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Research Article

Examination of Mathematics Course Contents Related to Proof in the Educational Informatics Network

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Abstract – This research aims to examine the mathematics course contents related to proof in the Educational Informatics Network. The document analysis method was used in this qualitative research. The data of the study consisted of 35-course contents related to proof in the Educational Informatics Network. These contents were subjected to content analysis. According to analysis results, it has been determined that there isn't enough proof in the contents of the Educational Informatics Network for all grade levels and in all learning domains. Although not in sufficient numbers, the proofs in these contents mostly consist of proofs belonging to the "Geometry" learning domain. The number of proofs in the "Numbers and Algebra" learning domain is limited. It is seen that no proof of the "Data, Counting, and Probability" learning domain is included. In line with the results of this research, it can be suggested to produce more content related to proof suitable for cognitive developments and grade levels by making the most of the potential of technology for all grade levels and all learning domains.

Keywords: COVID-19, distance education, mathematics learning media, online learning, proof teaching.

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Introduction

The World Health Organization (WHO) China Country Office was informed about the "cases of pneumonia of unknown etiology" in Wuhan City, Hubei Province, China, on the last day of 2019 (Ministry of Health [MoH], 2020, p. 5; World Health Organization [WHO], 2020a, p. 1). The number of reported cases from 31 December 2019 to 3 January 2020 was 44 (WHO, 2020a). On 7 January 2020, it was determined that the cause of these cases was a new coronavirus (2019-nCoV) (MoH, 2020; WHO, 2020a). On 13 January 2020, Thailand, on 15 January 2020, Japan, and on 20 January 2020, Korea reported the first cases of imported novel coronavirus (2019-nCoV) from Wuhan (WHO, 2020a). This disease, which affected the whole world in a short time (MoH, 2020; WHO, 2020a, 2023a), was accepted as a Public Health Emergency of International Concern (PHEIC) by WHO on January 30, 2020 (MoH, 2020; WHO, 2023a). On 11 February 2020, WHO Director-General Dr. Ghebreyesus first named the disease 2019-nCoV as **COVID-19** that is the acronym for **CO**rona, **VI**rüs, **D**isease, and 20**19** (MoH, 2020; New Jersey COVID-19 Information Hub, 2020; WHO, 2020b; Yaman, 2021). In the WHO's daily status reports, the disease was named COVID-19 for the first time in the 12 February 2020 report (WHO, 2020c). The causative virus of the disease was named severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) (MoH, 2020; WHO, 2020b). In a briefing on March 11, 2020, the WHO Director-General Dr. Ghebreyesus stated that COVID-19, which has been seen in 114 countries, can be characterized as a pandemic (MoH, 2020; WHO, 2020d, 2020e, 2023a). On 5 May 2023, the WHO announced that, although the COVID-19 pandemic wasn't over, it has ceased to be a global emergency for now and no longer complies with the definition of PHEIC (WHO, 2023a, 2023b). As of July 16, 2023, 6.9 million deaths and more than 768 million confirmed cases have been reported worldwide from this pandemic (WHO, 2023b).

Pandemics have serious social and economic impacts apart from deaths and important health problems (Yaman, 2021). One of the social institutions that has a close relationship with the field of health is the field of education (Yavuz & Toprakçı, 2021). Therefore, a series of measures were taken in the field of education, as in many other areas, to prevent the spread of coronavirus (Alanoğlu & Doğan-Atalan, 2021). Due to the COVID-19 pandemic, which was one of the biggest challenges facing education systems, many countries ended face-to-face education and switched to virtual training and online teaching (Daniel, 2020). The use of digital technology has also come into focus as the pandemic suddenly brought classrooms online around the world (Borba, 2021). Although distance and online learning concepts aren't new concepts, it has caused a deviation from the traditional classroom teaching model (Schneider & Council, 2021). Globally, 1.6 billion learners have been affected by this process (United Nations Educational, Scientific and Cultural Organization [UNESCO], 2021).

Following the announcement of the first case of COVID-19 in Türkiye on March 11, 2020 (MoH, 2020), the Ministry of National Education [MoNE] announced on March 13, 2020, that primary, secondary, and high school equivalent schools would be suspended for the week of Monday, March 16, 2020 - Friday, March 20, 2020 (Kırmızıgül, 2020; Türker & Dündar, 2020). In addition, MoNE decided to switch to distance education as of 23 March 2020 (Demir & Özdaş, 2020). With this decision, MoNE strengthened the infrastructure of the digital education platform Educational Informatics Network [EIN (EBA)], which has been serving since the 2011-2012 academic year, cooperated it with Turkish Radio and Television Corporation [TRT] and also implemented the psychosocial support system (Özer, 2020). As of August 31, 2020, schools were gradually opened to education with an application similar to the hybrid education model, but due to increasing cases, distance education was started again on November 17, 2020 (Yaman, 2021). Distance education, which was planned to continue until December 31, 2020, was extended until the beginning of the second semester, according to the course of the epidemic (Alanoğlu & Doğan-Atalan, 2021). Finally, as of September 6, 2021, face-to-face formal training was started for all levels, five days a week (Yaşa et al., 2022).

EIN, which has been on the air since 2012, and TRT EIN TV (TRT EBA TV) Primary School, TRT EIN TV Secondary School, and TRT EIN TV High School channels, which were established as a result of the cooperation with TRT, a public broadcaster, were used for education and training during the COVID-19 process (Türker & Dündar, 2020). In addition, distance education was supported with Zoom and similar applications (Balaman & Hanbay-Tiryaki, 2021). During the pandemic, EIN was a reliable meeting point where teachers interacted and communicated with their colleagues and students, and students with their teachers and classmates (EIN, 2020; Yapar et al., 2022). With the COVID-19 pandemic process, the content, functionality, usefulness, and current structure and status of such digital platforms have become even more important (Yapar et al., 2022). Based on this importance, there is an increasing tendency to study EIN-related studies in the literature. In the literature, there are studies examining the views of students (Atasoy & Yiğitcan-Nayir, 2019; Bahçeci & Efe, 2018; Durmuşçelebi & Temircan, 2017), pre-service mathematics teachers (Tuluk & Akyüz, 2019), mathematics teachers (Çavuş & Keskin-Yorgancı, 2020; Kepceoğlu & Ercan, 2019; Tuluk & Akyüz, 2019), and various branch teachers (Türker & Dündar, 2020) about EIN contents and the functionality and use of EIN. Apart from this, there are also studies in the literature that examine and evaluate the mathematics course contents in EIN from various perspectives (Dinler-Esim & Dinç-Artut, 2022; Günbaş & Öztürk, 2022; Poçan & Yaşaroğlu, 2017). For example, Dinler-Esim and Dinç-Artut (2022) examined mathematics course videos in the context of multimedia design principles. On the other hand, Poçan and Yaşaroğlu (2017) examined the contents of mathematics courses according to the seamless learning principles. There are also studies in the literature examining the effects of EIN use on students' cognitive or affective skills in various mathematics topics (Ertem-Akbaş, 2019; Özbey & Koparan, 2020). For example, Özbey and Koparan (2020) examined the effect of EIN-supported mathematics teaching on the motivation, attitude, and achievement of students on equality and equation. Unlike the studies in the literature, this study is aimed to examine the mathematics course contents related to proof in EIN. Like all learning environments, the mathematics learning media has also changed with the COVID-19 pandemic (Bullo, 2021; İlhan & Kırmızıgül, 2022). Distance education during the Covid-19 pandemic period has positively affected the use of digital technologies in mathematics education (Temel & Gür, 2022). How has the teaching of proof been affected by this change? Since proof is an important component of mathematics education and it is needed to improve mathematical understanding as it is at the center of mathematics (Ball et al., 2002); mathematics education researchers recommend that proof become part of students' mathematical experience for all grade levels (Stylianides, 2007). Despite this, mathematics education research shows that proof is very elusive to many students (Hoyles & Jones, 1998). In particular, the results of didactic studies show that students face many difficulties while approaching proving in the classroom (Olivero, 2002). It isn't difficult to foresee that students may experience difficulties in distance education in an area that has such difficulties even in face-to-face education. However, it may be possible to overcome these student difficulties with well-prepared course content. Because of all these reasons, answers to the following research questions were sought in this study:

- For which grade levels are the contents of mathematics courses related to proof are included in EIN?
- What kind of mathematics course contents related to proof are included in EIN?
- Which definitions are included in the contents about proof in EIN and how?

• Which proofs are included in the EIN, in which kind of contents, and how?

Method

This research was conducted following ethical principles and rules. Also, an ethics committee permission was obtained for the research. It was expressed in the official writing of the Kırklareli University Scientific Research and Publication Ethics Board dated 24 July 2023 and numbered E-35523585-302.99-91144 that this research didn't contain any ethical violations.

Research Design

This research included qualitative research, and document analysis (Creswell, 2014; Patton, 2001) was preferred as the research method.

Sampling and Data Collection

In qualitative research, data can be collected from written documents, audio or visual materials (Creswell, 2014). In this context, the data of this research were collected from course contents consisting of written documents, videos, or interactive activities related to proof in EIN.

The sample was determined by the criterion sampling technique, one of the purposeful sampling techniques (Creswell, 2014; Patton, 2001). For this, a set of inclusion and exclusion criteria (Creswell, 2014; Patton, 2001; Yıldırım & Şimşek, 2016) determined by the researcher were applied. The following criteria were determined as inclusion criteria: a) Mathematics course contents related to proof in EIN were included, and b) The contents that can be accessed in the spring term of the 2022/2023 school year were included. The following criteria were determined as exclusion criteria: a) Professional development contents and library contents were excluded, and b) Course contents other than mathematics were excluded.

Considering these criteria, the website of the EIN was accessed from https://www.eba.gov.tr/ (EIN, n.d.) on 25 July 2023. By typing proof in the EIN search engine, the mathematics course contents related to proof in EIN was surveyed (EIN, 2023). As a result of the search, a total of 51 pieces of content were found, including 39-course contents, 1 library content, and 11 professional development contents. Professional development content and library content were excluded from the scope of the study. Since these contents are related to the branches of Physics and Journalism, they were excluded from the scope. Of the remaining 39-course contents, four course contents of Turkish Language and Literature, Civil Law, and Physics were excluded from the scope. Thus, the remaining 35-course contents (EIN, 2023) were examined within the scope of the research. Only one of these contents is teacher-specific and the others are open to the access of both teachers and students.

Data Analysis

The data obtained from 35-course contents were analyzed by content analysis (Creswell, 2014; Julien, 2008; Yıldırım & Şimşek, 2016). In content analysis, data is analyzed in depth and codes are created so that previously non-existent categories or themes would be revealed (Creswell, 2014; Yıldırım & Şimşek, 2016). Qualitative analysis can be digitized in the report (Yıldırım & Şimşek, 2016) by counting the words, categories, or themes repeated in the document in the content analysis (Julien, 2008; Patton, 2001). In addition, one-to-one citations can be included in the reporting of qualitative data (Creswell, 2014).

In this research, in this direction, first of all, what kind of mathematics course contents related to proof in EIN was present for which grade levels were determined and codes were created. Based on these codes, frequency and percentage calculations were made and a cross table was created. Afterward, it was determined which definitions related to proof were included in which course contents and codes were created. The findings were presented with a frequency table. How these definitions are included in the contents were explained with examples and one-to-one quotations. Then, it was determined which proofs are included in which contents and codes were created. Again, the findings were presented with a frequency table. In addition, how these proofs are included in the course contents was explained with examples and one-to-one quotations.

Validity and reliability

In this research, the validity of the research was strived to be ensured by considering the criteria of confirmability, dependability, transferability, and credibility (Guba & Lincoln, 1982; Lincoln & Guba, 1985). The following points were taken into consideration for these criteria (Creswell, 2014; Guba & Lincoln, 982; Lincoln & Guba, 1985; Patton, 2001; Yıldırım & Şimşek, 2016). The contents in the EBA were analyzed impartially by the researcher to provide the confirmability criterion. The dependability criterion was tried to be provided by conducting all the processes of the research (accessing the contents, analyzing the contents, reaching the findings, etc.) in a way that is consistent with each other. The transferability criterion was tried to be taken into account by expressing which contents in the EBA were included in the research and which were excluded. Finally, the credibility criterion was tried to be taken into account by including detailed descriptions and raw data.

The reliability of this study was tried to be ensured by coding reliability (Miles & Huberman, 1994). Since this research is single-authored and the documents analyzed are voluminous, a time-dependent reliability study was conducted (Kirk & Miller, 1986; Yıldırım & Şimşek, 2016). All documents were coded three times by the researcher. The interval between the first two codings is fifteen days, and the interval between the second and third codings is approximately three months. No incompatibility was found between these three codings. In addition, some of the data was coded by another field expert to ensure consistency between coders (Miles & Huberman, 1994).

Findings

First, the findings of the “For which grade levels are the contents of mathematics courses related to proof are included in EIN?” and “What kind of mathematics course contents related to proof are included in EIN?” research questions are included. The frequency and percentage cross-table of the class levels and types of the course contents related to proof in EIN is given in Table 1.

Table 1 Frequency and Percentage Table for Class Levels and Types of Course Contents Related to Proof Included in EIN (EIN, 2023)

Grade level	Document (PDF) (f)	Interactive activity (f)	Video (f)	Total frequency (f)	Percentage (%)
Multi-grade (6^{th}, 7^{th}, 8^{th}) (f)	1	0	0	1	2.86
8^{th}-grade (f)	2	1	0	3	8.57
9^{th}-grade (f)	0	9	12	21	60.00
10^{th}-grade (f)	0	5	3	8	22.86
11^{th}-grade (f)	0	0	2	2	5.71
Total Frequency (f)	3	15	17	35	100.00
Percentage (%)	8.57	42.86	48.57	100.00	*

*Note. *Percentage values in the row and column have been rounded to the hundredths.*

As can be seen from Table 1, there is no course content at the primary school level. It is also observed that there is a very limited number of course contents at the secondary school level (11.43%). Most of the course contents related to proof included in EIN, namely, 21 of 35-course contents (60.00%), consists of 9^{th}-grade course content. This is followed by the

10th-grade course contents with 22.86%. It is seen that there are only 2 (5.71%) course contents belonging to the 11th-grade level.

Again, as seen in Table 1, 17 (48.57%) of the 35-course contents are videos, 15 (42.86%) are interactive activities, and only 3 (8.57%) are PDF documents. These course contents have been prepared by taking into account the learning gains in the mathematics course curriculum (MoNE, 2018a, 2018b, 2018c). Dynamic geometry software has been used in some video and interactive activities. Thanks to the features of dragging, animation, and dynamically adjusting such content, users have the chance to watch and interact repeatedly. 1 PDF document belonging to multi-grade level (6th, 7th, and 8th-grade) is a document specific to teachers. This document is an assessment-evaluation tool for proof methods for students in the special skills development program. Two PDF documents belonging to the 8th grade level are summary documents containing the same small historical pieces (Tzanakis & Arcavi, 2000) about the life of Pythagoras and two different proofs of the Pythagorean Theorem. The 8th grade interactive activity is the interactive activity of one of the proofs in the PDF documents. Elementary education level course contents are limited to these only.

There aren't PDF document in the content of the secondary education level courses. At the 9th-grade level, there are 9 interactive activities and 10 videos, which include definitions and examples of the components of the axiomatic system, and geometric proofs about angles in line, angle, side, and auxiliary elements in the triangle. Two of these videos are broadcast contents of EIN TV (EIN TV, 2020, 2021). There is also 1 video where De Morgan's Laws are proven in the truth table. Finally, there is 1 video with mathematical proofs to related multiplication and division operations in root numbers. At the 10th-grade level, there are 4 interactive activities and 2 videos with geometric proofs about rhombuses, parallelograms, trapeziums, and polygons. In addition, there are 1 video and 1 interactive activity with mathematical proofs about the roots of quadratic equations at the 10th-grade level. At the 11th-grade level, there are only 2 video contents. These two proofs also belong to trigonometry. The contents of the 12th-grade given within the scope of the Basic Proficiency Test [BPT] Mathematics course is a repetition of the topics of the previous grade levels. For example, the topic of "Propositions and Compound Propositions" of the 9th-grade is repeated within the scope of the 12th-grade BPT mathematics course.

Third, the findings of the "Which definitions are included in the contents about proof in EIN ? and how" research question is included. The frequencies of the definitions included in the contents are presented in Table 2.

Table 2 Frequency Table of the Definitions Included in the Course Contents Related to Proof in EIN (EIN, 2023)

Grade level	Learning domain	Sub-learning domain	Definitions	Interactive activity (f)	Video (f)	Total frequency (f)
9	Numbers and Algebra	Logic	Term	2	2	4
			Undefined term	1	1	2
			Defined term	1	1	2
			Definition	2	1	3
			Axiom (Postulate)	2	2	4
			Theorem	2	2	4
			Proof	1	2	3
			Hypothesis	2	1	3
			Conclusion	2	1	3
			Proposition	0	1	1
			Open proposition	0	2	2
			Conditional proposition	0	1	1
			Entailment	0	1	1
			Double entailment	0	1	1
			Quantifiers	0	1	1
			Truth set (Solution set)	0	1	1
			Negative of the open proposition	0	1	1

Note. While no definition is included in some contents, more than one definition is included in some contents.

As can be seen from this table, the definitions given in the contents related to proof in EIN are the definitions of the concepts belonging to the "Propositions and Compound Propositions" topic of the "Logic" sub-learning domain of the "Numbers and Algebra" learning domain in 9th-grade. Again, as can be seen from the table, the definitions of the concepts of "term", "axiom (postulate)", and "theorem" are the most common definitions in the contents related to proof in EIN. These are followed by the concepts of "definition", "proof", "hypothesis", and "conclusion".

From this point of view, it is seen that the definitions of the components of the axiomatic system, such as undefined term, defined term, definition, axiom (postulate), proposition, theorem, and proof are included in the course contents in EIN. However, it is seen that the definitions of components such as false propositions, disproof, lemma, and corollary aren't included in these 35-course contents. In addition, it is seen that the concepts given above have been defined by considering the criteria of being a definition. These definitions have been defined following the axiomatic structure and hierarchy. In addition, the necessary and sufficient features of the definitions have been given most economically. A specific example would be the definition of the axiom. The definition of axiom was defined as "Propositions whose truth is accepted and don't need to be proven are called the axiom" in the broadcast content on EIN TV (EIN TV, 2021, 22:03-22:14). In this definition, the concept of axiom is built on the concept of proposition following the hierarchical structure. In addition, all the features of the concept of axiom are given in a sentence most shortly.

These definitions have been given directly in the videos. This information is intended to be reinforced in interactive activities. For example, definitions of some of these concepts have been given in interactive activities. However, which concept a definition was referring to has been left blank. Users are asked to write which concept a definition was related to in these blanks. Users have a chance to interact more with control and retry buttons. In another example, in hypothesis and conclusion examples, a theorem has been given and it is aimed for users to be able to distinguish the hypothesis and conclusion of this theorem.

Finally, the findings of the "Which proofs are included in the EIN, as what kind of contents and how?" research question is included. The proofs at the 8^{th}, 9^{th}, 10^{th}, and 11^{th}-grade levels are tabulated separately. The frequency table of the proofs at the 8^{th} grade level is presented below.

Table 3 Frequency Table of Proofs Included in 8^{th}-Grade Level Course Contents in EIN (EIN, 2023)

Grade level	Learning domain	Sub-learning domain	Proofs	Document PDF (f)	Interactive activity (f)	Video (f)	Total frequency (f)
8	Geometry and Measurement	Triangles	The Pythagorean Theorem has been proven.	2	1	0	3

As can be seen from Table 3, the proofs at the 8^{th}-grade level in EIN are the proofs of the Pythagorean Theorem. Two PDF documents and an interactive activity about the Pythagorean Theorem are included in the "Triangles" sub-learning domain. Two of the many proofs of the Pythagorean Theorem (Nelsen, 1993) are included in these contents. While making these proofs, dynamic geometry software has been used. One of these two proofs is featured in both the video and the interactive activity. In the video and interactive activity, a right triangle with leg lengths a and b unit and hypotenuse length c unit has been given. A square has been formed on each side of this right triangle. And after, it has been proven that the sum of the areas of the small two squares formed by the legs equals the area of the large square formed by the hypotenuse, that is, $c^2 = a^2 + b^2$. In addition, it can be observed that even though the areas of the squares change when the corners of the triangle are dragged by the user in the interactive activity, this equality doesn't change. Then, the visual proof of this theorem is given. A similar 8th-grade visual drawn in GeoGebra, which is accessed from https://www.geogebra.org/ (GeoGebra., n.d.), is given below.

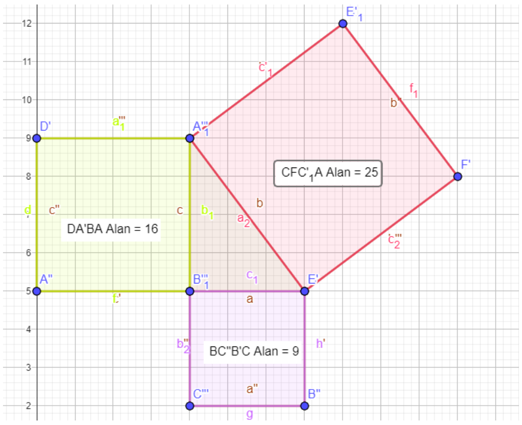

Figure 1 Any Proof of the Pythagorean Theorem with GeoGebra

In Figure 1, the relationship between the sides of the triangle, namely the Pythagorean Theorem, is easily seen from the relationship between the areas of the squares. Similar visual proofs are also included in the 8th-grade (Peker, 2021) and 9th-grade (Ayık, 2021a) textbooks. The frequency table of the proofs at the 9th-grade level is presented below.

Table 4 Frequency Table of Proofs Included in 9th-Grade Level Course Contents in EIN (EIN, 2023)

Grade level	Learning domains	Sub-learning domains	Proofs	Interactive activity (f)	Video (f)	Total frequency (f)
9	Numbers and Algebra	Logic	De Morgan's Laws have been proven.	0	1	1
		Equations and Inequalities	The equality $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ has been proven.	0	1	1
			The equality $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ has been proven.	0	1	1
9	Geometry	Triangles	It has been proven that the corresponding angles in a transversal of parallel lines are congruent.	1	0	1
			It has been proven that the alternate interior angles in a transversal of parallel lines are congruent.	1	0	1
			It has been proven that the alternate exterior angles in a transversal of parallel lines are congruent.	1	0	1
			It has been proven that the opposite interior angles in a transversal of parallel lines are supplementary.	1	0	1
			It has been proven that the opposite exterior angles in a transversal of parallel lines are supplementary.	1	0	1
			It has been proven that the sum of the measures of the interior angles of a triangle is 180°.	1	1	2
			It has been proven that the sum of the measures of the exterior angles of a triangle is 360^{0}.	0	1	1
			It has been proven that the sum of measures of two interior angles in a triangle is equal to the measure of non-adjacent exterior angles to these angles.	1	0	1
			The Interior Angle Bisector Theorem has been proven.	1	0	1
			It has been proven that if a point is on the bisector of an angle, then the point is equal distance from the sides of the angle.	1	0	1
			It has been proven that the centroid of triangles is the intersection of the medians.	0	1	1
			It has been proven that the centroids of triangles are at $\frac{2}{3}$ of the distance from the vertices to the mid-point of the sides.	0	1	1
			The Pythagorean Theorem has been proven.	1	2	3
			Euclid's Theorem has been proven.	0	1	1
			It has been proven that a point equal distance from the sides of an angle is on the bisector.	1	0	1
			It has been proven that in isosceles triangles, the bisector between the congruent sides cuts the opposite side perpendicularly.	1	0	1
			It has been proven that in equilateral triangles, each vertex angle bisector is the perpendicular bisector of the opposite sides.	1	0	1
			The Basic Proportionality Theorem has been proven.	0	1	1
			The Converse of Basic Proportionality Theorem has been proven.	0	1	1

As can be seen from Table 4, the proofs included in the EIN mostly belong to the 9th-grade level topics. While most of these proofs are related to the “Geometry” learning domain, proofs related to the “Numbers and Algebra” learning domain are also included. De Morgan’s Laws have been proven in the “Logic” sub-learning domain of the “Numbers and Algebra” learning domain. The equations $(p \vee q)' \equiv p' \wedge q'$ and $(p \wedge q)' \equiv p' \vee q'$, found by Augustus De Morgan (1806-1871), one of the founders of symbolic logic, have been proven by a truth table. These proofs can also be found in the 9th-grade textbooks (Ayık, 2021a; Gökbaş, Kaleci, Mutluoğlu, & Ballı, 2022; Ulualan, 2021). In addition, in the “Equations and Inequalities” sub-learning domain of the “Numbers and Algebra” learning domain, proofs about multiplication and division operations in root numbers with equal root degrees are included. These proofs in EIN are similar to the proofs in textbooks (Ayık, 2021a; Ulualan, 2021). In the “Triangles” sub-learning domain, there are proofs related to the topics “Basic Concepts of Triangles”, “Equality and Similarity in Triangles”, “Auxiliary Elements of Triangle”, and “Right Triangle and Trigonometry”.

As can be seen from the table, the proof that is mostly included in the course contents (f=3) is the proof of the Pythagorean Theorem, as it is at the 8th-grade level. Apart from that, it has been proven both in the video and in the interactive activity that the sum of the measures of the interior angles of the triangle is 180°. There is a visual proof of this theorem in the video. In this proof, the three interior angles of the triangle are folded to form a straight angle. A similar proof is given in Figure 2.

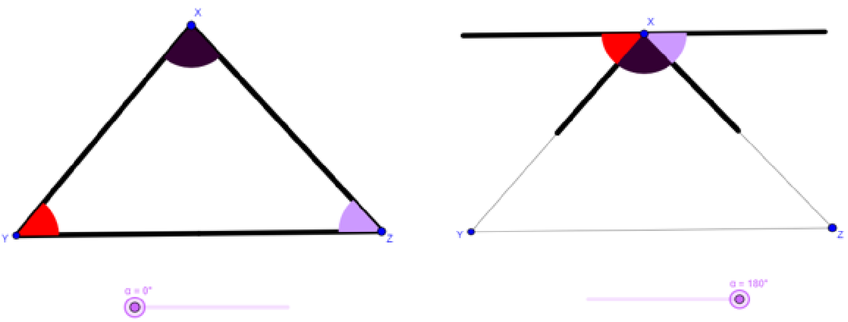

Figure 2 Any Proof with GeoGebra that the Sum of the Measures of the Interior Angles of a Triangle is 180°

The settings of the slider in the horizontal position, which is defined in terms of angle in Figure 2, are set as minimum=0°, maximum=180°, and increment=1°. It can be seen that the three interior angles of the triangle form a straight angle when the slider is dragged from 0° to 180°. The frequency table of the proofs at the 10th-grade level is presented below.

Table 5 Frequency Table of Proofs Included in 10th-Grade Level Course Contents in EIN (EIN, 2023)

Grade level	Learning domains	Sub-learning domains	Proofs	Interactive activity (f)	Video (f)	Total frequency (f)
10	Numbers and Algebra	Quadratic Equations	The relationship between the discriminant and the roots of quadratic equations has been proven.	0	1	1
			Root formulas of quadratic equations have been proven.	0	1	1
10	Geometry	Quadrilaterals and Polygons	It has been proven that the sum of the measures of the exterior angles of convex polygons is 360^0.	1	1	2
			It has been proven that consecutive angles in rhombuses are supplementary.	1	0	1
			It has been proven that opposite angles in rhombuses are congruent.	1	0	1
			It has been proven that diagonals in rhombuses are bisectors.	1	0	1
			It has been proven that in rhombuses, the diagonals intersect each other perpendicularly.	1	0	1
			It has been proven that opposite sides of parallelograms are congruent.	1	0	1
			It has been proven that the opposite angles of parallelograms are congruent.	1	0	1
			It has been proven that the diagonals of parallelograms are angle bisectors.	1	0	1
			It has been proven that consecutive angles in parallelograms are supplementary.	1	0	1
			It has been proven that the mid-segment of a trapezium is parallel to the bases.	1	0	1
			It has been proven that two adjacent angles between parallel sides in a trapezium are supplementary.	1	0	1
			It has been proven that the mid-segment length of a trapezium is equal to half the sum of the lengths of its parallel bases.	1	0	1
			It has been proven that the diagonals of a trapezium divide each other at the same rate.	1	0	1
			It has been proven that the trapezium is isosceles when the adjacent angles on the same base are congruent.	1	0	1
			It has been proven that the trapezium is isosceles when the diagonals divide each other into congruent line segments.	1	0	1

As can be seen from Table 5, proofs related to the "Geometry" learning field are mostly included at the 10th-grade level in EIN. Apart from this, proofs related to the “Numbers and Algebra” learning domain are also included. The relationship between the discriminant of quadratic equations and their roots, and accordingly, the root formulas have been proven by using real-life problems in the “Quadratic Equations” sub-learning domain of the “Numbers and Algebra” learning domain. It have been proven that for the equation $ax^2 + bx + c = 0$ when $a, b, c \in R$ and $a \neq 0$, if $\Delta < 0$ there is no real root of the equation, if $\Delta = 0$ there is a double real root of the equation and the formula is $x_1 = x_2 = -\frac{b}{2a}$, and if $\Delta > 0$ the equation has two different real roots and its formulas are $x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$ (Ayık, 2021b; MoNE, 2018b, 2018c; Ulualan, 2019). Proofs about the topics of “Polygons” and “Special Quadrilaterals” are included in the “Quadrilaterals and Polygons” sub-learning domain. It is seen that the proofs about the rhombuses, parallelograms, and trapeziums, which are special quadrilaterals, are included. As can be seen from the table, both the video and the interactive activity are included in the proof that the sum of the measures of the exterior angles of convex polygons is 360^0. A well-known algebraic proof, which is also included in 10th-grade textbooks (Ayık 2021b; Ulualan, 2019), is included in the video. The proof is started with the fact that the sum of the interior angles and exterior angles of n genes is $n \cdot 180^{\circ}$. It is also known that the sum of the interior angles of n genes is $(n-2) \cdot 180^{\circ}$. When the subtraction is done, $n \cdot 180^{\circ} - (n-2) \cdot 180^{\circ} = n \cdot 180^{\circ} - n \cdot 180^{\circ} + 360^{\circ} = 360^{\circ}$ is found. Thus, it has been proven that the sum of the measures of the exterior angles of n genes is 360. In the interactive activity, a visual proof of this theorem is included. In the interactive activity, users are asked to determine n corner points in the area reserved for visual proof. To complete the polygon, it is instructed to click on the first point determined, as in some dynamic geometry software. Thus, a polygon is formed, the sides of which form rays according to the number of points determined by the user. The exterior angles of the resulting polygon have been labeled. Users are then asked to drag the slider down. When the slider is dragged to the bottom, all exterior angles gather around a point and form a complete angle. It is possible to test this for all desired convex n genes. From this, it is possible to say visually that the sum of the measures of the exterior angles of any convex n gene is 360°. A similar proof for a convex pentagon is given in Figure 3.

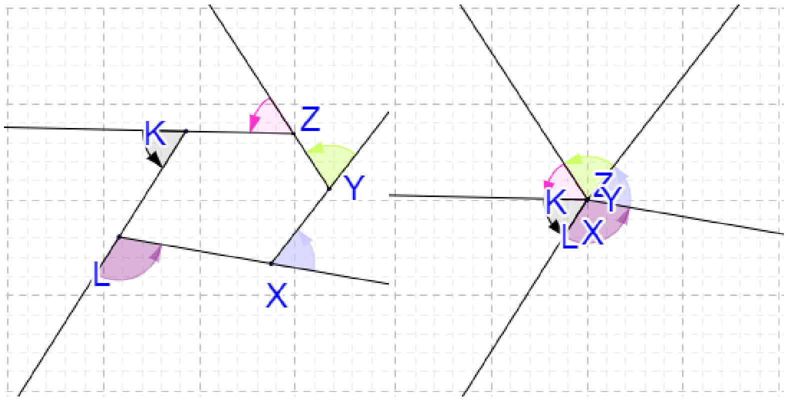

Figure 3 Any Proof with GeoGebra that the Sum of the Measures of the Exterior Angles of A Convex Pentagon is 360°

In Figure 3, it is seen that the exterior angles of a convex pentagon form one complete angle when the slider is moved. The frequency table of the proofs at the 11th-grade level is presented below.

Table 6 Frequency Table of Proofs Included in 11th-Grade Level Course Contents in EIN (EIN, 2023)

Grade level	Learning domain	Sub-learning domain	Proofs	Interactive activity (f)	Video (f)	Total frequency (f)
11	Geometry	Trigonometry	The Law of Cosines has been proven.	0	1	1
			The Law of Sines has been proven.	0	1	1

As can be seen from Table 6, only two proofs are included at the 11th-grade level in EIN. These proofs are proofs related to the "Trigonometric Functions" topic of the "Trigonometry" sub-learning domain of the "Geometry" learning domain. Proofs of Cosines Law and Sines Law are given in one video each. The Cosine Law has been proven by utilizing the real-life problem. In addition, the Sines Law has been proven by associating a triangle and the circumcircle of this triangle.

Conclusions, Discussions, and Suggestions

There are only 35-course contents related to proof in the EIN. It can be expected that this platform, which includes mathematics contents from elementary school to high school, will include more content about proof, which is an important component of mathematics education and is necessary in teaching every subject of mathematics. These course contents are mostly at the 9th-grade level. Contents related to proof are very limited in number until the 9th-grade of secondary education. There is no course content at the primary school level related to proof. The number of contents for the middle school level is also very limited. The number of proofs at the 11th and 12th-grades about proof in the EIN is also very limited. Because it is seen that there is no proof in the EIN regarding most subjects of these grade levels. Proof should be at the center of mathematics teaching for all grade levels (Ball et al., 2002). Accordingly, it can be recommended to produce contents suitable for the class level and cognitive development of students from the first grade of primary education to the last year of secondary education in EIN. By considering the changes in the meaning of proof as the individual matures from childhood to adulthood (Tall et al., 2012), the idea of proof in the contents produced can be expected to deepen.

Considering all grade levels, it is seen that no proof is included in the learning domain of the “Data, Counting, and Probability”. It is seen that a very limited number of proofs are included in the “Numbers and Algebra” learning domain. Considering the time and subject weight allocated to the learning domain the “Numbers and Algebra” in the curriculum (MoNE, 2018a, 2018b, 2018c), it is clear that the proofs included in EIN and presented in the tables in the findings are not sufficient for this learning domain. In this learning domain, which has a very broad subject content, only including proofs related to logic, equations and inequalities, and quadratic equations can be seen as a limitation. When all grade levels are considered, it is seen that proofs are given more place in the “Geometry” learning domain. Similarly, in the 9th-grade textbooks taught in Anatolian and Science High Schools in the 2022-2023 academic year (Ayık, 2021a; Gökbaş et al., 2022; Ulualan, 2021), the most proof has been made in the “Geometry” learning domain (Cihan, 2023). Although most learning gains in the 2018 high school mathematics curriculum have been allocated to the learning domain of the “Numbers and Algebra” (Cihan & Akkoç, 2023; MoNE, 2018b), it can be considered as a deficiency that there isn’t enough proof in the EIN about the topics in this learning domain. It is also thought-provoking that no proof is included in the “Data, Counting, and Probability” learning domain. The course contents in the EIN have been prepared by taking into account the learning gains in the mathematics curriculum. However, it is seen that enough content isn’t produced for every learning gain in every learning domain. From this point of view, it can be recommended to produce at least one written document, at least one video, and at least one interactive activity for each learning gains in each learning domain in terms of content richness. It shouldn’t be overlooked that interactive activities on digital platforms can serve as a process-oriented and constructivist approach to proof teaching. In process-oriented proof teaching, students in the role of active learners reach proof by doing it in the process (Yoo, 2008). Similarly, proof isn't presented as a product in interactive activities, the student reaches proof by interacting. Moreover, in interactive activities, students have the opportunity to make mistakes, see their mistakes and correct them.

Geometric proofs may require skills that are difficult to learn (Wong et al., 2011). For example, geometric proofs may require the use of shapes, transformations, diagrams, and inferences (Tall et al., 2012). Therefore, geometric proof teaching differs from mathematical proof teaching (Cihan, 2019). Also, dynamic visual proofs may be more effective than algebraic proofs for proof teaching (Štrausová & Hašek, 2013). At this point, the results of studies reporting the benefits of dynamic geometry environments for teaching geometric proof can be considered (Christou et al., 2004; Hoyles & Jones, 1998; Olivero, 2002). Dynamic geometry software has important functions for dynamically observing, discovering, generalizing, and proving any geometric feature (Özdemir-Erdoğan et al., 2020). By dragging the concrete object, users can interact with a dynamic figure where they can internalize abstract geometric ideas (Wong et al., 2011). It can be suggested that these potentials of dynamic geometry software be utilized more in proof-related videos and interactive activities.

The definitions included in the contents related to proof in the EIN are the definitions of the concepts belonging to the "Logic" sub-learning domain in 9th-grade. One of the learning gains aimed at students in the "Logic" sub-learning domain is "Explains the concepts of definition, axiom, theorem, and proof" (MoNE, 2018b, s. 18; MoNE, 2018c, s. 17). Therefore, definitions of these concepts are included in both video and interactive activities in EIN. It is seen that the definitions of the components of the axiomatic system such as undefined term, defined term, definition, axioms (postulate), proposition, theorem, and proof are included in the EIN, but the definitions of components such as a false proposition, disproof, lemma, and corollary aren't included. The components whose definitions are included and not included in the 9th-grade textbooks (Ayık, 2021a; Gökbaş et al., 2022; Ulualan, 2021) also agree with these results (Cihan, 2023).

Both the ebola, swine flu, bird flu, HIV/AIDS, MERS, SARS, and COVID pandemics seen in the last thirty years and the agreement of experts that new pandemics will emerge in the future also require preparation for the next pandemics (Öztek, 2020). The development of the contents of the EIN, which started to be used in 2012 and whose use and importance increased with the pandemic (Türker & Dündar, 2020; Özer, 2020), has now become an essential situation for every course. The same is true for higher education. In this context, it has been suggested in the literature for the Council of Higher Education to develop a national digital education platform that can be used in all universities in Türkiye (Durak et al., 2020). The best solution in extraordinary processes such as pandemics is distance education (Balaman & Hanbay-Tiryaki, 2021). Also, distance education platforms can also be used to supplement face-to-face education.

It may be thought that it is difficult for students to learn proof in the digital learning environment. However, having more than one access to course contents in EIN is among the advantages of EIN (Alanoğlu & Doğan-Atalan, 2021). The ability to watch videos related to proof more than once and to perform proof activities interactively by the user repeatedly can contribute to the teaching of proof. Because of all these, it can be predicted that increasing the quantity and quality of the contents related to proof may be beneficial in terms of teaching proof.

Suggestions regarding the aspects that teachers find lacking in EIN are available in the literature (Alanoğlu & Doğan-Atalan, 2021; Demir & Özdaş, 2020; Türker & Dündar, 2020; Yapar et al., 2022). EIN development studies can be carried out by taking these suggestions into account. In this study, the current situation regarding the proof has been revealed and suggestions have been made to EIN developers. It can be thought that these suggestions may be useful for teaching effective proof in online learning environments.

Compliance with Ethical Standards

Disclosure of potential conflicts of interest

No conflict of interest.

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CRediT author statement

This study was a single author and all processes of this research were carried out by the responsible author in accordance with ethical rules.

Research involving Human Participants and/or Animals

This research did not involve human participants and/or animals. This research was conducted following ethical principles and rules. Also, an ethics committee permission was obtained for the research. It was expressed in the official writing of the Kırklareli University Scientific Research and Publication Ethics Board dated 24 July 2023 and numbered E-35523585-302.99-91144 that this research didn't contain any ethical violations.

Eğitim Bilişim Ağındaki İspatla İlgili Matematik Ders İçeriklerinin İncelenmesi

Özet:

Bu araştırma Eğitim Bilişim Ağındaki ispatla ilgili matematik ders içeriklerini incelemeyi amaçlamaktadır. Bu nitel araştırmada doküman incelemesi yöntemi kullanılmıştır. Araştırmanın verilerini Eğitim Bilişim Ağındaki ispatla ilgili 35 ders içeriği oluşturmaktadır. Bu içerikler içerik analizine tabi tutulmuştur. Analiz sonuçlarına göre Eğitim Bilişim Ağındaki içeriklerde tüm sınıf düzeylerinde ve tüm öğrenme alanlarında yeterince ispat yapılmadığı saptanmıştır. Yeterli sayıda olmamakla birlikte bu içeriklerdeki ispatları çoğunlukla "Geometri" öğrenme alanına ait ispatlar oluşturmaktadır. "Sayılar ve Cebir" öğrenme alanına ait ispatlar sayıca sınırlı kalmaktadır. "Veri, Sayma ve Olasılık" öğrenme alanına ait hiçbir ispata yer verilmediği görülmektedir. Araştırmanın sonuçları doğrultusunda, tüm sınıf düzeylerinde ve tüm öğrenme alanlarında, teknolojinin potansiyellerinden en üst düzeyde yararlanılarak bilişsel gelişimlere ve sınıf seviyelerine uygun ispatla ilgili daha fazla içerik üretilmesi önerilebilir.

Anahtar kelimeler: COVID-19, çevrimiçi öğrenme, ispat öğretimi, matematik öğrenme ortamı, uzaktan eğitim.

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