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The General Parametric Equation of Pythagoras Theorem and The General Connectedness Theorem

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Abstract

In this article, the Quarter Squares Rule is used to show that it also satisfies the Pythagoras Theorem. By using this information, it will be defined that there are parametric equations among sides and the radius of inner circle of a right triangle. It is, accordingly, be proved that a quadratic equation which has some definite properties is connected with a right triangle. On the strength of utilizing this connection, it will be obtained the general parametric equation of The Pythagoras Theorem. Thus, it is going to be stated the general connectedness theorem. There is, also, an interesting example which is the relation between golden ratio and Earth's axial tilt angle.

Keys words: Pythagoras Theorem, Generalization, Parametric equation, Connectedness theorem

Öz:

Bu makalede, Çeyrek Kareler Kuralının, Pisagor Teoremini de sağladığını göstermek için kullanılmıştır. Bu bilgi kullanılarak, bir dik üçgenin kenarları ve iç teğet çemberinin yarıçapının aynı parametrelere bağlı olduğu tanımlanacaktır. Buna bağlı olarak, belirli özelliklere sahip ikinci dereceden bazı denklemlerin, dik üçgenlerle bağlantılı olduğu kanıtlanıyor. Bu bağlantının kullanılması gereğince, Pisagor Teoreminin, genel parametrik denklemi elde edilecektir. Böylece, genel bağlantılık teoremi açıklanacaktır. Altın oran ile Dünya'nın eksen eğiklik açısı arasındaki ilişkiyi gösteren, ilginç bir de örnek bulunmaktadır.

Anahtar sözcükler: Pisagor Teoremi, Genelleştirme, Parametrik denklem, Bağlantılık teoremi

1. Introduction

Generalization studies on the Pythagorean Theorem (PT) to date can be summarized with two main approaches. First one is the generalization of the methods of obtaining Pythagoras triples. Starting with Barning [1]; Alperin [2], Gollnick, Scheid and Zöllner [3], Hall [4], Jaeger [5], Kanga [6], Préau [7], Emelyanov [8], Romik [9] and Price [10] made generalizations which are Primitive Pythagoras Triples (PPT) trees, concerning the first approach. All of them are in R^2. Second approach is parameterization studies on the PT.

$$(s^2 - t^2, 2st , s^2 + t^2) \tag{1}$$

This equation (1) has been known for centuries (Hardy and Wright [11] mentioned, Theorem 225, Euclid's method). Moreover, Tanay Roy and F. Jaishmin Sonia [12], Benjamin Edun [13], Beaure- gard and Suryanarayan [14] introduced new parameterzation methods for PT. In [12], authors proved a new parameterzation system for all primitive and non-primitive triples in R^n. Yet, it has not been written parametrically. Furthermore, in Edun's article, parameterzations of PT in R^2 have been used for obtaining equations containing one and two variables. On the other hand, in [13], a^2 = x.y which is the same as in this article. However, they were derived from different equations. In [13], it has been obtained from equation (1), but I obtained from Quarter Square Rule (QSR) [15] which is;

$$\left(\frac{x+y}{2}\right)^2 = \left(\frac{x-y}{2}\right)^2 + xy$$

Additionally, the authors reached equation (1) with their parametric method in [14]. As it can be observed, all generalizations are in R^2 except [12] that is in R^n but not parametric, whereas in this article, PT has parametric generalization in R^n and it is different from all. This article has two main goals. Firstly, the general parametric equation of PT will be represented. Secondly, it will be verified the general connectedness theorem in accordance with the general parametric equation of PT. In the second section, the relationship between QSR and PT will be proved. Using this proof, it will be revealed that sides and inner radii of a right triangle are connected with each other parametrically. As a consequence of this connection, it is going to be concluded that Earth's axial tilt angle is almost equal to the angle defined by quadratic equation of golden ratio. Besides all these, in this section, this parameterization is used to attain for the general parametric equation of PT. In third section, the general connectedness theorem will be introduced.

2. The Parametric Equations

2.1.) The parametric equation of the Pythagoras Theorem and of inner radii of right triangles

Theorem 2.1.1. The QSR , $(\frac{x+y}{2})^2$ - $(\frac{x-y}{2})^2$ = xy, (x,y $\in \mathbb{R}^+$, x > y) satisfies The Pythagoras Theorem as composing the parametric equations of sides of a right triangle where a and b are legs, c is hypotenuse;

$$a^2(x,y) = x.y \text{ and } a(x,y) = \sqrt{x.y} \tag{2}$$

$$b^2(x,y) = (\frac{x-y}{2})^2 \text{ and } b(x,y) = \frac{x-y}{2} \tag{3}$$

$$c^2(x,y) = (\frac{x+y}{2})^2 \text{ and } c(x,y) = \frac{x+y}{2} \tag{4}$$

Proof of Theorem 2.1.1. $a^2 + b^2 = c^2$ is PT. Substituting equations (2), (3) and (4) in PT;

$$a^2(x,y) + b^2(x,y) = c^2(x,y)$$

$$(\sqrt{x.y})^2 + (\frac{x-y}{2})^2 = (\frac{x+y}{2})^2$$

$$x.y + \frac{x^2 - 2xy + y^2}{4} = \frac{x^2 + 2xy + y^2}{4}$$

$$\frac{4xy + x^2 - 2xy + y^2}{4} = \frac{x^2 + 2xy + y^2}{4}$$

$$\frac{x^2 + 2xy + y^2}{4} = \frac{x^2 + 2xy + y^2}{4}$$

Also, b (x,y) = $\sqrt{x.y}$ and a(x,y) = $\frac{x-y}{2}$ could be chosen. But, as it can be easily seen that the theorem 2.1.1 and its proof are not changed.

Theorem 2.1.2. a, x, y $\in \mathbb{R}^+$, x > y, a^2= x.y ; it is formed right triangles using ‘’a’’ value as any leg of a right triangle.

Proof of Theorem 2.1.2. According to theorem 2.1.1, using any multipliers of a^2 as parameters, we can obtain b and c values to create a right triangle.

Example 1. a = $5\sqrt{3}$, it is going to be found out some right triangle possibilities of ‘’a’’ value which is considered as any leg of a right triangle.

a^2 = 75 and some multipliers of 75 are;

75 = 1.75 = 3.25 = 5.15 = $10\sqrt{3}\,.\,\frac{5\sqrt{3}}{2}$ and according to the theorems 2.1.1 and 2.1.2;

x=75, y=1; b=37, c=38 so (a,b,c) = ($5\sqrt{3}$, 37, 38)

x=25, y=3; b=11, c=14 so (a,b,c) = ($5\sqrt{3}$, 11, 14)

x=15, y=5; b=5, c=10 so (a,b,c) = ($5\sqrt{3}$, 5, 10)

x=$10\sqrt{3}$, y =$\frac{5\sqrt{3}}{2}$; b = $\frac{15\sqrt{3}}{4}$, c = $\frac{25\sqrt{3}}{4}$ so (a,b,c) = ($5\sqrt{3}$, $\frac{15\sqrt{3}}{4}$, $\frac{25\sqrt{3}}{4}$)

Theorem 2.1.3. a=2n+1, n is a positive integer; there is at least one possibility for x and y values yields Pythagoras triples.

Proof of Theorem 2.1.3. If ''a'' value is prime; a=2n+1 , $a^2 = 4n^2 + 4n + 1 = 1.(4n^2 + 4n + 1)$ so it has to be x = $4n^2 + 4n + 1$ and y = 1.

Using parametric equations (2), ,(3) and (4);

$$b = \frac{(4n^2 + 4n + 1) - 1}{2} = \frac{(4n^2 + 4n)}{2} = 2n^2 + 2n \quad (5)$$

$$c = \frac{(4n^2 + 4n + 1) + 1}{2} = \frac{(4n^2 + 4n + 2)}{2} = 2n^2 + 2n + 1 \quad (6)$$

(a,b,c) = (2n+1, $2n^2 + 2n$, $2n^2 + 2n + 1$), depending on n, all values are positive integers and they compose Pythagoras triples.

This theorem was introduced by Euclid. I, only, proved it in a different parametric way.

Theorem 2.1.4. The parametric equation of inner radii of a right triangle is;

$$r(x,y) = \frac{\sqrt{x.y} - y}{2} = \frac{a(x,y) - y}{2} . \quad (7)$$

Proof of Theorem 2.1.4.

$r = \frac{a+b-c}{2}$ [16] is the formula of inner radii of a right triangle according to its sides. Substituting parametric equations (2), (3) and (4) in equation r;

$$r = \frac{\sqrt{x.y} + \left(\frac{x-y}{2}\right) - \left(\frac{x+y}{2}\right)}{2} \text{, we get } r = \frac{\sqrt{x.y} - y}{2} = \frac{a(x,y) - y}{2} .$$

2.2.) The general parametric equation of Pythagoras Theorem

Theorem 2.2.1. (n,i $\in \mathbb{N}$), $x_1, x_2, .., x_n$ are parameters, $a_1, a_2, .., a_n, a_{n+1}$ are components;

$a_1(x_1, x_2, \ldots, x_n)=$

$\sqrt{\frac{x_1}{(n-1)}.(x_2+..+x_n)+\frac{2.x_2}{(n-1)}.(x_3+..+x_n)+\frac{2.x_3}{(n-1)}.(x_4+..+x_n)+\cdots+\frac{2.x_{n-2}}{(n-1)}.(x_{n-1}+x_n)+\frac{2.x_{n-1}}{(n-1)}.x_n}$

$=\frac{1}{\sqrt{n-1}}.\sqrt{x_1.(\sum_{i=2}^{n} x_i)+2.(\sum_{i=2}^{n-1} x_i(\sum_{j=i+1}^{n} x_j))}$ where $2 \leq i \leq n$,

$$\left.\begin{aligned}
a_2(x_1, x_2, \ldots, x_n) &= \frac{x_1 - x_2 + x_3 + x_4 + .. + x_n}{2\sqrt{n-1}} \\
a_3(x_1, x_2, \ldots, x_n) &= \frac{x_1 + x_2 - x_3 + x_4 + .. + x_n}{2\sqrt{n-1}} \\
a_4(x_1, x_2, \ldots, x_n) &= \frac{x_1 + x_2 + x_3 - x_4 + .. + x_n}{2\sqrt{n-1}} \\
&\vdots \\
a_{n-1}(x_1, x_2, \ldots, x_n) &= \frac{x_1 + x_2 + x_3 + \cdots - x_{n-1} + x_n}{2\sqrt{n-1}} \\
a_n(x_1, x_2, \ldots, x_n) &= \frac{x_1 + x_2 + x_3 + \cdots + x_{n-1} - x_n}{2\sqrt{n-1}}
\end{aligned}\right\} 2 \leq i \leq n,\ a_i = \frac{1}{2\sqrt{n-1}}.(\textstyle\sum_{k=i-1}^{n} x_k - 2.x_i)$$

$$a_{n+1}(x_1, x_2, \ldots, x_n) = \frac{x_1 + x_2 + x_3 + x_4 + .. + x_n}{2} = \frac{1}{2}.\sum_{i=1}^{n} x_i \ . \quad (8)$$

The general parametric equation of Pythagoras Theorem by using equations (8) is;

$$a_1^2(x_1, x_2, \ldots, x_n) + a_2^2(x_1, x_2, \ldots, x_n) + \ldots + a_n^2(x_1, x_2, \ldots, x_n) = a_{n+1}^2(x_1, x_2, \ldots, x_n) \quad (9)$$

which means;

$$\left(\sqrt{\frac{x_1}{(n-1)}.(x_2+..+x_n)+\frac{2.x_2}{(n-1)}.(x_3+..+x_n)+\cdots+\frac{2.x_{n-2}}{(n-1)}.(x_{n-1}+x_n)+\frac{2.x_{n-1}}{(n-1)}.x_n}\right)^2$$
$$+\left(\frac{x_1 - x_2 + x_3 + x_4 + .. + x_n}{2\sqrt{n-1}}\right)^2+\left(\frac{x_1 + x_2 - x_3 + x_4 + .. + x_n}{2\sqrt{n-1}}\right)^2+\left(\frac{x_1 + x_2 + x_3 - x_4 + .. + x_n}{2\sqrt{n-1}}\right)^2+\ldots+$$
$$\left(\frac{x_1 + x_2 + x_3 + \cdots - x_{n-1} + x_n}{2\sqrt{n-1}}\right)^2+\left(\frac{x_1 + x_2 + x_3 + \cdots + x_{n-1} - x_n}{2\sqrt{n-1}}\right)^2=\left(\frac{x_1 + x_2 + x_3 + .. + x_n}{2}\right)^2 \quad (10)$$

Proof of Theorem 2.2.1.

i) In first step, it is going to be equaled to the number of term x_k^2 in left and right side of the equation (10). Firstly, on the left side, it is obvious that there is not any term x_k^2 in a_1^2. From a_2^2 to a_n^2, there are (n – 1), term $\frac{x_k^2}{4.(n-1)}$. Remark; $(x_k)^2 = (-x_k)^2 = x_k^2$. Hence, total number of the term x_k^2 on the left side is;

$$(n-1).\left(\frac{x_k^2}{4.(n-1)}\right) = \frac{x_k^2}{4}. \quad (11)$$

On the right side, it is also obvious that there is only 1 $\frac{x_k^2}{4}$ in a_{n+1}^2.
In conclusion, both sides are equal to $\frac{x_k^2}{4}$ in equation (10).

ii) Now, it is going to be equaled to the number of term $x_1.x_k$ which is multiplication of x_1 to other parameters on both sides of the equation (10). The parameter x_1 has no negative value on both sides. Yet, other parameters have 1 negative value on the left side. So, coefficient x_1 is always positive but the others not. Therefore, the number of x_1 is calculated separately. For x_1; on the left side, there is only one $\frac{x_1.x_k}{n-1}$ (k = 2,3, …, n) for each x_k getting from a_1^2 . Furthermore, there is one negative $\frac{2.x_1.x_k}{4.(n-1)}$ and there are (n-2) positive $\frac{2.x_1.x_k}{4.(n-1)}$ for each x_k getting from the square of other components. On the right side, there is one positive $\frac{2.x_1.x_k}{4}$ for each x_k getting from a_{n+1}^2. Thus;

$$\left(\frac{x_1.x_k}{n-1}\right) + (n-2).\left(\frac{2.x_1.x_k}{4.(n-1)}\right)-\left(\frac{2.x_1.x_k}{4.(n-1)}\right)= \frac{2.x_1.x_k}{4} \tag{12}$$

$$\left(\frac{2.2.x_1.x_k}{4.(n-1)}\right)+ (n-2).\left(\frac{2.x_1.x_k}{4.(n-1)}\right)-\left(\frac{2.x_1.x_k}{4.(n-1)}\right)= \frac{2.x_1.x_k}{4}$$

$$(n-2-1+2).\left(\frac{2.x_1.x_k}{4.(n-1)}\right)= (n-1).\left(\frac{2.x_1.x_k}{4.(n-1)}\right)= \frac{2.x_1.x_k}{4}$$

$$\frac{2.x_1.x_k}{4} = \frac{2.x_1.x_k}{4}.$$

i) At the end, it is going to be equaled to the number of term $x_i.x_j$ multiplication of parameters different from x_1 means i ≠ j ≠ 1, i =2,3,4,..,n-1 and j = 3,4,..,n on both sides . For the left side, there is one positive $\frac{2.x_i.x_j}{n-1}$ from a_1^2 . There are (n – 3) positive $\frac{2.x_i.x_j}{4.(n-1)}$ and 2 negative $\frac{2.x_i.x_j}{4.(n-1)}$. The reason for having two negative terms is also x_i and x_j both have 1 negative term. On the right side, there is only one positive $\frac{2.x_i.x_j}{4}$. Thus;

$$\frac{2.x_i.x_j}{n-1} + (n-3).\frac{2.x_i.x_j}{4.(n-1)} - \frac{2.2.x_i.x_j}{4.(n-1)} = \frac{2.x_i.x_j}{4} \tag{13}$$

$$\frac{4.2.x_i.x_j}{4.(n-1)} + (n-3)\,.\,\frac{2.x_i.x_j}{4.(n-1)} - \frac{2.2.x_i.x_j}{4.(n-1)} = \frac{2.x_i.x_j}{4}$$

$$(n-3-2+4)\,.\,\frac{2.x_i.x_j}{4.(n-1)} = (n-1)\,.\,\frac{2.x_i.x_j}{4.(n-1)} = \frac{2.x_i.x_j}{4}$$

$$\frac{2.x_i.x_j}{4} = \frac{2.x_i.x_j}{4}\,.$$

In conclusion, equation (10) is the general parametric equation of PT.

3. The Connection of The Pythagoras Theorem with Quadratic Equations

3.1. . The basic connection

Theorem 3.1.1. The quadratic equation, a.X^2 + b.X + c =0, which has properties

i) a = 1

ii) b $<$ 0

iii) c $>$ 0

iv) b^2 – 4c $>$ 0 where b,c $\in \mathbb{R}$

connects parametrically with a right triangle.

Proof of Theorem 3.1.1. The roots of a quadratic equation are;

$$X_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } X_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}\;;\; X_1 > X_2\,.$$

It has to be chosen that the parametric equation of hypotenuse (equation 4) is equal to X_1. This is because, it is obvious that the other possibilities (X_1 is equal to any of right sides) is resulted in contradiction to compose a right triangle. For X_2, there are two possibilities;

$$X_2 = \sqrt{x.y} \text{ and } X_2 = \frac{x-y}{2}\,.$$

For simplicity, when we choose;

$X_1 = \frac{x+y}{2}$, $X_2 = \frac{x-y}{2}$. So; $X_1 + X_2 = (\frac{x+y}{2}) + (\frac{x-y}{2}) = x = -b$ and

$$X_1 . X_2 = (\frac{x+y}{2}) . (\frac{x-y}{2}) = (\frac{x^2-y^2}{4}) = c.$$

So we get;

$$(X - X_1 (x,y)) . (X - X_2 (x,y)) = (X - (\frac{x+y}{2})) . (X - (\frac{x-y}{2})) = X^2 - x . X + (\frac{x^2-y^2}{4}) = 0. \quad (14)$$

This means, the parameters are determined with using coefficients of quadratic equation;

$$x = - b , c = (\frac{x^2-y^2}{4}) \text{ and we get; } y = \sqrt{x^2-4c} . \quad (15)$$

On the other hand, when the quadratic equation is solved using the other possibility which is $X_1 = \frac{x+y}{2}$, $X_2 = \sqrt{x.y}$, $X_1 + X_2 = (\frac{x+y}{2}) + (\sqrt{x.y}) = -b$ and $X_1 . X_2 = (\frac{x+y}{2}) . (\sqrt{x.y}) = c$ finding x and y parameters is more difficult than the first possibility since another quadratic equation will appear.

We can, eventually, write the roots of a quadratic equation which has properties a =1, b $<$ 0, c $>$ 0 , $b^2 - 4c > 0$, b,c $\in \mathbb{R}$ and a right triangle with the same parameters in R^2. For both possibilities, one will reach the same right triangle.

Corollary 3.1.2. According to the theorems 2.1.1 and 3.1.1, a quadratic equation connects with two different right triangles.

Proof of Corollary 3.1.2. When the roots x_1 and x_2 are matched with three parametric components; $\frac{x+y}{2}$, $\frac{x-y}{2}$ and $\sqrt{x.y}$, six possible situations occur.

1) $X_1 = \frac{x+y}{2}$, $X_2 = \frac{x-y}{2}$ 2) $X_1 = \frac{x+y}{2}$, $X_2 = \sqrt{x.y}$	Triangle 1: X_1 is the hypotenuse and X_2 is one of the leg of a triangle.
3) $X_1 = \frac{x-y}{2}$, $X_2 = \frac{x+y}{2}$ 4) $X_1 = \sqrt{x.y}$, $X_2 = \frac{x+y}{2}$	Contradiction with $X_1 > X_2$ and the theorem 2.1.1.
5) $X_1 = \frac{x-y}{2}$, $X_2 = \sqrt{x.y}$ 6) $X_1 = \sqrt{x.y}$, $X_2 = \frac{x-y}{2}$	Triangle 2: X_1 and X_2 both are the legs of a right triangle.

Corollary 3.1.3. According to the theorems 2.1.1 and 3.1.1, a right triangle connects with three different quadratic equations that are;

i) $\frac{x+y}{2} = X_1$ and $\frac{x-y}{2} = X_2$, we get;

$$(X - (\frac{x+y}{2})) .(X - (\frac{x-y}{2})) = X^2 - x .X + (\frac{x^2-y^2}{4}) = 0 \tag{16}$$

ii) $\frac{x+y}{2} = X_1$ and $\sqrt{x.y} = X_2$, we get;

$$(X - (\frac{x+y}{2})) . (X - (\sqrt{x.y})) = X^2 - ((\frac{x+y}{2}) + (\sqrt{x.y})) .X + ((\frac{x+y}{2}) . (\sqrt{x.y})) =0 \tag{17}$$

iii) $\frac{x-y}{2} = X_1$ and $\sqrt{x.y} = X_2$, we get;

$$(X - (\frac{x-y}{2})) . (X - (\sqrt{x.y})) = X^2 - ((\frac{x-y}{2}) + (\sqrt{x.y})) .X + ((\frac{x-y}{2}) . (\sqrt{x.y})) =0 \tag{18}$$

Example 2. (An interesting example)

We will find the roots of the quadratic equation: $X^2 - \sqrt{5} . X + 1 = 0$.

The parameters are (equations (15));

x = $\sqrt{5}$ and y = $\sqrt{x^2 - 4c} = \sqrt{(\sqrt{5})^2 - 4.1}$ = 1.

The roots are;

$X_1 = \frac{x+y}{2} = \frac{\sqrt{5}+1}{2} = \Phi$ which is golden ratio and $X_2 = \frac{x-y}{2} = \frac{\sqrt{5}-1}{2} = \frac{1}{\Phi}$.

With this, we use the parameters x and y to compose a right triangle;

$$b(x,y) = \frac{x-y}{2} = \frac{\sqrt{5}-1}{2}$$

$$c(x,y) = \frac{x+y}{2} = \frac{\sqrt{5}+1}{2}$$

$$a(x,y) = \sqrt{x.y} = \sqrt{\sqrt{5}.1} = \sqrt[4]{5}$$

The sides of the golden ratio right triangle are; ($\sqrt[4]{5}$, $\frac{\sqrt{5}-1}{2}$, $\frac{\sqrt{5}+1}{2}$).

The acute angles (α , β) of the golden ratio right triangle are;

$\sin\alpha = \frac{\sqrt[4]{5}}{\frac{\sqrt{5}+1}{2}} = \frac{1{,}49534878122122\ldots}{1{,}618033988749894\ldots}$ = 0,92417637183…, so;

α = 67, 5444848… and β = 90 – α = 22, 4555151…

The degree of Earth's axial tilt angle is $23{,}43\ldots^{o}$ which is almost equal to β = $22{,}45\ldots^{o}$.

Corollary 3.1.4. In accordence with theorem 3.1.1 and its proof, the roots of any kind of quadratic equation which has form; $X^2 + b.X + c = 0$ are found by using equations (15). Firstly, one finds x and y parameters and then one obtains roots parametrically.

3.2. The general connectedness theorem

Theorem 3.2.1. n ∈ ℝ and/or n ∈ ℂ, n numbers are connected with each other with the general parametric equations (8) of PT.
[The General Connectedness Theorem].

Proof of Theorem 3.2.1. Any of n numbers are matched with any of parametric equations (8) of $a_2, a_3, .., a_n, a_{n+1}$ one by one. When all n numbers are different from each other, there are n! equation possibilities. For simplicity a_1 term is neglected inasmuch as it is a square root term. In fact, a_{n+1} is prefered instead of a_1 to avoid a quadratic equation and we, thus, obtain n equations which have first degree with n unknowns. One can easily find $X_1, X_2, X_3, \ldots, X_n$ values by using Gaussian elimination method. Then, a_1 is determined as a hid- den component. Furthermore, if the pairing changes, for every different possibilities, the parametric $x_1, x_2, .., x_n$ values vary. Besides these, r_1 , r_2 , r_3 ,…, r_k express the number of numbers that are equal to each other where $r_1 + r_2 + r_3 + \ldots + r_k = n$; hence, there are $\frac{n!}{r_1!.\, r_2!.\, r_3! \ldots r_k!}$ equation possibilities.

4. Conclusion

In this article, it has been verified that all sides and inner radii of a right triangle are connected parametrically with each other. Also, un- der certain conditions, a quadratic equation is connected to a right tri- angle. On the other hand, the most important result obtained in this article has been the ability to compose the general connectedness the- orem (GCT) by obtaining the general parametric equation of PT. As all we know, it is very significant for mathematics and other branches of science. We also get parametric equations among sides and the radius of inner circle of a right triangle. Furthermore, it has been proved that a quadratic equation with some definite properties is connected with a right triangle. Besides all these, The GCT is going to reveal new opportunities to solve polynomials and differential equations. Finally, I should state the question: ‘’ Does the general connectedness theorem overcome Abel-Rufini’s impossibility theorem?’’

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