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Research Article

The Development of 7th Grade Students' Algebraic Thinking Through Task-assisted Instruction

7. Sınıf Öğrencilerinin Cebirsel Düşünmelerinin Görev Destekli Öğretim Yoluyla Geliştirilmesi

Nil Arabacı [1], Yeşim İmamoğlu[2], Hülya Kılıç[3]

[1]*Sorumlu Yazar, Doktora Öğrencisi, Boğaziçi Üniversitesi, arabacinil@gmail.com, (https://orcid.org/0000-0002-0701-5307)*
[2]*Doktor Öğretim Üyesi, Boğaziçi Üniversitesi, yesim.imamoglu@boun.edu.tr, (https://orcid.org/0000-0002-8790-3127)*
[3]*Doç. Dr., Yeditepe Üniversitesi, hulya.kilic@yeditepe.edu.tr, (https://orcid.org/0000-0003-1909-2016)*

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ABSTRACT

This study aims to investigate how the algebraic thinking skills of seventh-grade students develop with the task-assisted teaching approach. The study was conducted in a seventh-grade class at a public school in Istanbul. The tasks were designed to support the basic components of students' algebraic thinking processes - pattern recognition, writing algebraic expressions, constructing and solving equations. During the implementation, the students in the class were divided into groups of three and four, and a teacher candidate in each group was responsible for implementing the tasks. This paper focused on the pattern recognition component of algebraic thinking. Video analysis and students' responses showed that their algebraic thinking processes improved in the pattern recognition component, and furthermore, the pattern recognition component evaluation through qualitative analysis showed that there was an improvement in the students' algebraic thinking skills compared to their previous performance.

Keywords: Algebraic thinking, pattern tasks, task-assisted instruction, task design.

ÖZ

Bu çalışmanın amacı, yedinci sınıf öğrencilerinin cebirsel düşünme becerilerinin görev destekli öğretim yaklaşım ile nasıl geliştiğini araştırmaktır. Çalışma İstanbul'daki bir devlet okulunun yedinci sınıfındaki öğrenciler ile gerçekleştirilmiştir. Görevler, öğrencilerin cebirsel düşünme süreçlerinin temel bileşenlerini -örüntü tanıma, cebirsel ifadeleri yazma, denklem kurma ve çözme- desteklemek amacıyla tasarlanmıştır. Uygulama sırasında sınıftaki öğrenciler üçerli ve dörderli gruplara ayrılmış ve her grupta bir öğretmen adayı görevlerin uygulanmasından sorumlu olmuştur. Bu makalede cebirsel düşünmenin örüntü tanıma bileşenine odaklanılmıştır. Video analizi ve öğrencilerin yanıtları, onların cebirsel düşünme süreçlerinin örüntü tanıma bileşeninde gelişme gösterdiğini ve ayrıca, nitel analiz yoluyla yapılan örüntü tanıma bileşeni değerlendirmesi de, öğrencilerin cebirsel düşünme becerilerinde önceki performanslarına göre bir gelişim olduğunu göstermiştir.

Anahtar Kelimeler: Cebirsel düşünme, örüntü görevleri**,** görev destekli öğretim, görev tasarımı.

INTRODUCTION

Algebra plays a critical role in the academic achievement and future career opportunities of students (Adelman, 2006; Knuth et al., 2006). However, numerous studies have highlighted that students often face challenges and develop misunderstandings when it comes to learning algebraic concepts (e.g., Akkaya & Durmuş, 2006; Dede & Peker, 2007; Jupri et al., 2014; Lucariello et al., 2014; Welder, 2012). Consequently, researchers have worked to identify effective strategies for addressing these misconceptions and improving students' proficiency in algebraic thinking (e.g., Palabıyık & İspir, 2011; Lucariello et al., 2014).

The literature presents diverse perspectives on the investigation of algebraic thinking. Some researchers emphasize the importance of understanding relationships, while others highlight the ability to make generalizations. For instance, Smith (2003) and Kaput (1999) argue that algebraic thinking involves recognizing patterns and deriving general principles from them. Similarly, Driscoll (1999) asserts that it entails identifying and establishing rules for patterns. On a separate note, Steele (2005) underscores the connections between variables, stating that algebraic thinking revolves around comprehending variables and expressions and articulating relationships between these quantities. Additionally, Kreigler (2008) suggests that equations serve as a valuable tool for representing mathematical concepts in everyday situations. Fundamental elements of algebraic thinking encompass investigating patterns, formulating algebraic expressions, and understanding equivalence (Stephens & Ribeiro, 2012). Moreover, the organization of topics in the curriculum is a factor to consider, with patterns, algebraic expressions, and equations addressed in the 6th and 7th grades, respectively (Ministry of National Education (MoNE), 2018). Building upon the aforementioned literature, this study defines algebraic thinking through the following components: a) discerning the rule and pattern of the relationship between two variables, b) converting verbal expressions into algebraic expressions, and c) formulating and solving equations.

Researchers have put forth a range of strategies to enhance students' proficiency in algebraic thinking. These approaches encompass commencing algebra instruction at earlier grade levels (e.g., Carpenter et al., 2003), incorporating mathematical tasks (e.g., Lannin, 2005; Palabıyık & Ispir, 2011), utilizing concrete manipulatives (e.g., Saraswati & Putri, 2016), and establishing contexts that resonate with students (Walkington et al., 2013). Each of these strategies has demonstrated effectiveness in the instruction of algebra. In alignment with the literature, in this study, the tasks were thoughtfully designed to prompt students to utilize tangible materials like pattern blocks and to connect with real-world scenarios, such as building renovation projects.

Numerous scholars (e.g., Kaput, 1999; Lannin, 2005) have stressed the significance of comprehending the relationship between quantities and patterns in the realm of algebraic thinking. As outlined in the relevant literature, having an understanding of the connection between two quantities within patterns, and the ability to articulate and generalize the rules governing these relationships are crucial aspects of algebraic thinking. Given the importance of these elements, this paper specifically focuses on the initial facet of algebraic thinking: identifying the rule and pattern of a relationship between two variables. To enhance this aspect of algebraic thinking, tasks centered around patterns can be employed. For instance, Warren and Cooper (2005) underscored that pattern tasks can aid students in developing their reasoning skills in algebraic equivalence and equations. Amit and Neria (2007) conducted a study with children aged 11-13, using tasks involving growing patterns to instill algebraic concepts. Similarly, Store et al. (2010) carried out a teaching experiment with fifth-grade students, where students utilized pattern blocks to represent and discuss growing patterns. In both studies, the researchers found that pattern tasks were effective in introducing fundamental algebraic concepts, such as understanding relationships and using variables to express pattern rules. Therefore, in the present study, pattern tasks were employed to strengthen students’ proficiency in algebraic thinking.

The concept of a "task" is defined in various ways in the literature. In the current study, it is understood as a set of interconnected problems grounded in real-life contexts, to facilitate student learning. Watson and Mason (2007) underlined the difference between activity and task. They noted activity as all interaction between the students. On the contrary, the task includes the student activities, the extent to which students engage with and learn from these activities, and the teacher's guidance. Therefore, the task is related to student learning, implementation process, and interaction in the classroom which corresponds to a broader term than activity. For his study, the task is considered from the same perspective and covers all interactions among students and teachers and the implementation process of the task. Therefore, the term task is used rather than activity. Stein and Smith (1998) argued that how tasks are carried out directly impacts student learning, highlighting the substantial influence of the implementation phase on students' educational outcomes. Expanding on Stein and Smith's viewpoint, Liljedahl et al. (2007) emphasized the importance of scrutinizing the implementation process to assess the effectiveness of tasks in supporting student learning. Teachers may find it necessary to make adjustments and engage in reflective practices after task implementation. Essentially, task design can be seen as a cyclical process.

Furthermore, the tasks in this study were structured to create learning opportunities for students, aligning with the Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) framework as defined by Leatham and colleagues (Leatham et al., 2015). Leatham et al. (2015) pointed out that certain scenarios may lead to instances of MOST during instruction, including: "(a) a correct answer with novel reasoning, (b) an incorrect answer that involves a common or mathematically rich misconception, (c) a mathematical contradiction, (d) incomplete or incorrect reasoning, and (e) why or generalizing questions" (p. 100).

Different than to previous research, this study employs the Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) theoretical framework proposed by Leatham et al. (2015) to identify the cases to be analyzed arising from task-assisted instruction. Additionally, Sibgatullin et al. (2022) carried out a systematic review of earlier studies and concluded that achievement tests were generally used as assessment instruments. However, both as educators and scholars, it is crucial to delve into the process of students' algebraic thinking during problem-solving (Sibgatullin et al., 2022). Hence, this study focuses on students' algebraic thinking by analyzing their verbal expressions alongside their written work.

1.1. Theoretical framework

For task development, two theoretical frameworks were used. One of them was Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) defined by Leatham et al. (2015) The MOST framework was used in task design, task implementation, and data analysis. To be a MOST instance, a case needs to depend on students' mathematical knowledge, be mathematically significant, and be a pedagogical opportunity. The MOST framework has the potential to provide learning opportunities for the students. Tasks also include items that enable assessing students' misconceptions and difficulties in algebra. The researchers informed the pre-service teachers about the potential MOST cases during the discussions before the implementation of each task. In the next section, how the MOST framework was used in data analysis will be explained in detail.

Another framework was the Realistic Mathematics Education framework. The premises were reflected in the task design and task implementation process. As one of the premises, the reality principle was taken into consideration inclusion of daily life contexts such as the renovation of buildings and construction of rails. The interactivity and guidance principles were used in the task implementation process. The interactivity principle took place in group work of the students where students had the opportunity to interact with knowledge construction (Van den Heuevel-Panhuizen & Drijvers, 2020). Moreover, the pre-service teachers asked questions during the group discussions to elaborate on students' algebraic thinking aligned with the guidance principle.

This article presents a section of a larger research endeavor that delved into how students apply algebraic thinking within the context of task-assisted instruction. In this study, task-assisted instruction is defined as an educational approach wherein pre-service teachers employ mathematical tasks to explore students' algebraic thinking through group discussions. The broader research examined students' algebraic thinking, covering three aspects of algebraic thinking discerning the rule and pattern of the relationship between two variables, converting verbal expressions into algebraic expressions, and formulating and solving equations using eight tasks. The general research problem was to examine students' algebraic thinking during task-assisted instruction. In the aforementioned studies, numerous researchers (e.g., Kaput, 1999; Lannin, 2005) have emphasized the significance of comprehending the connection between quantities and patterns in the context of algebraic thinking. Recognizing the elevated importance of this aspect of algebraic thinking, this paper specifically focuses on discussing the qualitative findings regarding identifying the rule and pattern of the relationship between two variables, as highlighted in the pertinent literature. This study specifically delves into how students' algebraic thinking evolves through task-assisted instruction, employing a qualitative examination of algebraic thinking. Consequently, the following research question is explored:

How do students' performance in finding the rule and the terms of a pattern develop during task-assisted instruction?

METHODOLOGY

The basic qualitative research methodology is employed in this study to interpret how students' thinking developed through task-assisted instruction. Basic qualitative research is useful to analyze how learning in a classroom setting occurs (Merriam, 2009).

2.1. Participants

The researchers were involved in a TUBITAK-funded project (Project number: 215K049) led by one of the authors (Kılıç et al., 2019; Kılıç & Doğan, 2022). The researchers took the ethics committee approval of the university where the TUBITAK project was carried out. The researchers had a collaboration with a public middle school for the TUBITAK Project. The researchers also applied Istanbul District National Education Directorate for implementation allowance. The researchers have access to this school and used a convenient sampling technique for the data collection (Creswell et al., 2011). Thus, the tasks were administered in a 7th grade comprising 26 students, at the collaboration school in Istanbul. This particular school generally exhibits lower academic achievement compared to others in the surrounding area.

In the context of the TUBITAK project, a course on the application of mathematics tasks was opened. Eight senior pre-service mathematics teachers from the university also took part in the study by enrolling in the course. They voluntarily participated in the study and they were assigned responsibility for a group of 3 or 4 students during the task implementation. To facilitate a more comprehensive analysis of the evolution in students' algebraic thinking, a maximum variation sampling method was employed (Creswell et al., 2011), designating one pre-service teacher to each group. Consequently, 9 students from the class were selected for further examination. The selection of these students was based on several criteria, including representation from each group, the students' levels of achievement, and their communication skills. In terms of achievement, 3 students were chosen from each category (lower, middle, and higher) to observe how algebraic thinking developed among students at different levels of achievement.

The students selected for the qualitative analysis of algebraic thinking and learning were given the nicknames Alper, Burak, Doruk, Erdem, Harun, Gonca, Mert, Tansu, and Utku.

2.2.Tasks

A total of eight tasks were implemented to analyze students' proficiency in algebraic thinking and their overall achievement in algebra. Each week, one task was carried out. Out of these tasks, three were specifically focused on recognizing patterns: Task 1, Task 2, and Task 8. These tasks took place on the first, second, and eighth weeks of the implementation respectively. Task 2 was initially developed by Doğan and Dönmez (2016), while Task 1 and Task 8 were designed by the researchers based on the task literature and the aim of the study. For the validity of the tasks, the researchers consulted two mathematics educators' opinions on the tasks. Task 2 and Task 8 underwent a preliminary phase during the 2016-2017 academic year to evaluate their suitability for the students' level, taking into account their prior knowledge and the clarity of the questions. The preliminary phase was carried out in one of the 7th grade classes at the same school. Task 2 and Task 8 were implemented within a similar setting where pre-service teachers were responsible for one group of students and the researchers gave instructions for the implementation process. The preliminary phase was also video recorded and analyzed in terms of clarity of the task questions, and alignment with the research purpose. The analysis was also done through the transcripts of the videos and by examining students' algebraic thinking levels utilizing the same coding schema. The preliminary implementation phase also provided task validity since the questions were aligned with the aim of the research and beneficial to analyzing students' performance in finding the rule and the terms of the pattern. The tasks helped to elaborate students' algebraic thinking. After each task's implementation pre-service teachers' comments were also taken into consideration for possible refinements as a result of the preliminary phase. Based on the feedback and findings from this preliminary phase, minor adjustments were made to both Task 2 and Task 8. After the adjustments were done, the researchers again consulted two mathematics educators for the validity of the tasks. It was observed that students encountered difficulties in comprehending covariation and discerning the rule of the pattern, leading to the inclusion of a simpler task (Task 1) to better prepare students for Task 2. In the development of Task 1, input was solicited from mathematics teachers at the collaborating school to ensure that the questions were suitable for the student's level of understanding. Therefore, the researchers also took both the teacher's and experts' opinions on the tasks. The validity of the instruments was provided by expert opinion aligned with the premises of qualitative research.

During the construction of the tasks, various sources were consulted, including literature on algebraic thinking, the MOST Framework established by Leatham et al. (2015), and the task design cycle outlined by Liljedahl et al. (2007). Consequently, the tasks were designed with the following criteria in mind: they should 1) prompt students to contemplate a given situation and employ problem-solving techniques, 2) permit the use of manipulatives and hands-on materials, 3) facilitate communication among students (encourage collaborative work), 4) incorporate real-life contexts, 5) have the potential to evoke misconceptions in students, and 6) allow teachers to discern correct answers with innovative reasoning, as well as answers that may involve incomplete or incorrect reasoning.

The related questions in Task 1, Task 2, and Task 8 (given in Appendix 1), are analyzed and summarized in Table 1.

Table 1

Description of Tasks Related to Finding the Rule and the Terms of the Pattern

Task Number	Description of tasks
Task 1	There were two questions to indicate the relationship between dependent and independent variables. One question did not include a constant term. In the other question, two of the seats represented constant terms. The rules for these questions were *40n* and *3n+2*.
Task 2	To obtain similar pattern rules with Task 1, the first question represents a pattern rule without constant (*3y*) whereas the other two questions held a pattern with constants flats in the roof and initial street respectively. The rules for these questions were *4n+2* and *2m+1*. The additional question was a growing pattern utilizing a flower and the constant leaves.
Task 8	The question in this task was an example of a growing pattern where the middle square is constant and 4 squares were added each time. Therefore, the pattern rule was *4n+1*.

2.3. Implementation and data collection

The research setting utilized in this investigation had previously been employed in a TUBITAK-funded project (Project number: 215K049) led by one of the authors (see Kılıç et al., 2019; Kılıç & Doğan, 2022). A similar setup was established for the present study. Before implementing each task, the researcher carried out sessions with the pre-service teachers to explain the task implementation process, aiming to minimize variations among the groups. During these meetings, potential Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) instances were presented to the pre-service teachers for each task, ensuring they paid attention to crucial cases. For example, in Task 2, students might interpret the pattern for the first item as "n+3" instead of "3n" if they focus solely on the relationship between the numbers within one variable, rather than the relationship between the independent and dependent variables. In such cases, pre-service teachers were encouraged to support students in articulating their ideas. Furthermore, sample questions similar to those in the tasks were provided to the pre-service teachers to guide them on the types of questions they could pose to students during discussions. For instance, for Task 2, the following question was shared with the pre-service teachers: "How many families will move during the 40th year?" as students needed to discern a rule between the number of years and the families that moved in that question.

During these meetings, potential misconceptions held by students were also shared with the pre-service teachers. Throughout the implementation process, the researchers observed the interactions between pre-service teachers and students to ensure that the questions aligned with the suggested inquiries. After completing each task, the pre-service teachers were asked to highlight noteworthy instances to evaluate the effectiveness of the questions in student learning. Based on the feedback from pre-service teachers, the researchers adjusted scaffolding questions to further enhance their contribution to students' development of algebraic thinking.

Over 7 weeks, a total of 8 tasks were implemented. The entire implementation process was recorded on video to observe both individual and group work among students. Additionally, completed assignments from students were collected for analysis of their written work. Students' written artifacts and video recordings were collected for data triangulation which is essential for the validity of qualitative research (Merriam, 2009). The tasks were named based on the week in which they were administered. Specifically, three tasks (Task 1, Task 2, and Task 8) were focused on finding the rules and terms of the pattern as the first component of algebraic thinking. Task 3 was related to doing operations on algebraic expressions such as addition, subtraction, and division. Task 4 and Task 6 were focused on writing algebraic expressions in different contexts such as taking different roads. In these tasks, the students were expected to take some of the previous roads as reference points to write correct algebraic expressions. Therefore, Tasks 3, 4, and 6 were related algebraic expressions. Tasks 5 and 7 were about setting up and solving equations. In both Task 5 and Task 7, students had to convert verbal expressions to algebraic expressions and then write down the equations accordingly. In general, the tasks were ordered according to the difficulty level of questions and each group of the tasks included parallel questions.

Each task took place within an elective mathematics course at the school, with each session lasting 2 hours. The order of tasks was determined based on the complexity of the problems and their contexts. Task 8 presented a more intricate context and posed more challenging questions, thus it was implemented last. Throughout the implementation, students were organized into teams of 3 or 4, with deliberate efforts made to include students with varying performance records from the sixth-grade mathematics course in each group. Each group was supervised by a pre-service teacher, and the assignments of pre-service teachers to groups were determined randomly. At the beginning of each task, the pre-service teachers introduced the questions to their respective groups. After this, students were given 20 minutes to work independently, followed by a group discussion where students shared their solutions with each other. Finally, the pre-service teachers actively participated in the discussion, posing probing questions such as "How do you know that?" and "Why did you do this?" to encourage students to articulate their reasoning. When none of the students gave the correct answer, the pre-service teacher sought to elucidate the students' thought process by making simplifications on the given situation, making use of manipulative, or re-evaluating information provided in the problem statement. These questions exemplify scaffolding practices aimed at fostering students’ algebraic thinking, as scaffolding might facilitate an environment for discourse and deepen students' comprehension (Baxter & Williams, 2010; Kılıç & Doğan, 2022). For tasks centered around patterns, pattern blocks were utilized as manipulatives.

In related tasks, the pre-service teachers played a guiding role in helping students grasp the rationale behind the constant term and the coefficient term in the pattern's rule. They posed questions such as "What remains consistent in different shapes?", "What changes from one shape to another?", and "How did you arrive at the pattern's rule?" to gain insight into students’ thought processes during the tasks. When a student encountered difficulty in discerning the correct rule of the pattern, the pre-service teacher introduced pattern blocks as a visual aid. Students were instructed to construct the shapes outlined in the questions using pattern blocks, enabling them to analyze what elements remained consistent and what changed within the shapes.

In some cases, at least one student successfully determined the pattern’s rule. In these situations, the pre-service teachers encouraged them to explain their thought processes and offered assistance to other students. Through this approach, the pre-service teachers facilitated students’ reasoning using both manipulatives and within-group discussions, in line with the established criteria for the tasks.

2.4. Data analysis

The data sources were video recordings and written artifacts from the implementation process. In the data analysis part, transcripts from the implementation videos were meticulously analyzed with a keen focus on potential Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) instances. These instances were particularly interesting as they provided explicit insights into students’ algebraic thinking, facilitating a more straightforward assessment of their thought processes. Students’ expressions were extracted from the transcripts to evaluate the depth of their algebraic thinking following both individual work and group discussions. General terms and phrases were gathered from both settings to identify shared expressions among students. A coding scheme was developed to capture these commonalities, incorporating insights from the existing literature. For example, English and Warren (1998) emphasized the crucial role of understanding variables as a foundational concept for making generalizations. Similarly, Lew (2004) asserted that the concept of a variable is pivotal in comprehending algebraic concepts like patterns. Therefore, a solid grasp of variables was considered a prerequisite for engaging in algebraic thinking, denoted as Level 1 in the coding schema. Furthermore, deducing the rule of growing patterns requires an analysis of covariation, rather than paying attention to the change in one variable (Warren, 2005). Consequently, the coding schema also integrated the concept of simultaneous change in both the dependent and independent variables as another level of algebraic thinking. With these considerations in mind, the coding schema was refined and subsequently applied to evaluate students' responses in their worksheets, as well as their expressions following individual work and group discussions. The coding schema specifically designed for identifying the rules and terms of the pattern is outlined in Table 2.

Table 2

Algebraic Thinking Levels For Finding the Rule of the Pattern

Level	Explanation
NA	Student irrelevant answer or no answer
Level 1	Student finds a rule by paying attention to the changes in one (dependent) variable rather than focusing on the relationship between two (independent and dependent) variables OR student only understands that each of the letters n, m, a, b corresponds to a variable
Level 2	Student finds the rule by the trial-and-error process but does not what leading coefficient or constant term stands for OR student understands what leading coefficient and constant term represent but could not write the rule
Level 3	Student finds the rule by looking at the relationship between the independent and dependent variables and understanding what the leading coefficient and constant term represent stands for

In this study, students' algebraic thinking was analyzed through pattern tasks and compared in terms of their performance before and after group discussions. The primary unit of analysis was the level of students' algebraic thinking, which was evaluated for each task based on their verbal expressions from the implementation videos and their written work. Following the group discussion, pre-service teachers instructed students to write their solutions on the worksheets and articulate students' current thought processes through verbal expressions. These expressions were then coded according to the coding schema presented in Table 2. Additionally, the frequency of cases falling into the categories of NA, Level 1, Level 2, and Level 3 was determined. For each task, the total number of cases was proportionally compared to the occurrences of NA, Level 1, Level 2, and Level 3 within the same task, allowing for the calculation of percentages for each level of algebraic thinking. This approach unveiled the shifts in percentages for each level of algebraic thinking within the same task and across different tasks.

Students' algebraic thinking levels were coded by the researcher according to the levels given in Table 2. For the interrater reliability, two different researchers blindly coded 20% of the cases. After the coding process was completed, the researchers came together and agreed on every case when there were different levels of coding.

FINDINGS

In this section, the results will be proposed to dwell on how students' algebraic thinking changed through the task implementation. To conduct a more in-depth analysis of students' algebraic thinking, a sub-group of 9 students was selected based on the criteria outlined in the methodology section. From this sub-group, one student's algebraic thinking will be mentioned to exemplify the changes observed in each task (with a different student chosen for each task). This approach allows for an examination of the sub-group's progress in terms of algebraic thinking. A descriptive figure illustrating the evolution in the sub-group's algebraic thinking within the same task and from one task to another is presented in Figure 1. This figure includes percentages representing different levels of algebraic thinking (NA, Level 1, Level 2, Level 3).

Figure 1

The Change of Frequencies of Students' Algebraic Thinking In Component 1

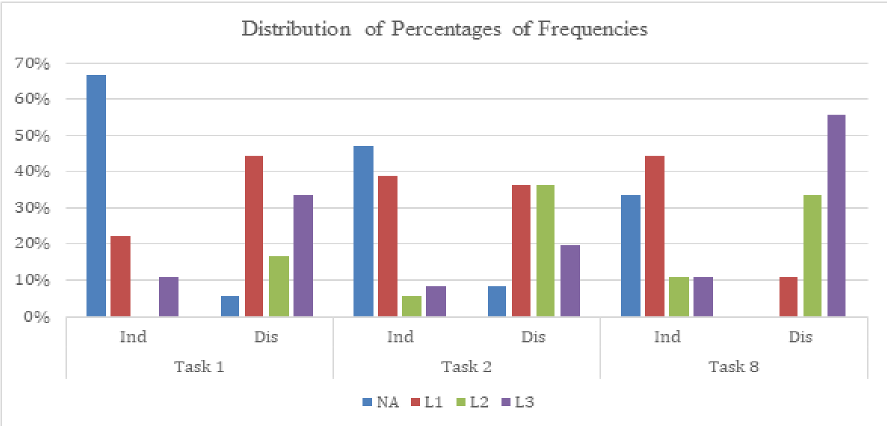


Figure 1 provides an overview of students' levels of algebraic thinking in the respective tasks, summarizing their proficiency in recognizing patterns. The analysis encompassed students' performance during individual work and after engaging in discussions with both students from the same group and pre-service teachers. The occurrences of NA, Level 1, Level 2, and Level 3 cases were proportionally compared to the total related cases within each task, and percentages were calculated for each level of algebraic thinking.

As depicted in the chart, there was a notable increase in students' levels of algebraic thinking after the discussions, compared to their individual work. This was evident from the relatively high percentages of Level 2 and Level 3 responses in the discussion phase across all tasks. For example, in the individual work portion of Task 8, Level 2 thinking was observed in only 10% of cases, whereas after the discussions, the percentage of cases exhibiting Level 2 thinking exceeded 30%. Similarly, the increase in Level 3 thinking from individual work to the discussion within the same task was 25%, 10%, and 45% in Task 1, Task 2, and Task 8 respectively.

Furthermore, it can be inferred that students' algebraic thinking progressed from Task 1 to Task 8. This is indicated by the decrease in percentages of NA and Level 1 responses, coupled with an increase in Level 2 and Level 3 responses throughout Task 1 to Task 8. In Task 1, seven out of nine students provided answers categorized as "no attempt" in their individual work, whereas in Task 8, this number was reduced to three. Moreover, nearly all of the student's answers in Task 8 were classified as Level 3 after the discussions.

In Task 2, certain questions saw an increase in students' algebraic thinking to Level 3 after the discussions, likely due to the use of pattern blocks by pre-service teachers to bolster students' algebraic thinking. For instance, in one question, Gonca initially derived an incorrect rule for the pattern, resulting in a Level 1 categorization. Following the pre-service teacher's guidance to reevaluate her rule using pattern blocks, Gonca realized the rule was flawed. She also identified that the roof represented the constant term, and the increase in the number of apartments each year corresponded to the coefficient of the rule. Consequently, Gonca's algebraic thinking was reclassified as Level 3 after the discussions. In two questions, the majority of students' algebraic thinking was enhanced with the aid of pattern blocks, leading to an elevation to Level 3 thinking in those specific questions. Additionally, in Task 8, no "no attempt" answers were recorded. Considering there were some instances of "no attempt" answers in the individual portions of Task 1 and Task 2, it can be inferred that even in their individual work, students were able to augment their algebraic thinking from Task 1 to Task 8.

In Task 1, during individual work, the majority of students struggled to provide an answer, resulting in their algebraic thinking being categorized as NA. For example, Alper focused solely on calculating the number of rails installed in one minute, failing to discern the underlying pattern. Consequently, his response was coded as NA. However, during the discussions, a student from Alper's group pointed out that the difference between the number of rails built on consecutive days was 40. When the pre-service teacher prompted them to determine the rule for the 10th day, this student correctly identified it as "40n." This realization prompted Alper to grasp that in the pattern's rule, the variable represents the number of days, leading to his thinking being coded as Level 1.

While working individually on the second question, Alper once again struggled to derive a rule. However, during the discussion phase for the second question, he was able to apply the insight gained from his earlier interaction with his friend. He correctly identified the difference between seats on consecutive days as the coefficient of the rule. Nonetheless, he still faced difficulty determining how to incorporate two constant seats into the rule. As a result, his algebraic thinking was coded as Level 2. In the second question, Alper demonstrated improvement in his thinking, thanks to the discussions from the first question.

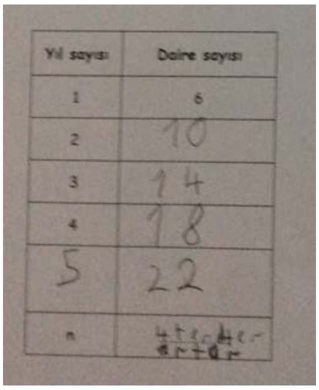
In this task, it was evident that all students experienced a shift in their thinking levels, moving up by at least one level after the discussions for each question. Likewise, when comparing the first question to the second question, it was observed that four students improved at least one level in their algebraic thinking. The remaining students' thinking levels remained unchanged, as two students were already operating at Level 3, while the others continued to concentrate solely on the change in the dependent variable by the end of the discussions.

In Task 2, there were a few instances where thinking levels were coded as NA (No Attempt or irrelevant answer) during the individual work of the students. Most students' algebraic thinking in their individual work was coded as Level 1. For instance, Harun considered only the increase in one variable rather than the relationship between the two variables. He correctly identified that the increase in the number of families moving each year was 3, leading to his thinking being categorized as Level 1. In the second question, Harun also found the change in the number of apartments in successive years. However, in the third question, he mistakenly derived a pattern rule by only examining how the number of weeks and streets related for the first row. Therefore, his algebraic thinking was categorized as NA. Similar to Harun's approach, other students primarily focused on the increase in the number of families, apartments, and streets during their individual work. However, during the discussion of the first question, the teacher redirected them to find a rule representing the relationship between the two variables, rather than focusing solely on changes in one variable. This prompted Harun to realize that this relationship could be identified through multiplication, and he arrived at the rule through a trial-and-error process. Consequently, after the discussion, his algebraic thinking was categorized as Level 2.

In the second question, the pre-service teacher instructed the students to construct the apartments with pattern blocks and initiated a discussion within the group. In the individual work, Harun paid attention to the increase in the dependent variable and wrote the number of apartments was increasing 4 by 4 as seen in Figure 2. Therefore, his algebraic thinking was coded as Level 1.

Figure 2

Harun's Answer From Individual Work



Yıl sayısı	Daire sayısı
1	6
2	10
3	14
4	18
5	22
n	[illegible]

Since Harun is part of the sub-group, the conversation between the pre-service teacher (PT) and Harun (H) will be provided below.

PT: What is the constant in these shapes?"

All of the students: The roof…

PT: The roof is constant right? Apart from the roof; in the first year there are four apartments, in the second year eight apartments, and in the third year 12 apartments, can you deduce the rule from there?".

Harun: 4 apartments are added, teacher.

PT: If 4 apartments are added, how you can express when you think as in the first question?

Harun: 4.n…

PT: How we can include the roof in my pattern rule?

Harun: Wouldn't be 6.n teacher?

PT asked him to try his rule.

PT: We have 4.n, you already said that. Since it goes 4 by 4, so how can I add the roof? Which operation did we use?

Harun: Adding…

PT: We used the addition operation.

Harun: 4.n+10…

PT: Where did 10 come from? In this first year, there are 4 apartments and what else?

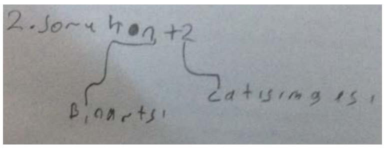
Harun: +2…

PT: Then, what is the rule?

Harun: 4.n+2…

Figure 3

Harun's Answer After The Group Discussions

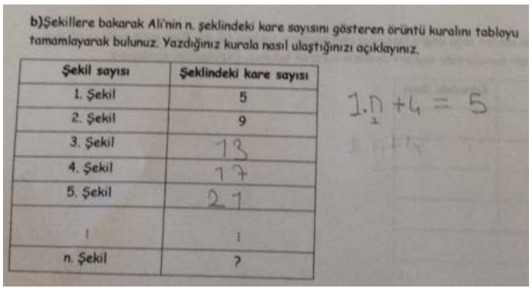


In his answer after the group discussions, Harun realized that 4n in the pattern rule comes from the increase in the number of buildings whereas the roof was presented by 2 which is the constant in the pattern rule. Harun was able to derive the pattern rule with the assistance of the teacher, resulting in his algebraic thinking thought as Level 2, similar to the first question. Likewise, in a similar vein to Harun, after the discussions of the questions, most students' algebraic thinking reached Level 2. This was because they were able to deduce the pattern rule with the guidance of their teachers and articulate where the coefficient or constant term originates.

In Task 8, the related question included growing patterns. Since the pattern was different from the patterns in Task 1 and Task 2, some of the students had difficulties in writing the rules of the pattern in their individual work. The majority of the students' algebraic thinking was coded as Level 1. For instance, Tansu focused on the increase in the number of squares in consecutive shapes and thought pattern rule as n+4 as seen in Figure 4.

Figure 4

Tansu's Answer From Individual Work



b)Şekillere bakarak Ali'nin n. şeklindeki kare sayısını gösteren örüntü kuralını tabloyu tamamlayarak bulunuz. Yazdığınız kurala nasıl ulaştığınızı açıklayınız.

Şekil sayısı	Şeklindeki kare sayısı
1. Şekil	5
2. Şekil	9
3. Şekil	13
4. Şekil	17
5. Şekil	21
⋮	⋮
n. Şekil	?

1.n+4 = 5

The following dialogue indicates the interaction between Tansu (T) and the pre-service teacher (PT) who was the responsible teacher from this group.

PT: How we can express the amount of increase in the group?

Tansu: n+4 since the number of squares increased 4 by 4.

PT: Can you try your rule?

Tansu: The rule is correct for the first pattern but is not provided for the others.

PT: What is the constant for these shapes?

Tansu: The middle… (she was pointing the middle square in the figures)

PT: What do you mean by middle? How we can express the middle?

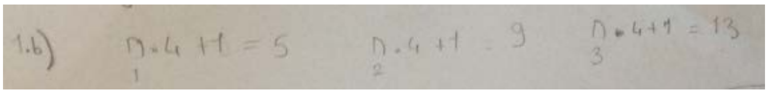
Tansu: +1…

PT: How many squares increased from one shape to the other? How can we state the increase in the rule?

Tansu: 4n+1…

Figure 5

Tansu's Answer After Group Discussions



As evidenced by the conversation and her written work, Tansu successfully derived the pattern rule through discussion with the pre-service teacher. She discerned that the constant square represented the constant term of the rule, while the number of square increases constituted the coefficient of the rule. Consequently, her algebraic thinking was classified as Level 3. At the end of the task, the majority of students demonstrated at least Level 2 algebraic thinking, with only one exception. This is noteworthy given the higher complexity of the question compared to those in Task 1 and Task 2. Furthermore, five students were able to independently derive the rule and articulate their reasoning, underscoring an advancement in their algebraic thinking compared to previous tasks.

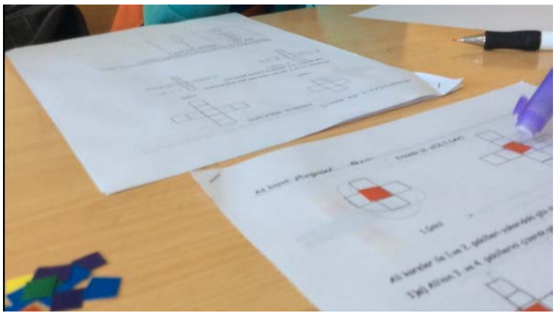
In certain groups, students initially struggled to determine the pattern rule on their own but received assistance from their peers in constructing it. Burak, who had encountered difficulties in prior tasks, continued paying attention to solely on the change in one variable. Conversely, within the same group, Erdem could examine how the dependent and independent variables were related even during individual work. Through discussions with Erdem, Burak was able to enhance his algebraic thinking and provide detailed explanations in his worksheet.

Throughout the task implementation, it was evident that some students derived greater benefit from the task-assisted instruction compared to others. By the conclusion of Task 8, 5 out of 9 students had elevated their algebraic thinking to Level 3. Among these students, three initially provided irrelevant answers during the early stages of task implementation. For example, one student was unable to determine the pattern rule in Task 1, resulting in their algebraic thinking being categorized as an NA answer. Similarly, another student's response did not pertain to the pattern rule, leading to the assignment of an NA level of thinking in Task 1. However, following the discussions held in Task 8, all three of these students demonstrated significant progress in their algebraic thinking. They came to understand the rationale behind the coefficient and constant term in the pattern rule. Of the five students who reached Level 3, Harun and Tansu initially possessed algebraic thinking levels of 1 and 3, respectively, at the commencement of the tasks. This indicates that they had some prior knowledge in finding the rule and terms of the pattern. Harun focused on discerning the difference in the number of rails each day, while Tansu was able to accurately write the pattern rule. Consequently, their levels of algebraic thinking exhibited less change compared to the other students.

On the contrary, in Task 8, after the discussions, some students' algebraic thinking reached at most Level 2. By the conclusion of the task implementation, three students were categorized as having Level 2 algebraic thinking. Two of these students, Burak and Mert, were from the same group, while the other two students had some learning difficulties. In light of this, Burak's group was restructured to provide additional support for his algebraic thinking. In the new group, with the assistance of his friend and the pre-service teacher, Burak was able to derive the pattern rule successfully. Similarly, the pre-service teacher guided Mert to recognize the constant and changing squares, achieved by posing questions about the differences between the shapes. With the researcher's help, Mert was able to state the pattern rule accurately. In some groups, students derived pattern rules together. As an example, in Doruk’s group the pre-service teacher what was the constant within the shapes and all students realized the red square was the constant one.

Figure 6

The Screenshot of One Student’s Pointing Out the Constant Red Square



The pre-service teacher also asked how many squares were added besides the red square in each shape Whole group stated 4, 8, 12, and 16 squares were added respectively. Then they continued the group discussion based on the rule of the pattern. Related group discussion is given below:

PT: You said 1 is always constant; what is the rule among 4,8,12,16 as changing ones?

S1: Increasing 4 by 4.

PT: I added 4 for the first shape, 8 for the second shape, and 12 for the third shape. What does this state for you?

S2: It increased four by four.

PT: 4 in the first shape, 8 in the second shape.

Doruk: 4×2

PT: How can you express in the third shape?

Doruk: 3×4

S1: 4 multiplied by 4, fixed 4…

PT: What is changing? You multiplied by 1, multiplied by 2…

S1: The number of shapes…

PT: What does the shape number represent?

S2: n…

PT: Then, what is the rule?

Doruk: 4n+1…

After the group discussion, all of the students wrote the correct rule of the pattern. Within the subgroup, only one student maintained their focus on the differences between the squares, even in Task 8. Consequently, his algebraic thinking was coded as Level 1. This student demonstrated improvement in their algebraic thinking by only one level throughout the tasks. In summary, 8 out of 9 students were able to enhance their algebraic thinking by at least two levels through task-assisted instruction.

Upon analyzing the interactions between students and teachers, it is evident that specific students derived notable benefits from task-assisted instruction, likely attributable to the probing inquiries posed by their instructors. When pre-service teachers effectively harnessed the opportunities outlined in the MOST Framework, they engaged students with questions aimed at bolstering their proficiency in algebraic thinking. According to Leatham et al. (2015), the timing and initiation of discussions play a pivotal role in defining a case as MOST. As a result, prospective teachers encouraged students to think more through additional questions. For example, in Task 2, Alper directed his attention toward the alteration in the dependent variable in two out of three questions. Subsequently, his teacher discerned the mathematical concepts evident in Alper's independent work and proceeded to inquire about the covariance of variables in both instances. This collaborative effort led to Alper grasping the underlying logic behind the pattern's rule, elevating his cognitive level to Level 2. Thus, it can be inferred that the tasks served as a conduit for revealing students' mathematical aptitude and effectively utilized the MOST scenario to stimulate algebraic thinking. In contrast, certain pre-service teachers failed to capitalize on these opportunities, even though the researcher had apprised them of potential MOST instances during discussions. Some were constrained by time limitations, resulting in the omission of certain questions or a preference for instructing higher-achieving students in solution methods rather than fostering broader discussions. Consequently, it may be beneficial to decrease the number of questions in tasks, allowing for more extensive discussion time.

DISCUSSION

Many researchers, such as Lannin (2005), have advocated the use of tasks to facilitate students' understanding of identifying the rule and terms within a pattern. Lannin (2005) concluded that students could derive generalizations about the pattern's rule through small group discussions. This study's findings also demonstrated an enhancement in students' algebraic thinking following these discussions. For instance, in Task 8, Erdem successfully tackled Level 2, even in his individual work, while Burak struggled to grasp the relationship between the variables. In the small group discussions, Erdem elucidated the pattern's rule to Burak, aiding him in recognizing the reasoning behind the constant term and coefficient in the pattern rule. Consequently, his algebraic thinking benefited from the small group discussions. Furthermore, Billings et al. (2007) and Moyer-Packenham (2005) employed growing patterns, akin to the first question in Task 8 of this study. Moyer-Packenham (2005) asserted that this type of pattern helps students discern the underlying rule, as they can perceive the incremental change from one shape to another. In this study, tasks involving growing patterns (as exemplified in Task 8) also proved beneficial in bolstering students' algebraic thinking. Notably, students like Tansu, Erdem, and Burak (as discussed in the later part) engaged in discussions where they recognized the evolving and constant aspects of growing patterns. In summary, the outcomes of prior studies align with the current study's findings regarding the positive impact of task-assisted instruction on students' proficiency in algebra.

The investigation of Gonca's responses in the findings section serves as compelling evidence for the advantages of incorporating manipulatives. When it came to finding the rule and terms of the pattern, Gonca demonstrated Level 3 thinking after engaging in discussions for two out of four questions in Task 2. The pre-service teacher, following the researcher's recommendations, employed manipulatives in these two questions. Gonca's articulations, elucidating the origins of the constant term and coefficient, provide clear indications of how she leveraged manipulatives to enhance her grasp of algebraic concepts, as she recognized that the constants corresponded to the constant term in the pattern's rule. The results of this study align with the findings of Saraswati and Putri (2016), who asserted that the utilization of algebra tiles can be a valuable tool in aiding students' comprehension.

The instructional approach followed a student-centered teaching model during task implementation. The pre-service teachers provided scaffolding by posing questions to guide students in their discussions. In Task 8, the researcher directed questions about the constant square and how to incorporate the constant term into the rule. Through these inquiries, Doruk independently arrived at his pattern rule and corrected his initial incorrect response from his individual work. This aligns with the study's objective, resulting in an elevation of his thinking to Level 3. Furthermore, the interactions between pre-service teachers and students Harun and Tansu were detailed for Task 2 and Task 8, respectively. The pre-service teachers recognized opportunities to foster students' algebraic thinking and appropriately intervened in the group discussions. Given that all pre-service teachers had prior discussions with the researcher regarding potential scenarios, they were well-informed about how to provide support for students. Consequently, it can be inferred that these discussions and the student-centered teaching approach effectively bolstered students' proficiency in discerning the rules of the pattern.

In the current study, the tasks were meticulously crafted to bring out the most illustrative instances, thereby offering robust support for students' development of algebraic thinking. The teacher-student interactions underscored that these highlighted instances played a pivotal role in eliciting and refining students' grasp of algebraic concepts. For example, one of the most illustrative instances highlighted in Task 2 emphasized focusing on the change in one variable rather than the covariation. In this scenario, the anticipated student response was n+4, given the consistent increase of 4 in the number of apartments over consecutive years. During the task implementation, these instances surfaced in multiple groups. Harun, for instance, honed in on the change in apartment numbers and initially defined the rule as n+4, even though the correct rule was 4n+2. The researcher intervened with strategic questions to help him discern the relationship between variables, ultimately guiding Harun to arrive at the correct answer. Furthermore, Harun applied this newfound understanding in the subsequent question that featured similar illustrative instances. This ease of knowledge transfer likely stemmed from the closely related contexts of the problems. In both cases, there existed constant terms within the rule, with the coefficient term representing the increment. To put it succinctly, these most illustrative instances not only serve as a scaffold for enhancing students' thinking but also allow them to rectify their misconceptions and deepen their comprehension of mathematical concepts and procedures.

CONCLUSION

The findings of the present study underscore the effectiveness of pattern tasks in bolstering students' proficiency in algebraic thinking. This was evident in their ability to apply previously acquired knowledge from one question to the subsequent tasks. Additionally, the instructional approach, which included elements like group work, utilization of pattern blocks, and the incorporation of scaffolding questions, proved instrumental in enhancing students' algebraic thinking. What sets this study apart from others is its analytical framework, which leveraged the MOST theoretical framework (Leatham et al., 2015) as a lens through which task-assisted instruction was evaluated. Consequently, the tasks were meticulously designed to not only have the potential to generate MOST instances but also to foster students' mathematical comprehension. To support this understanding, tasks were contextualized around scenarios relevant to students' immediate surroundings, whether close or far. Some tasks required the use of tangible materials and necessitated collaboration among students. While these attributes may not be entirely novel in research studies on tasks, the deliberate effort to create a setting conducive to generating MOST instances to enhance students' understanding and rectify misconceptions was a distinctive feature of this study. As a result, this study presents an opportunity for further exploration into the design of tasks that can generate MOST instances in a classroom setting, with the potential to significantly advance students' comprehension while addressing their misconceptions and challenges. This innovative approach holds promise for future research endeavors in this domain. As a suggestion, in further research, MOST can be used as a theoretical lens to elaborate students' comprehension of different mathematics topics. In addition, the effectiveness of the MOST framework can be investigated for the task implementation process in terms of support of students' mathematical knowledge and the timing of pedagogical opportunities.

LIMITATIONS AND IMPLICATIONS FOR FUTURE RESEARCH

Numerous scholars, such as Lannin (2005), have recommended the utilization of tasks to assist students in deducing the rules and terms of a pattern. Lannin (2005) determined that students can formulate generalizations about the pattern's rule through collaborative discussions in small groups. The outcomes of this research additionally illustrated an enhancement in students' proficiency in algebraic reasoning after such discussions. For instance, in Task 8, Erdem's individual work already exhibited a Level 2 understanding, whereas Burak encountered difficulties in comprehending the connection between the variables. During the small group deliberations, Erdem elucidated the pattern's rule to Burak and aided him in comprehending the basis of the constant term and coefficient. As a result, small group discussions provided crucial support for Burak's algebraic reasoning. Moreover, Billings et al. (2007) and Moyer-Packenham (2005) incorporated burgeoning patterns, as exemplified in the initial inquiry of the final task in this investigation. Moyer-Packenham (2005) argued that this pattern assists students in discerning the underlying rule, as they can apprehend the incremental change from one shape to another. To summarize, the preceding studies' discoveries correspond with the present study's findings, emphasizing the favorable influence of task-guided instruction on students' accomplishments in algebra.

The analysis of Gonca's responses in the results section presents compelling evidence in favor of integrating manipulatives. In the task of discerning the rules and terms of the pattern, Gonca exhibited Level 3 reasoning after participating in discussions for two out of four questions in Task 2. The teacher, following the researcher's recommendations, incorporated manipulatives in these two questions. Gonca's explanations, clarifying the origins of the constant term and coefficient, offer clear indications of how she utilized manipulatives to deepen her understanding of algebraic concepts. She recognized that the constants corresponded to the constant term in the pattern's rule. The outcomes of this investigation are consistent with the findings of Saraswati and Putri (2016), who emphasized the usefulness of algebra tiles in enhancing students' comprehension. In further studies, algebra tiles and manipulatives can be constructed in dynamic environments to elaborate students' algebraic thinking. These environments might allow them to create their own patterns and observe the constant terms among different patterns.

In this study, tasks were implemented using a student-centered teaching approach, where pre-service teachers employed scaffolding techniques through questioning during discussions. In Task 8, the researcher raised questions about the constant square and how to incorporate the constant term into the rule. Through these inquiries, Doruk was able to independently deduce the pattern rule and correct his initial incorrect response from his individual work. This aligns with the study's objective, resulting in an elevation of his thinking to Level 3. Furthermore, conversations between pre-service teachers and students Harun and Tansu were documented for Task 2 and Task 8, respectively. The pre-service teachers keenly identified opportunities to enhance students' algebraic thinking and effectively contributed to the group discussions. Since all pre-service teachers had prior discussions with the researcher regarding potential scenarios, they were well-equipped to offer appropriate support to the students. As a result, it can be concluded that these discussions and the student-centered teaching approach effectively enhanced students' proficiency in identifying the pattern rule.

Additionally, the tasks in this study were intentionally crafted to highlight the MOST instances, providing further support for students' development of algebraic thinking. Through interactions among teachers, researchers, and students, it became evident that these MOST instances played a crucial role in eliciting and refining students' understanding of algebraic concepts. For example, one of the MOST instances emphasized in Task 2 focused on isolating the change in one variable rather than the covariation. In this scenario, the expected student response was n+4, given the consistent increase of 4 in the number of apartments over successive years. This pattern was observed in many groups during task implementation. For instance, Harun initially concentrated on the change in the number of apartments and formulated the rule as n+4, even though the correct rule was 4n+2. The researcher used targeted questions to assist him in discerning the relationship between variables, ultimately guiding Harun to arrive at an accurate answer. Furthermore, Harun applied this newfound understanding in the subsequent question featuring similar illustrative instances. This ease of knowledge transfer likely stemmed from the closely related contexts of the problems. In both cases, there were constant terms within the rule, with the coefficient term representing the increment. In summary, these MOST instances not only serve as a support structure for enhancing students' thinking but also enable them to rectify their own misconceptions and deepen their comprehension of mathematical concepts and procedures.

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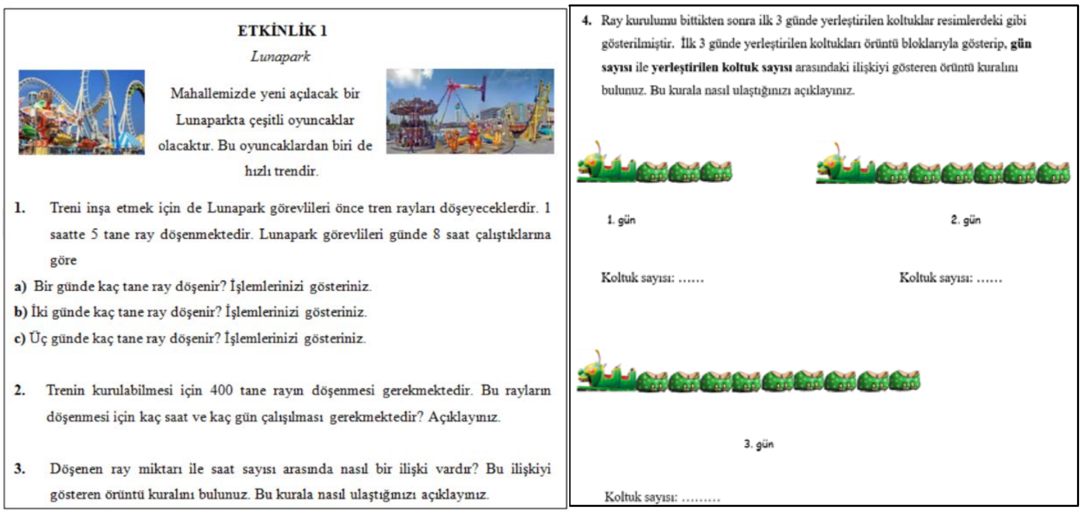
APPENDIX 1

Figure 7

Related Questions In Task 1



ETKİNLİK 1

Lunapark

Mahallemizde yeni açılacak bir Lunaparkta çeşitli oyuncaklar olacaktır. Bu oyuncaklardan biri de hızlı trendir.

1. Treni inşa etmek için de Lunapark görevlileri önce tren rayları döşeyeceklerdir. 1 saatte 5 tane ray döşenmektedir. Lunapark görevlileri günde 8 saat çalıştıklarına göre

a) Bir günde kaç tane ray döşenir? İşlemlerinizi gösteriniz.
b) İki günde kaç tane ray döşenir? İşlemlerinizi gösteriniz.
c) Üç günde kaç tane ray döşenir? İşlemlerinizi gösteriniz.

2. Trenin kurulabilmesi için 400 tane rayın döşenmesi gerekmektedir. Bu rayların döşenmesi için kaç saat ve kaç gün çalışılması gerekmektedir? Açıklayınız.

3. Döşenen ray miktarı ile saat sayısı arasında nasıl bir ilişki vardır? Bu ilişkiyi gösteren örüntü kuralını bulunuz. Bu kurala nasıl ulaştığınızı açıklayınız.

4. Ray kurulumu bittikten sonra ilk 3 günde yerleştirilen koltuklar resimlerdeki gibi gösterilmiştir. İlk 3 günde yerleştirilen koltukları örüntü bloklarıyla gösterip, **gün sayısı** ile **yerleştirilen koltuk sayısı** arasındaki ilişkiyi gösteren örüntü kuralını bulunuz. Bu kurala nasıl ulaştığınızı açıklayınız.

1. gün

Koltuk sayısı: ......

2. gün

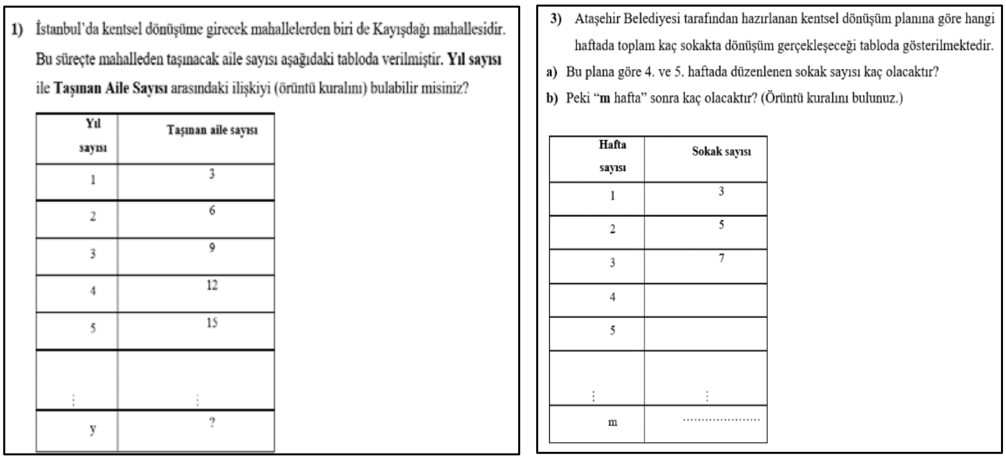
Koltuk sayısı: ......

3. gün

Koltuk sayısı: ....

Figure 8

The First and Third Questions of Task 2



1) İstanbul'da kentsel dönüşüme girecek mahallelerden biri de Kayışdağı mahallesidir. Bu süreçte mahalleden taşınacak aile sayısı aşağıdaki tabloda verilmiştir. **Yıl sayısı** ile **Taşınan Aile Sayısı** arasındaki ilişkiyi (örüntü kuralını) bulabilir misiniz?

Yıl sayısı	Taşınan aile sayısı
1	3
2	6
3	9
4	12
5	15
⋮	⋮
y	?

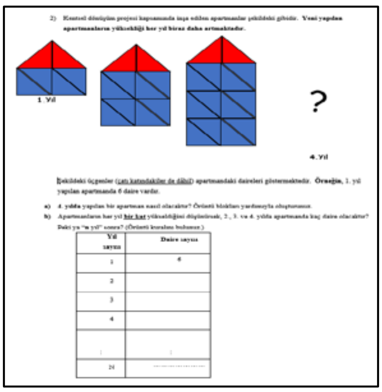
3) Ataşehir Belediyesi tarafından hazırlanan kentsel dönüşüm planına göre hangi haftada toplam kaç sokakta dönüşüm gerçekleşeceği tabloda gösterilmektedir.

a) Bu plana göre 4. ve 5. haftada düzenlenen sokak sayısı kaç olacaktır?
b) Peki "**m** hafta" sonra kaç olacaktır? (Örüntü kuralını bulunuz.)

Hafta sayısı	Sokak sayısı
1	3
2	5
3	7
4	
5	
⋮	⋮
m	....

Figure 9

The Second Question of Task 2



2) Kentsel dönüşüm projesi kapsamında inşa edilen apartmanlar şekildeki gibidir. **Yeni yapılan apartmanların yüksekliği her yıl biraz daha artmaktadır.**

1. Yıl

?

4. Yıl

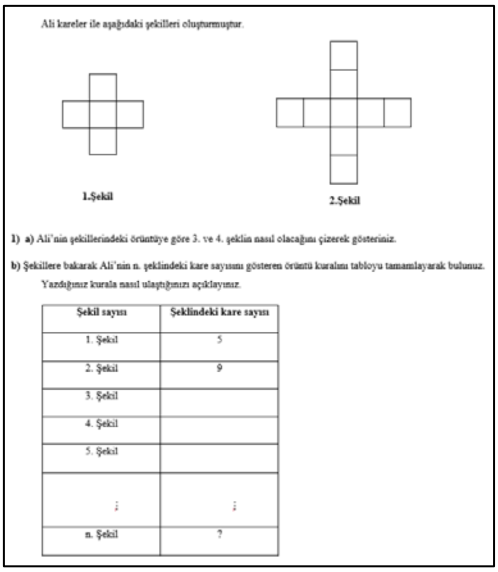
Şekildeki üçgenler (çatı katındakiler de dâhil) apartmandaki daireleri göstermektedir. **Örneğin,** 1. yıl yapılan apartmanda 6 daire vardır.

a) 4. yılda yapılan bir apartman nasıl olacaktır? Örüntü blokları yardımıyla oluşturunuz.
b) Apartmanların her yıl bir kat yükseldiğini düşünürsek, 2., 3. ve 4. yılda apartmanda kaç daire olacaktır? Peki ya "m yıl" sonra? (Örüntü kuralını bulunuz.)

Yıl sayısı	Daire sayısı
1	6
2	
3	
4	
⋮	⋮
m	....

Figure 10

The Related Question of Task 8



Ali kareler ile aşağıdaki şekilleri oluşturmuştur.

1.Şekil

2.Şekil

1) a) Ali'nin şekillerindeki örüntüye göre 3. ve 4. şeklin nasıl olacağını çizerek gösteriniz.

b) Şekillere bakarak Ali'nin n. şeklindeki kare sayısını gösteren örüntü kuralını tabloyu tamamlayarak bulunuz. Yazdığınız kurala nasıl ulaştığınızı açıklayınız.

Şekil sayısı	Şeklindeki kare sayısı
1. Şekil	5
2. Şekil	9
3. Şekil	
4. Şekil	
5. Şekil	
⋮	⋮
n. Şekil	?

GENİŞLETİLMİŞ ÖZET

Giriş

Cebir, matematiğin en önemli alanlarından biridir. Öğrenciler, cebirsel kavramları ve işlemleri anlamakta zorlanarak matematikte güçlük yaşayabilmektedir (Akkaya & Durmuş, 2006; Dede & Peker, 2007; Jupri vd., 2014; Lucariello vd., 2014; Welder, 2012). Bu konuda birçok araştırmacı, öğrencilerin cebirsel başarılarını ve kavrayışlarını artırabilecek eğitim ve öğretim yöntemleri üzerine çalışmalar yapmışlardır.

Literatürde değinilen eğitim öğretim yöntemlerinden biri, iyi planlanmış görevlerin uygulanmasıdır. Watson ve Mason (2007), bir etkinlik ve bir görev arasında bir ayrım olduğunu belirtmişlerdir. Bir etkinliği, öğrenciler arasındaki tüm iletişim olarak tanımlamışlardır. Bir görevi ise sadece öğrenciler tarafından gerçekleştirilen etkinlikler olarak değil, aynı zamanda etkinlikleri nasıl yorumladıkları, etkinliklerden nasıl yararlandıkları ve ne öğrendikleri, öğretmen tarafından sağlanan rehberliği de kapsayacak şekilde ifade etmişlerdir. Türkçe alanyazında hem "task" hem de "activity" kelimelerine karşılık olarak etkinlik kelimesinin kullanıldığı görülse da bu çalışmada literatürde belirtilen ayrımı vurgulamak için görev terimi tercih edilmiştir. Önceki çalışmalarda cebirde başarıyı artırmak isteyen araştırmacılar, örneğin Lannin (2005) ve Palabıyık ve İspir (2011), örüntü temelli görevleri tercih etmişlerdir. Walkington vd., (2013) ise günlük hayat bağlamlarını içeren görevleri araştırmalarında kullanmışlardır. Bu çalışmalar, örüntü temelli ve günlük yaşam bağlamı içeren görevlerin öğrencilerin cebir başarısına katkıda bulunduğunu gözlemlemişlerdir. Ayrıca, somut materyallerin, örneğin cebir karolarının, kullanımına da önem vermmişlerdir. Örneğin, Saraswati ve Putri (2016), cebir karolarının kullanıldığı çalışmalarında öğrencilerin tek değişkenli denklemleri daha iyi anlamalarına yardımcı olduğu sonucuna varmışlardır.

Carpenter ve Lehrer (1999) ile Lin (2004), matematik görevlerinin öğrencilerin matematiksel düşünme süreçlerine olumlu etkilerde bulunduğunu savunmuşlardır. Bu görevlerin sınıf ortamında öğretmenler tarafından kullanılması, öğrencilerin matematiksel düşünme becerilerini geliştirerek öğrenmelerine katkı sağlama fırsatı sunabilir. Leatham vd., (2015), bu öğrenme fırsatlarını "Öğrenci Düşünmesine Dayalı Matematiksel Önemli Pedagojik Fırsatlar" olarak tanımlamışlardır. Bu tanım, üç birbirine bağlı ve sıralı değişkeni içerir: öğrenci düşüncesine dayalı olma, matematiksel açıdan önemli olma ve pedagojik bir fırsat olma. Leatham vd., (2015), matematik görevlerinin bu fırsatları ortaya çıkaracak şekilde düzenlenerek, derslerin öğrenci matematiğine dayandırılabileceğini ve öğrencilerin matematik anlayışını değerlendirme fırsatı yakalayabileceğini belirtmişlerdir.

Yöntem

Bu çalışmanın amacı, alanyazındaki çalışmaları dikkate alarak, görev temelli öğretimin 7. sınıf öğrencilerinin cebirsel öğrenme ve düşünmelerini nasıl etkilediğini araştırmaktır. Araştırma, İstanbul'daki bir devlet okulunun 7. sınıf şubesinde gerçekleştirilmiştir. Bu sınıfta 7 hafta süresince görev temelli öğretim uygulanmıştır. Bu görevler, öğrencilerin cebirsel düşünce süreçlerini geliştirmek amacıyla örüntü kuralı ve terimlerini bulma, cebirsel ifadeler, denklem kurma ve çözme bileşenlerini destekleme üzere tasarlanmıştır. Bu makalede, örüntü kuralını bulma ile ilgili olan Görev 1, Görev 2 ve Görev 8'e dair bölümler raporlanacaktır.

Uygulamalar sırasında araştırmacı sınıfı 3 er ya da 4 er kişilik gruplara ayırmıştır. Öğretmen adayları her bir grupta uygulamadan sorumlu olmuştur. Öğretmen adayları, her bir görev öncesinde araştırmacı tarafından sağlanan yönergeler, uygulama prensipleri ve olası öğrenci hataları hakkında bilgilendirilmişlerdir. Ayrıca, öğretmen adaylarıyla görevlerin uygulama sürecinde karşılaşılabilecek Öğrenci Düşünmesine Dayalı Matematiksel Önemli Pedagojik Fırsatlar da paylaşılmıştır. Tüm uygulama süreci, öğrencilerin onayı alınarak kaydedilmiştir. Ayrıca, öğrencilerin görev kağıtları toplanarak veri analizinde kullanılmıştır.

Görev başlangıcında, öğrenciler ilk olarak 20 dakika boyunca bireysel olarak çalışarak cevaplarını görev kağıtlarına yazmışlardır. Bireysel çalışma sürecinin ardından, öğrencilerin fikirlerini paylaşmaları için grup tartışması gerçekleştirilmiş ve her bir öğrenci öğretmen adayları tarafından tek tek düşüncelerini açıklamaları için teşvik edilmiştir. Grup tartışması sırasında fikrini değiştiren öğrencilerden, yeni cevaplarını ek kağıtlara yazmaları istenmiştir. Öğretmen adayları, gerekli gördüklerinde öğrencilerden düşünme süreçlerini daha detaylı olarak açıklamalarını talep etmişlerdir. Eğer gruptaki hiçbir öğrenci doğru cevap verememişse, öğretmen adayları manipülatifler kullanarak ya da daha basit bir soru sorarak öğrencileri yönlendirmişlerdir. Bu şekilde, öğrencilerin düşünme süreçlerinin daha kapsamlı bir şekilde analiz edilmesine olanak sağlanmıştır.

Öğrencilerin cevapları ve görev uygulamaları sırasındaki sözlü ifadeleri, cebirsel düşünmelerini analiz etmek üzere incelenmiştir. Örüntü kuralını bulma bileşeni için 4 seviyeli bir kodlama şeması geliştirilmiştir. Hiç yanıt vermeyen veya ilgisiz cevaplar veren öğrencilerin cevapları "deneme yapılmayan cevap" olarak kodlanmıştır. Seviye 1, öğrencilerin sadece değişkeni anladıkları veya sadece bir değişkendeki değişime odaklandıklarını gösterirken, Seviye 2, öğrencilerin örüntü kuralını deneme yanılma yöntemiyle bulduklarını ya da kuralı bulsalar da kuraldaki katsayı/sabit terimin nereden geldiğini açıklayamadıklarını göstermektedir. Seviye 3 ise öğrencilerin bağımlı ve bağımsız değişken arasındaki ilişkiyi örüntü kuralına yansıttıklarını, katsayı ve sabit terimin nereden geldiğini açıklayabildiğini göstermektedir. Öğrencilerin cebirsel düşünmeleri, araştırmacı ve iki matematik eğitimcisi tarafından bağımsız bir şekilde kodlanmıştır. Kodlamaların uyuşma yüzdesi %97'dir.

Sonuçlar ve Tartışma

Video analizleri sonucunda öğrencilerin cebirsel düşünmelerinde üç bileşenin en az ikisinde ilerlediği görülmüştür. Özellikle örüntü kuralı bulma konusunda, Görev 1'den Görev 8'e geçerken öğrencilerin bireysel düşünmelerinde deneme yapılmayan cevap yüzdesinin azaldığı ve Seviye 2 ve Seviye 3 cevapların yüzdesinin arttığı gözlemlenmiştir. Ayrıca, her bir görev içerisinde bireysel çalışma sonrasındaki tartışmalarda cebirsel düşünme seviyesinin yükseldiği tespit edilmiştir.

Öğretmen adayları uygulama sırasında ortaya çıkan Öğrenci Düşünmesine Dayalı Matematiksel Önemli Pedagojik Fırsatları başarıyla değerlendirebilmişlerdir. Örneğin, Alper, Görev 2'de üç sorunun ikisinde sadece bağımlı değişkenin değişimine odaklanmıştır. Öğretmen adayı, bağımlı ve bağımsız değişkenlerin birlikte nasıl değiştiğine odaklanan sorular sormuştur. Ayrıca, soruları sorarken örüntü bloklarını manipülatif olarak kullanmıştır. Bu sayede, Alper örüntü kuralının arkasındaki mantığı anlamış ve bağımlı ve bağımsız değişkenin birlikte nasıl değiştiğine odaklanmıştır. Sonuç olarak, cebirsel düşünme seviyesi Seviye 2'ye yükselmiştir. Sonuçlar, öğrencilerin cebirsel düşünme becerilerini geliştirmek ve cebir öğrenimlerini desteklemek için görev temelli öğretimin etkili bir yöntem olabileceğini göstermektedir.

Çalışmanın elde ettiği sonuçlar, literatürdeki diğer çalışmalarla uyumlu görünmektedir. Örneğin, Saraswati ve Putri (2016) tarafından belirtildiği gibi, örüntü bloklarının manipülatif olarak kullanılması öğrencilerin cebirsel düşünme becerilerine olumlu etki yapmıştır. Ayrıca, Leatham vd., (2015) önerdiği Öğrenci Düşünmesine Dayalı Matematiksel Önemli Pedagojik Fırsatlar, bu çalışmada da öğretmen adayları tarafından değerlendirilmiştir. Bahsedilen bu fırsatlar, öğrenci düşüncesine dayandırıldığı için düşünme süreçlerini desteklemiş ve öğrencilerin cebirsel düşünme seviyelerinin artmasına katkıda bulunmuştur.