Modelling of Steady-State Seepage of an Embankment Dam Using Teaching-Learning Based Optimization Algorithm

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ABSTRACT

The goal of the this study is to investigate the applicability of the teaching-learning based optimization (TLBO) algorithm for modeling seepage in embankment dams. The input parameters selected for the models to be built are the values of permeability (k_s) , van Genuchten's suitability parameters α and n, whose effect on seepage has been investigated over the years due to their uncertainties. The validity of the TLBO was compared with that of conventional regression analysis (CRA) methods. Both methods were utilized with different regression forms. The parameters chosen as input are modeled as random variables with a log-normal distribution, and total discharge (Q) was obtained. Four statistical indices, that is, root mean square error, mean absolute error, average relative error and coefficient of determination, were used to evaluate the performance of the models. The equations obtained using TLBO algorithms can predict the total discharge in embankment dams better than CRA. In addition, the reliability of TLBO has been demonstrated by conducting analyses using the outputs of CRA as a benchmark.

Keywords: Monte Carlo Simulation, permeability, van genuchten parameters, seepage analysis, teaching-learning based optimization.

1. INTRODUCTION

The continuous and unimpeded movement of water from upstream to downstream of a dam is defined as seepage. The design of embankment dams aims to keep this movement within acceptable limits. In line with this objective, zoned embankment dams are designed using soils with low permeability (k) in the core section. However, the inevitable variability in the

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soil properties leads to uncertainties in performance [1,2]. These uncertainties mean that deterministic seepage analyses may produce results that can differ significantly from the measured seepage in the field. Therefore, probabilistic analyses that consider uncertainties often provide a more appropriate approach for seepage analysis of embankment dams.

Many researchers have used various geotechnical and hydraulic properties of the soil as random variables in these probabilistic analyses. Especially k, whose effect on seepage has already been clearly established, is a frequently used property in those studies [e.g.; 1, 3-9]. In addition to k, which is the most important parameter, α and n values, which are the suitability parameters of the van Genuchten water retention model, have also been among the parameters whose effect on seepage has been frequently investigated. Among these studies, Ahmed [4] investigated the seepage in embankment dams by probabilistic analysis, subjected the variable k to log-normal distribution and modelled the confined flow under a hydraulic structure using random field theorem. In the results of the study, it was determined that the amount of seepage was less than that calculated by deterministic methods for all values of coefficient of variation (COV) and fluctuation scale (Θ). Srivastava et al. [5] considered the value of k in a typical soil slope geometry as a log-normally distributed and spatially correlated random variable, and investigated the effect of this random variable on steadystate seepage flow and slope stability problems under steady-state seepage conditions. In the study of Le et al. [6], porosity and k were selected as random variables from heterogeneous material properties, and finite element analyses were performed by Monte Carlo (MC) simulation. Calamak [1] investigated the effect of soil variability on seepage in three different types of hypothetical embankment dams by taking hydraulic conductivity and Van Genuchten parameters as random variables. Tan et al. [7] numerically simulated saturatedunsaturated seepage by combining MC simulation and random field theory to investigate the effect of the variability of hydraulic parameters on the flow in earthfill dams. Sensitivity analyses revealed that the coefficients of variation of the soil-water relationship characteristic curve (SWCC) parameter n and k_s have a greater influence on the seepage flow rate than the SWCC parameter α .

Based on these considerations, this study presents a probabilistic seepage analysis where k, α , and *n* are modeled as random variables to determine their effect on total seepage (*O*). For this purpose, first, the statistical parameters mean (μ) and coefficient of variation (COV) were determined for k, α and n. Then, a hypothetical dam was created in accordance with the United States Bureau of Reclamation (USBR) criteria. Steady-state seepage analyses were performed on this hypothetical dam. The effect of the selected random variables on Q was investigated. Finally, the seepage within embankment dams is modeled by a new, simple, and robust optimization algorithm called teaching-learning based optimization (TLBO) and the conventional regression model (CRA) which were used in a number of previous studies in other fields of science and engineering [e.g., 10,11]. The TLBO algorithm is preferred because it has a small number of control parameters, and is therefore quite reliable. In addition, the fact that it gives relatively faster results compared to other swarm-based algorithms is also one of the reasons for its preference. Recently, this algorithm has started to be used in geotechnical problems involving retaining wall design and slope stability [12, 13]. This study distinguishes itself by pioneering an examination into the feasibility of employing the TLBO algorithm for modeling seepage in embankment dams, marking the first of its kind in this field.

2. CASE STUDY

A clay core embankment dam that was designed in accordance with the USBR criteria was employed for the analyses. The cross-section of the dam is given in Figure 1. The dam has a base length of 185 m, and a height of 30 m. The upstream and downstream slopes are 3:1 and 2.5:1, respectively. The core section has a width of 40 m, and slopes of 1:2. The typical geotechnical properties of the materials used for the upstream and downstream fill, and for the core are given in Table 1. Typical values from practice and literature were used when selecting deterministic material properties, except for permeability and van Genuchten parameters, which are modeled as random variables. In generating these random variables, particularly permeability, care was taken to ensure the values are plausible and acceptable in geotechnical and dam engineering practice. Detailed information on this consideration is provided in the random variable generation section. Note that using these material models and properties, Günay [14], in her study of probabilistic seepage at Büyükçekmece Dam, obtained results consistent with the measured seepage in the dam.



Figure 1 - Cross-section of the application dam

Parameter	Core	Fill	Unit
Soil model	Mohr-Coulomb	Mohr-Coulomb	-
Draninage type	Undrained(B)	Drained	-
¥	18	20	kN/m ³
Yunsat	16	16	kN/m ³
Groundwater classification type	User defined	Hypres	-
$\mathbf{k}_{\mathrm{x}} = \mathbf{k}_{\mathrm{y}}$	Random variable	1	m/day
α	Random variable	-	m ⁻¹
n	Random variable	-	-
E'ref	1,500	20,000	kN/m ²
C'ref	-	5	kN/m ²
Su,ref	5	-	kN/m ²
E'inc	300		kN/m²/m
SWCC fitting model	Van Genuchten	Van Genuchten	-

Table 1 - Material properties of the embankment dam

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3. METHODOLOGY

3.1. Finite Element Modelling

In this study, the finite element (FE) analyses for the seepage calculations were carried out using PLAXIS 2D Ultimate v22 [15]. Mohr-Coulomb soil model was deemed sufficient as the soil model, and "flow only" analysis type was used in the analyses. This type of analysis is more useful in problems that deal with fluid flow only. The finite element model was meshed to consist of 707 elements and 5,927 nodes. This mesh system is the finest mesh system (very fine) provided by PLAXIS 2D [15]. Van Genuchten [16] model and "user-defined" were employed for the SWCC curve of the materials. This allows α and n to be entered randomly. When determining the boundary conditions of the model, the bottom of the dam was completely closed to flow to focus solely on the flow within the dam body. Consequently, BoundaryXmin, BoundaryXmax, and BoundaryYmax were open to flow, while BoundaryYmin was closed to flow. The finite element model is shown in Figure 2.



Figure 2 - PLAXIS 2D finite element model

Stochastic analyses were utilized to explore the impact of uncertainties in core k_s and Van genuchten parameters α and n on Q. In these stochastic analyses, Python software [17] embedded in PLAXIS 2D was used to run the MC simulations. PLAXIS 2D v14 and later versions offers a Python scripting interaction interface that makes it possible for users to input data [18, 19, 20]. The interface from PLAXIS to Python is shown in Figure 3.



Figure 3 - Plaxis-Python connection [14]

3.2. Random Variable Generation

In the earlier stochastic seepage analyses, in addition to basic hydraulic and geotechnical properties such as k and k_s , Van Genuchten parameters (α and n) were also included in the analyses as random variables. In the current study, as in Li et al. [21], and Calamak [1], the k_s , α , and n were treated as random variables. Law [22], Bulnes [23], Warren and Price, [24], Bennion and Griffiths [25] show that k_s can be characterized by a log-normal distribution. In addition, Carsel and Parrish [26] show that α and n also follow a log-normal distribution. The essential statistical information, including the μ and the COV for the k_s , α , and n, was derived from Carsel and Parrish [26], which provides water retention relationships for twelve different soils. Specifically, these values for k_s were determined to be 0.062 m/day and 2.672, respectively [27]. Given the significant $COV(k_s)$, it is possible to produce a simulated k_s value that natural clay material would not typically exhibit. Therefore, values were initially generated with 0.5 COV(k_s). However, as the issue persists, it would be prudent, in accordance with the guidance provided by Casagrande [28], to limit the maximum k_s value to 10^{-4} cm/s, a value commonly associated with clays used in impervious sections of

Descriptive statistics	Random va	Reference		
	ks (m/day)	α (m ⁻¹)	n	Carsel and
COV	1.334	0.780	0.072	Parrish
μ	0.062	1.900	1.310	(1988)

Table 2 - Parameter values used in the study for k_s



Figure 4 - a, b: Random parameter generation and c, example of iteration output in Python

embankments. The mean value of 0.062 m/day represents a suitable k_s that aligns with the criteria proposed by Casagrande [28] for application in impermeable regions of dams and levees. In addition, related studies have shown that the $COV(k_s)$ value for k is in the range of 100-300% [27, 29, 30]. For the α , mean and COV values were taken as 1.90 m⁻¹ and 0.78, respectively. The value taken for the mean is between 0.21-2.46 values suggested by Qu et al. [31]. For the n value, mean and COV values were taken as 1.31 and 0.072, respectively. The value assumed for the mean n is between 1.05-1.35, which is the range proposed in Qu et al. [31]. All the obtained values are summarized in Table 2. The Python code for random variable generation using the values given in Table 2 and an example iteration output is given in Figure 4.

3.3. Teaching-Learning Based Algorithm (TLBO)

TLBO is a population-based stochastic optimization algorithm inspired by the teachinglearning process in a classroom developed by Rao et al [32]. This algorithm has been used in many studies such as modeling dissolved oxygen, estimating energy consumption and determining suspended sediment load [10, 33, 34]. In this study, it will be used for the first time on seepage analysis in dams. In the proposed algorithm, each candidate solution is characterized by a set of variables representing a student's results, consisting of grades in different subjects [35]. This algorithm includes teaching and learning phases. The student who best fits the solution is selected as the teacher for the teaching phase. The teaching phase



Figure 5 - Flow diagram of TLBO algorithm (Revised from [39])

is where students try to improve their results by getting information from the teacher. At the same time, the phase where students improve their performance by interacting with other students is called the learning phase [36]. The flow diagram of this algorithm is given in Figure 5. The reason why this algorithm is preferred over other algorithms such as the artificial bee colony algorithm or the ant colony algorithm, etc. is its simple digitized structure and independence from a set of control parameters to define the performance of the algorithm [10, 33, 37]. There are two control parameters in this algorithm. The first one is the population size, which is equal to the number of students. The other is the maximum number of cycles. The operation logic of this algorithm can be found in detail in Zou et al. [35] and [38].

The data is used as an input for the algorithm described above after being normalized using Eq.1. The objective function of the TLBO models is the sum square error (SSE). The regression equations have been evaluated by using data in the training set, and the best ones having the minimum SSE are determined. Also, performances of the TLBO and CRA models are evaluated using root mean square error (RMSE), mean absolute error (MAE), average relative error (ARE), and the coefficient of determination (R^2) for training and testing sets. SSE, RMSE, MAE, ARE and R^2 are obtained with Eqs. (2-6), respectively [40]. As the observed and estimated values converge, the Root Mean Square Error (RMSE), which is the standard deviation of the errors, decreases and approaches zero. The closer the RMSE is to zero, the better the correlation is in estimating the desired parameter [41]. In the literature, R^2 values between 0.9 and 1.0 indicate a perfect fit, while values between 0.75 and 0.9 indicate a very good fit [42].

Normalized value =
$$\left(\frac{Raw \ value - minimum \ value}{Maximum \ value - minimum \ value}\right) \times (0.9 - 0.1) + 0.1$$
 (1)

$$SSE = \sum_{i=1}^{N} (P_i - O_i)^2$$
(2)

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N} (P_i - O_i)^2\right]^{1/2}$$
(3)

$$MAE = \frac{1}{N} \sum_{i=1}^{N} |(P_i - O_i)|$$
(4)

$$ARE = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{(P_i - O_i)}{P_i} \right) \times 100$$
(5)

$$R^{2} = 1 - \left(\frac{\sum_{i=1}^{N} (P_{i} - O_{i})^{2}}{\sum_{i=1}^{N} (P_{i})^{2}}\right)$$
(6)

where;

N : the number of observations

O_i: the ith observed value

P_i : the ith estimated value for the regression functions

4. RESULTS AND DISCUSSION

FE analyses were conducted using the random variables k, α , and n, and the distribution of the resulting Q values is plotted in Figure 6. In this figure, the green line represents the deterministic Q result (obtained by keeping the random variables constant, Q_{det}). The red line represents the average Q value obtained probabilistically from 200 analyses. Note that the deterministically obtained value is less than the average of the probabilistically obtained values. Of the 200 Q values given in this histogram, 160 (80 %) were used in training and 40 (20 %) were used in testing. In the modeling phase, four regression functions, namely quadratic function (QF), exponential function (EF), linear function (LF), and hyperbolic function (HF), were used to estimate Q based on the analysis results. In the following, TLBO and CRA were used to optimize the unknown coefficients (w_i) of the independent variables (x_i).



Figure 6 - Histogram of Q values

Using the data obtained from the FE analysis, TLBO and CRA were applied to QF, EF, LF, and HF. One of the major challenges is to determine the best parameters of TLBO, since any change in the algorithm parameters affect the performance of the algorithm. For this reason, different scenarios for TLBO parameters were tested and the most successful features were used. Accordingly, the maximum number of iterations (NMI) = 3000 was set as one of the control parameters of TLBO. The other control parameter, the population size (SP), was set to SP = 100 for linear and hyperbolic regression functions, and SP = 200 for quadratic and exponential functions. Once the control parameters were set, 20 independent runs were performed for each regression equation using TLBO. The control parameter values of the TLBO models yielding the best results for the functions are given in Table 3. CRA analyses were performed with SPSS, version 11.5 for Windows. The optimal coefficients obtained for the functions in the analysis results are presented in Table 4.

The results obtained from the equations and the test set that the equations have never seen before were compared with the probabilistic FE results and the best-fitting equations were

determined. The comparison is based on performance indices such as RMSE, MAE, ARE and R^2 . The error values and R^2 for the training and testing sets using TLBO and CRA models are presented in Tables 5 and 6, respectively. After evaluating all equations, the best fitting equation is highlighted in bold for all error values.

The functions	TLBO parameters				
	SP	NMI			
Quadratic	200	3000			
Exponential	200	3000			
Linear	100	3000			
Hyperbolic	100	3000			

Table 3 - The control parameter values of the TLBO and models yielding the best results

	Coefficients									
	w_0	WI	W_2	W_3	W_4	W5	w ₆	<i>W</i> ₇	w_8	W9
TLBO	1.0084	0.9127	0.0069	-0.0037						
CRA	5.5860	0.8810	0.0450	-0.0050						
y hyperbolic	$= w_0 (x_1)^{w_1} (x_2)^{w_2} (x_1)^{w_1} (x_2)^{w_2} (x_2)^{w_2$	$(x_2)^{w^2}(x_3)^{w^3}$								
TLBO	0.0325	0.9818	0.0142	0.0011						
CRA	-0.0037	7.2321	0.0317	-0.0002						
$y_{linear} = v$	$w_0 + (x_1)w_1 + (x_2)w_1 + (x_1)w_2 + (x_2)w_1 + (x_2)w_2 + (x_$	$+(x_2)w_2+(x_3)w_3+(x_3)w_3+(x_3)w_3+(x_3)w_3+(x_3)w_3+(x_3)w_3+(x_3)w_3)$)w3							
TLBO CRA	-6.4455 -18.148	1.8711 2.8980	$0.1374 \\ 0.3900$	$\begin{array}{c} 0.0038\\ 0.0020\end{array}$	$0.0013 \\ 0.0001$					
y _{exponantia}	$w_0 + exp($	$(w_1 + (x_1)w_2)$	$+(x_2)w_3+(x_3)w_3)$	3)W4)						
TLBO	-0.0426	1.2226	-0.0003	0.1360	0.0455	-0.0578	0.0198	-0.2306	-0.023	-0.1233
CRA	-0.0500	8.7300	0.0690	0.0080	0.1910	-0.0410	-0.0070	-17.535	-0.0020	0.0010
yquadratic =	$y_{\text{quadratic}} = w_0 + (x_1)w_1 + (x_2)w_2 + (x_3)w_3 + (x_1x_2)w_4 + (x_1x_3)w_5 + (x_2x_3)w_6 + (x_1^2)w_7 + (x_2^2)w_8 + (x_3^2)w_9$									

 x_1 : k, m/day ; x_2 : n ; x_3 : α , m⁻¹ ; y : Q, m³/day/m

The	RMSE		MAE		ARE (%)		R ²	
functions	TLBO	CRA	TLBO	CRA	TLBO	CRA	TLBO	CRA
Hyperbolic	0.0031	0.0046	0.0033	0.0035	1.0101	1.0126	0.9992	0.9988
Linear	0.0070	0.0070	0.0058	0.0057	2.2710	2.9656	0.9968	0.9961
Exponential	0.0074	0.0082	0.0074	0.0073	3.2701	2.6687	0.9975	0.9973
Quadratic	0.0061	0.0070	0.0034	0.0035	1.4267	1.4808	0.9988	0.9989

Table 5 - The model results for training set

The	RM	ISE	M	AE	A	RE	R	2
functions	TLBO	CRA	TLBO	CRA	TLBO	CRA	TLBO	CRA
Hyperbolic	0.0023	0.0026	0.0040	0.0046	0.8906	0.8916	0.9998	0.9989
Linear	0.0093	0.0035	0.0079	0.0059	1.6767	1.8379	0.9984	0.9982
Exponential	0.0071	0.0039	0.0130	0.0072	2.1848	2.2585	0.9978	0.9986
Quadratic	0.0031	0.0038	0.0045	0.0035	0.8912	1.0263	0.9995	0.9992

Table 6 - The model results for testing set

It is clear from Tables 5 and 6 that the best-performing equations are obtained from hyperbolic functions using the TLBO algorithm. The minimum error value in the training and testing set was derived from HF with TLBO algorithm. The smallest ARE value for the training and testing sets were 1.0101% and 0.8906%, respectively in TLBO algorithm, and 1.0126% and 0.8916%, respectively in the CRA. According to the presented results, TLBO improved the performance of hyperbolic function by 32.6% in the training set, and by 11.54% in the testing set compared to CRA. Considering the error values for different function types (Tables 5, 6), it can be seen that the hyperbolic model gives the best result among all methods in both training and testing sets. Nonetheless, the alternative models also demonstrated exceptionally high performance, as evidenced by all R² values exceeding 0.99 with both TLBO and CRA methodologies.

The functions	RMSE	MAE	ARE	R ²
Hyperbolic	0.0339	0.0032	1.7598	0.9499
Linear	0.0345	0.0041	2.0790	0.9400
Exponential	0.0344	0.0055	2.4532	0.9373
Quadratic	0.0321	0.0035	1.7883	0.9479

Table 7 - The model results for training set with CRA results as a benchmark

Table 8 - The model results for testing set with CRA results as a benchmark

The functions	RMSE	MAE	ARE	R ²	
Hyperbolic	0.0027	0.0022	0.6570	0.9808	
Linear	0.0018	0.0009	0.8072	0.9794	
Exponential	0.0060	0.0031	1.1121	0.9693	
Quadratic	0.0049	0.0045	1.0773	0.9793	

In addition, the results from CRA were utilized as a benchmark to validate the effectiveness of TLBO, and the outcomes obtained using TLBO were compared accordingly. Utilizing the results of CRA, which is more widely used than TLBO, provided more logical and reliable outcomes. The results of these analyses are presented in Tables 7 and 8 for the training and test sets, respectively.

Tables 7 and 8 clearly demonstrate that the best performance is again achieved using hyperbolic functions in the analyses where CRA results were used as a benchmark. The minimum error values in both the training and test sets were obtained from HF. Although a slight decrease in R^2 values is observed in the training set, the values remain between 0.93 and 0.95, indicating the model's robustness

5. CONCLUSIONS

In this study, for the first time, the ability of the Teaching-Learning-Based Optimization (TLBO) algorithm to predict total seepage (Q) in an embankment dam, based on the hydraulic and geotechnical properties of the clay core specifically saturated permeability (k_s) and van Genuchten parameters (α and n) is investigated. The main conclusions that can be drawn from the present study are as follows:

- The comparison of results using various performance indices clearly indicates that the best fit equations for each parameter are obtained from the hyperbolic function.
- The comparison of results demonstrates that the TLBO algorithm outperforms the CRA algorithm in predicting *Q*, as evidenced by a higher *R*² value and lower error metrics. For the training set, there was a 32% improvement in RMSE, a 5.7% improvement in MAE, and a 19.84% improvement in ARE. Additionally, the *R*² value increased by 4.5%. For the testing set, there was an 11.54% improvement in RMSE and a 13% improvement in MAE.
- To evaluate the reliability of TLBO, additional analyses were conducted using CRA results as a benchmark, comparing the performance of TLBO against CRA. The high R² values, ranging between 0.93 and 0.95, confirmed the model's accuracy.
- The equations derived using the TLBO algorithms successfully predict Q. Given this achievement, TLBO can serve as an effective optimization algorithm in seepage problems. Thus, a reasonable and reliable approximation for Q can be provided made by the equation obtained via TLBO.
- Based on the coefficients obtained, it is inferred that the probability distribution parameters of α and *n* have a lesser impact on *Q* compared to the parameter k_s .

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Author's Contributions

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Data Availability

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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