_2, \varrho_3, \eta) + A_2\eta^{-\beta}(1-\eta)^{-\frac{ikr_e}{\alpha}} \times {}_2F_1(\varrho_1 - \varrho_3 + 1, \varrho_2 - \varrho_3 + 1, 2 - \varrho_3, \eta) \quad (43)$$

To gain a regular solution of the Eq. (43) and derive the relations that give the necessary physical quantities describing the scattering states, we have to consider the behavior of wave functions at their boundary points (Flügge, 1999):

i) If $r \to 0$, the $R_{nl}(\eta)$ must be get finite value. With the applying first condition, we conclude that the normalization coefficient $A_2 = 0$, so the wave functions is found as:

$$R_{nl}(\eta) = A_1\eta^{\beta}(1-\eta)^{-\frac{ikr_e}{\alpha}}\, {}_2F_1(\varrho_1, \varrho_2, \varrho_3, \eta) \quad 44)$$

ii) If $r \to \infty$, the attitude of the $R_{nl}(\eta)$ at infinity smust be as follows (Landau and Lifshitz, 1977)

$$R_{nl}(\infty) \to 2\sin(kr - \frac{1}{2}l\pi + \phi) \quad (45)$$

where ϕ is the phase shift. Under this condition, Eq. (44) takes the following form

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ISSN: 2149-0910

Research article/Araştırma makalesi
DOI: 10.29132/ijpas.1369826

$$R_{nl}(r\to\infty)\to$$

$$A_1\left(-\frac{1}{\lambda}\right)^{-\frac{ikr_e}{\alpha}} e^{ikr}\ {}_2F_1(\varrho_1,\varrho_2,\varrho_3,1-\frac{1}{1-\lambda\, e^{-\left(\frac{\alpha}{r_e}\right)r}}) \tag{46}$$

We can write the recurrence relation given for the Hypergeometric function in this equation (Flügge, 1999) and taking ${}_2F_1(\varrho_1,\varrho_2,\varrho_3,0)=1$ for $r\to\infty$, we get

$${}_2F_1(\varrho_1,\varrho_2,\xi\varrho_3,1)\xrightarrow{r\to\infty}$$

$$\Gamma(\varrho_3)\left\{\begin{array}{c}\frac{\Gamma(\varrho_3-\varrho_2-\varrho_1)}{\Gamma(\varrho_3-\varrho_1)\Gamma(\varrho_3-\varrho_2)}\\ +\left(-\frac{1}{\lambda}\right)^{\frac{2ikr_e}{\alpha}} e^{-2ikr}\frac{\Gamma(\varrho_1+\varrho_2-\varrho_3)}{\Gamma(\varrho_1)\Gamma(\varrho_2)}\end{array}\right\} \tag{47}$$

When the following relations are used

$$\varrho_1+\varrho_2-\varrho_3=-\frac{2ikr_e}{\alpha}=(\varrho_3-\varrho_2-\varrho_1)^*,$$

$$\varrho_3-\varrho_1=\beta+\frac{ikr_e}{\alpha}-\sigma+\frac{1}{2}=(\varrho_2)^*,$$

$$\varrho_3-\varrho_2=\beta+\frac{ikr_e}{\alpha}+\sigma+\frac{1}{2}=(\varrho_1)^* \tag{48}$$

and required calculations are taken, we find

$${}_2F_1(\varrho_1,\varrho_2,\varrho_3,1)\xrightarrow{r\to\infty}$$

$$\Gamma(\varrho_3)\left(-\frac{1}{\lambda}\right)^{\frac{ikr_e}{\alpha}}\left\{\begin{array}{c}\frac{\Gamma(\varrho_3-\varrho_2-\varrho_1)\left(-\frac{1}{\lambda}\right)^{-\frac{ikr_e}{\alpha}}}{\Gamma(\varrho_3-\varrho_1)\Gamma(\varrho_3-\varrho_2)}\\ +e^{-2ikr}\left[\frac{\Gamma(\varrho_3-\varrho_2-\varrho_1)\left(-\frac{1}{\lambda}\right)^{-\frac{ikr_e}{\alpha}}}{\Gamma(\varrho_3-\varrho_1)\Gamma(\varrho_3-\varrho_2)}\right]^*\end{array}\right\}. \tag{49}$$

By using relations below

$$\frac{\Gamma(\varrho_3-\varrho_2-\varrho_1)}{\Gamma(\varrho_3-\varrho_1)\Gamma(\varrho_3-\varrho_2)}=\left|\frac{\Gamma(\varrho_3-\varrho_2-\varrho_1)}{\Gamma(\varrho_3-\varrho_1)\Gamma(\varrho_3-\varrho_2)}\right|e^{i\vartheta_1},$$

$$\left(-\frac{1}{\lambda}\right)^{-\frac{ikr_e}{\alpha}}=\left|\left(-\frac{1}{\lambda}\right)^{-\frac{ikr_e}{\alpha}}\right|e^{i\vartheta_2}, \tag{50}$$

and inserting these expressions in Eq.(49), we have

$${}_2F_1(\varrho_1,\varrho_2,\varrho_3,1)\xrightarrow{r\to\infty} 2\Gamma(\varrho_3)\left(-\frac{1}{\lambda}\right)^{\frac{ikr_e}{\alpha}}e^{-ikr}$$

$$\times\left|\frac{\Gamma(\varrho_3-\varrho_2-\varrho_1)\left(-\frac{1}{\lambda}\right)^{-\frac{ikr_e}{\alpha}}}{\Gamma(\varrho_3-\varrho_1)\Gamma(\varrho_3-\varrho_2)}\right| sin(kr+\vartheta_1+\vartheta_2+\frac{\pi}{2}) \tag{51}$$

The replacement of the Eq.(51) into the Eq.(46) yields

$$R_{nl}(r)\xrightarrow{r\to\infty} 2A_1\Gamma(\varrho_3)\left|\frac{\Gamma(\varrho_3-\varrho_2-\varrho_1)\left(-\frac{1}{\lambda}\right)^{-\frac{ikr_e}{\alpha}}}{\Gamma(\varrho_3-\varrho_1)\Gamma(\varrho_3-\varrho_2)}\right|$$

$$\times sin(kr+\vartheta_1+\vartheta_2+\frac{\pi}{2}) \tag{52}$$

Matching the Eq. (52) with the Eq. (45), we get:

$$A_1=\frac{1}{\Gamma(\varrho_3)}\left|\frac{\Gamma(\varrho_3-\varrho_1)\Gamma(\varrho_3-\varrho_2)\left(-\frac{1}{\lambda}\right)^{\frac{ikr_e}{\alpha}}}{\Gamma(\varrho_3-\varrho_2-\varrho_1)}\right| \tag{53}$$

$$\phi=\frac{1}{2}(l+1)\pi+arg\,\Gamma(\varrho_3-\varrho_2-\varrho_1)-arg\,\Gamma(\varrho_3-\varrho_1)-arg\,\Gamma(\varrho_3-\varrho_2)+arg\left(-\frac{1}{\lambda}\right)^{-\frac{ikr_e}{\alpha}} \tag{54}$$

If the scattering amplitude goes to infinity ($A_1\to\infty$), continuum states occur and thus we can obtain bound state energy eigenvalues. By using the following property of Gamma functions as

$$\Gamma(\chi)=\frac{\Gamma(\chi+1)}{\chi}=\frac{\Gamma(\chi+2)}{\chi(\chi+1)}$$

$$=\frac{\Gamma(\chi+3)}{\chi(\chi+1)(\chi+2)}=\ldots\ \chi=0,-1,-2,-3,\ldots \tag{55}$$

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One can easily obtain the singular pole points Eq. (53) as $\varrho_3 - \varrho_1 = n$ or $\varrho_3 - \varrho_2 = n$. Hence, the condition becomes

$$\beta - \frac{ikr_e}{\alpha} - \sigma + \frac{1}{2} = -n \tag{56}$$

By using these equation and definition given in Eq.(39) and Eq. (41), the energy eigenvalues is found as follows:

$$E_{n,l}^2 - m_0^2 - \frac{E_{n,l}(e^{\alpha}-\lambda)(e^{\alpha}-\lambda+2)D_e}{\lambda^2} - \frac{(C_0\lambda^2-C_1\lambda+C_2)l(l+1)}{\lambda^2 r_e^2} + \frac{\alpha^2}{4r_e^2}\left[1+2n+\sqrt{\frac{\alpha^2\lambda^2+4l(l+1)C_2+4E_{n,l}(e^{\alpha}-\lambda)^2D_e r_e^2}{\alpha^2\lambda^2}} -2\sqrt{\frac{l(l+1)\lambda^2C_0+\left((E_{n,l}^2-m_0^2)\lambda^2-2E_{n,l}(e^{\alpha}-\lambda)(\lambda-1)D_e\right)r_e^2}{\alpha^2\lambda^2}}\right]^2 = 0 \tag{57}$$

CONCLUSION

The DKP equation with Sun interaction field model is discussed for the first time in this study. Scattering state solutions of vector bosons were obtained. The suitability of the approach used for centrifugal term is given in Figure 1. By using the wave function, the phase shift equation and scattering amplitude relation were derived. In addition, by using scattering amplitude relation, the relation giving the bound state energy eigenvalues was directly obtained. Phase shift and energy eigenvalues were calculated numerically through the Mathematica software program.

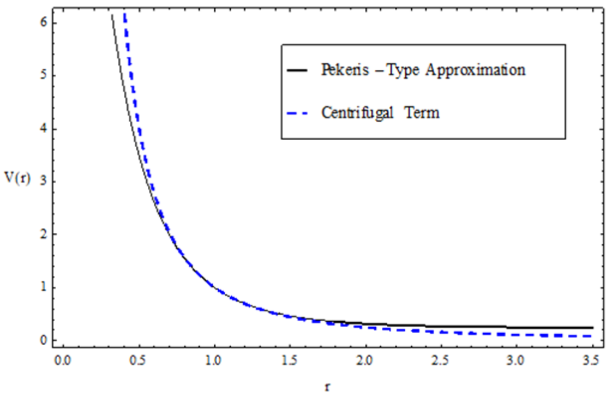

Figure 1. Graphical representation of the approximation applied to the centrifugal term for α=2, r_e=1 fm and λ=0.1.

These results are presented in Table 1. and Table 2. As seen in Table 1, if the angular momentum eigenvalues increase, phase shift values increase. Also, the same situation applies if the alpha parameter increases. It can be easily seen in Table 2 that the energy eigenvalues increase if the n and l values increase.

Table 1. *Calculated phase shift values for* $D_e = 2\,fm^{-1}$, $\alpha = 2$, $r_e = 0.8\,fm^{-1}$, $m = 1\,fm^{-1}$, $\lambda = 0.5$ $E = 3.651\,fm^{-1}$ *and* $\hbar = c = 1$

l	$\phi(\alpha = 1.6)$	$\phi(\alpha = 2)$	$\phi(\alpha = 2.4)$
1	1.03030	1.82089	8.63096
2	2.62303	3.41576	10.2273
3	4.22675	5.02269	11.8364
4	5.84148	6.64172	13.4584
5	7.46728	8.27289	15.0932
6	9.10419	16.1994	16.741
7	10.7523	17.8551	18.4017
8	12.4116	18.3186	22.3612
9	16.326	20.0368	25.8844
10	21.586	28.8247	34.9442

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DOI: 10.29132/ijpas.1369826

Table 2. *Calculated energy eigenvalues for* $D_e = 2\,fm^{-1}$, $\alpha = 2$, $r_e = 0.8\,fm^{-1}$, $m = 1\,fm^{-1}$, $\lambda = 0.5$ *and* $\hbar = c = 1$

n	l	E_{nl} (fm^{-1})	n	l	E_{nl} (fm^{-1})
1	0	0.129231	5	0	1.91933
2	0	0.377758	5	1	2.03664
2	1	0.487010	5	2	2.26547
3	0	0.754972	5	3	2.59785
3	1	0.867171	5	4	3.02697
3	2	1.079500	6	0	3.54854
4	0	1.26622	6	1	2.84562
4	1	1.38089	6	2	3.08231
4	2	1.60201	6	3	3.42908
4	3	1.91954	6	4	3.88014

The effects of the parameters in the interaction field on the energy values are shown in Figure 2 and Figure 3. If the drawings in Figure 2 are examined carefully, Energy values increase for values of the $\boldsymbol{\alpha}$ parameter in the range $\mathbf{1} < \boldsymbol{\alpha} < \mathbf{1.22}$, and decrease for values in the range $\mathbf{2.22} < \boldsymbol{\alpha}$. This situation becomes more evident as large n and l values increase. In addition, for values of the alpha parameter in the range $\boldsymbol{\alpha} < \mathbf{1}$, the energy values for all quantum states approach each other and go to zero. In the other drawings, it is seen that if the $\boldsymbol{r_e}$ values increase, the energy values decrease. While the difference between the energy values corresponding to different quantum states is large at small values of the $\boldsymbol{r_e}$ parameter, for $\boldsymbol{r_e} > \mathbf{1\,fm^{-1}}$ the difference decreases and the energy values approach zero. Looking at the drawings in Figure 3, it is very easy to say that the $\boldsymbol{D_e}$ parameter behaves like the $\boldsymbol{r_e}$ parameter, but the opposite is true for the $\boldsymbol{\lambda}$ parameter. If the lambda parameter increases, the energy values increase and the energy values for different quantum states diverge. For $\boldsymbol{\lambda} < \mathbf{0.2}$, it is seen that the energy values decrease and go to zero for each quantum state.

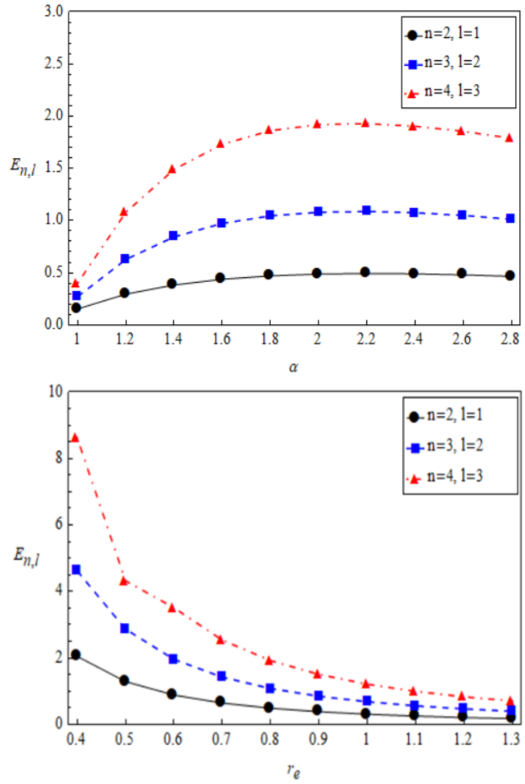

Figure 2. Variation of bound state energy eigenvalues according to α parameter for $D_e = 2\,fm^{-1}$, $r_e = 0.8\,fm^{-1}$, $m = 1\,fm^{-1}$, $\lambda = 0.5$, and $\boldsymbol{r_e}$ parameter for $D_e = 2\,fm^{-1}$, $\alpha = 2$, $m = 1\,fm^{-1}$, $\lambda = 0.5$.

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ISSN: 2149-0910

Research article/Araştırma makalesi
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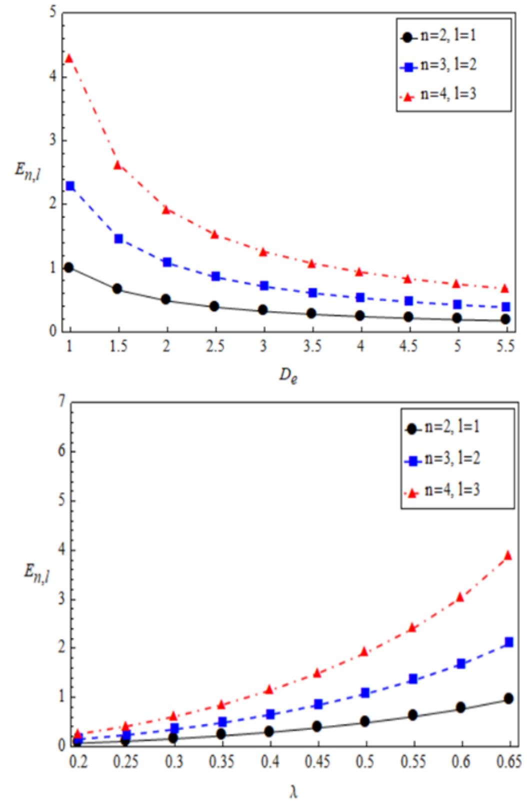

Figure 3. Variation of bound state energy eigenvalues according to D_e parameter for $\lambda = 0.5,\ \alpha = 2,\ r_e = 0.8\ fm^{-1},\ m = 1\ fm^{-1}$ and λ parameter for $D_e = 2\ fm^{-1},\ \alpha = 2,\ r_e = 0.8\ fm^{-1},\ m = 1\ fm^{-1}$.

This study will be benefical for researchers working in this field as the Sun potential has successfully described many diatomic structures and there has been no previous study for vector bosons in the presence of this model.

CONFLICT OF INTEREST

The Author report no conflict of interest relevant to this article

RESEARCH AND PUBLICATION ETHICS STATEMENT

The author declares that this study complies with research and publication ethics.

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ISSN: 2149-0910

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ISSN: 2149-0910

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DOI: 10.29132/ijpas.1369826

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