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Soft intersection almost ideals of semigroups

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ABSTRACT

This study aims to present the notion of soft intersection almost left (respectively, right) ideal of a semigroup which is a generalization of the nonnull soft intersection left (respectively, right) ideal of a semigroup, and investigate the related properties in detail. We show that every idempotent soft intersection almost (left/right) ideal is a soft intersection almost subsemigroup. Besides, we acquire remarkable relationships between almost left (respectively, right) ideals and soft intersection almost left (respectively, right) ideals of a semigroup as regards minimality, primeness, semiprimeness, and strongly primeness.

I. INTRODUCTION

In the fields of theoretical computer science, automata, coding theory, and formal languages, as well as in the solutions of graph theory and optimization theory, semigroups serve as the fundamental algebraic structure. Ideals are crucial to the advanced study of algebraic structures and their applications. Further study of algebraic structures requires the generalization of ideals in algebraic structures. Numerous mathematicians demonstrated significant findings and characterized algebraic structures by introducing various extensions of the idea of ideals in algebraic structures. The idea of almost left, right, and two-sided ideals of semigroups were first presented by Grosek and Satko [1] in 1980. When there are no proper left, right, or two-sided ideals in a semigroup, they investigated how these ideals are characterized. As an extension of bi-ideals, Bogdanovic [2] developed the idea of almost bi-ideals in semigroups later in 1981. In 2018, Wattanatripop et al. [3] proposed the notion of almost quasi-ideals by utilizing the notions of quasi-ideals of semigroups and almost ideals. Using the ideas of almost ideals and interior ideals of semigroups, Kaopusek et al. [4], in 2020, proposed the notions of almost interior ideals and weakly almost interior ideals of semigroups and examined their features. Iampan et al. [5] in 2021; Chinram and Nakkhasen [6], in 2022; Gaketem [7] in 2022; introduced the notion of almost subsemigroups of semigroups; almost bi-quasi interior ideals of semigroups; almost bi-interior ideal of semigroups, respectively. Additionally, different types of almost ideals' fuzzification were studied by many researchers in [3, 5-9].

Molodtsov [10], in 1999, proposed the idea of the soft set as a function from the parameter set E to the power set of U to model uncertainty. Since then, soft set has attracted the attention of researchers in many fields. In [11-26],

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soft set operations, the basic concept of the theory, are studied. Çağman and Enginoğlu [27] modified the definition of soft set and soft set operations. Moreover, several soft algebraic structures were inspired by the notion of soft intersection groups, introduced by Çağman et al. [28]. The usage of soft sets in semigroups came up with the notion of soft intersection substructures of semigroups. Sezer et al. [29, 30] introduced and studied soft intersection subsemigroups, left (right/two-sided ideals), (generalized) bi-ideals, interior ideals, and quasi-ideals of semigroups. Sezgin and Orbay [31] characterized semisimple semigroups, duo semigroups, right (left) zero semigroups, right (left) simple semigroups, semilattice of left (right) simple semigroups, semilattice of left (right) groups and semilattice of groups in terms of soft intersection substructures of semigroups. Soft sets were studied as as wide range of algebraic structures in [32-43]. Recently, Rao [44-47] introduced some new types of ideals of semigroups such as bi-interior ideals, bi-quasi ideals, quasi-interior ideals, weak-interior ideals, and bi-quasi-interior ideals, respectively. Baupradist [48] defined essential ideals of semigroups.

In this study, we introduced the notion of soft intersection almost left (resp. right) ideals, which is a generalization of the nonnull soft intersection left (resp. right) ideals of a semigroup. Furthermore, we show that every idempotent soft intersection almost (left/right) ideal is a soft intersection almost subsemigroup. We obtain that the collection of soft intersection almost left (resp. right) ideals of a semigroup constructs a semigroup under the binary operation of union for soft sets, but not under the binary of operation of intersection for soft sets. Furthermore, we demonstrated the connection between almost left (resp. right) ideals and soft intersection almost left (resp. right) ideals of a semigroup corresponding with minimality, primeness, semiprimeness, and strongly primeness.

II. PRELIMINARY

In this section, we review several fundamental notions related to semigroups and soft sets.

Definition 2.1. Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U and $K \subseteq E$. A soft set f_K over U is a set-valued function such that $f_K\colon E \to P(U)$ such that for all $x \notin K$, $f_K(x) = \emptyset$. A soft set over U can be represented by the set of ordered pairs

$$f_K = \{(x, f_K(x)) \colon x \in E, f_K(x) \in P(U)\}$$

[10, 27]. Throughout this paper, the set of all the soft sets over U is designated by $S_E(U)$.

Definition 2.2. Let $f_A \in S_E(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then f_A is called a null soft set and denoted by $\emptyset_E$. If $f_A(x) = U$ for all $x \in E$, then f_A is called an absolute soft set and denoted by U_E [27].

Definition 2.3. Let $f_A, f_B \in S_E(U)$. If for all $x \in E$, $f_A(\mathrm{x}) \subseteq f_B(\mathrm{x})$, then f_A is a soft subset of f_B and denoted by $f_A \tilde{\subseteq} f_B$. If $f_A(x) = f_B(x)$ for all $x \in E$, then f_A is called soft equal to f_B and denoted by $f_A = f_B$ [27].

Definition 2.4. Let $f_A, f_B \in S_E(U)$. The union of f_A and f_B is the soft set $f_A \tilde{\cup} f_B$, where $(f_A \tilde{\cup} f_B)(x) = f_A(x) \cup f_B(x)$ for all $x \in E$. The intersection of f_A and f_B is the soft set $f_A \tilde{\cap} f_B$, where $(f_A \tilde{\cap} f_B)(x) = f_A(x) \cap f_B(x)$ for all $x \in E$ [27].

Definition 2.5. For a soft set f_A, the support of f_A is defined by

$$supp(f_A) = \{x \in A : f_A(x) \neq \emptyset\} \text{ [15]}$$

It is obvious that a soft set with an empty support is a null soft set, otherwise the soft set is nonnull.

Note 2.6. If $f_A \tilde{\subseteq} f_B$, then $supp(f_A) \subseteq supp(f_B)$ [49].

A semigroup S is a nonempty set with an associative binary operation and throughout this paper, S stands for a semigroup and all the soft sets are the elements of $S_S(U)$ unless otherwise specified.

Definition 2.7. A nonempty subset A of S is called,

(1) a subsemigroup of S if $AA \subseteq A$,
(2) a right ideal of S if $AS \subseteq A$; and a left ideal of S if $SA \subseteq A$; and an ideal of S when is both a left ideal of S and a right ideal of S,
(3) an almost subsemigroup of S if $AA \cap A \neq \emptyset$,
(4) an almost left ideal of S if $sA \cap A \neq \emptyset$ for all $s \in S$; and an almost right ideal of S if $As \cap A \neq \emptyset$ for all $s \in S$; and an almost ideal of S when is both an almost left ideal of S and an almost right ideal of S.

Definition 2.8. An almost left (resp. right) ideal A of S is called minimal almost left (resp. right) ideal of S if for any almost left (resp. right) ideal B of S if whenever $B \subseteq A$, then $A = B$.

Definition 2.9. Let P be an almost left (resp. right) ideal of S. Then P is called,

(1) a prime almost left (resp. right) ideal of S if for any almost left (resp. right) ideals A and B of S such that $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$,
(2) a semiprime almost left (resp. right) ideal of S if for any almost left (resp. right) ideal A of S such that $AA \subseteq P$ implies that $A \subseteq P$,
(3) a strongly prime almost left (resp. right) ideal of S if for any almost left (resp. right) ideals A and B of S such that $AB \cap BA \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 2.10. Let f_S and g_S be soft sets over the common universe U. Then, soft intersection product $f_S \circ g_S$ is defined by [29]

$$(f_S \circ g_S)(x) = \begin{cases} \bigcup_{x=yz} \{f_S(y) \cap g_S(z)\}, & if\ \exists y, z \in S\ such\ that\ x = yz \\ \emptyset, & otherwise \end{cases}$$

Theorem 2.11. Let $f_S, g_S, h_S \in S_S(U)$. Then,

i) $(f_S \circ g_S) \circ h_S = f_S \circ (g_S \circ h_S)$.

ii) $f_S \circ g_S \neq g_S \circ f_S$, $generally$.

iii) $f_S \circ (g_S \tilde{\cup} h_S) = (f_S \circ g_S) \tilde{\cup} (f_S \circ h_S)$ and $(f_S \tilde{\cup} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cup} (g_S \circ h_S)$.

iv) $f_S \circ (g_S \tilde{\cap} h_S) = (f_S \circ g_S) \tilde{\cap} (f_S \circ h_S)$ and $(f_S \tilde{\cap} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cap} (g_S \circ h_S)$.

v) If $f_S \tilde{\subseteq} g_S$, then $f_S \circ h_S \tilde{\subseteq} g_S \circ h_S$ and $h_S \circ f_S \tilde{\subseteq} h_S \circ g_S$.

vi) If $t_S, k_S \in S_S(U)$ such that $t_S \tilde{\subseteq} f_S$ and $k_S \tilde{\subseteq} g_S$, then $t_S \circ k_S \tilde{\subseteq} f_S \circ g_S$ [29].

Lemma 2.12. Let f_S and g_S be soft sets over U. Then, $f_S \circ g_S = \emptyset_S \Leftrightarrow f_S = \emptyset_S$ or $g_S = \emptyset_S$.

Definition 2.13. Let A be a subset of S. We denote by S_A the soft characteristic function of A and define as

$$S_A(x) = \begin{cases} U, & if\ x \in A \\ \emptyset, & if\ x \in S \backslash A \end{cases}$$

The soft characteristic function of A is a soft set over U, that is, $S_A: S \longrightarrow P(U)$ [29].

Corollary 2.14. $supp(S_A) = A$ [49].

Theorem 2.15. Let X and Y be nonempty subsets of S. Then, the following properties hold [29]:

i) If $X \subseteq Y$ if and only if $S_X \tilde{\subseteq} S_Y$

ii) $S_X \tilde{\cap} S_Y = S_{X \cap Y}$ and $S_X \tilde{\cup} S_Y = S_{X \cup Y}$

iii) $S_X \circ S_Y = S_{XY}$

Proof: In [29], (i) is given as if $X \subseteq Y$, then if $S_X \tilde{\subseteq} S_Y$. In [49], it was also shown that if $S_X \tilde{\subseteq} S_Y$, then $X \subseteq Y$. Let $S_X \tilde{\subseteq} S_Y$ and $x \in X$. Then, $S_X(x) = U$ and this implies that $S_Y(x) = U$ since $S_X \tilde{\subseteq} S_Y$, Hence, $x \in Y$ and so $X \subseteq Y$. Now let $x \notin Y$. Then, $S_Y(x) = \emptyset$, and this implies that $S_X(x) = \emptyset$ since $S_X \tilde{\subseteq} S_Y$. Hence, $x \notin X$ and so $Y' \subseteq X'$, implying that $X \subseteq Y$.

Definition 2.16. Let x be an element in S. We denote by S_x the soft characteristic function of x and define as

$$S_x(y) = \begin{cases} U, & if\ \ y = x \\ \emptyset, & if\ \ y \neq x \end{cases}$$

The soft characteristic function of x is a soft set over U, that is, $S_x: S \longrightarrow P(U)$ [49].

Corollary 2.17. Let $x \in S$, f_S and S_x be soft sets over U. Then,

$$f_S \circ S_x = \emptyset_S \Leftrightarrow f_S = \emptyset_S \ (S_x \circ f_S = \emptyset_S \Leftrightarrow f_S = \emptyset_S).$$

Proof: By Lemma 2.12, $f_S \circ S_x = \emptyset_S \Leftrightarrow f_S = \emptyset_S$ or $S_x = \emptyset_S$. By Definition 2.16, $S_x \neq \emptyset_S$; hence the rest of the proof is obvious.

Definition 2.18. A soft set f_S over U is called a soft intersection subsemigroup of S over U if $f_S(xy) \supseteq f_S(x) \cap f_S(y)$ for all $x, y \in S$; and is called a soft intersection left (resp. right) ideal of S over U if $f_S(xy) \supseteq f_S(y)$ $(f_S(xy) \supseteq f_S(x))$ for all $x, y \in S$. A soft set f_S over U is called a soft intersection ideal of S if it is both a soft intersection left ideal of S over U and a soft intersection right ideal of S over U [29].

It is easy to see that if $f_S(x) = U$ for all $x \in S$, then f_S is a soft intersection (left/right) ideal. We denote such a kind of soft intersection (left/right) ideal by $\tilde{\mathbb{S}}$. It is obvious that $\tilde{\mathbb{S}} = S_S$, that is, $\tilde{\mathbb{S}}(x) = U$ for all $x \in S$ [29].

Theorem 2.19. Let f_S be a soft set over U. Then, f_S is a soft intersection subsemigroup of S over U if and only if $f_S \circ f_S \tilde{\subseteq} f_S$; and f_S is a soft intersection left (resp. right) ideal of S over U if and only if $\tilde{\mathbb{S}} \circ f_S \tilde{\subseteq} f_S$ ($f_S \circ \tilde{\mathbb{S}} \tilde{\subseteq} f_S$) [29].

Definition 2.20. Let f_S be a soft set over U. Then, f_S is a soft intersection almost subsemigroup of S over U if $(f_S \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} f_S$ [49].

Inspired by the divisibility of determinants, we refer to [50] for the considerations of graph applications and network analysis.

III. RESULTS ON SOFT INTERSECTION ALMOST IDEALS OF SEMIGROUPS

Definition 3.1. Let f_S be a soft set over U. f_S is called a soft intersection almost left ideal of S if for all $x \in S$,

$$(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$$

Definition 3.2. Let f_S be a soft set over U. f_S is called a soft intersection almost right ideal of S if for all $x \in S$,

$$(f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$$

Definition 3.3. Let f_S be a soft set over U. f_S is called a soft intersection almost ideal of S if for all $x \in S$,

$$(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S \text{ and } (f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$$

Hereafter, for brevity, soft intersection almost left ideal and soft intersection almost right ideal of S are denoted by SI-almost L-ideal and SI-almost R-ideal, respectively. The similar arguments and abbreviations are valid for almost left and right ideals and soft intersection left and right ideals of S.

Example 3.4. Let $S = \{\ell, g, n\}$ be the semigroup with the following Cayley Table.

	ℓ	g	n
ℓ	ℓ	g	n
g	g	n	ℓ
n	n	ℓ	g

Let f_S and h_S be soft sets over $U = \left\{\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \middle| a \in \mathbb{Z}_4\right\}$ as follows:

$$f_S = \left\{\left(\ell, \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}\right)\right\}$$

$$h_S = \left\{\left(\ell, \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right\}\right)\right\}$$

and let g_S be soft set over $U = D_3 = \{< x, y >: x^3 = y^2 = e,\ xy = yx^2\} = \{e, x, x^2, y, yx, yx^2\}$ as follows:

$$g_S = \{(\ell, \{e\}), (g, \{x, x^2\}), (n, \{y, yx, yx^2\})\}$$

Here, f_S and h_S are both SI-almost ideals. Let's first show that f_S is an SI-almost L-ideal, that is, for all $x \in S$, $(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$:

Let's start with S_ℓ:

$[(S_\ell \circ f_S) \tilde{\cap} f_S](\ell) = (S_\ell \circ f_S)(\ell) \cap f_S(\ell) = \left[\left(S_\ell(\ell) \cap f_S(\ell)\right) \cup \left(S_\ell(g) \cap f_S(n)\right) \cup \left(S_\ell(n) \cap f_S(g)\right)\right] \cap f_S(\ell) = f_S(\ell) = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right\}$

$[(S_\ell \circ f_S) \tilde{\cap} f_S](g) = (S_\ell \circ f_S)(g) \cap f_S(g) = \left[\left(S_\ell(\ell) \cap f_S(g)\right) \cup \left(S_\ell(g) \cap f_S(\ell)\right) \cup \left(S_\ell(n) \cap f_S(n)\right)\right] \cap f_S(g) = f_S(g) = \left\{\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}$

$[(S_\ell \circ f_S) \tilde{\cap} f_S](n) = (S_\ell \circ f_S)(n) \cap f_S(n) = \left[\left(S_\ell(\ell) \cap f_S(n)\right) \cup \left(S_\ell(g) \cap f_S(g)\right) \cup \left(S_\ell(n) \cap f_S(\ell)\right)\right] \cap f_S(n) = f_S(n) = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}$

Thus,

$$(S_\ell \circ f_S) \tilde{\cap}\ f_S = \left\{\left(\ell, \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

Let's continue with S_g:

$\left[(S_g \circ f_S) \tilde{\cap} f_S\right](\ell) = \left(S_g \circ f_S\right)(\ell) \cap f_S(\ell) = \left[\left(S_g(\ell) \cap f_S(\ell)\right) \cup \left(S_g(g) \cap f_S(n)\right) \cup \left(S_g(n) \cap f_S(g)\right)\right] \cap f_S(\ell) = f_S(n) \cap f_S(\ell) = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

$\left[(S_g \circ f_S) \tilde{\cap} f_S\right](g) = \left(S_g \circ f_S\right)(g) \cap f_S(g) = \left[\left(S_g(\ell) \cap f_S(g)\right) \cup \left(S_g(g) \cap f_S(\ell)\right) \cup \left(S_g(n) \cap f_S(n)\right)\right] \cap f_S(g) = f_S(\ell) \cap f_S(g) = \left\{\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right\}$

$\left[(S_g \circ f_S) \tilde{\cap} f_S\right](n) = \left(S_g \circ f_S\right)(n) \cap f_S(n) = \left[\left(S_g(\ell) \cap f_S(n)\right) \cup \left(S_g(g) \cap f_S(g)\right) \cup \left(S_g(n) \cap f_S(\ell)\right)\right] \cap f_S(n) = f_S(g) \cap f_S(n) = \left\{\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}$

Thus,

$$(S_g \circ f_S) \tilde{\cap}\ f_S = \left\{\left(\ell, \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

Let's continue with S_n:

$[(S_n \circ f_S) \tilde{\cap} f_S](\ell) = (S_n \circ f_S)(\ell) \cap f_S(\ell) = \left[\left(S_n(\ell) \cap f_S(\ell)\right) \cup \left(S_n(g) \cap f_S(n)\right) \cup \left(S_n(n) \cap f_S(g)\right)\right] \cap f_S(\ell) = f_S(g) \cap f_S(\ell) = \left\{\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right\}$

$[(S_n \circ f_S) \tilde{\cap} f_S](g) = (S_n \circ f_S)(g) \cap f_S(g) = [(S_n(\ell) \cap f_S(g)) \cup (S_n(g) \cap f_S(\ell)) \cup (S_n(n) \cap f_S(n))] \cap f_S(g) = f_S(n) \cap f_S(g) = \left\{\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}$

$[(S_n \circ f_S) \tilde{\cap} f_S](n) = (S_n \circ f_S)(n) \cap f_S(n) = [(S_n(\ell) \cap f_S(n)) \cup (S_n(g) \cap f_S(g)) \cup (S_n(n) \cap f_S(\ell))] \cap f_S(n) = f_S(\ell) \cap f_S(n) = \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right\}$

Consequently,

$$(S_n \circ f_S) \tilde{\cap} f_S = \left\{\left(\ell, \left\{\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

Therefore, for all $x \in S$, $(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, so f_S is an SI-almost L-ideal. Now let's show that f_S is an SI-almost R-ideal, that is, for all $x \in S$, $(f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$.

Let's start with S_ℓ:

$[(f_S \circ S_\ell) \tilde{\cap} f_S](\ell) = (f_S \circ S_\ell)(\ell) \cap f_S(\ell) = [(f_S(\ell) \cap S_\ell(\ell)) \cup (f_S(g) \cap S_\ell(n)) \cup (f_S(n) \cap S_\ell(g))] \cap f_S(\ell) = f_S(\ell) = \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right\}$

$[(f_S \circ S_\ell) \tilde{\cap} f_S](g) = (f_S \circ S_\ell)(g) \cap f_S(g) = [(f_S(\ell) \cap S_\ell(g)) \cup (f_S(g) \cap S_\ell(\ell)) \cup (f_S(n) \cap S_\ell(n))] \cap f_S(g) = f_S(g) = \left\{\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}, \begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}$

$[(f_S \circ S_\ell) \tilde{\cap} f_S](n) = (f_S \circ S_\ell)(n) \cap f_S(n) = [(f_S(\ell) \cap S_\ell(n)) \cup (f_S(g) \cap S_\ell(g)) \cup (f_S(n) \cap S_\ell(\ell))] \cap f_S(n) = f_S(n) = \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}$

Thus,

$$(f_S \circ S_\ell) \tilde{\cap} f_S = \left\{\left(\ell, \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}, \begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

Let's continue with S_g:

$[(f_S \circ S_g) \tilde{\cap} f_S](\ell) = (f_S \circ S_g)(\ell) \cap f_S(\ell) = [(f_S(\ell) \cap S_g(\ell)) \cup (f_S(g) \cap S_g(n)) \cup (f_S(n) \cap S_g(g))] \cap f_S(\ell) = f_S(n) \cap f_S(\ell) = \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right\}$

$[(f_S \circ S_g) \tilde{\cap} f_S](g) = (f_S \circ S_g)(g) \cap f_S(g) = [(f_S(\ell) \cap S_g(g)) \cup (f_S(g) \cap S_g(\ell)) \cup (f_S(n) \cap S_g(n))] \cap f_S(g) = f_S(\ell) \cap f_S(g) = \left\{\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right\}$

$[(f_S \circ S_g) \tilde{\cap} f_S](n) = (f_S \circ S_g)(n) \cap f_S(n) = [(f_S(\ell) \cap S_g(n)) \cup (f_S(g) \cap S_g(g)) \cup (f_S(n) \cap S_g(\ell))] \cap f_S(n) = f_S(g) \cap f_S(n) = \left\{\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}$

Hence,

$$(f_S \circ S_{g}) \,\tilde{\cap}\, f_S = \left\{\left(\ell, \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

Let's continue with S_n:

$[(f_S \circ S_n) \,\tilde{\cap}\, f_S](\ell) = (f_S \circ S_n)(\ell) \cap f_S(\ell) = \left[\left(f_S(\ell) \cap S_n(\ell)\right) \cup \left(f_S(g) \cap S_n(n)\right) \cup \left(f_S(n) \cap S_n(g)\right)\right] \cap f_S(\ell) = f_S(g) \cap f_S(\ell) = \left\{\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right\} \neq \emptyset$

$[(f_S \circ S_n) \,\tilde{\cap}\, f_S](g) = (f_S \circ S_n)(g) \cap f_S(g) = \left[\left(f_S(\ell) \cap S_n(g)\right) \cup \left(f_S(g) \cap S_n(\ell)\right) \cup \left(f_S(n) \cap S_n(n)\right)\right] \cap f_S(g) = f_S(n) \cap f_S(g) = \left\{\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\} \neq \emptyset$

$[(f_S \circ S_n) \,\tilde{\cap}\, f_S](n) = (f_S \circ S_n)(n) \cap f_S(n) = \left[\left(f_S(\ell) \cap S_n(n)\right) \cup \left(f_S(g) \cap S_n(g)\right) \cup \left(f_S(n) \cap S_n(\ell)\right)\right] \cap f_S(n) = f_S(\ell) \cap f_S(n) = \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right\} \neq \emptyset$

Consequently,

$$(f_S \circ S_n) \,\tilde{\cap}\, f_S = \left\{\left(\ell, \left\{\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

Therefore, for all $x \in S$, $(f_S \circ S_x) \,\tilde{\cap}\, f_S \neq \emptyset_S$, so f_S is an SI-almost R-ideal, thus f_S is an SI-almost ideal.

Similarly, h_S is an SI-almost ideal. In fact;

$$(S_\ell \circ h_S) \,\tilde{\cap}\, h_S = \left\{\left(\ell, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

$$(S_{g} \circ h_S) \,\tilde{\cap}\, h_S = \left\{\left(\ell, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right)\right\} \neq \emptyset_S,$$

$$(S_n \circ h_S) \,\tilde{\cap}\, h_S = \left\{\left(\ell, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

Hence, h_S is an SI-almost L-ideal. And,

$$(h_S \circ S_\ell) \,\tilde{\cap}\, h_S = \left\{\left(\ell, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

$$(h_S \circ S_{g}) \,\tilde{\cap}\, h_S = \left\{\left(\ell, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

$$(h_S \circ S_n) \,\tilde{\cap}\, h_S = \left\{\left(\ell, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right), \left(g, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right), \left(n, \left\{\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\right\}\right)\right\} \neq \emptyset_S$$

Thus, h_S is an SI-almost R-ideal, thus h_S is an SI-almost ideal.

One can also show that g_S is neither an SI-almost L-ideal nor an SI-almost R-ideal. In fact;

$\left[(S_{g} \circ g_S) \,\tilde{\cap}\, g_S\right](\ell) = (S_{g} \circ g_S)(\ell) \cap g_S(\ell) = \left[\left(S_{g}(\ell) \cap g_S(\ell)\right) \cup \left(S_{g}(g) \cap g_S(n)\right) \cup \left(S_{g}(n) \cap g_S(g)\right)\right] \cap g_S(g) = g_S(n) \cap g_S(\ell) = \emptyset$

$[(S_{g} \circ g_S) \tilde{\cap} g_S](g) = (S_{g} \circ g_S)(g) \cap g_S(g) = \left[\left(S_{g}(\ell) \cap g_S(g)\right) \cup \left(S_{g}(g) \cap g_S(\ell)\right) \cup \left(S_{g}(n) \cap g_S(n)\right)\right] \cap g_S(g) = g_S(\ell) \cap g_S(g) = \emptyset$

$[(S_{g} \circ g_S) \tilde{\cap} g_S](n) = (S_{g} \circ g_S)(n) \cap g_S(n) = \left[\left(S_{g}(\ell) \cap g_S(n)\right) \cup \left(S_{g}(g) \cap g_S(g)\right) \cup \left(S_{g}(n) \cap g_S(\ell)\right)\right] \cap g_S(n) = g_S(g) \cap g_S(n) = \emptyset$

Hence, for $g \in S$; $(S_{g} \circ g_S) \tilde{\cap} g_S = \{(\ell, \emptyset), (g, \emptyset), (n, \emptyset)\} = \emptyset_S$, thus g_S is not an SI-almost L-ideal. Similarly, for $n \in S$; $(g_S \circ S_n) \tilde{\cap} g_S = \{(\ell, \emptyset), (g, \emptyset), (n, \emptyset)\} = \emptyset_S$, thus, g_S is not an SI-almost R-ideal. It is obvious that g_S is not an SI-almost ideal.

From now on, the proofs are given for only SI-almost L-ideal, since the proofs for SI-almost R-ideal can be shown similarly.

Proposition 3.5. If f_S is an SI-L (resp. R)-ideal such that $f_S \neq \emptyset_S$, then f_S is an SI-almost L (resp. R)-ideal.

Proof: Let $f_S \neq \emptyset_S$ be an SI-L-ideal, thus $\tilde{\mathbb{S}} \circ f_S \tilde{\subseteq} f_S$. Since $f_S \neq \emptyset_S$, by Corollary 2.17 it follows that $S_x \circ f_S \neq \emptyset_S$. We need to show that for all $x \in S$,

$$(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S.$$

Since $S_x \circ f_S \tilde{\subseteq} \tilde{\mathbb{S}} \circ f_S \tilde{\subseteq} f_S$, it follows that $S_x \circ f_S \tilde{\subseteq} f_S$. Thus,

$$(S_x \circ f_S) \tilde{\cap} f_S = S_x \circ f_S \neq \emptyset_S$$

implying that f_S is an SI-almost L-ideal.

Here it is obvious that $\emptyset_S$ is an SI-L-ideal as $\tilde{\mathbb{S}} \circ \emptyset_S \tilde{\subseteq} \emptyset_S$; but it is not SI-almost L-ideal since $(S_x \circ \emptyset_S) \tilde{\cap} \emptyset_S = \emptyset_S \tilde{\cap} \emptyset_S = \emptyset_S$.

Here note that if f_S is an SI-almost L (resp. R)-ideal, then f_S needs not to be an SI-L (resp. R)-ideal as shown in the following example:

Example 3.6. In Example 3.4, it is shown that f_S and h_S are SI-almost L (resp. R)-ideals; however f_S and h_S are not SI-L (resp. R)-ideals. In fact,

$$\begin{aligned}(\tilde{\mathbb{S}} \circ f_S)(\ell) &= \left[\left(\tilde{\mathbb{S}}(\ell) \cap f_S(\ell)\right) \cup \left(\tilde{\mathbb{S}}(g) \cap f_S(n)\right) \cup \left(\tilde{\mathbb{S}}(n) \cap f_S(g)\right)\right] \\ &= f_S(\ell) \cup f_S(n) \cup f_S(g) \nsubseteq f_S(\ell)\end{aligned}$$

thus, f_S is not an SI-L-ideal. Similarly,

$$\begin{aligned}(f_S \circ \tilde{\mathbb{S}})(\ell) &= \left[\left(f_S(\ell) \cap \tilde{\mathbb{S}}(\ell)\right) \cup \left(f_S(g) \cap \tilde{\mathbb{S}}(n)\right) \cup \left(f_S(n) \cap \tilde{\mathbb{S}}(g)\right)\right] \\ &= f_S(\ell) \cup f_S(g) \cup f_S(n) \nsubseteq f_S(\ell)\end{aligned}$$

thus, f_S is not an SI-R-ideal. It is obvious that f_S is not an SI-ideal.

Similarly,

$$\begin{aligned}(\widetilde{\mathbb{S}} \circ h_S)(\ell) &= \left[\left(\widetilde{\mathbb{S}}(\ell) \cap h_S(\ell)\right) \cup \left(\widetilde{\mathbb{S}}(g) \cap h_S(n)\right) \cup \left(\widetilde{\mathbb{S}}(n) \cap h_S(g)\right)\right] \\ &= h_S(\ell) \cup h_S(n) \cup h_S(g) \nsubseteq h_S(\ell)\end{aligned}$$

thus, h_S is not an SI-L-ideal. Similarly,

$$\begin{aligned}(h_S \circ \widetilde{\mathbb{S}})(\ell) &= \left[\left(h_S(\ell) \cap \widetilde{\mathbb{S}}(\ell)\right) \cup \left(h_S(g) \cap \widetilde{\mathbb{S}}(n)\right) \cup \left(h_S(n) \cap \widetilde{\mathbb{S}}(g)\right)\right] \\ &= h_S(\ell) \cup h_S(g) \cup h_S(n) \nsubseteq h_S(\ell)\end{aligned}$$

thus, h_S is not an SI-R-ideal. It is clear that h_S is not an SI-ideal.

Proposition 3.7. Let f_S be an idempotent soft set. If f_S is an SI-almost (L/R)-ideal, then f_S is an SI-almost subsemigroup.

Proof: Assume that f_S is an idempotent SI-almost L-ideal, then $f_S \circ f_S = f_S$ and $(S_x \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$. We need to show that f_S is an SI-almost subsemigroup, that is $(f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$.

$$\begin{aligned}(S_x \circ f_S) \widetilde{\cap} f_S &= [(S_x \circ f_S) \widetilde{\cap} f_S] \widetilde{\cap} f_S \\ &= [(S_x \circ f_S) \widetilde{\cap} (f_S \circ f_S)] \widetilde{\cap} f_S \\ &\widetilde{\subseteq} (f_S \circ f_S) \widetilde{\cap} f_S\end{aligned}$$

Since $(S_x \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$. Thus, f_S is an SI-almost subsemigroup.

Theorem 3.8. Let $f_S \widetilde{\subseteq} h_S$ such that f_S is an SI-almost L (resp. R)-ideal, then h_S is an SI-almost L (resp. R)-ideal.

Proof: Assume that f_S is an SI-almost L-ideal. Hence, for all $x \in S$, $(S_x \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$. We need to show that $(S_x \circ h_S) \widetilde{\cap} h_S \neq \emptyset_S$. In fact,

$$(S_x \circ f_S) \widetilde{\cap} f_S \widetilde{\subseteq} (S_x \circ h_S) \widetilde{\cap} h_S.$$

Since $(S_x \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(S_x \circ h_S) \widetilde{\cap} h_S \neq \emptyset_S$. This completes the proof.

Theorem 3.9. Let f_S and h_S be SI-almost L (resp. R)-ideals. Then, $f_S \widetilde{\cup} h_S$ is an SI-almost L (resp. R)-ideal.

Proof: Since f_S is an SI-almost L-ideal by assumption and $f_S \widetilde{\subseteq} f_S \widetilde{\cup} h_S$, $f_S \widetilde{\cup} h_S$ is an SI-almost L-ideal by Theorem 3.8.

Corollary 3.10. The finite union of SI-almost L (resp. R)-ideals is an SI-almost L (resp. R)-ideal.

Corollary 3.11. Let f_S or h_S be SI-almost L (resp. R)-ideal. Then, $f_S \widetilde{\cup} h_S$ is an SI-almost L (resp. R) ideal.

Here note that if f_S and h_S are SI-almost L (resp. R)-ideals, then $f_S \widetilde{\cap} h_S$ needs not to be an SI-almost L (resp. R)-ideal.

Example 3.12. Consider the SI-almost L (resp. R)-ideals f_S and h_S in Example 3.4. Since,

$$f_S \tilde{\cap} h_S = \{(\ell, \emptyset), (g, \emptyset), (n, \emptyset)\} = \emptyset_S$$

$f_S \tilde{\cap} h_S$ is not an SI-almost L (resp. R)-ideal.

Now, we give the relationship between almost L (resp. R)-ideal and SI-almost L (resp. R)-ideal of S. But first of all, we give the following lemma to use it in Theorem 3.14.

Lemma 3.13. Let $x \in S$ and Y be nonempty subset of S. Then, $S_x \circ S_Y = S_{xY}$.

Proof: Let $s \in S$ such that $s \in xY$. Then, $s = xn$ for some $n \in Y$ and $x \in S$. Thus,

$$\begin{aligned}(S_x \circ S_Y)(s) &= \bigcup_{s=xq} \{S_x(x) \cap S_Y(q)\} \\ &\supseteq S_x(x) \cap S_Y(n) \\ &= U \cap U \\ &= U\end{aligned}$$

so $(S_x \circ S_Y)(s) = U$. Since $s = xn \in xY$, it follows that $S_{xY}(s) = U$. Hence, $S_x \circ S_Y = S_{xY}$.

In another case, let $s \in S$ such that $s \notin xY$. Then, if $s = mn$ for some $m, n \in S$, we have that $m \neq x$ or $n \notin Y$. Consider the following equations:

$$(S_x \circ S_Y)(s) = \bigcup_{s=mn} \{S_x(m) \cap S_Y(n)\};$$

Case 1: Let $m \neq x$ and $n \in Y$. Then,

$$\bigcup_{s=mn} \{S_x(m) \cap S_Y(n)\} = \bigcup_{s=mn} \{\emptyset \cap U\} = \emptyset$$

Case 2: Let $m = x$ and $n \notin Y$. Then,

$$\bigcup_{s=mn} \{S_x(m) \cap S_Y(n)\} = \bigcup_{s=xn} \{U \cap \emptyset\} = \emptyset$$

Case 3: Let $m \neq x$ and $n \notin Y$, Then,

$$\bigcup_{s=mn} \{S_x(m) \cap S_Y(n)\} = \bigcup_{s=mn} \{\emptyset \cap \emptyset\} = \emptyset.$$

In all cases, $(S_x \circ S_Y)(s) = \emptyset$ when $s \notin xY$. Since $s \notin xY$, it follows that $S_{xY}(s) = \emptyset$. Hence $S_x \circ S_Y = S_{xY}$.

When X is a nonempty subset of S and $y \in S$, then it is obvious that $S_X \circ S_y = S_{Xy}$.

Theorem 3.14. Let A be a subset of S. Then, A is an almost L (resp. R)-ideal if and only if S_A, the soft characteristic function of A, is an SI-almost L (resp. R)-ideal, where $\emptyset \neq A \subseteq S$.

Proof: Assume that $\emptyset \neq A$ is an almost L-ideal. Then, $xA \cap A \neq \emptyset$ for all $x \in S$, and so there exist $k \in S$ such that $k \in xA \cap A$. Since,

$$\big((S_x \circ S_A) \tilde{\cap} S_A\big)(k) = (S_{xA} \tilde{\cap} S_A)(k) = (S_{xA\cap A})(k) = U \neq \emptyset$$

it follows that $(S_x \circ S_A) \tilde{\cap} S_A \neq \emptyset_S$. Thus, S_A is an SI-almost L-ideal.

Conversely assume that S_A is an SI-almost L-ideal. Hence, we have $(S_x \circ S_A) \tilde{\cap} S_A \neq \emptyset_S$ for all $x \in S$. To show that A is an almost L-ideal, we should prove that $A \neq \emptyset$ and $xA \cap A \neq \emptyset$ for all $x \in S$. $A \neq \emptyset$ is obvious from assumption. Now,

$$\begin{aligned}
\emptyset_S \neq (S_x \circ S_A) \tilde{\cap} S_A &\Rightarrow \exists k \in S\,;\ \big((S_x \circ S_A) \tilde{\cap} S_A\big)(k) \neq \emptyset \\
&\Rightarrow \exists k \in S\,;\ (S_{xA} \tilde{\cap} S_A)(k) \neq \emptyset \\
&\Rightarrow \exists k \in S\,;\ (S_{xA\cap A})(k) \neq \emptyset \\
&\Rightarrow \exists k \in S\,;\ (S_{xA\cap A})(k) = U \\
&\Rightarrow k \in xA \cap A
\end{aligned}$$

Hence, $xA \cap A \neq \emptyset$. Consequently, A is an almost L-ideal.

Lemma 3.15. Let f_S be a soft set over U. Then, $f_S \tilde{\subseteq} S_{supp(f_S)}$ [49].

Theorem 3.16. If f_S is an SI-almost L (resp. R)-ideal, then $supp(f_S)$ is an almost L (resp. R)-ideal.

Proof: Assume that f_S is an SI-almost L-ideal. Thus, $(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$. To show that $supp(f_S)$ is an almost L-ideal, by Theorem 3.14, it is enough to show that $S_{supp(f_S)}$ is an SI-almost L-ideal. By Lemma 3.15,

$$(S_x \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} \big(S_x \circ S_{supp(f_S)}\big) \tilde{\cap} S_{supp(f_S)}$$

and $(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, it implies that $\big(S_x \circ S_{supp(f_S)}\big) \tilde{\cap} S_{supp(f_S)} \neq \emptyset_S$. Consequently, $S_{supp(f_S)}$ is an SI-almost L-ideal and by Theorem 3.14, $supp(f_S)$ is an almost L-ideal.

Here note that the converse of Theorem 3.16 is not true in general as shown in the following example.

Example 3.17. We know that g_S is not an SI-almost L-ideal in Example 3.4. and it is obvious that $supp(g_S) = \{\ell, g, n\} = S$. Since,

$$[\{\ell\}supp(g_S)] \cap supp(g_S) = \{\ell\}\{\ell, g, n\} \cap \{\ell, g, n\} = \{\ell, g, n\} \neq \emptyset$$

$$[\{g\}supp(g_S)] \cap supp(g_S) = \{g\}\{\ell, g, n\} \cap \{\ell, g, n\} = \{\ell, g, n\} \neq \emptyset$$

$$[\{n\}supp(g_S)] \cap supp(g_S) = \{n\}\{\ell, g, n\} \cap \{\ell, g, n\} = \{\ell, g, n\} \neq \emptyset.$$

It is seen that $[\{x\}supp(g_S)] \cap supp(g_S) \neq \emptyset$ for all $x \in S$. That is to say, $supp(g_S)$ is an almost L-ideal; although g_S is not an SI-almost L-ideal.

Definition 3.18. Let f_S and h_S be SI-almost L (resp. R)-ideals such that $h_S \tilde{\subseteq} f_S$. If $supp(h_S) = supp(f_S)$, then f_S is called a minimal SI-almost L (resp. R)-ideal.

Theorem 3.19. A is a minimal almost L (resp. R)-ideal if and only if S_A, the soft characteristic function of A, is a minimal SI-almost L (resp. R)-ideal, where $\emptyset \neq A \subseteq S$.

Proof: Assume that A is a minimal almost L-ideal. Thus, A is an almost L-ideal, and so S_A is an SI-almost L-ideal by Theorem 3.14. Let f_S be an SI-almost L-ideal such that $f_S \tilde{\subseteq} S_A$. By Theorem 3.15, $supp(f_S)$ is an almost L-ideal, and by Note 2.6 and Corollary 2.14,

$$supp(f_S) \subseteq supp(S_A) = A.$$

Since A is a minimal almost L-ideal, $supp(f_S) = supp(S_A) = A$. Thus, S_A is a minimal SI-almost L-ideal by Definition 3.18.

Conversely, let S_A be a minimal SI-almost L-ideal. Thus, S_A is an SI-almost L-ideal, and A is an almost L-ideal by Theorem 3.14. Let B be an almost L-ideal such that $B \subseteq A$. By Theorem 3.14, S_B is an SI-almost L-ideal, and by Theorem 2.15 (i), $S_B \tilde{\subseteq} S_A$. Since S_A is a minimal SI-almost L-ideal,

$$B = supp(S_B) = supp(S_A) = A$$

by Corollary 2.14. Thus, A is a minimal almost L-ideal.

Definition 3.20. Let f_S, g_S and h_S be any SI-almost L (resp. R)-ideals. If $h_S \circ g_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-prime almost L (resp. R)-ideal.

Definition 3.21. Let f_S and h_S be any SI-almost L (resp. R)-ideals. If $h_S \circ h_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$, then f_S is called an SI-semiprime almost L (resp. R)-ideal.

Definition 3.22. Let f_S, g_S, and h_S be any SI-almost L (resp. R)-ideals. If $(h_S \circ g_S) \tilde{\cap} (g_S \circ h_S) \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-strongly prime almost L (resp. R)-ideal.

It is obvious that every SI-strongly prime almost L (resp. R)-ideal is an SI-prime almost L (resp. R)-ideal and every SI-prime almost L (resp. R)-ideal is an SI-semiprime almost L (resp. R)-ideal.

Theorem 3.23. If S_P, the soft characteristic function of P, is an SI-prime almost L (resp. R)-ideal, then P is a prime almost L (resp. R)-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-prime almost L-ideal. Thus, S_P is an SI-almost L-ideal and thus, P is an almost L-ideal by Theorem 3.14. Let A and B be almost L-ideals such that $AB \subseteq P$. Thus, by Theorem 3.14, S_A and S_B are SI-almost L-ideals, and by Theorem 2.15 (i) and (iii), $S_A \circ S_B = S_{AB} \tilde{\subseteq} S_P$. Since S_P is an SI-prime almost L-ideal and $S_A \circ S_B \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Therefore, by Theorem 2.15 (i), $A \subseteq P$ or $B \subseteq P$. Consequently, P is a prime almost L-ideal.

Theorem 3.24. If S_P, the soft characteristic function of P, is an SI-semiprime almost L (resp. R)-ideal, then P is a semiprime almost L (resp. R)-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-semiprime almost L-ideal. Thus, S_P is an SI-almost L-ideal and thus, P is an almost L-ideal by Theorem 3.14. Let A be an almost L-ideal such that $AA \subseteq P$. Thus, by Theorem 3.14, S_A is an SI-almost L-ideal, and by Theorem 2.15 (i) and (iii), $S_A \circ S_A = S_{AA} \tilde{\subseteq} S_P$. Since S_P is an SI-semiprime almost L-ideal and $S_A \circ S_A \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$. Therefore, by Theorem 2.15 (i), $A \subseteq P$. Consequently, P is a semiprime almost L-ideal.

Theorem 3.25. If S_P, the soft characteristic function of P, is an SI-strongly prime almost L (resp. R)-ideal, then P is a strongly prime almost L (resp. R)-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-strongly prime almost L-ideal. Thus, S_P is an SI-almost L-ideal and thus, P is an almost L-ideal by Theorem 3.14. Let A and B be almost L-ideals such that $AB \cap BA \subseteq P$. Thus, by Theorem 3.14, S_A and S_B are SI-almost L-ideal, and by Theorem 2.15, $(S_A \circ S_B) \tilde{\cap} (S_B \circ S_A) = S_{AB} \tilde{\cap} S_{BA} = S_{AB\cap BA} \tilde{\subseteq} S_P$. Since S_P is an SI-strongly prime almost L-ideal and $(S_A \circ S_B) \tilde{\cap} (S_B \circ S_A) \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Thus, by Theorem 2.15 (i), $A \subseteq P$ or $B \subseteq P$. Therefore, P is a strongly prime almost L-ideal.

IV. CONCLUSIONS

In this study, as a generalization of the nonnull soft intersection left (resp. right) ideal of a semigroup, we introduced the idea of the soft intersection almost left (resp. right) ideal of a semigroup. We showed that every idempotent soft intersection almost (left/right) ideal of a semigroup is a soft intersection almost subsemigroup. We obtained that a semigroup can be constructed under the binary operation of union, but not under the binary of operation intersection for soft sets, given the collection of almost (left/right) ideals of a semigroup. Moreover, we showed the relation between soft intersection almost left (resp. right) ideals of a semigroup and almost left (resp. right) ideals of a semigroup in accordance with minimality, primeness, semiprimeness, and strongly primeness. In future studies, many types of soft intersection almost ideals, including quasi-ideal, interior ideal, bi-ideal, bi-interior ideal, bi-quasi-ideal, quasi-interior ideal, and bi-quasi-interior ideal of semigroups can be examined.

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