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Nonlinear thermal analysis of serrated fins by using homotopy perturbation method

Homotopi pertürbasyon yöntemi kullanılarak kesikli dairesel kanatların lineer olmayan ısıl analizi

İshak Gökhan AKSOY[1*]

[1]Department of Mechanical Engineering, Engineering Faculty, Inonu University, Malatya, Turkey.
gokhan.aksoy@inonu.edu.tr

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Abstract

Thermal analysis of serrated fins which are consist of annular and plain sections are investigated. Serrated fin's thermal conductivity is assumed to change linearly with temperature. Nonlinear differential equations are obtained by applying the energy balance equation for both sections of the serrated fin and these equations are solved by applying homotopy perturbation method. Insulated fin tip, constant fin base temperature and common boundary conditions between the interface of two sections are considered. Serrated fin radii ratio (ε), segment height ratio (δ), thermo-geometric fin parameter (ψ) and thermal conductivity parameter (β) effecting the thermal performance and temperature distribution are investigated. The results showed that the homotopy perturbation is a reliable method for the solutions of such nonlinear differential equations. A very good agreement with the homotopy perturbation method and the numerical finite difference method are obtained. It is seen that, serrated fin efficiency lays between annular and rectangular fins and increases with the increase of segment height ratio and thermal conductivity parameter. Such as, fin efficiency values under the condition of $\varepsilon = 2$, $\psi_1 = 1.0$ and $\beta = 0$ for $\delta = 0$, 0.5, and 1 are 0.692, 0.718, and 0.762, respectively.

Keywords: Homotopy perturbation method, Serrated fin, Variable thermal conductivity.

Öz

Bu çalışmada, dairesel ve düz kısımlardan oluşan kesikli dairesel kanatçıkların ısıl performansları incelenmiştir. Kanatın ısı iletim katsayısının lineer olarak sıcaklığa bağlı olduğu kabul edilmiştir. Doğrusal olmayan diferansiyel denklemler, kesikli dairesel kanadın her iki bölümü için enerji dengesi denklemi uygulanarak elde edilmiş ve bu denklemler homotopi pertürbasyon yöntemi uygulanarak çözülmüştür. Yalıtılmış kanat ucu, sabit kanat taban sıcaklığı ve iki bölümün ara yüzü arasındaki ortak sınır koşulları göz önünde bulundurulmuştur. Isıl performansı ve sıcaklık dağılımını etkileyen kesik kanat yarıçap oranı (ε), kesik kanat yükseklik oranı (δ), termo-geometrik kanat parametresi (ψ) ve ısıl iletkenlik parametresi (β) incelenmiştir. Sonuçlar, homotopi pertürbasyon yönteminin, bu tür doğrusal olmayan diferansiyel denklemlerin çözümleri için güvenilir bir yöntem olduğunu göstermiştir. Homotopi pertürbasyon yönteminin sonuçları ile sayısal sonlu farklar yönteminin sonuçları arasında çok iyi bir uyum elde edilmiştir. Kesikli dairesel kanat veriminin dairesel ve dikdörtgen kanatçıklar arasında yer aldığı ve kesik kanat yükseklik oranının artmasıyla arttığı görülmektedir. Örneğin $\varepsilon = 2, \psi_1 = 1.0$ ve $\beta = 0$ durumunda $\delta = 0$, 0.5 ve 1 için kanat verimi değerleri sırasıyla 0.692, 0.718 ve 0.762'dir.

Anahtar kelimeler: Homotopi pertürbasyon yöntemi, Kesikli dairesel kanat, Değişken ısıl iletkenlik.

1 Introduction

Fins enhance the heat transfer between the solid and ambient fluid by increased surface area. Increased heat transfer surface areas can also be observed in nature, for example, the large ears of African elephants and rabbits, dolphin's dorsal fins, and flukes help to control body temperature by releasing excess heat. Heat exchangers have extensively used in the heating and cooling applications, for example in air conditioning, space heating and waste heat recovery systems, power and industrial plants, refrigerators, cooling of electronic devices. Various fin geometries such as rectangular, pin, helical, disk type, annular or radial plain fins are used in heat exchangers as an extended surface. An extensive review on the analysis of heat transfer in expanded surfaces has been presented by Kraus et al. [1]. Heat exchangers manufactured by using the serrated finned tubes are widely used for cooling and heating gases in cross-flow heat exchangers. Serrated fin geometry consists of two sections, namely, annular and segmented segments. There are many advantages to use serrated fined tubes over the other common plain or solid fined tubes [2]. Higher heat transfer convection coefficient required less heat transfer area for the same amount of heat transfer, better geometry to achieve turbulence regime, higher heat transfer, and hence lighter, smaller weight are some of the advantages of the serrated fins. An analytical study on the efficiency of the serrated fin is firstly performed by Hashizume et al [3]. Fin efficiency was derived analytically in terms of modified Bessel functions. In their analysis, the side edges of the segmented section of the serrated fin are assumed to be insulated and a uniform heat transfer coefficient is considered. They also performed experiments to determine the effect of assumptions. An experimental correction factor was determined to obtain actual fin efficiency.

Fin problems with temperature dependent properties are highly nonlinear problems. In most cases, temperature dependent thermal conductivity generally is assumed in problem formulation. Depending on the variation of temperature, the flow of free electrons and the amplitude of lattice vibrations change in solids such as metals and alloys. Therefore, the thermal conductivity increases or decreases

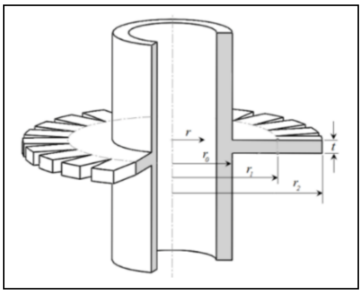
*Corresponding author/Yazışılan Yazar

with increasing in the temperature. Exact analytical solutions of such nonlinear fin problems are not possible, semi-analytical, approximation or numerical methods are used in these cases. Adomian decomposition method, variational iteration method, homotopy analysis method, homotopy perturbation method, variation of parameters method are some of these approximation methods. From these methods, homotopy perturbation method (HPM) is among the effective methods used to solve these type nonlinear differential equations. HPM is first proposed by He in 1999. It is a semi-analytical method which problem solution is obtained as a power series. Advantages of this method over other methods, it is not requiring a small parameter. HPM are used to solve many different non-linear and linear differential equations or problems [4]-[7]. Ganji [8] compared the HPM results with numerical and the perturbation methods in some heat transfer equations. Two different cases, namely cooling of a lumped system and temperature distribution in a thick rectangular fin are considered in the analysis. In their analysis, a variable specific heat capacity is considered. It is found that there is a noticeable difference between the HPM and perturbation method results when the effect of the nonlinear term is not negligible. Rajabi et al. [9] determined the temperature distribution in a lumped system of combined radiation-convection and a non-linear equation of the steady conduction in a slab with variable thermal conductivity by using HPM. The obtained results are compared with perturbation method and it is seen that nearly the same results were obtained in both methods. Hosseini et al. [10] applied homotopy perturbation method to obtain temperature distribution within a radiating rectangular fin with constant emissivity and variable thermal conductivity. Obtained results are compared with the results of Adomian decomposition method and a very good agreement is found. Chowdhury and Hashim [11] determined the temperature distribution of a rectangular fin by using HPM. It is considered a power-law temperature dependent surface heat flux. Obtained results with six terms are compared to Adomian decomposition solution with 13 terms. Domairry and Nadim [12] applied homotopy analysis method to solve nonlinear differential heat transfer equations. Results from homotopy analysis method are compared with the numerical and HPM results. Arslantürk [13] proposed a modified fin geometry with a change in thickness for better utilization of fin material. Temperature distribution inside the fin is estimated by using HPM. An optimum geometry has been also found for a given volume to maximizes the heat transfer. Ganji et al. [14] analyzed the temperature distribution of the annular fin by using HPM. Saedodin and Shahbabaei [15] applied HPM to analyze the performance of a porous rectangular fin. The dependence of temperature distribution on the convection parameter and porous parameter are investigated. Roy et al. [16] analyzed the heat transfer rates and local temperature distribution in a convective radiative fin by using HPM. In their analysis, both thermal conductivity and surface emissivity were assumed to vary with temperature. Cuce and Cuce [17] determined dimensionless temperature distribution, fin effectiveness, and efficiency expressions for rectangular porous fin via HPM as a function of convection and porosity parameters. It is found that porous fin temperature quickly decreases and fin rapidly reaches the ambient temperature when the convection and porous parameter increases. Arslantürk [18] proposed correlation equations for the rectangular profile annular fins. Optimization calculation was obtained by solving the nonlinear fin equation with the variation of parameters. Venkitesh and Mallick [19] studied the thermal characteristics of annular porous fins with hyperbolic and rectangular cross sections and internal heat generation by using HPM. Heat transfer through porous media is modelled by employed Darcy's model. It is found that the annular fin with a hyperbolic cross-sectional profile was more efficient than the fin with a uniform cross-section.

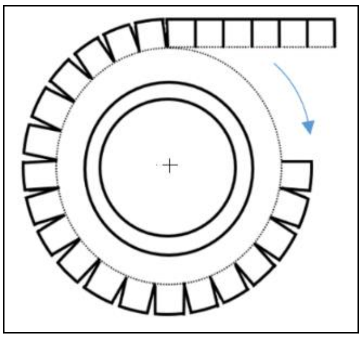
Within the scope of this study, thermal analyses of the serrated fin with temperature dependent of thermal conductivity are analyzed by using homotopy perturbation method. Dimensionless temperature distribution and fin efficiency are determined based on dimensionless parameters, that are serrated fin radius ratio, segment height ratio, thermal conductivity parameter and thermo-geometric parameter. Since serrated fin consists of annular and segmented sections, the problem formulation is performed in two steps. Two non-linear differential equations obtained from problem formulation are solved via HPM by defining appropriate boundary conditions. After that effects of some dimensionless geometric and thermal parameters on the thermal performance of fin are investigated. Results obtained for HPM are also compared with numerical finite difference results with variable thermal parameters and exact solution with constant thermal conductivity. Obtained results from HMP is consistent with numerical and exact results. It is also observed that dimensionless fin parameters have an important effect on the thermal performance of fin. The main contribution of this study is also to show how to apply the HPM to a system of nonlinear differential equations with common boundary condition.

2 Problem formulation

Schematics of serrated fin geometry is shown in Figure 1. Since serrated fin geometry consists of two sections, namely annular and plain sections as seen in Figure 1 (a)-(b), problem formulation has been carried out in two parts by considering these sections.



(a)



(b)

Figure 1. Schematics of serrated fin geometry. (a): Perspective view and (b): Top view.

The following assumptions are made in mathematical model of the serrated fin problem:

- One dimensional, steady-state heat conduction,
- Constant fin base temperature and thickness,
- No internal heat generation,
- Thermal conductivity is linearly dependent on temperature,
- No heat loss edge sides of serrated sections,
- Constant heat transfer coefficient and surrounding temperature,
- Convection from the fin surface.

$$\frac{d}{dr}\left(kA_{c1}\frac{dT_1}{dr}\right)dr-2h(T_1-T_a)dA_{s1}=0,\quad r_o\le r\le r_1 \tag{1}$$

$$\frac{d}{dr}\left(kA_{c2}\frac{dT_2}{dr}\right)dr-2h(T_2-T_a)dA_{s2}=0,\quad r_1\le r\le r_2 \tag{2}$$

and boundary conditions can be expressed as,

$$\begin{aligned} &T_1(r)=T_b \quad at \quad r=r_o\\ &T_1(r)=T_2(r)\ \ and\ \ \frac{dT_1}{dr}=\frac{dT_2}{dr}\quad at\quad r=r_1\\ &\frac{dT_2}{dr}=0\quad at\quad r=r_2\end{aligned} \tag{3}$$

Where, k is the thermal conductivity depended on temperature, dA_{c2}, dA_{s2}, A_{c1} and A_{s1}, are the elemental surface and cross sectional areas of the annular and rectangular sections, respectively. T_b is the base temperature of the serrated fin. Surface and cross-sectional areas will be $dA_{s1}=2\pi r dr, dA_{s2}=2\pi r_1 dr,\ A_{c1}(r)=2\pi rt, A_{c2}=2\pi r_1 t$.

Thermal conductivity is assumed to change linearly with temperature as,

$$k(T)=k_a(1+(T-T_a)\,\lambda) \tag{4}$$

Where λ is a parameter which indicates the variation of thermal conductivity and k_a is the serrated fin's thermal conductivity at ambient temperature.

Dimensionless variables can be defined as,

$$\begin{aligned} &\theta=\frac{T_1-T_a}{T_b-T_a},\quad \phi=\frac{T_2-T_a}{T_b-T_a},\quad \beta=\lambda(T_b-T_a)\\ &\xi=\frac{r-r_o}{r_o},\quad \zeta=\frac{r-r_1}{r_1},\quad \gamma_1=\frac{r_1}{r_o},\quad \gamma_2=\frac{r_2}{r_1}\\ &\psi_1^2=\frac{2hr_o^2}{tk_a},\quad \psi_2^2=\frac{2hr_1^2}{tk_a}=\psi_1^2\gamma_1^2\end{aligned} \tag{5}$$

and fin radii ratio and segment height ratio are defined respectively as,

$$\epsilon=\frac{r_2}{r_o}\equiv\gamma_1\gamma_2,\qquad \delta=\frac{r_2-r_1}{r_2-r_o}\equiv\frac{\gamma_1(\gamma_2-1)}{\gamma_1\gamma_2-1} \tag{6}$$

Thermo-geometric fin parameter (ψ_2) can also be written in terms of dimensionless parameters as

$$\psi_2\equiv\psi_1(\epsilon-\delta(\epsilon-1)) \tag{7}$$

Substituting dimensionless variables into Eqs. (1-2) and Eq.(3), the energy equation and boundary conditions reduces to,

$$\frac{d^2\theta}{d\xi^2}+\beta\theta\frac{d^2\theta}{d\xi^2}+\frac{1}{\xi+1}\frac{d\theta}{d\xi}+\frac{\beta}{\xi+1}\theta\frac{d\theta}{d\xi}+\beta\left(\frac{d\theta}{d\xi}\right)^2-\psi_1^2\theta=0\,,\quad 0\le\xi\le\gamma_1-1 \tag{8}$$

$$\frac{d^2\phi}{d\zeta^2}+\beta\phi\frac{d^2\phi}{d\zeta^2}+\beta\left(\frac{d\phi}{d\zeta}\right)^2-\psi_2^2\phi=0\ ,\quad 0\le\zeta\le\gamma_2-1 \tag{9}$$

$$\begin{aligned} &\theta=1\quad at\quad \xi=0\\ &\theta=\phi\quad and\quad \gamma_1\frac{d\theta}{d\xi}=\frac{d\phi}{d\zeta}\quad at\\ &\xi=\gamma_1-1\quad and\quad \zeta=0\\ &\frac{d\phi}{d\zeta}=0\quad at\quad \zeta=\gamma_2-1\end{aligned} \tag{10}$$

3 Theory of homotopy perturbation method

To describe the concept of HPM, consider the nonlinear differential equation in the form [4]:

$$A(u)-f(r)=0,\ \ r\in\Omega \tag{11}$$

With the boundary condition of

$$B\left(u,\frac{\partial u}{\partial n}\right)=0,\ \ r\in\Gamma \tag{12}$$

Where A is a general differential operator, B is a boundary operator, *f(r)* is a known analytic function and Γ is the boundary of the domain Ω.

The differential operator A may be expressed in two parts in terms of linear *L(u)* and nonlinear operators *N(u)* as follows:

$$A(u)=L(u)+N(u) \tag{13}$$

Then, the Eq. (11) may be written as:

$$L(u)+N(u)-f(r)=0 \tag{14}$$

A homotopy $v(r,p):\Omega\times[0,1]\to\mathcal{R}$ can be defined which satisfies the following equations as:

$$H(v,p)=(1-p)[L(v)-L(u_o)]+p[A(v)-f(r)]=0 \tag{15}$$

or

$$H\,(v,p)=L(v)-L(u_o)+pL(u_o)+p[N(v)-f(r)]=0 \tag{16}$$

Where $p\in[0,1]$ is an embedded homotopy parameter and u_o is the first approximation of Eq. (11) that satisfy the boundary conditions.

Eq. (17) and Eq. (18) can be written by using Eq. (15) and Eq. (16) as,

$$H(v,0)=L(v)-L(u_o) \tag{17}$$

and

$$H(v,1) = A(v) - f(r) \tag{18}$$

The changing of parameter p from $p=0$ to $p=1$, the $v(r,p)$ solution series is changing from $u_o(r)$ to $u(r)$ solution of equation. In topology, this is called deformation. Solution of equation may be written as power series of the powers of p.

$$v = v_o + pv_1 + p^2v_2 + p^3v_3 + \cdots \tag{19}$$

and p parameter approaches 1, the approximation solution can be expressed as follows:

$$u = \lim_{p\to 1} v = v_o + v_1 + v_2 + v_3 + \cdots \tag{20}$$

4 Serrated fin temperature distribution

Eq. (8) can be rearranged as:

$$(\xi+1)\frac{d^2\theta}{d\xi^2} + \beta(\xi+1)\theta\frac{d^2\theta}{d\xi^2} + \frac{d\theta}{d\xi} + \beta\theta\frac{d\theta}{d\xi} + \beta(\xi+1)\left(\frac{d\theta}{d\xi}\right)^2 - \psi_1^2(\xi+1)\theta = 0 \tag{21}$$

Then, linear and non-linear parts of Eq. (21) can be defined as,

$$L(\theta) = (\xi+1)\frac{d^2\theta}{d\xi^2} + \frac{d\theta}{d\xi} \tag{22}$$

$$N(\theta) = \beta(\xi+1)\theta\frac{d^2\theta}{d\xi^2} + \beta\theta\frac{d\theta}{d\xi} + \beta(\xi+1)\left(\frac{d\theta}{d\xi}\right)^2 - \psi_1^2(\xi+1)\theta \tag{23}$$

Also, linear and non-linear parts of Eq. (9) can be written as

$$L(\phi) = \frac{d^2\phi}{d\zeta^2} \tag{24}$$

$$N(\phi) = \beta\phi\frac{d^2\phi}{d\zeta^2} + \beta\left(\frac{d\phi}{d\zeta}\right)^2 - \psi_2^2\phi \tag{25}$$

According to the homotopy perturbation method, Eq. (21) can be expressed by using Eq. (16) as:

$$L(\theta) + pL(\theta_o) - L(\theta_o) + p\left[\beta(\xi+1)\theta\frac{d^2\theta}{d\xi^2} + \beta\theta\frac{d^2\theta}{d\xi^2} + \beta(\xi+1)\left(\frac{d\theta}{d\xi}\right)^2 - \psi_1^2(\xi+1)\theta\right] = 0 \tag{26}$$

With boundary conditions at the fin base,

$$\theta = 1 \quad at \quad \xi = 0, \qquad \frac{d\theta}{d\xi} = a \quad at \quad \xi = 0 \tag{27}$$

Where, a is a constant represents dimensionless temperature gradient at the fin base.

Substitution of θ in power form as in Eq. (19) into Eq. (26) and rewriting based on power of p-terms we have

p^0:

$$(\xi+1)\frac{d^2\theta_0}{d\xi^2} + \frac{d\theta_0}{d\xi} = 0 \tag{28}$$

$$\theta_0 = 1 \quad at \quad \xi = 0, \qquad \frac{d\theta_0}{d\xi} = a \quad \text{at} \quad \xi = 0 \tag{29}$$

p^1:

$$(\xi+1)\frac{d^2\theta_1}{d\xi^2} + \frac{d\theta_1}{d\xi} + \beta(\xi+1)\theta_o\frac{d^2\theta_o}{d\xi^2} + \beta\theta_o\frac{d\theta_o}{d\xi} + \beta(\xi+1)\left(\frac{d\theta_o}{d\xi}\right)^2 - (\xi+1)\psi_1^2\theta_o = 0 \tag{30}$$

$$\theta_1 = 0 \quad at \quad \xi = 0, \qquad \frac{d\theta_1}{d\xi} = 0 \quad \text{at} \quad \xi = 0 \tag{31}$$

p^2:

$$(\xi+1)\frac{d^2\theta_2}{d\xi^2} + \frac{d\theta_2}{d\xi} + \beta(\xi+1)\theta_o\frac{d^2\theta_1}{d\xi^2} + \beta(\xi+1)\theta_1\frac{d^2\theta_0}{d\xi^2} + \beta\theta_o\frac{d\theta_1}{d\xi} + \beta\theta_1\frac{d\theta_0}{d\xi} + 2\beta(\xi+1)\left(\frac{d\theta_o}{d\xi}\right)\left(\frac{d\theta_1}{d\xi}\right) - (\xi+1)\psi_1^2\theta_1 = 0 \tag{32}$$

$$\theta_2 = 0 \quad at \quad \xi = 0, \qquad \frac{d\theta_2}{d\xi} = 0 \quad at \quad \xi = 0 \tag{33}$$

p^3:

$$(\xi+1)\frac{d^2\theta_3}{d\xi^2} + \frac{d\theta_3}{d\xi} + \beta(\xi+1)\theta_o\frac{d^2\theta_2}{d\xi^2} + \beta(\xi+1)\theta_1\frac{d^2\theta_1}{d\xi^2} + \beta(\xi+1)\theta_2\frac{d^2\theta_0}{d\xi^2} + \beta\theta_o\frac{d\theta_2}{d\xi} + \beta\theta_1\frac{d\theta_1}{d\xi} + \beta\theta_2\frac{d\theta_0}{d\xi} + \beta(\xi+1)\left[2\left(\frac{d\theta_o}{d\xi}\right)\left(\frac{d\theta_2}{d\xi}\right) + \left(\frac{d\theta_1}{d\xi}\right)^2\right] - (\xi+1)\psi_1^2\theta_2 = 0 \tag{34}$$

$$\theta_3 = 0 \quad at \quad \xi = 0, \qquad \frac{d\theta_3}{d\xi} = 0 \quad \text{at} \quad \xi = 0 \tag{35}$$

⋮

$\theta_o, \theta_1, \theta_2, \ldots$ can be obtained by solving the Eqs. (28)-(35). When $p \to 1$, dimensionless temperature distribution of the annular section of the serrated fin can be expressed as:

$$\theta(\xi) = 1 + a\,ln(1+\xi) - \frac{1}{2}\left[a^2\beta\,ln(1+\xi)^2 - \frac{1}{2}\psi_1^2(a\xi^2 + 2a\xi + 2a - 2) + \frac{1}{2}\psi_1^2(a-1)(1+\xi)\,\xi\right] \pm \cdots \tag{36}$$

If the same procedure is followed in Eq. (9), it can be write,

$$L(\phi)+pL(\phi_o)-L(\theta_o)+p\left[\beta\phi\frac{d^2\phi}{d\zeta^2}+\beta\left(\frac{d\phi}{d\zeta}\right)^2-\psi_2^2\phi\right]=0 \quad (37)$$

$$\phi=b \quad at \quad \zeta=0, \quad \frac{d\phi}{d\zeta}=c \quad at \quad \zeta=0 \quad (38)$$

Where b and c are constants represents the dimensionless temperature and dimensionless temperature gradient between the annular and plain sections.

Based on power of p-terms we have,

p^0:

$$\frac{d^2\phi_0}{d\zeta^2}=0 \quad (39)$$

$$\phi_0=b \quad at \quad \zeta=0, \quad \frac{d\phi_0}{d\zeta}=c \quad at \quad \zeta=0 \quad (40)$$

p^1:

$$\frac{d^2\phi_1}{d\zeta^2}+\beta\phi_0\frac{d^2\phi_0}{d\zeta^2}+\beta\left(\frac{d\phi_0}{d\zeta}\right)^2-\psi_2^2\phi_0=0 \quad (41)$$

$$\phi_1=0 \quad at \quad \zeta=0, \quad \frac{d\phi_1}{d\zeta}=0 \quad at \quad \zeta=0 \quad (42)$$

p^2:

$$\frac{d^2\phi_2}{d\zeta^2}+\beta\phi_0\frac{d^2\phi_1}{d\zeta^2}+\beta\phi_1\frac{d^2\phi_0}{d\zeta^2}+2\beta\left(\frac{d\phi_0}{d\zeta}\right)\left(\frac{d\phi_1}{d\zeta}\right)-\psi_2^2\phi_1=0 \quad (43)$$

$$\phi_2=0 \quad at \quad \zeta=0, \quad \frac{d\phi_2}{d\zeta}=0 \quad at \quad \zeta=0 \quad (44)$$

p^3:

$$\frac{d^2\phi_3}{d\zeta^2}+\beta\phi_0\frac{d^2\phi_2}{d\zeta^2}+\beta\phi_1\frac{d^2\phi_1}{d\zeta^2}+\beta\phi_2\frac{d^2\phi_0}{d\zeta^2}+2\beta\left(\frac{d\phi_0}{d\zeta}\right)\left(\frac{d\phi_2}{d\zeta}\right)+\beta\left(\frac{d\phi_1}{d\zeta}\right)^2-\psi_2^2\phi_2=0 \quad (45)$$

$$\phi_3=0 \quad at \quad \zeta=0, \quad \frac{d\phi_3}{d\zeta}=0 \quad at \quad \zeta=0 \quad (46)$$

⋮

$\phi_o,\phi_1,\phi_2,\ldots$ can also be obtained by solving the Eqs. (39)-(46). When $p\to 1$, dimensionless temperature distribution of the plain section of the serrated fin can be expressed as:

$$\begin{aligned}\phi(\zeta)=b+c\zeta&+\frac{1}{2}(b^5\beta^4\psi_2^2-b^4c^2\beta^5-b^4\beta^3\psi_2^2+b^3c^2\beta^4\\&+b^3\beta^2\psi_2^2-b^2c^2\beta^3-b^2\beta\psi_2^2\\&+bc^2\beta^2+b\psi_2^2-c^2\beta)\zeta^2\\&+\frac{1}{6}(13b^4c\beta^4\psi_2^2-12b^3c^3\beta^5\\&-10b^3c\beta^3\psi_2^2+9b^2c^3\beta^4\\&+7b^2c\beta^2\psi_2^2-6bc^3\beta^3-4bc\beta\psi_2^2\\&+3c^3\beta^2+c\psi_2^2)\zeta^3+\cdots\end{aligned} \quad (47)$$

The constants, namely, a, b and c in Eq. (36) and Eq. (47) are determined with help of boundary conditions by applying Newton-Raphson method.

5 Fin efficiency

The ratio of the actual heat transfer rate to the ideal heat transfer rate from the fin is defined as fin efficiency. Ideal heat transfer is actually the state of maximum heat transfer which occurs when the entire fin surface is at the fin's base temperature, T_b. Serrated fin efficiency can be written as:

$$\eta=\frac{q_f}{q_{max}}=\frac{-kA_c\left.\frac{dT}{dr}\right|_{r=r_0}}{2\pi h[(r_1^2-r_0^2)+2r_1(r_2-r_1)](T_b-T_a)} \quad (48)$$

Fin efficiency can also be expressed in non-dimensioning form as:

$$\eta=\frac{2(1+\beta)\left.\frac{d\theta}{d\xi}\right|_{\xi=0}}{\psi_1^2(\epsilon-1)(\delta^2\epsilon-\delta^2-\epsilon-1)} \quad (49)$$

6 Results and analysis

Homotopy perturbation method reveals an analytical approximation solution to solving nonlinear differential equations formulated from the serrated fin problem. Accuracy of the results depend on the term's number taken in the solution. To get the more accurate results, the first five or six terms are considered in the solution. Obtained results are compared with the numerical finite difference method (FDM) and exact solutions results to verify the reliability of HPM. A comparison of the HPM results with the numerical and exact solution results are given in Table 1. Numerical results are obtained by using Maple software which uses finite difference method based on the Richardson extrapolation technique.

Table 1. The comparison of the HPM results with FDM and exact solution (in the case of $\epsilon=2, \delta=0.5, \psi_1=0.4$).

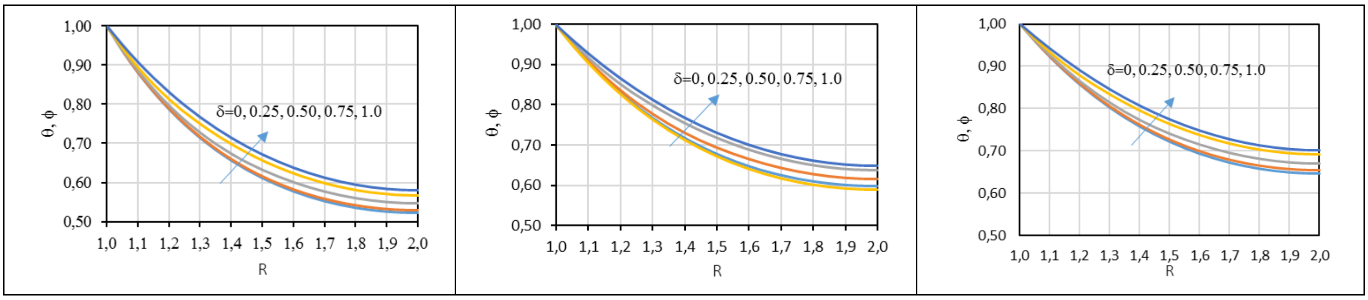
	$\beta=-0.3$		$\beta=0$			$\beta=0.3$	
R	HPM	FDM	HPM	FDM	EXACT	HPM	FDM
1.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1.1	0.973815	0.973805	0.981095	0.981095	0.981095	0.985221	0.985221
1.2	0.952246	0.952229	0.965338	0.965338	0.965338	0.972851	0.972853
1.3	0.934608	0.934586	0.952327	0.952327	0.952327	0.962601	0.962604
1.4	0.920381	0.920355	0.941749	0.941749	0.941749	0.954244	0.954247
1.5	0.909164	0.909137	0.933357	0.933357	0.933357	0.947597	0.947601
1.6	0.900066	0.900328	0.926733	0.926733	0.926733	0.942341	0.942344
1.7	0.893544	0.893518	0.921592	0.921592	0.921592	0.938255	0.938258
1.8	0.888702	0.888676	0.917926	0.917926	0.917926	0.935338	0.935341
1.9	0.885805	0.885779	0.915728	0.915728	0.915728	0.933588	0.933591
2.0	0.884841	0.884815	0.914996	0.914996	0.914996	0.933005	0.933008

Nonlinear ordinary equation system are solved by using dsolve function with the numeric option in Maple as a real-valued two-point boundary value problem (BVP). The dimensionless temperature distribution of the annular and planar sections was calculated dependent on dimensionless coordinates (ξ and ζ). While showing the temperature distribution in the Table-1 or other graphs, two dimensionless coordinates are normalized to a single dimensionless coordinate (R), and the temperature distribution is given depending on normalized radial R coordinate. As seen in Table 1, dimensionless temperature distribution results obtained from homotopy perturbation method are in accordance with the FDM results. In addition, results obtained from HPM is validated with the exact results under the condition of $\beta = 0$. Exact solution for serrated fin under the condition of constant thermal conductivity ($\beta = 0$) are obtained by Hashizume et al. (2002) in terms of Bessel functions. As seen in Table 1, almost the same results are obtained from all three solutions.

The dimensionless temperature change along the serrated fin on condition of $\psi_1 = 1$ and $\epsilon = 2$ against the different values of segment height ratio (δ) are shown in Figure 2(a)-(c) for different thermal conductivity parameter (β) values. Segment height ratio is the ratio of segment height to the total fin height. The segment height ratio equals to zero ($\delta = 0$) means that the fin is completely annular fin, and this ratio equals to one ($\delta = 1$) means that fin is the rectangular fin. The segment height ratio of the serrated fin varies between 0 and 1. From the figures, it is noted that temperature gradient throughout the fins decreases monotonically towards the fin tip. For all cases of β, temperature gradient throughout the serrated fin decreases with the increasing the segment height ratio. From the figures, it is also observed that, temperature gradient throughout the fin decreases with the increases conductivity parameter, β. This is a consequence of the nonlinearity of temperature dependent thermal conductivity. In the case of segment height ratio $\delta = 0.5$, fin tip dimensionless temperatures for the $\beta = -0.3, 0$ and 0.3 values are $0.547, 0.616$ and 0.671, respectively.

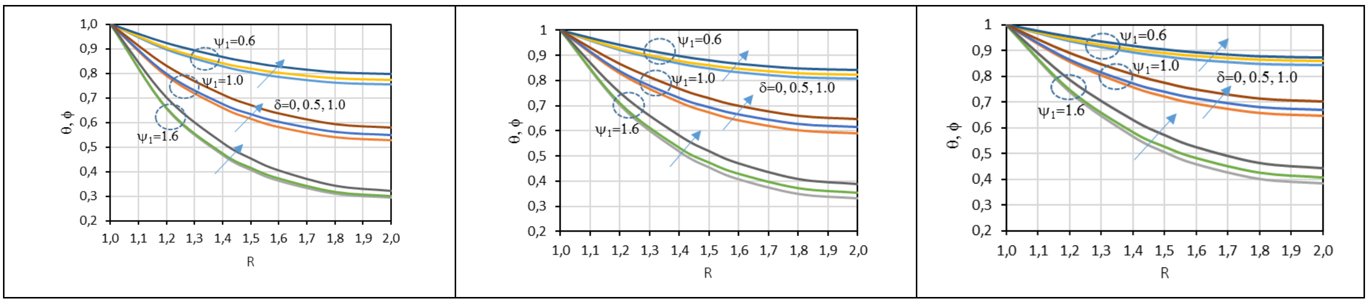
The influence of the thermo-geometric fin variable on the non-dimensional temperature distribution throughout the serrated fin under the condition of $\epsilon = 2$ against different values of segment height ratio (δ) are shown in Figure 3 (a)-(c) for different thermal conductivity parameter β values. Increasing thermo-geometric fin parameter (ψ) causes to increase the temperature gradient throughout the fin. The reason is that increases thermo-geometric fin parameter leads to a decrease in the fin's thermal conductivity. Therefore, internal resistance of the fin to conduction increases. As seen in figures, dimensionless temperature gradient stay between the annular ($\delta = 0$) and rectangular ($\delta = 1$) fins. In the case of ψ =1 and $\beta = -0.3$, the fin tip non-dimensional temperatures for the $\delta = 0, 0.5$ and 1 values are $0.526, 0.547$ and 0.579, respectively. As mentioned before, temperature gradient throughout the fin decreases with the increase of conductivity parameter, β. Such as, fin tip dimensionless temperatures in the case of $\psi_1 = 1$ and $\beta = 0.3$ for the $\delta = 0, 0.5$ and 1 is calculated as $0.646, 0.671$ and 0.701, respectively.

Serrated fin efficiency variation with respect to thermo-geometric fin parameter (ψ) at different values of segment height ratios (δ) and fin radius ratio (ε) are shown in Figure 4 (a)-(c) in the case of $\beta = -0.3, \beta = 0$ and $\beta = 0.3$, respectively. From the figures, it is observed that fin efficiency decreases with the increase of thermo- geometric fin parameter values, ψ for a specified ε, δ and β. Also fin efficiency increases with increasing values of segment height ratio for a specified ε, ψ and β.



(a) (b) (c)

Figure 2. Non-dimensional temperature distribution along the serrated fin for $\psi_1 = 1, \epsilon = 2$ and different values of β.
(a): $\beta = -0.3$, (b): $\beta = 0$, (c): $\beta = 0.3$.



(a) (b) (c)

Figure 3. Non-dimensional temperature distribution along the serrated fin for $\varepsilon = 2$ and different values of β.
(a): $\beta = -0.3$, (b): $\beta = 0$, (c): $\beta = 0.3$.

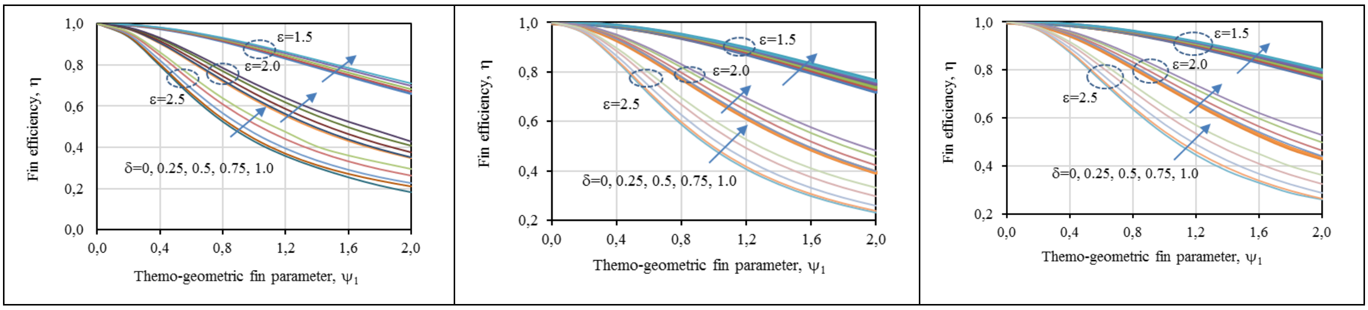


Figure 4. Serrated fin efficiency variation with respect to thermo-geometric fin parameter. (a): $\beta = -0.3$, (b): $\beta = 0$, (c): $\beta = 0.3$.

Such as, fin efficiency values under the condition of $\varepsilon = 2, \psi_1 = 1.2$ and $\beta = -0.3$ for $\delta = 0, 0.25, 0.5, 0.75$ and 1 are 0.552, 0.563, 0.586, 0.618 and 0.639, respectively. The minimum value of the fin efficiency reached under the condition of annular fin, $\delta = 0$. The highest value of the fin efficiency reached under the condition of rectangular fin, $\delta = 1$. Serrated fin efficiency lays between annular and rectangular fins depending on segment height ratio, δ. It is also observed that, efficiency of the fin decreases with increasing fin's radius ratio (ε) for a stated δ, ψ and β. Fin efficiency values under the condition of $\delta = 0.5, \psi_1 = 0.8$ and $\beta = 0.3$ for $\varepsilon = 1.5, 2.0$ and 2.5 are 0.956, 0.831 and 0.672, respectively. Increasing the value of β for a specified ε, ψ and δ values causes the fin efficiency to increase. For example, fin efficiency values for $\beta = -0.3, 0$ and 0.3 in the case of $\delta = 0.5, \psi_1 = 0.8$ and $\varepsilon = 2$ are 0.745, 0.796 and 0.831, respectively. It is also noted that, the exact solution results in terms of Bessel functions are closely same with the results of HPM under the condition of $\beta = 0$ constant thermal conductivity.

7 Discussion and conclusion

In the present work, homotopy perturbation method has been used for the nonlinear analysis of serrated fins with variable thermal conductivity. In problem formulation, serrated fin's thermal conductivity is taken as a linear function of temperature. Dimensionless temperature distribution and fin efficiency are determined based on dimensionless parameters, that are serrated fin radius ratio, segment height ratio, thermal conductivity parameter and thermo-geometric parameter.

The effect of these variables on the performance of serrated fin is investigated and given by graphs. It is observed that dimensionless parameters have an important effect on the dimensionless temperature variation along the fin and fin efficiency. HPM results are compared to the results obtained from the FDM, finite difference method. The results are also compared with the analytic exact solution in a specific case of constant thermal conductivity. A very good agreement between the HPM and FDM and exact results are found. The following outcomes of this study can be written as follows:

- Non-dimensional temperature gradient throughout the fin decreases with the increasing the segment height ratio and thermal conductivity parameter,
- Increasing thermo-geometric fin parameter causes the dimensionless temperature gradient along the fin to increases,
- Serrated fin efficiency lays between annular and rectangular fins depending on segment height ratio. Such as, fin efficiency values under the condition of $\varepsilon = 2$, $\psi_1 = 1.0$ and $\beta = 0$ for $\delta = 0, 0.5$, and 1 are 0.692, 0.718, and 0.762, respectively,
- Serrated fin efficiency decreases with increasing the thermo-geometric fin parameter and fin radii ratio. Such as, fin efficiency values under the condition of $\varepsilon = 2$, $\delta = 0.5$ and $\beta = 0.3$ for $\psi_1 = 0.2, 0.8$, and 1.2 are 0.987, 0.831, and 0.690, respectively,
- Serrated fin efficiency increases with increasing the thermal conductivity parameter and segment height ratio. Such as, fin efficiency values under the condition of $\varepsilon = 2, \delta = 0.5$ and $\psi_1 = 1.0$ for $\beta = -0.3, 0$ and 0.3 are 0.662, 0.718, and 0.761, respectively.

It should be expressed that, homotopy perturbation method can be applied to this type non-linear engineering problem.

8 Nomenclatures

A_c	:	Cross-sectional area (m^2),
dA_s	:	Elemental surface area (m^2),
h	:	Heat transfer coefficient (W/m^{2}K),
k	:	Thermal conductivity (W/mK),
r	:	Radial coordinate, radius (m),
r_o	:	Serrated fin base radius (m),
r_1	:	Serrated fin interface radius (m),
r_2	:	Serrated fin tip radius (m),
t	:	Fin thickness (m),
T	:	Temperature (^{o}C),
T_a	:	Ambient temperature (^{o}C),
T_b	:	Fin base temperature (^{o}C),
β	:	Thermal conductivity parameter
γ_1, γ_2	:	Radius ratio of the annular and rectangular segments,
δ	:	Segment height ratio,
ε	:	Fin radii ratio,
ξ	:	Dimensionless coordinate of the annular section,
ζ	:	Dimensionless coordinate of the rectangular section,
η	:	Serrated fin efficiency,
θ	:	annular section dimensionless temperature,
ϕ	:	Rectangular section dimensionless temperature,
λ	:	Parameter describing the variation of thermal conductivity,
ψ_1, ψ_2	:	Thermo-geometric fin parameter of the annular and rectangular sections,

R : Dimensionless normalized radial coordinate ($R = 1 + \xi$ for annular section, $R = \gamma_1 + (2 - \gamma_1)\zeta/(\gamma_2 - 1)$ for plain section).

9 Author contribution statement

İshak Gökhan AKSOY contributed to the formation of the idea, literature review, analysis and interpretation of the results, and writing of the article.

10 Ethics committee approval and conflict of interest statement

"There is no need to obtain permission from the ethics committee for the article prepared". "There is no conflict of interest with any person/institution in the article prepared."

11 References

[1] Kraus AD, Aziz A, Welty J. *Extended Surface Heat Transfer*. New York, USA, Wiley, 2001.

[2] Reid DR, Taborek J. "Selection criteria for plain and segmented finned tubes for heat recovery systems". *ASME Journal of Engineering for Gas Turbines and Power*, 116(2), 406-410, 1994.

[3] Hashizume K, Morikawa R, Koyama T, Matsue T. "Fin Efficiency of Serrated Fins". *Heat Transfer Engineering*, 23(2), 6-14, 2002.

[4] He JH. "Homotopy perturbation technique". *Computer Methods in Applied Mechanics and Engineering*, 178(3-4), 257-262, 1999).

[5] He JH. "A coupling method of a homotopy technique and a perturbation technique for non-linear problems". *International Journal of Non-Linear Mechanics*, 35(1), 37-43, 2000.

[6] He JH. "Homotopy perturbation method: A new nonlinear analytical technique". *Applied Mathematics and Computation*, 135(1), 73-79, 2003.

[7] He JH. "Homotopy Perturbation Method for Solving Boundary Value Problems". *Physics Letters A*, 350(1-2), 87-88, 2006).

[8] Ganji DD. "The application of He's homotopy perturbation method to nonlinear equations arising in heat transfer". *Physics Letters A*, 355(4-5), 337-341, 2006.

[9] Rajabi A, Ganji DD, Taherian H. "Application of homotopy perturbation method in nonlinear heat conduction and convection equations". *Physics Letters A*, 360(3-4), 570-573, 2007.

[10] Hosseini MJ, Gorji M, Ghanbarpour M. "Solution of temperature distribution in a radiating fin using homotopy perturbation method". *Mathematical Problems in Engineering*, 2009, 1-8, 2009.

[11] Cowdhury MSH, Hashim I. "Analytical solutions to heat transfer equations by homotopy perturbation method revisited". *Physics Letters A*, 372(8), 1240-1243, 2008.

[12] Domairry G, Nadim N. "Assessment of homotopy analysis method and homotopy perturbation method in non-linear heat transfer equation". *International Communications in Heat and Mass Transfer*, 35(1), 93-102, 2008.

[13] Arslantürk C. "Optimization of straight fins with a step change in thickness and variable thermal conductivity by homotopy perturbation method". *Journal of Thermal Science and Technology*, 30(2), 09-19, 2010.

[14] Ganji DD, Ganji ZZ, Ganji HD. "Determination of temperature distribution for annual fins with temperature-dependent thermal conductivity by HPM". *Thermal Science*, 15(1), 111-115, 2011.

[15] Saedodin S, Shahbabaei M. "Thermal Analysis of Natural Convection in Porous Fins with Homotopy Perturbation Method (HPM)". *Arabian Journal for Science and Engineering*, 38(8), 2227-2231, 2013.

[16] Roy PK, Das A, Mondal H, Mallick, A. "Application of homotopy perturbation method for a conductive-radiative fin with temperature dependent thermal conductivity and surface emissivity". *Ain Shams Engineering Journal*, 6(3), 1001-1008, 2015.

[17] Cuce E, Cuce PM. "A successful application of homotopy perturbation method for efficiency and effectiveness assessment of longitudinal porous fins". *Energy Conversion and Management,* 93, 92-99, 2015.

[18] Arslantürk C. "Variation of parameters method for optimizing annular fins with variable thermal properties". *Pamukkale University Journal of Engineering Sciences*, 24(1), 1-7, 2018.

[19] Venkitesh V, Mallick A. "Thermal analysis of a convective-conductive-radiative annular porous fin with variable thermal parameters and internal heat generation*". Journal of Thermal Analysis and Calorimetry*, 147, 1519-1533, 2022.