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PLANE KINEMATICS IN HOMOTHETIC MULTIPLICATIVE CALCULUS

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Abstract. In this study, pole points of motion, pole trajectories, velocities, accelerations and relations between velocities and accelerations are obtained. In addition we gave some new theorems

1. Introduction

Grossman and Katz introduced multiplicative calculus which is also called Non-Newtonian calculus. They defined derivative and integral in the multiplicative sense. We refer to Grossman and Katz [11], Stanley [18], Campbell [9], Grossman [12, 13], Jane Grossman [14, 15] for different kinds of Non-Newtonian calculus and its practices. Bashirov et al [3] given the entire mathematical definition of multiplicative calculus. An extension of multiplicative calculus to functions of complex variables can be found in [1, 2, 19, 20, 21]. Çakmak and Başar [8], characterized matrix transformations in sequence spaces based on multiplicative calculus. K. Boruah and B. Hazarika [5], have given the real number line and perpendicular axes system in multiplicative calculus. Gurefe [16], defined vector spaces, inner products and operations on matrices. K. Boruah and B. Hazarika [22] have given some conclusions about geometry. Selahattin Aslan et al. [23], gave geometric 3-space and multiplicative quaternions. Semra Kaya Nurkan et al. [24], gave vector properities of geometric calculus. Es [25], gave some basic concepts on the one-parameter motions witih multiplicative calculus.

2. BASIC CONCEPTS

Sentence $\mathbb{R}(G)$ can be defined as follows

$$\mathbb{R}(G) = \{\exp(p) = e^p : p \in \mathbb{R}\} \tag{2.1}$$

with the multiplicative addition

$$e^p \oplus e^r = e^{p+r} \tag{2.2}$$

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and the multiplicative multiplication

$$e^p \otimes e^r = e^{pr} \tag{2.3}$$

all $e^p, e^r \in \mathbb{R}(G)$. On the $\mathbb{R}(G)$ sentence, we can define addition $\oplus$ and multiplication $\otimes$, respectively (i.e.,$(\mathbb{R}(G), \oplus, \otimes)$), and it is a field with multiplicative zero $e^0 = 1$ and multiplicative identity $e^1 = e$. The connection between simple multiplicative operations and common arithmetic operations for each p, r elements of $\mathbb{R}(G)$ can be given as follows.

$$\begin{aligned}
p \oplus r &= p.r, \\
p \ominus r &= \frac{p}{r}, \\
p \otimes r &= p^{\ln r} = r^{\ln p}, \\
p \oslash r &= p^{\frac{1}{\ln r}},\ p \neq 1, \\
\sqrt{p}^G &= e^{(\ln p)^{\frac{1}{2}}}, \\
p^{-1_G} &= e^{\frac{1}{\log p}}, \\
\sqrt{p^{2_G}}^G &= |p|^G, \\
p^{2_G} &= p \otimes p = p^{\ln p}, \\
p \otimes e &= p,\ p \oplus 1 = p,
\end{aligned}$$

and thus we can write

$$\begin{gathered}
e^p \otimes e^r = e^{pr}, e^p \oplus e^r = e^{p+r}, \\
e^p \ominus e^r = e^{p-r}, e^p \oslash e^r = e^{\frac{p}{r}}, \\
\sqrt{e^p}^G = e^{\sqrt{p}}.
\end{gathered}$$

Positive and negative multiplicative real numbers can be defined as follows

$$\mathbb{R}^+(G) = \{m \in \mathbb{R}(G) : m > 1\}$$

and

$$\mathbb{R}^-(G) = \{m \in \mathbb{R}(G) : 0 < m < 1\},$$

respectively, [16, 20, 22].

The sentence $\mathbb{R}^2(G)$ is defined as follows

$$\mathbb{R}^2(G) = \{p^\circ = (e^{p_1}, e^{p_1}) : e^{p_1}, e^{p_1} \in \mathbb{R}(G)\} \subset \mathbb{R}^2$$

$$\begin{aligned}
p^\circ \oplus r^\circ &= (e^{p_1}, e^{p_2}) \oplus (e^{r_1}, e^{r_2}) \\
&= (e^{p_1} \oplus e^{r_1}, e^{p_2} \oplus e^{r_2}) \\
&= \left(e^{p_1+r_1}, e^{p_2+r_2}\right)
\end{aligned}$$

and the multiplicative scalar multiplication as

$$\begin{aligned}
e^c \otimes p^\circ &= e^c \otimes (e^{p_1}, e^{p_2}) \\
&= (e^c \otimes e^{p_1}, e^c \otimes e^{p_2}) \\
&= (e^{cp_1}, e^{cp_2}),
\end{aligned}$$

where $e^c \in \mathbb{R}(G),\ p^\circ,\ r^\circ \in \mathbb{R}^2(G)$.

Definition 1. *We can define multiplicative calculus absolute value as follows*

$$|p|^G = \begin{cases} p & , \quad p > 1, \\ 1 & , \quad p = 1, \\ p^{-1} & , \quad p < 1, \end{cases}$$

where $p \in \mathbb{R}(G)$ [20].

Definition 2. *The relationship between the multiplicative derivative and the classical derivative is as*

$$f^{*(n)}(x) = e^{(\ln f(x))^{(n)}}.$$

[1, 2, 3, 7, 10, 16].

Definition 3. *The relationship between trigonometry and multiplicative trigonometry is as* $\sin_g \theta = e^{\sin \theta}, \cos_g \theta = e^{\cos \theta}, \tan_g \theta = e^{\tan \theta} = \frac{\sin_g \theta}{\cos_g \theta}$ [4, 22, 23, 25].

Definition 4. *An* 2×2 *multiplicative matrix is defined by*

$$D = \begin{bmatrix} e^{d_{11}} & e^{d_{12}} \\ e^{d_{21}} & e^{d_{22}} \end{bmatrix}$$

where $e^{d_{11}}, e^{d_{12}}, e^{d_{21}}, e^{d_{22}} \in \mathbb{R}(G)$. *Let* D *and* G *be two multiplicative matrices and* $D \otimes G = E$ *be the multiplication of these matrices, where*

$$E = \begin{bmatrix} e^{d_{11}g_{11}+d_{12}g_{21}} & e^{d_{11}g_{12}+d_{12}g_{22}} \\ e^{d_{21}g_{11}+d_{22}g_{21}} & e^{d_{21}g_{12}+d_{22}g_{22}} \end{bmatrix}.$$

Definition 5. 2×2 *type identity matrix in multiplicative calculus is*

$$I = \begin{bmatrix} e & 1 \\ 1 & e \end{bmatrix}.$$

If matrix F *is a* 2×2 *type matrix and* $F^T \otimes F = F \otimes F^T = I$, *then* F *is called a multiplicative orthogonal matrix.*

3. PLANE KINEMATICS IN HOMOTHETIC MULTIPLICATIVE CALCULUS

Definition 6. *The inner product of* $\mathbb{R}^2(G)$ *in multiplicative plane is*

$$\langle \alpha, \beta \rangle^G = e^{\alpha_1 \beta_1 + \alpha_2 \beta_2}, \tag{3.1}$$

where $\alpha = (\alpha_1, \alpha_2)$, $\beta = (\beta_1, \beta_2) \in \mathbb{R}^2(G)$ [16, 23, 24, 25].

Definition 7. *The norm of a multiplicative vector* $\alpha = (\alpha_1, \alpha_2)$ *is*

$$\|\alpha\|^G = \sqrt{\langle \alpha, \alpha \rangle^G}^G = e^{\sqrt{\alpha_1^2 + \alpha_2^2}} \tag{3.2}$$

[16, 23, 24, 25].

Definition 8. *The multiplicative unit circle* $S^1(G)$ *in* $\mathbb{R}^2(G)$ *can be defined as*

$$\begin{aligned} S^1(G) &= \left\{ p^\circ = (e^{p_1}, e^{p_1}) \in \mathbb{R}^2(G) : \langle p^\circ, p^\circ \rangle^G = e \right\} \\ &= (\cos_g \theta, \sin_g \theta) = \left(e^{\cos \theta}, e^{\sin \theta} \right). \end{aligned} \tag{3.3}$$

Definition 9. *Let $u = (e^{u_1}, e^{u_2})$ and $v = (e^{v_1}, e^{v_2})$ be unit vectors in $\mathbb{R}^2(G)$. Then the equation*

$$\begin{bmatrix} e^{\cos\theta} & e^{-\sin\theta} \\ e^{\sin\theta} & e^{\cos\theta} \end{bmatrix} \otimes \begin{bmatrix} e^{u_1} \\ e^{u_2} \end{bmatrix} = \begin{bmatrix} e^{v_1} \\ e^{v_2} \end{bmatrix} \tag{3.4}$$

represents a rotation in $\mathbb{R}^2(G)$ of the multiplicative vector u by a multiplicative angle $\theta \in \mathbb{R}$ in positive direction around the origin $O = (1,1)$ of the Cartesian coordinate system of $\mathbb{R}^2(G)$. We will call this rotation as multiplicative planar rotation. After this rotation multiplicative vector u turns to the multiplicative vector v. Here $A(\theta) = \begin{bmatrix} e^{\cos\theta} & e^{-\sin\theta} \\ e^{\sin\theta} & e^{\cos\theta} \end{bmatrix}$ is a rotation matrix in multiplicative plane.

Definition 10. *In the multiplicative plane, a parameter homothetic multiplicative calculus motion is defined as*

$$\begin{bmatrix} Y \\ e \end{bmatrix} = \begin{bmatrix} h\otimes A & C \\ 1 & e \end{bmatrix} \otimes \begin{bmatrix} X \\ e \end{bmatrix} \tag{3.5}$$

where, $B = h \otimes A$, $A \in SO(2)_G$, A is a positive orthogonal matrix. Here $h = h(t)$, $A = A(t)$ and $C = C(t)$ are functions that can be differentiated with respect to the time parameter t to any order. Y, X and C are $2\times$ real matrices, and$Y$, X and $C \in \mathbb{R}^2_1(G)$. Equation 3.5 can be also given as

$$Y(t) = B(t) \otimes X(t) \oplus C(t) \tag{3.6}$$

$$Y = \begin{bmatrix} e^{v_1} \\ e^{v_2} \end{bmatrix}, X = \begin{bmatrix} e^{x_1} \\ e^{x_2} \end{bmatrix}, C = \begin{bmatrix} e^{a} \\ e^{b} \end{bmatrix},$$

where Y and X are the position vectors of the same point B, respectively, for the multiplicative fixed and multiplicative moving systems, and C is the multiplicative translation vector. By taking the derivatives with respect to t in 3.6, we get

$$Y^{\star} = B^{*} \otimes X \oplus B \otimes X^{*} \oplus C^{*} \tag{3.7}$$

Here $V_a = Y^{\star}, V_f = B^{*} \otimes X \oplus C^{*}$ and $V_r = B \otimes X^{*}$ are named absolute, sliding, and relative velocities of the multiplicative motion, respectively. These motions in multiplicative plane $R^2(G)$ are indicated by $B_1 = M/M'$ where M' and M are fixed and moving multiplicative planes, respectively. If the equation 3.7 is differentiated with respect to parameter t, we get

$$Y^{\star\star} = B^{**} \otimes X \oplus e^2 \otimes (B^{*} \otimes X^{*}) \oplus B \otimes X^{**} \oplus C^{**}, \tag{3.8}$$

$$b_a = b_r \oplus b_c \oplus b_f \tag{3.9}$$

where the velocities

$$b_a = Y^{\star\star}, b_f = B^{**} \otimes X \oplus C^{**}, b_r = B \otimes X^{**} \text{ and } b_c = e^2 \otimes (B^{*} \otimes X^{*}) \tag{3.10}$$

are named absolute acceleration, sliding acceleration, relative acceleration and Coriolis accelerations, respectively.

Definition 11. *The velocity vector V_r of the point X according to the moving plane M is called the relative velocity vector of X.*

Definition 12. *The velocity vector V_a of the point X according to the fixed plane M' is called the absolute velocity vector of X. Thus from equation 3.7 the relation between V_a, V_f, and V_r velocities is*

$$V_a = V_f \oplus V_r \tag{3.11}$$

If X is a fixed point in multiplicative moving plane M, then we have $V_a = V_f$, because $V_r = 1$. The equality 3.11 is said to be the velocity law of the motion $B_1 = M\,/M'$. Based on this information, we can state the following theorem.

Theorem 1. *In homothetic multiplicative motion, the absolute velocity vector is equal to the sum of the sliding velocity vector and the relative velocity vectors. So it is*

$$V_a = V_f \oplus V_r.$$

4. POLES OF ROTATING AND ORBIT IN HOMOTHETIC MULTIPLICATIVE CALCULUS

Definition 13. *In a homothetic parameter motion in the Euclidean sense, the points (where the sliding velocity V_f at each moment t is multiplicative zero for a fixed point X in space) are moving and fixed points on the fixed plane. These points are the pole points of the motion.*

Theorem 2. *In a motion $B_1 = M\,/M'$ whose multiplicative angular velocity is not multiplicative zero, there is a single point that remains fixed in both multiplicative fixed plane and multiplicative moving plane at each time t.*

Proof. Since point X is fixed in both the moving and fixed planes, $V_r = 1$ and $V_f = 1$. Therefore, for such points, if $V_f = 1$, then,

$$B^* \otimes X \oplus C^* = 1, \tag{4.1}$$

and

$$X = e^{-1} \otimes (B^*)^{m-inv} \otimes C^*,$$

where $(B^*)^{m-inv}$ is the multiplicative inverse of B^*. Since

$$B = e^h \otimes \begin{bmatrix} e^{\cos\theta} & e^{-\sin\theta} \\ e^{\sin\theta} & e^{\cos\theta} \end{bmatrix} = \begin{bmatrix} e^{h\cos\theta} & e^{-h\sin\theta} \\ e^{h\sin\theta} & e^{h\cos\theta} \end{bmatrix}, C = \begin{bmatrix} e^a \\ e^b \end{bmatrix},$$

$$B^* = \begin{bmatrix} e^{h'\cos\theta - h\theta'\sin\theta} & e^{-h'\sin\theta - h\theta'\cos\theta} \\ e^{h'\sin\theta + h\theta'\cos\theta} & e^{h'\cos\theta - h\theta'\sin\theta} \end{bmatrix}, C^* = \begin{bmatrix} e^{a'} \\ e^{b'} \end{bmatrix},$$

we get $\det^G(B^*) = e^{(h')^2 + (h\theta')^2} \neq 1$. Thus B^* is regular and

$$(B^*)^{m-inv} = e^{\frac{1}{(h')^2 + (h\theta')^2}} \otimes \begin{bmatrix} e^{h'\cos\theta - h\theta'\sin\theta} & e^{h'\sin\theta + h\theta'\cos\theta} \\ e^{-h'\sin\theta - h\theta'\cos\theta} & e^{h'\cos\theta - h\theta'\sin\theta} \end{bmatrix}. \tag{4.2}$$

□

Therefore, the equation $V_f = 1$ has a unique solution X. Point X is the pole point in plane M. Consequently from 3.1;

$$X = P = e^{\frac{1}{(h')^2 + (h\theta')^2}} \otimes \begin{bmatrix} e^{(-a'h' - b'h\theta')\cos\theta + (a'h\theta' - b'h')\sin\theta} \\ e^{(a'h\theta' - b'h')\cos\theta + (a'h' + b'h\theta')\sin\theta} \end{bmatrix}, \tag{4.3}$$

and the pole point in the fixed plane is given as

$$(P)' = B \otimes P \oplus C \tag{4.4}$$

If the necessary calculations are carried out it can be obtained

$$(P)' = e^{\frac{1}{(h')^2 + (h\theta')^2}} \otimes \begin{bmatrix} e^{-a'h'h - h^2 b'\theta'} \\ e^{h^2 a'\theta' - h'hb'} \end{bmatrix} \oplus \begin{bmatrix} e^a \\ e^b \end{bmatrix} \tag{4.5}$$

$$(P)' = \begin{bmatrix} e^{\frac{-a'h'h-h^2b'\theta'}{(h')^2+(h\theta')^2}+a} \\ e^{\frac{h^2a'\theta'-h'hb'}{(h')^2+(h\theta')^2}+b} \end{bmatrix}, \tag{4.6}$$

or as a vector

$$(P)' = \left(e^{\frac{-a'h'h-h^2b'\theta'}{(h')^2+(h\theta')^2}+a}, e^{\frac{h^2a'\theta'-h'hb'}{(h')^2+(h\theta')^2}+b} \right). \tag{4.7}$$

Here we assume that $\theta'(t) \neq 1$, for every t, i.e., non zero angular velocity. In this situation, there is only one pole point in each of the moving and fixed planes of each moment t.

Corollary 1. *If $\theta(t) = t$, then equation 4.3 will be obtained as*

$$X = P = e^{\frac{1}{(h')^2+h^2}} \otimes \begin{bmatrix} e^{(-a'h'-b'h)\cos\theta+(a'h-b'h')\sin\theta} \\ e^{(a'h-b'h')\cos\theta+(a'h'+b'h)\sin\theta} \end{bmatrix}.$$

Corollary 2. *For $\theta(t) = t$ and $h(t) = 1$, then equation 4.3 will be obtained as*

$$X = P = \begin{bmatrix} e^{a'\sin\theta-b'\cos\theta} \\ e^{a'\cos\theta+b'\cos\theta} \end{bmatrix}.$$

Corollary 3. *Let $\theta(t) = t$, then equation 4.7 will be obtained as*

$$P' = \left(e^{\frac{-a'h'h-h^2b'}{(h')^2+h^2}+a}, e^{\frac{h^2a'-h'hb'}{(h')^2+h^2}+b} \right).$$

Corollary 4. *For $\theta(t) = t$ and $h(t) = 1$, then equation 4.7 will be obtained as*

$$P' = \left(e^{-b'+a}, e^{a'+b} \right).$$

Definition 14. *In multiplicative plane motion, the point $P = (p_1, p_2)$ at time t is called the multiplicative pole of rotation or the center of sudden rotation.*

Theorem 3. *The relationship between the sliding velocity vector V_f and the pole passing from pole P to point X at every time t is as follows*

$$\|V_f\|^G \otimes \cos_g\theta = h^* \otimes \|P'Y\|^G$$

Proof. The pole point in multiplicative moving plane $Y = B \otimes X \oplus C$ implies that

$$X = (B)^{m-inv} \otimes \left(Y \oplus (e^{-1}) \otimes C)\right), \tag{4.8}$$

$$V_f = B^* \otimes X \oplus C^* \text{ and } B^* \otimes X \oplus C^* = 1$$

that leads to

$$X = P = e^{-1} \otimes (B^*)^{m-inv} \otimes C^*. \tag{4.9}$$

Now let us find pole points in multiplicative fixed plane. We have from equation

$$Y = B \otimes X \oplus C \tag{4.10}$$

$$Y' = P' = B \otimes \left(e^{-1} \otimes (B^*)^{m-inv} \otimes C^*) \oplus C\right),$$

Hence, we get

$$C^* = B^* \otimes (B)^{m-inv} \otimes \left(C \oplus (e^{-1} \otimes P')\right)$$

□

If we substitute this values in the equation $V_f = B^* \otimes X \oplus C^*$ we have $V_f = B^* \otimes (B)^{m-inv} \otimes P'Y$. Now let us calculate the value of $B^* \otimes (B)^{m-inv} \otimes P'Y$, where $P'Y = (e^{y_1-p_1}, e^{y_2-p_2})$, then

$$V_f = \begin{bmatrix} e^{\frac{h'}{h}(y_1-p_1)-\theta'(y_2-p_2)} \\ e^{\theta'(y_1-p_1)+\frac{h'}{h}(y_2-p_2)} \end{bmatrix} \tag{4.11}$$

or as a vector

$$V_f = \left(e^{\frac{h'}{h}(y_1-p_1)-\theta'(y_2-p_2)}, e^{\theta'(y_1-p_1)+\frac{h'}{h}(y_2-p_2)}\right), \tag{4.12}$$

hence we obtain

$$\begin{aligned}\langle V_f, P'Y\rangle^G &= \left\langle e^{\frac{h'}{h}(y_1-p_1)-\theta'(y_2-p_2)}, e^{\theta'(y_1-p_1)+\frac{h'}{h}(y_2-p_2)}, e^{y_1-p_1}, e^{y_2-p_2}\right\rangle^G \\ &= e^{\frac{h'}{h}\|P'Y\|^2}\end{aligned} \tag{4.13}$$

on the other hand we know that

$$\langle V_f, P'Y\rangle^G = \|V_f\|^G \otimes \|P'Y\|^G \otimes \cos_g \theta, \tag{4.14}$$

Thus, from the equalities in 4.13 and 4.14 we have that

$$\|V_f\|^G \otimes \cos_g \theta = h^* \otimes \|P'Y\|^G. \tag{4.15}$$

Corollary 5. *If the scalar matrix h is constant, the sliding velocity vector V_f is perpendicular to the pole ray passing from the pole P to vector X.*

Corollary 6. *In a $B_1 = M / M'$ multiplicative motion, the focus of the point X of M is an orbit, which it's normals pass through the rotation pole P.*

Theorem 4. *The norm of the sliding velocity vector is as*

$$\|V_f\|^G = \exp\left(\sqrt{\left(\frac{h'}{h}\right)^2 + (\theta')^2}\,\|P'Y\|\right). \tag{4.16}$$

Proof.

$$V_f = \left(e^{\frac{h'}{h}(y_1-p_1)-\theta'(y_2-p_2)}, e^{\theta'(y_1-p_1)+\frac{h'}{h}(y_2-p_2)}\right),$$

hence

$$\|V_f\|^G = \exp\left(\sqrt{\left(\frac{h'}{h}\right)^2 + (\theta')^2}\,\|P'Y\|\right).$$

□

Corollary 7. *If h is constant, the norm of the sliding velocity vector is*

$$\|V_f\|^G = \exp\left(|\theta'|\,\|P'Y\|\right). \tag{4.17}$$

Theorem 5. *The speed that occurs when drawing the curve (P) at point M at X is called V_r. At the same time, V_a is the speed that occurs when drawing the $(P)'$ curve of this point in the plane M'. These velocities are equal to each other at time t.*

Proof. $V_a = V_f \oplus V_r$, since $V_f = 1, V_a = V_r$. □

Definition 15. *The absolute acceleration vector of point X according to the plane M' is V_a. This vector V_a is determined by b_a. Since $V_a = Y^*$ then $b_a = V_a^* = Y^{**}$.*

Definition 16. *Let X be a fixed point on the moving plane M. This acceleration vector of the point X according to the fixed plane M' is called the sliding acceleration vector and is determined by b_f. Since acceleration of the multiplicative sliding acceleration X is a fixed point of M, then $b_f = V_f^* = B^{**} \otimes C^{**}$.*

5. ACCELERATIONS AND UNION OF ACCELERATIONS IN HOMOTHETIC MULTIPLICATIVE CALCULUS

Definition 17. *If the derivative of the vector $V_r = B \otimes X^*$ is taken, the vector $V_r^* = b_r = B \otimes X^{**}$ is obtained. The vector is called multiplicative relative acceleration vector and will be denoted by b_r. Considering point X as a moving point in M, matrix B is taken as constant*

Theorem 6. *Let X be a point moving in the plane M according to a parameter t. The relation between multiplicative acceleration formulas of this point is as*

$$b_a = b_r \oplus b_c \oplus b_f,$$

where $b_c = e^2 \otimes B^* \otimes X^*$ is denoted multiplicative Corilois acceleration.

Corollary 8. *If point X is a fixed point of multiplicative moving plane, multiplicative sliding acceleration of point X is equal to multiplicative absolute acceleration of that point.*

Proof. Note that

$$V_a = B^* \otimes X \oplus B \otimes X^* \oplus C^*,$$

Differentiating the both sides we have

$$V_a^* = B^{**} \otimes X \oplus e^2 \otimes (B^* \otimes X^*) \oplus B \otimes X^{**} \oplus C^{**},$$

since the point X is constant its derivative is 1. Hence

$$\begin{aligned} b_a &= V_a^* \\ &= B^{**} \otimes X \oplus C^{**} \\ &= b_f. \end{aligned}$$

□

Theorem 7. *The relationship between V_r and b_c can be given as*

$$\langle b_c, V_r \rangle^G = \exp\left(2hh'({x_1'}^2 + {x_1'}^2\right).$$

Proof.

$$\begin{aligned} V_r &= B \otimes X^*, \\ b_c &= e^2 \otimes (B^* \otimes X^*), \end{aligned}$$

So it is obvious that

$$\langle b_c, V_r \rangle^G = \exp\left(2hh'({x_1'}^2 + {x_1'}^2\right).$$

□

Corollary 9. *If h is a constant, then the Coriolis acceleration b_c is perpendicular to the relative velocity vector V_r at each instant moment t.*

6. THE ACCELERATION POLES ON THE MOTIONS

The solution of the equation $b_f = V_f^* = B^{**} \otimes X \oplus C^{**}$ gives us multiplicative acceleration pole of multiplicative motion.$V_f^* = B^{**} \otimes X \oplus C^{**}$ implies $X = e^{-1} \otimes (B^{**})^{m-inv} \otimes C^{**}$. Now calculating the matrices $e^{-1} \otimes (B^{**})^{m-inv}$ and C^{**}, and setting these in $X = P_1 = e^{-1} \otimes (B^{**})^{m-inv} \otimes C^{**}$, we obtain

$$X = P_1 = \begin{bmatrix} e^{\frac{1}{T}\left(a''(-r\cos\theta + z\sin\theta) - b''(r\sin\theta + z\cos\theta)\right)} \\ e^{\frac{1}{T}\left(a''(r\sin\theta + z\cos\theta) + b''(-r\cos\theta + z\sin\theta)\right)} \end{bmatrix}, \tag{6.1}$$

Here, the first-order pole curve of the plane M is denoted by P_1. If the pole curve of the plane M' plane is represented by P_1', then

$$P_1' = B \otimes P_1 \oplus C \tag{6.2}$$

Hence

$$P_1' = \begin{bmatrix} e^{\frac{1}{T}\left(-hra'' - hzb''\right) + a} \\ e^{\frac{1}{T}\left(hza'' - hrb''\right) + b} \end{bmatrix} \tag{6.3}$$

or as a vector

$$P_1' = \left(e^{\frac{1}{T}\left(-hra'' - hzb''\right) + a}, e^{\frac{1}{T}\left(hza'' - hrb''\right) + b}\right), \tag{6.4}$$

where $r = h'' - h(\theta')^2, z = 2h'\theta' + h\theta'', \quad T = r^2 + z^2$.

Corollary 10. *If $\theta(t) = t$, then equation 6.1 will be obtained as*

$$X = P_1 = \begin{bmatrix} e^{\frac{1}{(h''-h)^2 + 4(h')^2}\left(a''(-(h''-h)\cos\theta + 2h'\sin\theta) - b''((h''-h)\sin\theta + 2h'\cos\theta)\right)} \\ e^{\frac{1}{(h''-h)^2 + 4(h')^2}\left(a''((h''-h)\sin\theta + 2h'\cos\theta) + b''(-(h''-h)\cos\theta + 2h'\sin\theta)\right)} \end{bmatrix} \tag{6.5}$$

Corollary 11. *If $\theta(t) = t$ and $h(t) = 1$, then equation 6.1 will be obtained as*

$$X = P_1 = \begin{bmatrix} e^{a''\cos\theta + b''\sin\theta} \\ e^{-a''\sin\theta + b''\cos\theta} \end{bmatrix} \tag{6.6}$$

Corollary 12. *If $\theta(t) = t$, then equation 6.4 will be obtained as*

$$P_1' = \left(e^{\frac{1}{(h''-h)^2 + 4(h')^2}\left(-h(h''-h)a'' - 2hh'b''\right) + a}, e^{\frac{1}{(h''-h)^2 + 4(h')^2}\left(2hh'a'' - h(h''-h)b''\right) + b}\right). \tag{6.7}$$

Corollary 13. *If $\theta(t) = t$ and $h(t) = 1$, then equation 6.4,will be obtained as*

$$P_1' = \left(e^{-a'' + a}, e^{b'' + b}\right). \tag{6.8}$$

7. CONCLUSIONS

In multiplicative homothetic motions, velocities in plane motion, the relationship between velocities, pole points, and pole curves are given. Additionally, multiplicative accelerations and multiplicative acceleration combinations have been found.

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This study does not be necessary ethical committee permission or any special permission.

The Declaration of Research and Publication Ethics
The author declared that they comply with the scientific, ethical, and citation rules of Journal of Universal Mathematics in all processes of the study and that they do not make any falsification on the data collected. Besides, the author declared that Journal of Universal Mathematics and its editorial board have no responsibility for any ethical violations that may be encountered and this study has not been evaluated in any academic publication environment other than Journal of Universal Mathematics.

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