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A New Type of Extended Soft Set Operations: Complementary Extended Difference Operation

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Abstract

Soft set theory has many theoretical and practical applications. It was first introduced by Molodtsov in 1999 as a way to represent specific situations including uncertainty. The fundamental building blocks of soft set theory are soft set operations. Since its debut, several types of soft set operations have been defined and utilized in diverse contexts. To further the theory, a new soft set operation known as the complementary extended difference operation is defined in this paper. Its properties are thoroughly discussed, with particular attention to how it differs from the difference operation in classical sets. Additionally, the distribution of this operation over other types of soft set operations is examined to determine how this operation relates to other soft set operations.

Keywords: *Soft set, Conditional complements, Complementary extended difference operation.*

Yeni Bir Esnek Küme İşlemi: Tümleyenli Genişletilmiş Fark İşlemi

Özet

Esnek küme teorisinin birçok teorik ve pratik uygulaması vardır. İlk kez 1999 yılında Molodtsov tarafından belirsizlik durumlarını temsil etmenin bir yolu olarak tanıtıldı. Esnek küme teorisinin temel yapı taşları esnek küme işlemleridir. İlk çıkışından bu yana, çeşitli bağlamlarda esnek küme işlemlerinin çeşitli türleri tanımlanmış ve kullanılmıştır. Teoriyi ilerletmek amacıyla bu çalışmada tümleyenli genişletilmiş fark işlemi olarak isimlendirilen yeni bir esnek küme işlemi tanımlanmıştır. Özellikleri, klasik kümelerdeki fark işlemi ile kıyaslanarak kapsamlı bir şekilde tartışılmıştır. Ayrıca, bu işlemin diğer esnek küme işlemleri ile nasıl bir ilişkisi olduğunu belirlemek amacıyla bu işlemin diğer esnek küme işlemlerine dağılımı da incelenmiştir.

Anahtar Kelimeler: *Esnek küme, koşullu tümleyenler, tümleyenli genişletilmiş işlemler.*

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1. Introduction

It might be difficult to remark on and explain some of the events that happen in our lives. Some terms are ambiguous and change depending on the individual, such as "big bike," "quality bag," and "hot weather." Due to the uncertainty, they include, these phrases, circumstances, and occurrences are frequently arbitrary and contingent on people, places, and times. Numerous scientific disciplines, including mathematics, are susceptible to uncertainties. In many scientific disciplines, researchers work to find solutions to challenging issues, yet they have also encountered modeling uncertainties. Since uncertainties come in a variety of forms, it has been necessary to eradicate these uncertainties using ways other than classical procedures—methods that also assess uncertainty. As a result, scientists have developed a wide range of hypotheses to explain uncertainty and offer remedies.

Some of the most popular and widely applied mathematical theories for modeling uncertainty include fuzzy set theory, interval mathematics, statistics, and probability theory. Among these ideas, Zadeh's fuzzy set theory [1] is one of the most well-known. Other hypotheses have been required since this hypothesis has some structural flaws. It is well known that a fuzzy set's membership function defines it. The nature of the membership function is extremely personalized since it is challenging to design a membership function for every situation. As a result, a set theory independent of the membership function's creation has been required. Molodstov [2] introduced the Soft Set Theory, which has solved the membership function issues. Molodstov has introduced soft set theory into several mathematical fields. Soft set theory has been effectively applied in the fields of operations research, game theory, probability, measurement theory, continuously differentiable functions, Riemann's integration, and Perron's integration.

Since studies on soft algebraic structures and soft decision-making techniques depend on soft set operations, soft set operations form the foundation of soft set theory. Maji et al. [3] initiated the influential research on soft set operations in this area. Pei and Miao [4] suggested a definition of soft subset that is more often accepted than the definition provided by Maji et al. [5]. Soft set operations fall into two categories: restricted and extended operations [3, 5-9].

A novel form of soft set operation was proposed by Eren and Çalışıcı [10] and later on, by Sezgin and Çalışıcı [11], who enhanced the work of Eren and Çalışıcı [10] by examining the characteristics of the soft binary piecewise difference operation and contrasting it with the difference operation in classical sets. Aybek [12] extended the study of novel binary set operations by Çağman [13] and Sezgin et al. [14] to soft sets. Furthermore, several researchers [15-29] have presented novel types of soft set operations that differ from the restricted and extended forms of soft set operations. [30-43] are some additional applications of soft sets with relation to algebraic structures that we can refer to.

One of the most crucial mathematical problems in algebra is to categorize algebraic structures by examining the characteristics of the operation specified on a set. To conceptually contribute to the literature on soft sets, we provide a new class of soft set operations in this paper, which we name complementary extended difference operations, and we go into great detail about its properties. We try to find the analogies that of the difference operation in classical sets. The distribution of complementary extended difference operations over other types of soft set operations, such as restricted and extended soft set operations and soft binary piecewise operations, is examined to ascertain the relationship between the operation and other soft set operations. Many intriguing results are obtained.

2. Preliminaries

Definition 2.1. [2] Let E be the parameter set, U be the universal set, $P(U)$ be the power set of U, and $M \subseteq E$. A pair (F, M) is called a soft set over U. Here, F is a function given by $F: M \to P(U)$.

$S_E(U)$ denotes the set of all the soft sets over U throughout this paper. Let M be a fixed subset of E, then the set of all soft sets over U with M is indicated by $S_M(U)$. In other words, in the collection $S_M(U)$, only soft sets with the parameter set M are included, while in the collection $S_E(U)$, soft sets over U with any parameter set can be included.

Definition 2.2. [5, 7] Let (F, M) be a soft set over U. For all $v \in M$, if $F(v) = \emptyset$, then the soft set (F, M) is called a null soft set with respect to M, indicated by $\emptyset_M$. For all $v \in M$, if $F(v) = U$, then the soft set (F, M) is called a whole soft set with respect to M, indicated by U_M. The relative whole soft set U_E with respect to E is called the absolute soft set over U. A soft set with an empty parameter set is indicated by $\emptyset_\emptyset$, called as empty soft set, and $\emptyset_\emptyset$ is the only soft set with an empty parameter set.

Definition 2.3. [4] For two soft sets and (F, M) and (G, Y), we say that (F, M) is a soft subset of (G, Y), and it is indicated by $(F, M) \widetilde{\subseteq} (G, Y)$, if $M \subseteq Y$ and $F(v) \subseteq G(v)$, for all $v \in M$. Two soft sets (F, M) and (G, Y) are said to be soft equal if (F, M) is a soft subset of (G, Y) and (G, Y) is a soft subset of (F, M).

Definition 2.4. [5] The relative complement of a soft set (F, M), indicated by $(F, M)^r$, is defined by $(F, M)^r = (F^r, M)$, where $F^r: M \to P(U)$ is a mapping given by $(F, M)^r = U \setminus F(v)$, for all $v \in M$. From now on, $U \setminus F(v) = [F(v)]'$ will be designated by $F'(v)$ for the sake of designation.

Çağman [13] defined two new complements as inclusive and exclusive complements. $+$ and θ denote inclusive and exclusive complements, respectively. Let M and N be two sets. Then, these binary operations are defined as follows: $M + N = M' \cup N$, $M \theta N = M' \cap N'$. Sezgin et al. [14] analyzed the relations between these two operations and also defined three new binary operations and examined their relations with each other. Let M and N be two sets. Then, $M * N = M' \cup N'$, $M \gamma N = M' \cap N$, and $M \lambda N = M \cup N'$.

Let $\circledast$ denote $\cap, \cup, \setminus, \Delta$ (symmetric difference), $\lambda, \gamma, \theta, +, *$. Then, all the types of soft set operations can be given with the following generalized forms:

Definition 2.5. [5-9, 12] Let $(F, M), (G, Y) \in S_E(U)$. The restricted $\circledast$ operation of (F, M) and (G, Y) is the soft set $(H, Ƶ)$ denoted by $(F, M) \circledast_{\Re} (G, Y) = (H, Ƶ)$, where $Ƶ = M \cap Y \neq \emptyset$, and $H(v) = F(v) \circledast G(v)$, for all $v \in Ƶ$. Here, if $Ƶ = M \cap Y = \emptyset$, then $(F, M) \circledast_{\Re} (G, Y) = \emptyset_\emptyset$.

Definition 2.6. [3-5, 8-9, 12] Let $(F, M), (G, Y) \in S_E(U)$. The extended $\circledast$ operation (F, M) and (G, Y) is the soft set $(H, Ƶ)$, indicated by $(F, M) \circledast_{\varepsilon} (G, Y) = (H, Ƶ)$, where $Ƶ = M \cup Y$, and for all $v \in Ƶ$,

$$H(v) = \begin{cases} F(v), & v \in M \setminus Y \\ G(v), & v \in Y \setminus M \\ F(v) \circledast G(v), & v \in M \cap Y \end{cases}$$

Definition 2.7. [16-18]
Let $(F, M), (G, Y) \in S_E(U)$. The complementary extended ⊛ operation (F, M) and (G, Y) is the soft set (H, Z), indicated by $(F, M) \overset{*}{\circledast}_{\varepsilon} (G, Y) = (H, Z)$, where $Z = M \cup Y$, and for all $v \in Z$,

$$H(v) = \begin{cases} F'(v), & v \in M - Y \\ G'(v), & v \in Y - M \\ F(v) \circledast G(v), & v \in M \cap Y \end{cases}$$

Definition 2.8. [10-11,15,28] Let $(F, M), (G, Y) \in S_E(U)$. The soft binary piecewise ⊛ of (F, M) and (G, Y) is the soft set (H, M), indicated by $(F, M) \underset{\circledast}{\sim} (G, Y) = (H, M)$, where for all $v \in M$,

$$H(v) = \begin{cases} F(v), & v \in M \setminus Y \\ F(v) \circledast G(v), & v \in M \cap Y \end{cases}$$

Definition 2.9. [20-26,29] Let $(F, M), (G, Y) \in S_E$. The complementary soft binary piecewise ⊛ of (F, M) and (G, Y) is the soft set (H, M), indicated by $(F, M) \overset{*}{\underset{\circledast}{\sim}} (G, Y) = (H, M)$, where for all $v \in M$,

$$H(v) = \begin{cases} F'(v), & v \in M \setminus Y \\ F(v) \circledast G(v), & v \in M \cap Y \end{cases}$$

Definition 2.10. [44] Let X be a set, " $\star$ " be a binary operation on X, and 0 be an element of the set X. Then, X is called a BCI-algebra if the following conditions are satisfied for the triple $(X; \star, 0)$,

BCI-1 $((\curlyvee \star \wp) \star (\curlyvee \star z)) \star (z \star \wp) = 0$

BCI-2 $(\curlyvee \star (\curlyvee \star \wp)) \star \wp = 0$

BCI-3 $\curlyvee \star \curlyvee = 0$

BCI-4 $\curlyvee \star \wp = 0$ and $\wp \star \curlyvee = 0$ implies $\curlyvee = \wp$

for all $\curlyvee, \wp, z \in X$. If a BCI-algebra satisfies the following additional condition, it is called a BCK-algebra:

BCK-5 $0 \star \curlyvee = 0$.

If there exists an element $1 \in X$ such that $\curlyvee \star 1 = 0$ for every $\curlyvee \in X$, then X is called a bounded BCK algebra. An element $\curlyvee \in X$ is called an involution for a BCK algebra if it satisfies the condition $1 \star (1 \star \curlyvee) = \curlyvee$. For the possible future graph applications and network analysis as regards soft sets, we refer to Pant et al. [44] which is motivated by the divisibility of determinants.

3. Complementary Extended Difference Operation

In this section, the algebraic properties of the soft set operation called the complementary extended difference operation are examined comparatively with the properties of the difference operation in classical sets. It is investigated which algebraic structure these operations constitute in the collection of soft sets with a fixed parameter set, and distributive rules are examined to see the relationships of this operation with other operations, and similar results to the distributions in classical sets are obtained.

Definition 3.1. Let (F,T) and $(G,Ƶ)$ be two soft sets over U. The complementary extended difference operation of (F,T) and $(G,Ƶ)$ is the soft set (H,C), indicated by $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,Ƶ)=(H,C)$, where $C=T\cup Ƶ$, and for all $ω\in C$,

$$H(ω)=\begin{cases}F'(ω), & ω\in T\backslash Ƶ\\ G'(ω), & ω\in Ƶ\backslash T\\ F(ω)\backslash G(ω), & ω\in T\cap Ƶ\end{cases}$$

Example 3.2. Let $E=\{e_1,e_2,e_3,e_4\}$ be the parameter set, $T=\{e_1,e_3\}$ and $Ƶ=\{e_2,e_3,e_4\}$ be two subsets of E and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set.
Assume that $(F,T)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$, and $(G,Ƶ)=\{(e_2,\{h_1,h_4,h_5\}),(e_3,\{h_2,h_3,h_4\}),(e_4,\{h_3,h_5\})\}$ be soft sets over U. Let $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,Ƶ)=(H,T\cup Ƶ)$, where

$$H(ω)=\begin{cases}F'(ω), & ω\in T\backslash Ƶ\\ G'(ω), & ω\in Ƶ\backslash T\\ F(ω)\backslash G(ω), & ω\in T\cap Ƶ\end{cases}$$

for all $ω\in T\cup Z$. Hence, $H(e_1)=F'(e_1)=\{h_1,h_3,h_4\}$, $H(e_2)=G'(e_2)=\{h_2,h_3\}$, $H(e_4)=G'(e_4)=\{h_1,h_2,h_4\}$, and $H(e_3)=F(e_3)\backslash G(e_3)=\{h_1,h_5\}$. Thus, $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,Ƶ)=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_2,h_3\}),(e_3,\{h_1,h_5\}),(e_4,\{h_1,h_2,h_4\})\}$.

Theorem 3.3.

1) $\overset{*}{\backslash}_{\varepsilon}$ is closed in $S_E(U)$.

Proof: $\overset{*}{\backslash}_{\varepsilon}: S_E(U)xS_E(U)\to S_E(U)$

$((F,T),(G,Ƶ))\to(F,T)\overset{*}{\backslash}_{\varepsilon}(G,Ƶ)=(H,T\cup Z)$

Similarly,

$\overset{*}{\backslash}_{\varepsilon}: S_T(U)xS_T(U)\to S_T(U)$

$((F,T),(G,T))\to(F,T)\overset{*}{\backslash}_{\varepsilon}(G,T)=(H,T\cup T)$

That is, when T is a fixed subset of the set E, (F,T) and (G,T) are elements of $S_T(U)$, then so is $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,T)$. Namely, $S_T(U)$ is closed under the operation $\overset{*}{\backslash}_{\varepsilon}$ as well.

2) $[(F,T)\overset{*}{\backslash}_{\varepsilon}(G,Ƶ)]\overset{*}{\backslash}_{\varepsilon}(H,M)\neq(F,T)\overset{*}{\backslash}_{\varepsilon}[(G,Ƶ)]\overset{*}{\backslash}_{\varepsilon}(H,M)]$

Proof: Firstly, let's consider the left hand side (LHS). Suppose that $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,Ƶ)=(S,T\cup Z)$, where

$$S(ω)=\begin{cases}F'(ω), & ω\in T\backslash Ƶ\\ G'(ω), & ω\in Ƶ\backslash T\\ F(ω)\backslash G(ω), & ω\in T\cap Ƶ\end{cases}$$

for all $ω\in T\cup Z$. Let $(S,T\cup Z)\overset{*}{\backslash}_{\varepsilon}(H,M)=(L,(T\cup Z)\cup M))$, where

$$L(\omega)=\begin{cases} S'(\omega), & \omega\in(T\cup Z)\setminus M\\ H'(\omega), & \omega\in M\setminus(T\cup M)\\ S(\omega)\setminus H(\omega), & \omega\in(T\cup Z)\cap M\end{cases}$$

for all $\omega\in(T\cup Z)\cup M$. Thus,

$$L(\omega)=\begin{cases} F(\omega), & \omega\in(T\setminus Z)\setminus M=T\cap Z'\cap M'\\ G(\omega) & \omega\in(Z\setminus T)\setminus M=T'\cap Z\cap M'\\ F'(\omega)\cup G(\omega) & \omega\in(T\cap Z)\setminus M=T\cap Z\cap M'\\ H'(\omega), & \omega\in M\setminus(T\cup Z)=T'\cap Z'\cap M\\ F'(\omega)\cap H'(\omega), & \omega\in(T\setminus Z)\cap M=T\cap Z'\cap M\\ G'(\omega)\cap H'(\omega), & \omega\in(Z\setminus T)\cap M=T'\cap Z\cap M\\ [F'(\omega)\cup G(\omega)]\cap H'(\omega) & \omega\in(T\cap Z)\cap M=T\cap Z\cap M\end{cases}$$

Now, let's consider the right hand side (RHS). Suppose that $(G,Z)\overset{*}{\setminus}_{\varepsilon}(H,M)=(R,Z\cup M)$, where

$$R(\omega)=\begin{cases} G'(\omega), & \omega\in Z\setminus M\\ H'(\omega), & \omega\in M\setminus Z\\ G(\omega)\setminus H(\omega), & \omega\in Z\cap M\end{cases}$$

for all $\omega\in Z\cup M$. Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(R,Z\cup M)=(N,(T\cup(Z\cup M))$, where

$$N(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus(Z\cup M)\\ R'(\omega), & \omega\in(Z\cup M)\setminus T\\ G(\omega)\setminus H(\omega), & \omega\in T\cap(Z\cup M)\end{cases}$$

for all $\omega\in T\cup(Z\cup M)$. Thus,

$$N(\omega)=\begin{cases} F(\omega), & \omega\in T\setminus(Z\cup M)=T\cap Z'\cap M'\\ G(\omega) & \omega\in(Z\setminus M)\setminus T=T'\cap Z\cap M'\\ H(\omega), & \omega\in(M\setminus Z)\setminus T=T'\cap Z'\cap M\\ G'(\omega)\cup H(\omega), & \omega\in(Z\cap M)\setminus T=T'\cap Z\cap M\\ F(\omega)\cap G(\omega), & \omega\in T\cap(Z\setminus M)=T\cap Z\cap M'\\ F(\omega)\cap H(\omega), & \omega\in T\cap(M\setminus Z)=T\cap Z'\cap M\\ F(\omega)\setminus[G(\omega)\setminus H(\omega)], & \omega\in T\cap(Z\cap M)=T\cap Z\cap M\end{cases}$$

It is seen that $(L,(T\cup Z)\cup M)\neq(N,T\cup(Z\cup M))$. That is, in the set $S_E(U)$, $\overset{*}{\setminus}_{\varepsilon}$ is not associative.

3) $[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)]\overset{*}{\setminus}_{\varepsilon}(H,T)\neq(F,T)\overset{*}{\setminus}_{\varepsilon}[(G,T)]\overset{*}{\setminus}_{\varepsilon}(H,T)]$

Proof: Since $[F(\omega)\setminus G(\omega)]\setminus H(\omega)\neq F(\omega)\setminus[G(\omega)\setminus H(\omega)]$, $\overset{*}{\setminus}_{\varepsilon}$ is not associative in the set $S_T(U)$, where T is a fixed subset of E.

4) $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,Z)\neq(G,Z)\overset{*}{\setminus}_{\varepsilon}(F,T)$

Proof: Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,Z)=(H,T\cup Z)$, where

$$H(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus Z\\ G'(\omega), & \omega\in Z\setminus T\\ F(\omega)\setminus G(\omega), & \omega\in T\cap Z\end{cases}$$

for all $\omega \in T \cup Z$. Let $(G,Z) \overset{*}{\backslash_{\varepsilon}} (F,T) = (S, Z \cup T)$, where

$$S(\omega) = \begin{cases} G'(\omega), & \omega \in Z \backslash T \\ F'(\omega), & \omega \in T \backslash Z \\ G(\omega) \backslash F(\omega), & \omega \in Z \cap T \end{cases}$$

for all $\omega \in Z \cup T$. Thus, $(F,T) \overset{*}{\backslash_{\varepsilon}} (G,Z) \neq (G,Z) \overset{*}{\backslash_{\varepsilon}} (F,T)$. If $Z \cap T = \emptyset$, then $(F,T) \overset{*}{\backslash_{\varepsilon}} (G,Z) = (G,Z) \overset{*}{\backslash_{\varepsilon}} (F,T)$. It is also obvious that $(F,T) \overset{*}{\backslash_{\varepsilon}} (G,T) \neq (G,T) \overset{*}{\backslash_{\varepsilon}} (F,T)$. Thereby, $\overset{*}{\backslash_{\varepsilon}}$ is commutative neither in $S_E(U)$ nor in $S_T(U)$.

5) $(F,T) \overset{*}{\backslash_{\varepsilon}} (F,T) = \emptyset_T$

Proof: Let $(F,T) \overset{*}{\backslash_{\varepsilon}} (F,T) = (H,T)$ where $H(\omega) = F(\omega) \cap F'(\omega) = \emptyset$, for all $\omega \in T$, Thus, $(H,T) = \emptyset_T$. That is, $\overset{*}{\backslash_{\varepsilon}}$ is not idempotent in $S_E(U)$.

6) $(F,T) \overset{*}{\backslash_{\varepsilon}} \emptyset_T = (F,T)$

Proof: Let $\emptyset_T = (S,T)$ and $(F,T) \overset{*}{\backslash_{\varepsilon}} (S,T) = (H,T)$. Then, $S(\omega) = \emptyset$ and $H(\omega) = F(\omega) \cap S'(\omega) = F(\omega) \cap U = F(\omega)$, for all $\omega \in T$. Thus, $(H,T) = (F,T)$.

That is, in $S_T(U)$, the right identity element of $\overset{*}{\backslash_{\varepsilon}}$ is the soft set $\emptyset_T$.

7) $\emptyset_T \overset{*}{\backslash_{\varepsilon}} (F,T) = \emptyset_T$

Proof: Let $\emptyset_T = (S,T)$ and $(S,T) \overset{*}{\backslash_{\varepsilon}} (F,T) = (H,T)$. Then, $S(\omega) = \emptyset$ and $H(\omega) = S(\omega) \cap F'(\omega) = \emptyset \cap F'(\omega) = \emptyset$, for all $\omega \in T$. Therefore, $(H,T) = \emptyset_T$.
That is, the left absorbing element of $\overset{*}{\backslash_{\varepsilon}}$ in $S_T(U)$ is the soft set $\emptyset_T$.

8) $(F,T) \overset{*}{\backslash_{\varepsilon}} \emptyset_{\emptyset} = (F,T)^r$

Proof: Let $\emptyset_{\emptyset} = (S,\emptyset)$ and $(F,T) \overset{*}{\backslash_{\varepsilon}} (S,\emptyset) = (H, T \cup \emptyset)$. Then,

$$H(\omega) = \begin{cases} F'(\omega), & \omega \in T \backslash \emptyset = T \\ S'(\omega), & \omega \in \emptyset \backslash T = \emptyset \\ F(\omega) \backslash S(\omega), & \omega \in \emptyset \backslash T = \emptyset \end{cases}$$

for all $\omega \in T$. Thus, $H(\omega) = F'(\omega)$, for all $\omega \in T$. Hence, $(H,T) = (F,T)^r$.

9) $\emptyset_{\emptyset} \overset{*}{\backslash_{\varepsilon}} (F,T) = (F,T)^r$

Proof: Let $\emptyset_{\emptyset} = (S,\emptyset)$ and $(S,\emptyset) \overset{*}{\backslash_{\varepsilon}} (F,T) = (H, \emptyset \cup T)$, where

$$H(\omega) = \begin{cases} S'(\omega), & \omega \in \emptyset \backslash T = \emptyset \\ F'(\omega), & \omega \in T \backslash \emptyset = T \\ S(\omega) \backslash F(\omega), & \omega \in \emptyset \cap T = \emptyset \end{cases}$$

for all $\omega \in T$. Thus, $H(\omega) = F'(\omega)$, for all $\omega \in T$. Thereby, $(H,T) = (F,T)^r$.

10) $(F,T)\overset{*}{\setminus_\varepsilon} U_E = \emptyset_E$

Proof: Let $U_E = (L,E)$ and $(F,T)\overset{*}{\setminus_\varepsilon}(L,E) = (H,T\cup E)$. Then, $L(\omega) = U$, and

$$H(\omega) = \begin{cases} F'(\omega), & \omega \in T\setminus E = \emptyset \\ L'(\omega), & \omega \in E\setminus T = T' \\ F(\omega)\setminus L(\omega), & \omega \in T\cap E = T \end{cases}$$

for all ω∈E. Hence,

$$H(\omega) = \begin{cases} F'(\omega), & \omega \in T\setminus E = \emptyset \\ \emptyset, & \omega \in E\setminus T = T' \\ \emptyset, & \omega \in T\cap E = T \end{cases}$$

for all $\omega\in E$. Thereby, $H(\omega) = \emptyset$, for all $\omega\in E$. Consequently, $(H,E) = \emptyset_E$.

11) $(F,T)\overset{*}{\setminus_\varepsilon} U_T = \emptyset_T$

Proof: Let $U_T = (K,T)$ and $(F,T)\overset{*}{\setminus_\varepsilon}(K,T) = (K,T)$. Then, $K(\omega) = U$ and $H(\omega) = F(\omega)\cap K'(\omega) = F(\omega)\cap \emptyset = \emptyset$, for all $\omega\in T$. Therefore, $(K,T) = \emptyset_T$.

12) $U_T\overset{*}{\setminus_\varepsilon}(F,T) = (F,T)^r$

Proof: Let $U_T = (K,T)$ and $(K,T)\overset{*}{\setminus_\varepsilon}(F,T) = (H,T)$. Then, $K(\omega) = U$ and $H(\omega) = T(\omega)\cap F'(\omega) = U\cap F'(\omega) = F'(\omega)$, for all $\omega\in T$. Thus, $(H,T) = (F,T)^r$.

13) $(F,T)\overset{*}{\setminus_\varepsilon}(F,T)^r = (F,T)$

Proof: Let $(F,T)^r = (H,T)$ and $(F,T)\overset{*}{\setminus_\varepsilon}(H,T) = (L,T)$. Then, $H(\omega) = F'(\omega)$ and $L(\omega) = F(\omega)\cap H'(\omega) = F(\omega)\cap F(\omega) = F(\omega)$, for all $\omega\in T$. Thus, $(L,T) = (F,T)$.
That is, in $S_E(U)$, the complement of every element is its own right identity for $\overset{*}{\setminus_\varepsilon}$.

14) $(F,T)^r\overset{*}{\setminus_\varepsilon}(F,T) = (F,T)^r$

Proof: Let $(F,T)^r = (H,T)$ and $(H,T)\overset{*}{\setminus_\varepsilon}(F,T) = (L,T)$. Then, $H(\omega) = F'(\omega)$ and $T(\omega) = H(\omega)\cap F'(\omega) = F'(\omega)\cap F'(\omega) = F'(\omega)$, for all $\omega\in T$. Thus, $(L,T) = (F,T)^r$.
That is, in $S_E(U)$, the complement of every element is its own left absorbing element for $\overset{*}{\setminus_\varepsilon}$.

15) $[(F,T)\overset{*}{\setminus_\varepsilon}(G,Z)]^r = (F,T) +_\varepsilon (G,Z)$

Proof: Let $(F,T)\overset{*}{\setminus_\varepsilon}(G,Z) = (H,T\cup Z)$, where

$$H(\omega) = \begin{cases} F'(\omega), & \omega \in T\setminus Z \\ G'(\omega), & \omega \in Z\setminus T \\ F(\omega)\setminus G(\omega), & \omega \in T\cap Z \end{cases}$$

for all $\omega\in T\cup Z$. Let $(H,T\cup Z)^r = (K,T\cup Z)$, where

$$K(\omega)=\begin{cases} F(\omega), & \omega\in T\backslash Z \\ G(\omega), & \omega\in Z\backslash T \\ F'(\omega)\cup G(\omega), & \omega\in T\cap Z \end{cases}$$

for all $\omega\in T\cup Z$. Thus, $(K,T\cup Z)=(F,T)+_{\varepsilon}(G,Z)$.

16) $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,T)=U_T \Leftrightarrow (F,T)=U_T$ and $(G,T)=\emptyset_T$

Proof: Let $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,T)=(K,T)$ and $(K,T)=U_T$. Then, $K(\omega)=F(\omega)\cap G'(\omega)=U$, for all $\omega\in T\Leftrightarrow F(\omega)=U$ and $G'(\omega)=U$, for all $\omega\in T \Leftrightarrow F(\omega)=U$ and $G(\omega)=\emptyset$, for all $\omega\in T\Leftrightarrow$ $(F,T)=U_T$ and $(G,T)=\emptyset_T$.

17) $\emptyset_T \,\tilde{\subseteq}\, (F,T)\overset{*}{\backslash}_{\varepsilon}(G,Z)$, $\emptyset_Z \,\tilde{\subseteq}\, (F,T)\overset{*}{\backslash}_{\varepsilon}(G,Z)$, and $\emptyset_Z \,\tilde{\subseteq}\, (G,Z)\overset{*}{\backslash}_{\varepsilon}(F,T)$, $\emptyset_T \,\tilde{\subseteq}\, (G,Z)\overset{*}{\backslash}_{\varepsilon}(F,T)$. Moreover, $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,Z) \,\tilde{\subseteq}\, U_{T\cup Z}$ and $(G,Z)\overset{*}{\backslash}_{\varepsilon}(F,T) \,\tilde{\subseteq}\, U_{Z\cup T}$

18) $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,T) \,\tilde{\subseteq}\, (G,T)^r$ and $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,T) \,\tilde{\subseteq}\, (F,T)$

Proof: Let $(F,T)\overset{*}{\backslash}_{\varepsilon}(F,T)=(H,T)$, where $H(\omega)=F(\omega)\cap G'(\omega)$, for all $\omega\in T$. Since $H(\omega)=F(\omega)\cap G'(\omega)\subseteq F(\omega)$ and $H(\omega)=F(\omega)\cap G'(\omega)\subseteq G'(\omega)$, for all $\omega\in T$, the rest of the proof is obvious.

19) If $(F,T) \,\tilde{\subseteq}\, (G,T)$, then $(H,Z)\overset{*}{\backslash}_{\varepsilon}(G,T) \,\tilde{\subseteq}\, (H,Z)\overset{*}{\backslash}_{\varepsilon}(F,T)$ and $(F,T)\overset{*}{\backslash}_{\varepsilon}(H,Z) \,\tilde{\subseteq}\, (G,T)\overset{*}{\backslash}_{\varepsilon}(H,Z)$

Proof: Let $(F,T) \,\tilde{\subseteq}\, (G,T)$. Then, $F(\omega)\subseteq G(\omega)$ and $G'(\omega)\subseteq F'(\omega)$, for all $\omega\in T$. Let $(H,Z)\overset{*}{\backslash}_{\varepsilon}(G,T)=(Y,Z\cup T)$, where

$$Y(\omega)=\begin{cases} F'(\omega), & \omega\in T\backslash Z \\ G'(\omega), & \omega\in Z\backslash T \\ F(\omega)\backslash G(\omega), & \omega\in T\cap Z \end{cases}$$

for all $\omega\in Z\cup T$. Let $(H,Z)\overset{*}{\backslash}_{\varepsilon}(F,T)=(W,Z\cup T)$, where

$$W(\omega)=\begin{cases} H'(\omega), & \omega\in Z\backslash T \\ F'(\omega), & \omega\in T\backslash Z \\ H(\omega)\backslash F(\omega), & \omega\in Z\cap T \end{cases}$$

for all $\omega\in Z\cup T$. If $\omega\in Z\setminus T$, then $Y(\omega)=H'(\omega)\subseteq H'(\omega)=W(\omega)$. If $\omega\in T\setminus Z$, then $Y(\omega)=G'(\omega)\subseteq F'(\omega)=W(\omega)$, and if $\omega\in Z\cap T$, then $Y(\omega)=H(\omega)\cap G'(\omega)\subseteq H(\omega)\cap F'(\omega)=W(\omega)$. Therefore, $Y(\omega)\subseteq W(\omega)$, for all $\omega\in Z\cup T$. Consequently, $(H,Z)\overset{*}{\backslash}_{\varepsilon}(G,T) \,\tilde{\subseteq}\, (H,Z)\overset{*}{\backslash}_{\varepsilon}(F,T)$. Similarly, one can show that if $(F,T) \,\tilde{\subseteq}\, (G,T)$, then $(F,T)\overset{*}{\backslash}_{\varepsilon}(H,Z) \,\tilde{\subseteq}\, (G,T)\overset{*}{\backslash}_{\varepsilon}(H,Z)$.

20) If $(H,Z)\overset{*}{\backslash}_{\varepsilon}(G,T) \,\tilde{\subseteq}\, (H,Z)\overset{*}{\backslash}_{\varepsilon}(F,T)$, then $(F,T) \,\tilde{\subseteq}\, (G,T)$ needs not be true. Similarly, if $(F,T)\overset{*}{\backslash}_{\varepsilon}(H,Z) \,\tilde{\subseteq}\, (G,T)\overset{*}{\backslash}_{\varepsilon}(H,Z)$, then $(F,T) \,\tilde{\subseteq}\, (G,T)$ needs not be true.

Proof: Let $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$ be the parameter set, $T=\{e_1,e_3\}$ and $Z=\{e_1,e_3,e_5\}$ be the subsets of E, and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal and (F,T), (G,T), and (H,Z) be the soft sets over U as follows: $(F,T)=\{(e_1,U),(e_3,U)\}$, $(G,T)=\{(e_1,\{h_1,h_2\}),(e_3,\{h_3,h_4\})\}$, and $(H,Ƶ)=\{(e_1,\emptyset),(e_3,\emptyset),(e_5,\{h_5\})\}$. Let $(H,Ƶ)\overset{*}{\setminus}_{\varepsilon}(G,T)=(L,Ƶ\cup T)$, where $L(e_1)=H(e_1)\cap G'(e_1)=\emptyset$, $L(e_3)=H(e_3)\cap G'(e_3)=\emptyset$, and $L(e_5)=H'=\{h_1,h_2,h_3,h_4\}$, for all $\omega\in Ƶ\cup T=\{e_1,e_3,e_5\}$. Thereby, $(H,Ƶ)\overset{*}{\setminus}_{\varepsilon}(G,T)=\{(e_1,\emptyset),(e_3,\emptyset),(e_5,\{h_1,h_2,h_3,h_4\})\}$.

Let $(H,Ƶ)\overset{*}{\setminus}_{\varepsilon}(F,T)=(W,Ƶ\cup T)$, where $W(e_1)=H(e_1)\cap F'(e_1)=\emptyset$, $W(e_3)=H(e_3)\cap F'(e_3)=\emptyset$, and $W(e_5)=H'(e_5)=\{h_1,h_2,h_3,h_4\}$, for all $\omega\in Ƶ\cup T=\{e_1,e_3,e_5\}$. Thus, $(H,Ƶ)\overset{*}{\setminus}_{\varepsilon}(F,T)=\{(e_1,\emptyset),(e_3,\emptyset),(e_5,\{h_1,h_2,h_3,h_4\})\}$. Hence, $(H,Ƶ)\overset{*}{\setminus}_{\varepsilon}(G,T)\,\tilde{\subseteq}\,(H,Ƶ)\overset{*}{\setminus}_{\varepsilon}(F,T)$, but (F,T) is not a soft subset of (G,T).

Similarly, one can show that $(F,T)\overset{*}{\setminus}_{\varepsilon}(H,Ƶ)\,\tilde{\subseteq}\,(G,T)\overset{*}{\setminus}_{\varepsilon}(H,Ƶ)$ does not imply that $(F,T)\,\tilde{\subseteq}\,(G,T)$ by taking $(F,T)=\{(e_1,U),(e_3,U)\}$, $(G,T)=\{(e_1,\{h_1,h_2\}),(e_3,\{h_3,h_4\})\}$, and $(H,Ƶ)=\{(e_1,U),(e_3,U),(e_5,\{h_5\})\}$.

21) If $(F,T)\,\tilde{\subseteq}\,(G,T)$ and $(K,T)\,\tilde{\subseteq}\,(L,T)$, then $(F,T)\overset{*}{\setminus}_{\varepsilon}(L,T)\,\tilde{\subseteq}\,(G,T)\overset{*}{\setminus}_{\varepsilon}(K,T)$ and $(K,T)\overset{*}{\setminus}_{\varepsilon}(G,T)\,\tilde{\subseteq}\,(L,T)\overset{*}{\setminus}_{\varepsilon}(F,T)$

Proof: Let $(F,T)\,\tilde{\subseteq}\,(G,T)$ and $(K,T)\,\tilde{\subseteq}\,(L,T)$, Then, $F(\omega)\subseteq G(\omega)$ and $K(\omega)\subseteq L(\omega)$, for all $\omega\in T$. Thereby, $G'(\omega)\subseteq F'(\omega)$ and $L'(\omega)\subseteq K'(\omega)$, for all $\omega\in T$. Hence, $F(\omega)\cap L'(\omega)\subseteq G(\omega)\cap K'(\omega)$ and $K(\omega)\cap G'(\omega)\subseteq L(\omega)\cap F'(\omega)$, for all $\omega\in T$. Consequently, $(F,T)\overset{*}{\setminus}_{\varepsilon}(L,T)\,\tilde{\subseteq}\,(G,T)\overset{*}{\setminus}_{\varepsilon}(K,T)$ and $(K,T)\overset{*}{\setminus}_{\varepsilon}(G,T)\,\tilde{\subseteq}\,(L,T)\overset{*}{\setminus}_{\varepsilon}(F,T)$

22) $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(F,T)\overset{*}{\cap}_{\varepsilon}(G,T)^r$

Proof: Let $(F,T)\overset{*}{\cap}_{\varepsilon}(G,T)^r=(H,T)$, where $H(\omega)=F(\omega)\cap G'(\omega)=F(\omega)\setminus G(\omega)$, for all $\omega\in T$. Thereby, $(H,T)=(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)$.

In classical sets $T\subseteq Ƶ\Leftrightarrow T\setminus Ƶ=\emptyset$, as an analogy we have:

23) $(F,T)\,\tilde{\subseteq}\,(G,T)\Leftrightarrow(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=\emptyset_T$

Proof: Let $(F,T)\,\tilde{\subseteq}\,(G,T)$ and $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(H,T)$. Then, $F(\omega)\subseteq G(\omega)$ and $H(\omega)=F(\omega)\setminus G(\omega)$, for all $\omega\in T$. Since $F(\omega)\subseteq G(\omega)$, it implies that $H(\omega)=F(\omega)\setminus G(\omega)=\emptyset$, for all $\omega\in T$. Therefore $(H,T)=\emptyset_T$. Conversely, let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=\emptyset_T$. Then, $F(\omega)\setminus G(\omega)=\emptyset$, and so $F(\omega)\subseteq G(\omega)$, for all $\omega\in T$. Thereby, $(F,T)\,\tilde{\subseteq}\,(G,T)$.

In classical sets, $T\setminus(T\setminus Ƶ)=T\cap Ƶ$. As an analogy we have:

24) $(F,T)\overset{*}{\setminus}_{\varepsilon}[(F,T)\setminus_R(G,Ƶ)]=(F,T)\overset{*}{\underset{\cap}{\sim}}(G,Ƶ)$

Proof: Let $(F,T)\setminus_R(G,Ƶ)=(K,T\cap Ƶ)$ and $(F,T)\overset{*}{\setminus}_{\varepsilon}(K,T\cap Ƶ)=(S,T\cup(T\cap Ƶ))=(S,T)$. Then, $K(\omega)=F(\omega)\setminus G(\omega)$, for all $\omega\in T\cap Ƶ$, and

$$S(ɷ)=\begin{cases} F'(ɷ), & ɷ \in T\setminus(T\cap Ƶ)=T\setminus Ƶ \\ K'(ɷ), & ɷ \in (T\cap Ƶ)\setminus T=\emptyset \\ F(ɷ)\setminus K(ɷ), & ɷ \in T\cap(T\cap Ƶ)=T\cap Ƶ \end{cases}$$

for all $ɷ \in T\cup(T\cap Ƶ)$. Thereby,

$$S(ɷ)=\begin{cases} F'(ɷ), & ɷ \in T\setminus Ƶ \\ F(ɷ)\setminus[F(ɷ)\setminus G(ɷ)], & ɷ \in T\cap Ƶ \end{cases}$$

Therefore

$$S(ɷ)=\begin{cases} F'(ɷ), & ɷ \in T\setminus Ƶ \\ F(ɷ)\cap G(ɷ), & ɷ \in T\cap Ƶ \end{cases}$$

Consequently, $(S,T)=(F,T)\overset{*}{\underset{\cap}{\sim}}(G,Ƶ)$.

In classical sets, $T\setminus(T\cap Ƶ)=T\setminus Ƶ$. As an analogy we have:

25) $(F,T)\setminus_{\varepsilon}^{*}[(F,T)\cap_R(G,Ƶ)=(F,T)\overset{*}{\underset{\setminus}{\sim}}(G,Ƶ)$

Proof: Let $(F,T)\cap_R(G,Ƶ)=(K,T\cap Ƶ)$ and $(F,T)\setminus_{\varepsilon}^{*}(K,T\cap Ƶ)=(S,T\cup(T\cap Ƶ)=(S,T)$. Then, $K(ɷ)=F(ɷ)\cap G(ɷ)$, for all $ɷ\in T\cap Ƶ$, and

$$S(ɷ)=\begin{cases} F'(ɷ), & ɷ \in T\setminus(T\cap Ƶ)=T\setminus Ƶ \\ K'(ɷ), & ɷ \in (T\cap Ƶ)\setminus T=\emptyset \\ F(ɷ)\setminus K(ɷ), & ɷ \in T\cap(T\cap Ƶ)=T\cap Ƶ \end{cases}$$

for all $ɷ \in T\cup(T\cap Ƶ)$. Thus,

$$S(ɷ)=\begin{cases} F'(ɷ), & ɷ \in T\setminus Ƶ \\ F(ɷ)\setminus[F(ɷ)\cap G(ɷ)], & ɷ \in T\cap Ƶ \end{cases}$$

Thereby,

$$S(ɷ)=\begin{cases} F'(ɷ), & ɷ \in T\setminus Ƶ \\ F(ɷ)\setminus G(ɷ), & ɷ \in T\cap Ƶ \end{cases}$$

Therefore, $(S,T)=(F,T)\overset{*}{\underset{\setminus}{\sim}}(G,Ƶ)$.

In classical sets, if $T\cap Ƶ=\emptyset$, then $T\setminus Ƶ=T$. As an analogy, we have:

26) If $(F,T)\cap_{\varepsilon}^{*}(G,T)=\emptyset_T$, then $(F,T)\setminus_{\varepsilon}^{*}(G,T)=(F,T)$

Proof: Let $(F,T)\cap_{\varepsilon}^{*}(G,T)=(K,T)=\emptyset_T$. Then, $K(\omega)=F(\omega)\cap G(\omega)=\emptyset$ for all $\omega\in T$. Let $(F,T)\setminus_{\varepsilon}^{*}(G,T)=(L,T)$, where $L(\omega)=F(\omega)\setminus G(\omega)$, for all $\omega\in T$. Since $F(\omega)\cap G(\omega)=\emptyset$, this implies that $L(\omega)=F(\omega)\setminus G(\omega)=F(\omega)$. Thereby, $(L,T)=(F,T)$.

In classical sets, $(T\setminus Ƶ)\cap Ƶ=\emptyset$. As an analogy, we have:

27) $[(F,T)\setminus_{\varepsilon}^{*}(G,T)]\cap_{\varepsilon}^{*}(G,T)=\emptyset_T$

Proof: Let $(F,T)\setminus_{\varepsilon}^{*}(G,T)=(K,T)$, and $(K,T)\cap_{\varepsilon}^{*}(G,T)=(L,T)$. Then, $K(\omega)=F(\omega)\setminus G(\omega)$ and $L(\omega)=K(\omega)\cap G(\omega)=[F(\omega)\setminus G(\omega)]\cap G(\omega)=\emptyset$, for all $\omega\in T$. Thereby, $(L,T)=\emptyset_T$.

In classical sets, since $(T\setminus Z)\cap Z=\emptyset$, thus $(T\setminus Z)\setminus Z=T\setminus Z$. As as an analogy, we have:

28) $[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)]\overset{*}{\setminus}_{\varepsilon}(G,T)=(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)$

Proof: Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(K,T)$ and $(K,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(L,T)$. Then, $K(\omega)=F(\omega)\setminus G(\omega)$ and $L(\omega)=K(\omega)\setminus G(\omega)=[F(\omega)\setminus G(\omega)]\setminus G(\omega)=F(\omega)\setminus G(\omega)$, for all $\omega\in T$. Thereby, $(L,T)=(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)$.

In classical sets, $(T\setminus Z)\cap(Z\setminus T)=\emptyset$. As an analogy, we have:

29) $[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)]\overset{*}{\cap}_{\varepsilon}[(G,T)\overset{*}{\setminus}_{\varepsilon}(F,T)]=\emptyset_T$

Proof: Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(K,T)$, $(G,T)\overset{*}{\setminus}_{\varepsilon}(F,T)=(L,T)$, and $(K,T)\overset{*}{\cap}_{\varepsilon}(L,T)=(S,T)$, Then, $K(\omega)=F(\omega)\setminus G(\omega)$, $L(\omega)=G(\omega)\setminus F(\omega)$, and $S(\omega)=K(\omega)\cap L(\omega)=[F(\omega)\setminus G(\omega)]\cap[G(\omega)\setminus F(\omega)]=\emptyset$, for all $\omega\in T$. Therefore, $(S,T)=\emptyset_T$.

In classical sets, since $(T\setminus Z)\cap(Z\setminus T)=\emptyset$, $(T\setminus Z)\setminus(Z\setminus T)=T\setminus Z$. As an analogy, we have:

30) $[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)]\overset{*}{\cap}_{\varepsilon}[(G,T)\overset{*}{\setminus}_{\varepsilon}(F,T)]=(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)$

Proof: Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(K,T)$, $(G,T)\overset{*}{\setminus}_{\varepsilon}(F,T)=(L,T)$, and $(K,T)\overset{*}{\cap}_{\varepsilon}(L,T)=(S,T)$. Then, $K(\omega)=F(\omega)\setminus G(\omega)$, $L(\omega)=G(\omega)\setminus F(\omega)$, and $S(\omega)=K(\omega)\setminus L(\omega)=[F(\omega)\setminus G(\omega)]\setminus[G(\omega)\setminus F(\omega)]=F(\omega)\setminus G(\omega)$, for all $\omega\in T$. Thereby, $(S,T)=(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)$.

In classical sets, $(T\setminus Z)\cap(T\cap Z)=\emptyset$. As an analogy, we have:

31) $[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)]\overset{*}{\cap}_{\varepsilon}[(F,T)\overset{*}{\cap}_{\varepsilon}(G,T)]=\emptyset_T$

Proof: Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(K,T)$, $(F,T)\overset{*}{\cap}_{\varepsilon}(G,T)=(L,T)$, and $(K,T)\overset{*}{\cap}_{\varepsilon}(L,T)=(S,T)$. Then, $K(\omega)=F(\omega)\setminus G(\omega)$, $L(\omega)=F(\omega)\cap G(\omega)$, and $S(\omega)=K(\omega)\cap L(\omega)=[F(\omega)\setminus G(\omega)]\cap[F(\omega)\cap G(\omega)]=\emptyset$, for all $\omega\in T$. Thereby, $(S,T)=\emptyset_T$.

In classical sets, since $(T\setminus Z)\cap(T\cap Z)=\emptyset$, $(T\setminus Z)\setminus(T\cap Z)=T\setminus Z$. As an analogy, we have:

32) $[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)]\overset{*}{\setminus}_{\varepsilon}[(F,T)\overset{*}{\cap}_{\varepsilon}(G,T)]=(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)$

Proof: Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(K,T)$, $(F,T)\overset{*}{\cap}_{\varepsilon}(G,T)=(L,T)$, and $(K,T)\overset{*}{\setminus}_{\varepsilon}(L,T)=(S,T)$. Then, $K(\omega)=F(\omega)\setminus G(\omega)$, $L(\omega)=F(\omega)\cap G(\omega)$ and $S(\omega)=K(\omega)\setminus L(\omega)=[F(\omega)\setminus G(\omega)]\setminus[F(\omega)\cap G(\omega)]=F(\omega)\setminus G(\omega)$, for all $\omega\in T$. Therefore, $(S,T)=(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)$.

In classical sets, $T\cap(Z\setminus T)=\emptyset$. As an analogy, we have:

33) $(F,T)\overset{*}{\cap}_{\varepsilon}[(G,T)\overset{*}{\setminus}_{\varepsilon}(F,T)]=\emptyset_T$

Proof: Let $(G,T)\overset{*}{\setminus}_{\varepsilon}(F,T)=(K,T)$ and $(F,T)\overset{*}{\cap}_{\varepsilon}(K,T)=(L,T)$. Then, $K(\omega)=G(\omega)\setminus F(\omega)$ and $L(\omega)=F(\omega)\cap K(\omega)=F(\omega)\cap[G(\omega)\setminus F(\omega)]=\emptyset$, for all $\omega\in T$. Thereby, $(L,T)=\emptyset_T$.

In classical sets, since $T\cap(Z\setminus T)=\emptyset$, $T\setminus(Z\setminus T)=T$. As an analogy, we have:

34) $(F,T)\overset{*}{\setminus}_{\varepsilon}[(G,T)\overset{*}{\setminus}_{\varepsilon}(F,T)]=(F,T)$.

Proof: Let $(G,T)\overset{*}{\setminus}_{\varepsilon}(F,T)=(K,T)$ and $(F,T)\overset{*}{\setminus}_{\varepsilon}(K,T)=(L,T)$. Then, $K(\omega)=G(\omega)\setminus F(\omega)$ and $L(\omega)=F(\omega)\setminus K(\omega)=F(\omega)\setminus[G(\omega)\setminus F(\omega)]=F(\omega)$, for all $\omega\in T$. Thus, $(L,T)=(F,T)$.

In classical sets, $T=(T\setminus Z)\cup(T\cap Z)$. As an analogy, we have:

35) $(F,T)=[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)]\overset{*}{\cup}_{\varepsilon}[(F,T)\overset{*}{\cap}_{\varepsilon}(G,T)]$

Proof: Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(K,T)$, $(F,T)\overset{*}{\cap}_{\varepsilon}(G,T)=(L,T)$, and $(K,T)\overset{*}{\cup}_{\varepsilon}(L,T)=(S,T)$. Then, $K(\omega)=F(\omega)\setminus G(\omega)$, $L(\omega)=F(\omega)\cap G(\omega)$, and $S(\omega)=K(\omega)\cup L(\omega)=(F(\omega)\setminus G(\omega))\cup(F(\omega)\cap G(\omega))=F(\omega)$, for all $\omega\in T$. Thereby, $(S,T)=(F,T)$.

In classical sets, $T\cup Z=(T\setminus Z)\cup Z$ *ve* $T\cup Z=(Z\setminus T)\cup T$. As an analogy, we have:

36) $(F,T)\overset{*}{\cup}_{\varepsilon}(G,T)=[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)]\overset{*}{\cup}_{\varepsilon}(G,T)$ and $(F,T)\overset{*}{\cup}_{\varepsilon}(G,T)=[(G,T)\overset{*}{\setminus}_{\varepsilon}(F,T)]\overset{*}{\cup}_{\varepsilon}(F,T)$

Proof: Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(K,T)$ and $(K,T)\overset{*}{\cup}_{\varepsilon}(G,T)=(L,T)$. Then, $K(\omega)=F(\omega)\setminus G(\omega)$, and $L(\omega)=K(\omega)\cup G(\omega)=(F(\omega)\setminus G(\omega))\cup G(\omega)=F(\omega)\cup G(\omega)$, for all $\omega\in T$. Thus, $(L,T)=F,T)\overset{*}{\cup}_{\varepsilon}(G,T)$.

In classical sets, $T\cup Z=(T\setminus Z)\cup(Z\setminus T)\cup(T\cap Z)$. As an analogy, we have:

37) $(F,T)\overset{*}{\cup}_{\varepsilon}(G,Z)=[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,Z)]\overset{*}{\cup}_{\varepsilon}[(G,Z)\overset{*}{\setminus}_{\varepsilon}(F,T)]\overset{*}{\cup}_{\varepsilon}[(F,T)\overset{*}{\cap}_{\varepsilon}(G,Z)]$

Proof: Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,Z)=(H,T\cup Z)$, $(G,Z)\overset{*}{\setminus}_{\varepsilon}(F,T)=(K,T\cup Z)$, and $(F,T)\overset{*}{\cap}_{\varepsilon}(G,Z)=(S,T\cup Z)$. Then,

$$H(\omega)=\begin{cases}F'(\omega), & \omega\in T\setminus Z\\ G'(\omega), & \omega\in Z\setminus T\\ F(\omega)\setminus G(\omega), & \omega\in T\cap Z\end{cases}$$

$$K(\omega)=\begin{cases}G'(\omega), & \omega\in Z\setminus T\\ F'(\omega), & \omega\in T\setminus Z\\ G(\omega)\setminus F(\omega), & \omega\in Z\cap T\end{cases}$$

and

$$S(\omega)=\begin{cases}F'(\omega), & \omega\in T\setminus Z\\ G'(\omega), & \omega\in Z\setminus T\\ F(\omega)\cap G(\omega), & \omega\in T\cap Z\end{cases}$$

for all $\omega\in T\cup Z$. Let $(H,T\cup Z)\overset{*}{\cup}_{\varepsilon}(K,Z\cup T)=(M,T\cup Z)$, where

$$M(\omega)=\begin{cases}H'(\omega), & \omega\in(T\cup Z)\setminus(Z\cup T)=\emptyset\\ K'(\omega), & \omega\in(Z\cup T)\setminus(T\cup Z)=\emptyset\\ H(\omega)\setminus K(\omega), & \omega\in(T\cup Z)\cap(Z\cup T)=T\cup Z\end{cases}$$

for all $\omega\in T\cup Z$. Therefore,

$$M(\omega)=\begin{cases} F'(\omega)\cup G'(\omega), & \omega\in(T\setminus Z)\cap(Z\setminus T)=\emptyset \\ F'(\omega)\cup F'(\omega), & \omega\in(T\setminus Z)\cap(T\setminus Z)=T\setminus Z \\ F'(\omega)\cup[G(\omega)\setminus F(\omega)], & \omega\in(T\setminus Z)\cap(Z\cap T)=\emptyset \\ G'(\omega)\cup G'(\omega), & \omega\in(Z\setminus T)\cap(Z\setminus T)=Z\setminus T \\ G'(\omega)\cup F'(\omega), & \omega\in(Z\setminus T)\cap(T\setminus Z)=\emptyset \\ G'(\omega)\cup[G(\omega)\setminus F(\omega)], & \omega\in(Z\setminus T)\cap(T\setminus Z)=\emptyset \\ [F(\omega)\setminus G(\omega)]\cup G'(\omega), & \omega\in(T\cap Z)\cap(Z\setminus T)=\emptyset \\ [F(\omega)\setminus G(\omega)]\cup F'(\omega), & \omega\in(T\cap Z)\cap(T\setminus Z)=\emptyset \\ [F(\omega)]\setminus G(\omega)\cup[G(\omega)\setminus F(\omega)], & \omega\in(T\cap Z)\cap(Z\cap T)=T\cap Z \end{cases}$$

Thereby,

$$M(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus Z \\ G'(\omega), & \omega\in Z\setminus T \\ [F(\omega)\setminus G(\omega)]\cup[G(\omega)\setminus F(\omega)], & \omega\in T\cap Z \end{cases}$$

for all $\omega\in T\cup Z$. Let $(M,T\cup Z)\overset{*}{\cup}_{\varepsilon}(S,T\cup Z)=(W,T\cup Z)$, where

$$W(\omega)=\begin{cases} M'(\omega), & \omega\in(T\cup Z)\setminus(Z\cup T)=\emptyset \\ S'(\omega), & \omega\in(Z\cup T)\setminus(T\cup Z)=\emptyset \\ M(\omega)\setminus S(\omega), & \omega\in(T\cup Z)\cap(Z\cup T)=T\cup Z \end{cases}$$

for all $\omega\in T\cup Z$. Hence,

$$W(\omega)=\begin{cases} F'(\omega)\cup F'(\omega), & \omega\in(T\setminus Z)\cap(T\setminus Z)=T\setminus Z \\ F'(\omega)\cup G'(\omega), & \omega\in(T\setminus Z)\cap(Z\setminus T)=\emptyset \\ F'(\omega)\cup[G(\omega)\cap G(\omega)], & \omega\in(T\setminus Z)\cap(T\cap Z)=\emptyset \\ G'(\omega)\cup F'(\omega), & \omega\in(Z\setminus T)\cap(T\setminus Z)=\emptyset \\ G'(\omega)\cup G'(\omega), & \omega\in(Z\setminus T)\cap(Z\setminus T)=Z\setminus T \\ G'(\omega)\cup[F(\omega)\cap G(\omega)], & \omega\in(Z\setminus T)\cap(T\cap Z)=\emptyset \\ [F(\omega)\setminus G(\omega)]\cup[G(\omega)\setminus F(\omega)]\cup F'(\omega), & \omega\in(T\cap Z)\cap(T\setminus Z)=\emptyset \\ [F(\omega)\setminus G(\omega)]\cup[G(\omega)\setminus F(\omega)]\cup G'(\omega), & \omega\in(T\cap Z)\cap(Z\setminus T)=\emptyset \\ [F(\omega)\setminus G(\omega)]\cup[G(\omega)\setminus F(\omega)]\cup[F(\omega)\cap G(\omega)], & \omega\in(T\cap Z)\cap(T\cap Z)=T\cap Z \end{cases}$$

Therefore,

$$W(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus Z \\ G'(\omega), & \omega\in Z\setminus T \\ F(\omega)\cup G(\omega), & \omega\in T\cap Z \end{cases}$$

for all $\omega\in T\cup Z$. Therefore, $(W,T\cup Z)=(F,T)\overset{*}{\cup}_{\varepsilon}(G,Z)$.

Theorem 3.4. $(S_T(U),\overset{*}{\setminus}_{\varepsilon},\emptyset_T)$ is a BCK-algebra whose all elements are involution.

Proof: Let $(F,T),(G,T),(H,T)\in S_T(U)$. Thereby,

BCI-1 $\{[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)]\overset{*}{\setminus}_{\varepsilon}[(F,T)\overset{*}{\setminus}_{\varepsilon}(H,T)]\}\overset{*}{\setminus}_{\varepsilon}[(H,T)\overset{*}{\setminus}_{\varepsilon}(G,T)]=\emptyset_T$

Indeed, let $(F,T)\overset{*}{\setminus}_{\varepsilon}(G,T)=(W,T)$, $(F,T)\overset{*}{\setminus}_{\varepsilon}(H,T)=(M,T)$, and $(W,T)\overset{*}{\setminus}_{\varepsilon}(M,T)=(L,T)$. Then, $W(\omega)=F(\omega)\setminus G(\omega)$, $M(\omega)=F(\omega)\setminus H(\omega)$ and $L(\omega)=W(\omega)\setminus M(\omega)=[F(\omega)\setminus G(\omega)]\setminus[F(\omega)\setminus H(\omega)]$, for all $\omega\in T$.

Let $(H,T)\overset{*}{\backslash}_{\varepsilon}(G,T) = (S,T)$and $(L,T)\overset{*}{\backslash}_{\varepsilon}(S,T) = (X,T)$. Then, $S(\omega) = H(\omega)\backslash G(\omega)$ and $X(\omega) = L(\omega)\backslash S(\omega) = [F(\omega)\backslash G(\omega)]\backslash [F(\omega)\backslash H(\omega)]\backslash [H(\omega)\backslash G(\omega)]) = \emptyset$, for all $\omega \in T$. Hence, (X,T))]$= \emptyset_T$.

BCI-2 $[(F,T)\overset{*}{\backslash}_{\varepsilon}[(F,T)\overset{*}{\backslash}_{\varepsilon}(G,T)]]\overset{*}{\backslash}_{\varepsilon}(G,T) = \emptyset_T$. Indeed, let $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,T) = (K,T)$ and $(F,T)\overset{*}{\backslash}_{\varepsilon}(K,T) = (M,T)$. Then, $K(\omega) = F(\omega)\backslash G(\omega)$ and $M(\omega) = F(\omega)\backslash K(\omega) = F(\omega)\backslash[F(\omega)\backslash G(\omega)] = F(\omega)\cap G(\omega)$, for all $\omega \in T$. Let $(M,T)\overset{*}{\backslash}_{\varepsilon}(G,T) = (L,T)$. Then, $L(\omega) = M(\omega)\backslash G(\omega) = [F(\omega)\cap G(\omega)]\backslash G(\omega) = \emptyset$, for all $\omega \in T$. Therefore, $(L,T) = \emptyset_T$.

BCI-3 By Theorem 3.3 (5), $(F,T)\overset{*}{\backslash}_{\varepsilon}(F,T) = \emptyset_T$.

BCI-4 By Theorem 3.3 (24), $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,T) = \emptyset_T \Rightarrow (F,T) \,\tilde{\subseteq}\, (G,T)$ and $(G,T)\overset{*}{\backslash}_{\varepsilon}(F,T) = \emptyset_T \Rightarrow (G,T) \,\tilde{\subseteq}\, (F,T)$. Thereby, $(F,T) = (G,T)$.

BCK-5 By Theorem 3.3 (7), $\emptyset_T\overset{*}{\backslash}_{\varepsilon}(F,T) = \emptyset_T$.

Hence, $(S_T(U), \overset{*}{\backslash}_{\varepsilon}, \emptyset_T)$ is a BCK-algebra. By Theorem 3.4 (11), $(F,T)\overset{*}{\backslash}_{\varepsilon} U_T = \emptyset_T$ for all $(F,T) \in S_T(U)$. Hence, $(S_T(U), \overset{*}{\backslash}_{\varepsilon}, \emptyset_T)$ is a bounded BCK-algebra. Moreover, since $U_T\overset{*}{\backslash}_{\varepsilon}[U_T\overset{*}{\backslash}_{\varepsilon}(F,T)] = (F,T)$ for all $(F,T) \in S_T(U)$, (As by Theorem 3.3 (12), $U_T\overset{*}{\backslash}_{\varepsilon}(F,T) = (F,T)^r$, and so $U_T\overset{*}{\backslash}_{\varepsilon}(F,T)^r = (F,T)$, each element of $S_T(U)$ is an involution.

Theorem 3.5. Let (F,T), $(G,\mathbb{Z})$ and (H,M) be soft sets over U. The complementary extended difference operation has the following distributions over other soft set operations:

Theorem 3.5.1 Let (F,T), $(G,\mathbb{Z})$ and (H,M) be soft sets over U. The complementary extended difference operation has the following distributions over restricted soft set operations:

i) LHS Distribution

1) If $T\cap(\mathbb{Z}\Delta M) = \emptyset$, then $(F,T)\overset{*}{\backslash}_{\varepsilon}[(G,\mathbb{Z}) \cup_R (H,M)] = [(F,T)\overset{*}{\backslash}_{\varepsilon}(G,\mathbb{Z})] \cap_R [(F,T)\overset{*}{\backslash}_{\varepsilon}(H,M)]$

Proof: Let's first handle the RHS. Let $(G,\mathbb{Z}) \cup_R (H,M) = (M,\mathbb{Z}\cap M)$, where $M(\omega) = G(\omega)\cup H(\omega)$ for all $\omega \in \mathbb{Z}\cap M$. Let $(F,T)\overset{*}{\backslash}_{\varepsilon}(M,\mathbb{Z}\cap M) = (N,T\cup(\mathbb{Z}\cap M))$, where

$$N(\omega) = \begin{cases} F'(\omega), & \omega \in T\backslash(\mathbb{Z}\cap M) \\ M'(\omega), & \omega \in (\mathbb{Z}\cap M)\backslash T \\ F(\omega)\backslash M(\omega), & \omega \in T\cap(\mathbb{Z}\cap M) \end{cases}$$

for all $\omega \in T\cup(\mathbb{Z}\cap M)$. Therefore,

$$N(\omega) = \begin{cases} F'(\omega), & \omega \in T\backslash(\mathbb{Z}\cap M) \\ G'(\omega)\cap H'(\omega), & \omega \in (\mathbb{Z}\cap M)\backslash T \\ F(\omega)\backslash[G(\omega)\cup H(\omega)], & \omega \in T\cap(\mathbb{Z}\cap M) \end{cases}$$

Now consider the RHS. Let $(F,T)\overset{*}{\backslash_{\varepsilon}}(G,Z) = (M,T\cup Z)$, where

$$M(\omega) = \begin{cases} F'(\omega), & \omega \in T\backslash Z \\ G'(\omega), & \omega \in Z\backslash T \\ F(\omega)\backslash G(\omega), & \omega \in T \cap Z \end{cases}$$

for all $\omega\in T\cup Z$. Let $(F,T)\overset{*}{\backslash_{\varepsilon}}(H,M) = (K,T\cup M)$, where

$$K(\omega) = \begin{cases} F'(\omega), & \omega \in T\backslash M \\ H'(\omega), & \omega \in M\backslash T \\ F(\omega)\backslash H(\omega), & \omega \in T \cap M \end{cases}$$

for all $\omega\in T\cup M$. Let $(M,T\cup Z)\cap_R(K,T\cup M)=(W,(T\cup Z)\cap(T\cup M))$, where $W(\omega)=M(\omega)\cap K(\omega)$, for all $\omega\in(T\cup Z)\cap(T\cup M)$. Hence,

$$W(\omega) = \begin{cases} F'(\omega) \cap F'(\omega), & \omega \in (T\backslash Z) \cap (T\backslash M) = T \cap Z' \cap M \\ F'(\omega) \cap H'(\omega), & \omega \in (T\backslash Z) \cap (M\backslash T) = \emptyset \\ F'(\omega) \cap [F(\omega)\backslash H(\omega)], & \omega \in (T\backslash Z) \cap (T \cap M) = T \cap Z' \cap M \\ G'(\omega) \cap F'(\omega), & \omega \in (Z\backslash T) \cap (T\backslash M) = \emptyset \\ G'(\omega) \cap H'(\omega), & \omega \in (Z\backslash T) \cap (M\backslash T) = T' \cap Z \cap M \\ G'(\omega) \cap [F(\omega)\backslash H(\omega)], & \omega \in (Z\backslash T) \cap (T \cap M) = \emptyset \\ [F(\omega)\backslash G(\omega)] \cap F'(\omega), & \omega \in (T \cap Z) \cap (T\backslash M) = T \cap Z \cap M' \\ [F(\omega)\backslash G(\omega)] \cap H'(\omega), & \omega \in (T \cap Z) \cap (M\backslash T) = \emptyset \\ [F(\omega)\backslash G(\omega)] \cap [F(\omega)\backslash H(\omega)], & \omega \in (T \cap Z) \cap (T \cap M) = T \cap Z \cap M \end{cases}$$

Therefore,

$$W(\omega) = \begin{cases} F'(\omega), & \omega \in T \cap Z' \cap M \\ \emptyset, & \omega \in T \cap Z' \cap M \\ G'(\omega) \cap H'(\omega), & \omega \in T' \cap Z \cap M \\ \emptyset, & \omega \in T \cap Z \cap M' \\ [F(\omega)\backslash G(\omega)] \cap [F(\omega)\backslash H(\omega)], & \omega \in T \cap Z \cap M \end{cases}$$

Here, when considering the $T\backslash(Z\cap M)$ in the function N, since $T\backslash(Z\cap M)=T\backslash(Z\cap M)'$ if an element is in the complement of $Z\cap M$, it is either in $Z\backslash M$ or in $M\backslash Z$ or in $(Z\cup M)'$. Therefore, if $\omega \in T\backslash(Z\cap M)$, then $\omega\in T\cap Z\cap M'$ or $\omega\in T\cap Z'\cap M$ or $\omega\in T\cap Z'\cap M'$. Hence, $N=W$ under the condition $T\cap Z'\cap M=T\cap Z\cap M'=\emptyset$. It is obvious that the condition $T\cap Z'\cap M=T\cap Z\cap M'=\emptyset$ is equivalent to the condition $T\cap(Z\Delta M)=\emptyset$.

2) If $T\cap(Z\Delta M)=\emptyset$, then $(F,T)\overset{*}{\backslash_{\varepsilon}}[(G,Z)\cap_R(H,M)] = [(F,T)\overset{*}{\backslash_{\varepsilon}}(G,Z)]\cup_R[(F,T)\overset{*}{\backslash_{\varepsilon}}(H,M)]$

ii) RHS Distributions:

1) If $T\cap Z\cap M'=\emptyset$, then $[(F,T)\cup_R(G,Z)]\overset{*}{\backslash_{\varepsilon}}(H,M)=[(F,T)\overset{*}{\backslash_{\varepsilon}}(H,M)]\cup_R[(G,Z)\overset{*}{\backslash_{\varepsilon}}(H,M)]$

Proof: Let's first handle the LHS. Let $(F,T)\cup_R(G,Z)=(R,T\cap Z)$, where $R(\omega)=F(\omega)\cup G(\omega)$, for all $\omega\in T\cap Z$. Let $(R,T\cap Z)\overset{*}{\backslash_{\varepsilon}}(H,M)=(L,(T\cap Z)\cup M)$, where

$$L(\omega) = \begin{cases} R'(\omega), & \omega \in (T \cap Z)\backslash M \\ H'(\omega), & \omega \in M\backslash(T \cap Z) \\ R(\omega)\backslash H(\omega), & \omega \in (T \cap Z) \cap M \end{cases}$$

for all $\omega \in (T \cap Z) \cup M$. Hence,

$$L(\omega) = \begin{cases} F'(\omega) \cap G'(\omega), & \omega \in (T \cap Z) \setminus M \\ H'(\omega), & \omega \in M \setminus (T \cap Z) \\ [F(\omega) \cup G(\omega)] \setminus H(\omega), & \omega \in (T \cap Z) \cap M \end{cases}$$

Now consider the RHS, i.e., $[(F,T) \overset{*}{\setminus}_{\varepsilon} (H,M)] \cup_R [(G,Z) \overset{*}{\setminus}_{\varepsilon} (H,M)]$. Let $(F,T) \overset{*}{\setminus}_{\varepsilon} (H,M) = (S, T \cup M)$, where

$$S(\omega) = \begin{cases} R'(\omega), & \omega \in T \setminus M \\ H'(\omega), & \omega \in M \setminus T \\ F(\omega) \setminus H(\omega), & \omega \in T \cap M \end{cases}$$

for all $\omega \in T \cup M$. Let $(G,Z) \overset{*}{\setminus}_{\varepsilon} (H,M) = (K, Z \cup M)$, where

$$K(\omega) = \begin{cases} G'(\omega), & \omega \in Z \setminus M \\ H'(\omega), & \omega \in M \setminus Z \\ F(\omega) \setminus H(\omega), & \omega \in Z \cap M \end{cases}$$

for all $\omega \in Z \cup M$. Let $(S, T \cup Z) \cup_R (K, Z \cup M) = (W, (T \cup Z) \cap (Z \cup M))$, where $W(\omega) = S(\omega) \cup K(\omega)$, for all $\omega \in (T \cup Z) \cap (Z \cup M)$. Hence,

$$W(\omega) = \begin{cases} F'(\omega) \cup G'(\omega), & \omega \in (T \setminus M) \cap (Z \setminus M) = T \cap Z \cap M' \\ F'(\omega) \cup H'(\omega), & \omega \in (T \setminus M) \cap (M \setminus Z) = \emptyset \\ F'(\omega) \cup [G(\omega) \setminus H(\omega)], & \omega \in (T \setminus M) \cap (Z \cap M) = \emptyset \\ H'(\omega) \cup G'(\omega), & \omega \in (M \setminus T) \cap (Z \setminus M) = \emptyset \\ H'(\omega) \cup H'(\omega), & \omega \in (M \setminus T) \cap (M \setminus Z) = T' \cap Z' \cap M \\ H'(\omega) \cup [G(\omega) \setminus H(\omega)], & \omega \in (M \setminus T) \cap (Z \cap M) = T' \cap Z \cap M \\ [F(\omega) \setminus H(\omega)] \cup G'(\omega), & \omega \in (T \cap M) \cap (Z \setminus M) = \emptyset \\ [F(\omega) \setminus H(\omega)] \cup H'(\omega), & \omega \in (T \cap M) \cap (M \setminus Z) = T \cap Z' \cap M \\ [F(\omega) \setminus H(\omega)] \cup [G(\omega) \setminus H(\omega)], & \omega \in (T \cap M) \cap (Z \cap M) = T \cap Z \cap M \end{cases}$$

Therefore,

$$W(\omega) = \begin{cases} F'(\omega) \cup G'(\omega), & \omega \in T \cap Z \cap M' \\ H'(\omega), & \omega \in T' \cap Z' \cap M \\ H'(\omega) \cup [G(\omega) \setminus H(\omega)], & \omega \in T' \cap Z \cap M \\ [F(\omega) \setminus H(\omega)] \cup H'(\omega), & \omega \in T \cap Z' \cap M \\ [F(\omega) \setminus H(\omega)] \cup [G(\omega) \setminus H(\omega)], & \omega \in T \cap Z \cap M \end{cases}$$

Here, when considering the $M \setminus (T \cap Z)$ in the function L, since $M \setminus (T \cap Z) = M \cap (T \cap Z)'$ if an element is in the complement of $T \cap Z$, it is either in $T \setminus Z$ or in $Z \setminus T$ or in $(T \cup Z)'$. Thus, if $\omega \in M \setminus (T \cap Z)$, then $\omega \in M \cap T \cap Z'$ or $\omega \in M \cap Z \cap T'$ or $\omega \in M \cap T' \cap Z'$. Therefore, $N = T$ under the condition $T \cap Z \cap M' = \emptyset$.

2) If $(T \Delta Z) \cap M = T \cap Z \cap M' = \emptyset$, then $[(F,T) \cap_R (G,Z)] \overset{*}{\setminus}_{\varepsilon} (H,M) = [(F,T) \overset{*}{\setminus}_{\varepsilon} (H,M)] \cap_R [(G,Z) \overset{*}{\setminus}_{\varepsilon} (H,M)]$

Theorem 3.5.2. Let (F,T), (G,Z), and (H,M) be soft sets over U. The complementary extended difference operation has the following distributions over extended soft set operations:

i) LHS Distributions

1) If $T\cap(Z\Delta M)=\emptyset$, then $(F,T)\overset{*}{\backslash}_{\varepsilon}[(G,Z)\cap_{\varepsilon}(H,M)]=[(F,T)\overset{*}{\backslash}_{\varepsilon}(G,Z)]\cup_{\varepsilon}[(F,T)\overset{*}{\backslash}_{\varepsilon}(H,M)]$

Proof: Let's consider first the LHS. Let $(G,Z)\cap_{\varepsilon}(H,M)=(R,Z\cup M)$, where

$$R(\omega)=\begin{cases} G(\omega), & \omega\in Z\setminus M\\ H(\omega), & \omega\in M\setminus Z\\ G(\omega)\setminus H(\omega), & \omega\in Z\cap M\end{cases}$$

for all $\omega\in Z\cup M$. Let $(R,Z\cup M)=(N,(T\cup(Z\cup M))$, where

$$N(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus(Z\cup M)\\ R'(\omega), & \omega\in(Z\cup M)\setminus T\\ F(\omega)\setminus R(\omega), & \omega\in T\cap(Z\cup M)\end{cases}$$

for all $\omega\in T\cup(Z\cup M)$. Hence,

$$N(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus(Z\cup M)=T\cap Z'\cap M'\\ G'(\omega), & \omega\in(Z\setminus M)\setminus T=T'\cap Z\cap M'\\ H'(\omega), & \omega\in(M\setminus Z)\setminus T=T'\cap Z'\cap M\\ G'(\omega)\cup H'(\omega), & \omega\in(Z\cap M)\setminus T=T'\cap Z\cap M\\ F(\omega)\setminus G(\omega), & \omega\in T\cap(Z\setminus M)=T\cap Z\cap M'\\ F(\omega)\setminus H(\omega), & \omega\in T\cap(M\setminus Z)=T\cap Z'\cap M\\ [F(\omega)\setminus[G(\omega)\cap H(\omega)], & \omega\in T\cap(Z\cap M)=T\cap Z\cap M\end{cases}$$

Let's consider the RHS. Let $(F,T)\overset{*}{\backslash}_{\varepsilon}(G,Z)=(K,T\cup Z)$, where

$$K(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus Z\\ G'(\omega), & \omega\in Z\setminus T\\ F(\omega)\setminus G(\omega), & \omega\in T\cap Z\end{cases}$$

for all $\omega\in T\cup Z$. Let $(F,T)\overset{*}{\backslash}_{\varepsilon}(H,M)=(S,T\cup M)$, where

$$S(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus M\\ H'(\omega), & \omega\in M\setminus T\\ F(\omega)\setminus H(\omega), & \omega\in T\cap M\end{cases}$$

for all $\omega\in T\cup M$. Let $(K,T\cup Z)\cup_{\varepsilon}(S,T\cup M)=(L,(T\cup Z)\cup(T\cup M))$, where

$$L(\omega)=\begin{cases} K(\omega), & \omega\in(T\cup Z)\setminus(T\cup M)\\ S(\omega), & \omega\in(T\cup M)\setminus(T\cup Z)\\ K(\omega)\cup S(\omega), & \omega\in(T\cup Z)\cap(T\cup M)\end{cases}$$

for all $\omega\in(T\cup Z)\cup(T\cup M)$. Hence,

$$L(\omega)=\begin{cases} G'(\omega), & \omega\in T'\cap Z\cap M'\\ H'(\omega), & \omega\in T'\cap Z'\cap M\\ F'(\omega), & \omega\in T\cap Z'\cap M'\\ F'(\omega)\cup H'(\omega), & \omega\in T\cap Z'\cap M\\ G'(\omega)\cup H'(\omega), & \omega\in T'\cap Z\cap M\\ [F(\omega)\setminus G(\omega)]\cup F'(\omega), & \omega\in T\cap Z\cap M'\\ [F(\omega)\setminus G(\omega)]\cup[F(\omega)\setminus H(\omega)], & \omega\in T\cap Z\cap M\end{cases}$$

It is observed that $N=L$, where $T\cap Z\cap M' = T\cap Z'\cap M=\emptyset$. It is obvious that the condition $T\cap Z'\cap M=T\cap Z\cap M'=\emptyset$is equivalent to the condition $T\cap(Z\Delta M)=\emptyset$.

2) If $T\cap(Z\Delta M)$, then $(F,T)\overset{*}{\setminus}_{\varepsilon}[(G,Z)\cup_{\varepsilon}(H,M)]=[(F,T)\overset{*}{\setminus}_{\varepsilon}(G,Z)]\cap_{\varepsilon}[(F,T)\overset{*}{\setminus}_{\varepsilon}(H,M)]$

ii) RHS Distributions

1) If $T\cap Z\cap M'=\emptyset$, then $[(F,T)\cap_{\varepsilon}(G,Z)]\overset{*}{\setminus}_{\varepsilon}(H,M)=[(F,T)\overset{*}{\setminus}_{\varepsilon}(H,M)]\cap_{\varepsilon}[(G,Z)\overset{*}{\setminus}_{\varepsilon}(H,M)]$

Proof: First let's consider the LHS. Let $(F,T)\cap_{\varepsilon}(G,Z)=(R,T\cup Z)$, where

$$R(\omega)=\begin{cases}F(\omega), & \omega\in T\setminus Z\\ G(\omega), & \omega\in Z\setminus T\\ F(\omega)\cap G(\omega), & \omega\in T\cap Z\end{cases}$$

for all $\omega\in T\cup Z$, Let $(R,T\cup Z)\overset{*}{\setminus}_{\varepsilon}(H,M)=(N,(T\cup Z)\cup M)$, where

$$N(\omega)=\begin{cases}R'(\omega), & \omega\in(T\cup Z)\setminus M\\ H'(\omega), & \omega\in M\setminus(T\cup Z)\\ R(\omega)\setminus H(\omega), & \omega\in(T\cup Z)\cap M\end{cases}$$

for all $\omega\in(T\cup Z)\cup M$. Hence,

$$N(\omega)=\begin{cases}F'(\omega), & \omega\in(T\setminus Z)\setminus M=T\cap Z'\cap M'\\ G'(\omega), & \omega\in(Z\setminus T)\setminus M=T'\cap Z\cap M'\\ F'(\omega)\cup G'(\omega), & \omega\in(T\cap Z)\setminus M=T\cap Z\cap M'\\ H'(\omega), & \omega\in M\setminus(T\cup Z)=T'\cap Z'\cap M\\ F(\omega)\setminus H(\omega), & \omega\in(T\setminus Z)\cap M=T\cap Z'\cap M\\ G(\omega)\setminus H(\omega), & \omega\in(Z\setminus T)\cap M=T'\cap Z\cap M\\ [F(\omega)\cap G(\omega)]\setminus H(\omega), & \omega\in T\cap(Z\cap M)=T\cap Z\cap M\end{cases}$$

Consider the RHS. Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(H,M)=(K,T\cup M)$, where

$$K(\omega)=\begin{cases}F'(\omega), & \omega\in T\setminus M\\ H'(\omega), & \omega\in M\setminus T\\ F(\omega)\setminus H(\omega), & \omega\in T\cap M\end{cases}$$

for all $\omega\in T\cup M$. Let $(G,Z)\overset{*}{\setminus}_{\varepsilon}(H,M)=(S,Z\cup M)$, where

$$S(\omega)=\begin{cases}G'(\omega), & \omega\in Z\setminus M\\ H'(\omega), & \omega\in M\setminus Z\\ G(\omega)\setminus H(\omega), & \omega\in Z\cap M\end{cases}$$

for all $\omega\in Z\cup M$. Let $(K,T\cup M)\cap_{\varepsilon}(S,Z\cup M)=(L,(T\cup M)\cup(Z\cup M))$, where

$$L(\omega)=\begin{cases}K(\omega), & \omega\in(T\cup M)\setminus(Z\cup M)\\ S(\omega), & \omega\in(Z\cup M)\setminus(T\cup M)\\ K(\omega)\cap S(\omega), & \omega\in(T\cup M)\cap(Z\cup M)\end{cases}$$

for all $\omega\in(T\cup M)\cup(Z\cup M)$. Hence,

$$\mathrm{L}(\omega)=\begin{cases} F'(\omega), & \omega\in T\cap Z'\cap M' \\ G'(\omega), & \omega\in T'\cap Z\cap M' \\ F'(\omega)\cap G'(\omega), & \omega\in T\cap Z\cap M' \\ H'(\omega), & \omega\in T'\cap Z'\cap M \\ F(\omega)\setminus H(\omega), & \omega\in T\cap Z'\cap M \\ G(\omega)\setminus H(\omega), & \omega\in T'\cap Z\cap M \\ [F(\omega)\cap G(\omega)]\setminus H(\omega), & \omega\in T\cap Z\cap M \end{cases}$$

It is observed that $N=L$, where $T\cap Z\cap M'=\emptyset$.

2) If $(T\Delta Z)\cap M = T\cap Z\cap M'=\emptyset$, then $[(F,T)\cup_\varepsilon(G,Z)]\overset{*}{\setminus}_\varepsilon(H,M) = [(F,T)\overset{*}{\setminus}_\varepsilon(H,M)]\cup_\varepsilon[(G,Z)\overset{*}{\setminus}_\varepsilon(H,M)]$

Theorem 3.5.3. Let (F,T), (G,Z), and (H,M) be soft sets over U. The complementary extended difference operation has the following distributions over soft binary piecewise operations:

i) LHS Distributions

1)If $T\cap(Z\Delta M)=\emptyset$, then $(F,T)\overset{*}{\setminus}_\varepsilon[(G,Z)\overset{\sim}{\cap}(H,M)]=[(F,T)\overset{*}{\setminus}_\varepsilon(G,Z)]\overset{\sim}{\cup}[(F,T)\overset{*}{\setminus}_\varepsilon(H,M)]$

***Proof**:* First let's consider the LHS. Let $(G,Z)\overset{\sim}{\cap}(H,M)=(R,Z)$, where

$$R(\omega)=\begin{cases} G(\omega), & \omega\in Z\setminus M \\ G(\omega)\cap H(\omega), & \omega\in Z\cap M \end{cases}$$

for all $\omega\in Z$. Let $(F,T)\overset{*}{\setminus}_\varepsilon(R,Z)=(N,T\cup Z)$, where

$$N(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus Z \\ R'(\omega), & \omega\in Z\setminus T \\ F(\omega)\setminus R(\omega), & \omega\in T\cap Z \end{cases}$$

for all $\omega\in T\cup Z$. Hence,

$$\mathrm{L}(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus Z \\ G'(\omega), & \omega\in (Z\setminus M)\setminus T = T'\cap Z\cap M' \\ G'(\omega)\cup H'(\omega), & \omega\in (Z\cap M)\setminus T = T'\cap Z\cap M \\ F(\omega)\setminus G(\omega), & \omega\in T\cap(Z\setminus M) = T\cap Z\cap M' \\ F(\omega)\setminus[G(\omega)\cap H(\omega)], & \omega\in T\cap(Z\cap M) = T\cap Z\cap M \end{cases}$$

Consider the RHS. Let $(F,T)\overset{*}{\setminus}_\varepsilon(G,Z)=(K,T\cup Z)$, where

$$K(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus Z \\ G'(\omega), & \omega\in Z\setminus T \\ F(\omega)\setminus G(\omega), & \omega\in T\cap Z \end{cases}$$

for all $\omega\in T\cup Z$. $(F,T)\overset{*}{\setminus}_\varepsilon(H,M)=(S,T\cup M)$, where

$$S(\omega)=\begin{cases} F'(\omega), & \omega\in T\setminus M \\ H'(\omega), & \omega\in M\setminus T \\ F(\omega)\setminus H(\omega), & \omega\in T\cap M \end{cases}$$

for all $\omega \in T \cup M$. Let $(K, T \cup Z) \tilde{\cup} (S, T \cup M) = (L, (T \cup Z) \cup (T \cup M))$, where

$$L(\omega) = \begin{cases} K(\omega), & \omega \in (T \cup Z) \setminus (T \cup M) \\ K(\omega) \cup S(\omega), & \omega \in (T \cup Z) \cap (T \cup M) \end{cases}$$

for all $\omega \in (T \cup Z) \cup (T \cup M)$. Hence,

$$L(\omega) = \begin{cases} F'(\omega), \omega \in (T \setminus Z) \setminus (T \cup M) = \emptyset \\ G'(\omega), \omega \in (Z \setminus T) \setminus (T \cup M) = T' \cap Z \cap M' \\ F(\omega) \setminus G(\omega), \omega \in (T \cap Z) \setminus (T \cup M) = \emptyset \\ F'(\omega) \cup F'(\omega), \omega \in (T \setminus Z) \cap (T \setminus M) = T \cap Z' \cap M' \\ F'(\omega) \cup H'(\omega), \omega \in (T \setminus Z) \cap (M | T) = \emptyset \\ F'(\omega) \cup [F(\omega) \setminus H(\omega)], \omega \in (T \setminus Z) \cap (T \cap M) = T \cap Z' \cap M \\ G'(\omega) \cup F'(\omega), \omega \in (Z \setminus T) \cap (T \setminus M) = \emptyset \\ G'(\omega) \cup H'(\omega), \omega \in (Z \setminus T) \cap (M \setminus T) = T' \cap Z \cap M \\ G'(\omega) \cup [F(\omega) \setminus H(\omega)], \omega \in (Z \setminus T) \cap (T \cap M) = \emptyset \\ [F(\omega) \setminus G(\omega)] \cup F'(\omega), \omega \in (T \cap Z) \cap (T \setminus M) = T \cap Z \cap M' \\ [F(\omega) \setminus G(\omega)] \cup H'(\omega), \omega \in (T \cap Z) \cap (M \setminus T) = \emptyset \\ [F(\omega) \setminus G(\omega)] \cup [F(\omega) \setminus H(\omega)], \omega \in (T \cap Z) \cap (T \cap M) = T \cap Z \cap M \end{cases}$$

Hence,

$$L(\omega) = \begin{cases} G'(\omega), & \omega \in T' \cap Z \cap M' \\ F'(\omega), & \omega \in T \cap Z' \cap M' \\ F'(\omega \cup H'(\omega), & \omega \in T \cap Z' \cap M \\ G'(\omega) \cup H'(\omega), & \omega \in T' \cap Z \cap M \\ G(\omega) \cup F'(\omega), & \omega \in T \cap Z \cap M' \\ [F(\omega) \setminus G(\omega)] \cup [F(\omega) \setminus H(\omega)] & \omega \in T \cap Z \cap M \end{cases}$$

Here, if we consider $T \setminus Z$ in the function N, since $T \setminus Z = T \cap Z''$ if an element is in the complement of Z, the element is either in $M \setminus Z$ or in $(M \cup Z)'$. Thus, if $\omega \in T \setminus Z$, then $\omega \in T \cap M \cap Z'$ or $\omega \in T \cap M' \cap Z'$. Therefore, $N = L$, where $T \cap Z' \cap M = T \cap Z \cap M' = \emptyset$. It is obvious that the condition $T \cap Z' \cap M = T \cap Z \cap M' = \emptyset$ is equivalent to the condition $T \cap (Z \Delta M) = \emptyset$.

2) If $T \cap (Z \Delta M) = \emptyset$, then $(F, T) \overset{*}{\setminus}_{\varepsilon} [(G, Z) \tilde{\cup} (H, M)] = [(F, T) \overset{*}{\setminus}_{\varepsilon} (G, Z)] \tilde{\cap} [(F, T) \overset{*}{\setminus}_{\varepsilon} (H, M)]$

ii) RHS Distributions:

1)If $T \cap (Z \Delta M) = \emptyset$, then $[(F, T) \tilde{\cup} (G, Z)] \overset{*}{\setminus}_{\varepsilon} (H, M) = [(F, T) \overset{*}{\setminus}_{\varepsilon} (H, M)] \tilde{\cup} [(G, Z) \overset{*}{\setminus}_{\varepsilon} (H, M)]$

Proof: Let's first consider the LHS. Let $(F, T) \tilde{\cup} (G, Z) = (R, T)$, where

$$R(\omega) = \begin{cases} F(\omega), & \omega \in T \setminus Z \\ F(\omega) \cup G(\omega), & \omega \in T \cap Z \end{cases}$$

for all $\omega \in T$. Let $(R, T) \overset{*}{\setminus}_{\varepsilon} (H, M) = (N, T \cup M)$, where

$$N(\omega) = \begin{cases} R'(\omega), & \omega \in T \setminus M \\ H'(\omega), & \omega \in M \setminus T \\ R(\omega) \setminus H(\omega), & \omega \in T \cap M \end{cases}$$

for all $\omega \in T \cup M$. Hence,

$$\mathrm{N}(\omega)=\begin{cases}F'(\omega), & \omega\in T\cap Z'\cap M'\\ F'(\omega)\cap G'(\omega), & \omega\in T\cap Z\cap M'\\ H'(\omega), & \omega\in M\setminus T\\ F(\omega)\setminus H(\omega) & \omega\in T\cap Z'\cap M\\ [F(\omega)\cup G(\omega)]\setminus H(\omega) & \omega\in T\cap Z\cap M\end{cases}$$

Now consider the RHS. Let $(F,T)\overset{*}{\setminus}_{\varepsilon}(H,M)=(K,T\cup M)$, where

$$K(\omega)=\begin{cases}F'(\omega), & \omega\in T\setminus M\\ H'(\omega), & \omega\in M\setminus T\\ F(\omega)\setminus H(\omega), & \omega\in T\cap M\end{cases}$$

for all $\omega\in T\cup M$. Let $(G,Z)\overset{*}{\setminus}_{\varepsilon}(H,M)=(S,Z\cup M)$, where

$$S(\omega)=\begin{cases}G'(\omega), & \omega\in Z\setminus M\\ H'(\omega), & \omega\in M\setminus Z\\ G(\omega)\setminus H(\omega), & \omega\in Z\cap M\end{cases}$$

for all $\omega\in Z\cup M$. Let $(K,T\cup M)\,\tilde{\cup}\,(S,Z\cup M)=(L,(T\cup M)\cup(Z\cup M))$, where

$$L(\omega)=\begin{cases}K(\omega), & \omega\in(T\cup M)\setminus(Z\cup M)\\ K(\omega)\cup S(\omega), & \omega\in(T\cup M)\cap(Z\cup M)\end{cases}$$

for all $\omega\in(T\cup M)\cup(Z\cup M)$. Thereby,

$$L(\omega)=\begin{cases}F'(\omega), \omega\in(T\setminus M)\setminus(Z\cup M)=T\cap Z'\cap M'\\ H'(\omega), \omega\in(M\setminus T)\setminus(Z\cup M)=\emptyset\\ F(\omega)\setminus H(\omega), \omega\in(T\cap M)\setminus(Z\cup M)=\emptyset\\ F'(\omega)\cup G'(\omega), \omega\in(T\setminus M)\cap(Z\setminus M)=T\cap Z\cap M'\\ F'(\omega)\cup H'(\omega), \omega\in(T\setminus M)\cap(M\setminus Z)=\emptyset\\ F'(\omega)\cup[G(\omega)\setminus H(\omega)],\ \omega\in(T\setminus M)\cap(Z\cap M)=\emptyset\\ H'(\omega)\cup G'(\omega), \omega\in(M\setminus T)\cap(Z\setminus M)=\emptyset\\ H'(\omega)\cup H'(\omega), \omega\in(M\setminus T)\cap(M\setminus Z)=T'\cap Z'\cap M\\ H'(\omega)\cup[G(\omega)\setminus H(\omega)], \omega\in(M\setminus T)\cap(Z\cap M)=T'\cap Z\cap M\\ [F(\omega)\setminus H(\omega)]\cup G'(\omega), \omega\in(T\cap M)\cap(Z\setminus M)=\emptyset\\ [F(\omega)\setminus H(\omega)]\cup H'(\omega), \omega\in(T\cap M)\cap(M\setminus Z)=T\cap Z'\cap M\\ [F(\omega)\setminus H(\omega)]\cup[G(\omega)\setminus H(\omega)], \omega\in(T\cap M)\cap(Z\cap M)=T\cap Z\cap M\end{cases}$$

for all $\omega\in(T\cup M)\cup(Z\cup M)$. Thus,

$$\mathrm{L}(\omega)=\begin{cases}F'(\omega), & \omega\in T\cap Z'\cap M'\\ F'(\omega)\cup G'(\omega), & \omega\in T\cap Z\cap M'\\ H'(\omega), & \omega\in T'\cap Z'\cap M\\ H'(\omega), & \omega\in T'\cap Z\cap M\\ H'(\omega), & \omega\in T\cap Z'\cap M\\ [F(\omega)\setminus H(\omega)]\cup[F(\omega)\setminus H(\omega)] & \omega\in T\cap Z\cap M\end{cases}$$

Here, if we consider $M\setminus T$ in the function N, since $M\setminus T=M\cap T'$, then if an element is in the complement of T, then it is either in $Z\setminus T$ or in $(Z\cup T)'$. Thus, $N=L$, where $T\cap Z'\cap M=T\cap Z\cap M'=\emptyset$. It is obvious that the condition $T\cap Z'\cap M=T\cap Z\cap M'=\emptyset$ is equivalent to the condition $T\cap(Z\Delta M)=\emptyset$.

2) $If\,(T\Delta M) \cap Ƶ = \emptyset$, then $[(F,T) \widetilde{\cap} (G,Ƶ)] \overset{*}{\backslash_{\varepsilon}} (H,M) = [(F,T) \overset{*}{\backslash_{\varepsilon}} (H,M)] \widetilde{\cap} [(G,Ƶ) \overset{*}{\backslash_{\varepsilon}} (H,M)]$

4. Conclusion

The most essential building component of soft set theory for its advancement in both theoretical and practical domains is soft set operations. Numerous restricted and expanded operations have been introduced since the theory's 1999 introduction. The complementary extended difference operation is a novel soft set operation that is proposed and its algebraic properties are studied in this study. We address the distributions of complementary extended difference operations over other different kinds of soft set operations. We believe that this work contributes to the literature of both classical algebra and soft set theory as the ideas associated with soft set operations are as important for soft sets as fundamental operations from classical set theory. Specifically, studying the algebraic structures of soft sets in relation to new soft set operations gives us a thorough understanding of their application as well as new examples of algebraic structures. Many types of complemented extended soft set operations may be examined in future studies together with their distributions and characteristics to find out what algebraic structures are formed in the classes of soft sets with a fixed parameter set or over the universe.

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Author’s Contribution

The contribution of the authors is equal.

Conflict of Interest

The authors have declared that there is no conflict of interest.

References

1. Zadeh LA. Fuzzy set theory, Inf. Control, 1965; 8(3): 338-353.
2. Molodtsov D. Soft set theory—first results, Comput Math Appl, 1999; 37(1): 19-31.
3. Maji PK, Biswas R and Roy AR. Soft set theory, Comput Math Appl, 2003; 45(1): 555-562.
4. Pei D and Miao D. From soft sets to information systems. In: Proceedings of Granular Computing IEEE, 2005; 2: 617-621.
5. Ali MI, Feng F, Liu X, Min WK, Shabir M. On some new operations in soft set theory, Comput Math Appl, 2009; 57(9): 1547-1553.
6. Sezgin A and Atagün AO. On operations of soft sets. Comput Math Appl, 2011; 61(5): 1457-1467.
7. Ali MI, Shabir M and Naz M. Algebraic structures of soft sets associated with new operations, Comput Math Appl 2011; 61: 2647–2654.
8. Sezgin A, Shahzad A and Mehmood A. New operation on soft sets: Extended difference of soft sets. J New Theory, 2019; (27): 33-42.

9. Stojanovic NS. A new operation on soft sets: Extended symmetric difference of soft sets. Military Technical Courier, 2021; 69(4): 779-791.
10. Eren ÖF and Çalışıcı H. On some operations of soft sets, The Fourth International Conference on Computational Mathematics and Engineering Sciences; 2019 Apr 20-22; Antalya, Türkiye.
11. Sezgin A and Çalışıcı H. A comprehensive study on soft binary piecewise difference operation, Eskişehir Teknik Üniversitesi Bilim ve Teknoloji Dergisi B - Teorik Bilimler, 2024;12(1): 32-54.
12. Aybek FN. New restricted and extended soft set operations, MSc Thesis, Amasya University, Amasya, Türkiye, 2024.
13. Çağman N. Conditional complements of sets and their application to group theory, J New Results Sci, 2021; 10(3): 67-74.
14. Sezgin A, Çağman N, Atagün AO and Aybek FN. Complemental binary operations of sets and their application to group theory, Matrix Science Mathematic, 2023; 7(2): 114-121.
15. Yavuz E. Soft binary piecewise operations and their properties, MSc Thesis, Amasya University, Amasya, Türkiye, 2024.
16. Akbulut E. New type of extended operations of soft set: Complementary extended difference and lambda operation, MSc Thesis, Amasya University, Amasya, Türkiye, 2024.
17. Demirci AM. A new type of extended operations of soft set: Complementary extended union, plus and theta operation, MSc Thesis, Amasya University, Amasya, Türkiye, 2024.
18. Sarıalioğlu M. A new type of extended operations of soft set: Complementary extended intersection, gamma and star operation, MSc Thesis, Amasya University, Amasya, Türkiye, 2024.
19. Sezgin A and Atagün AO. A new soft set operation: Complementary soft binary piecewise plus operation, Matrix Science Mathematic, 2023; 7(2): 125-142.
20. Sezgin A and Aybek FN. A new soft set operation: Complementary soft binary piecewise gamma operation, Matrix Science Mathematic, 2023; 7(1): 27-45.
21. Sezgin A, Aybek FN and Güngör NB. A new soft set operation: Complementary soft binary piecewise union operation, Acta Informatica Malaysia, 2023; 7(1): 38-53.
22. Sezgin A, Aybek FN and Atagün AO. A new soft set operation: Complementary soft binary piecewise intersection operation, BSJ Eng Sci, 2023; 6(4): 330-346.
23. Sezgin A and Çağman N. A new soft set operation: Complementary soft binary piecewise difference operation, Osmaniye Korkut Ata Üniv Fen Biliml Derg, 2024; 7(1): 58-94.
24. Sezgin A and Demirci AM. A new soft set operation: Complementary soft binary piecewise star operation, Ikonion Journal of Mathematics, 2023; 5(2): 24-52.
25. Sezgin A and Sarıalioğlu M. A new soft set operation: Complementary soft binary piecewise theta operation, Journal of Kadirli Faculty of Applied Sciences, 2024; 4(2): 325-357.
26. Sezgin A and Yavuz E. A new soft set operation: Complementary Soft Binary Piecewise Lambda Operation, Sinop University Journal of Natural Sciences, 2023; 8(2): 101-133.
27. Sezgin A and Yavuz E. A new soft set operation: Soft binary piecewise symmetric difference operation, Necmettin Erbakan University Journal of Science and Engineering, 2023; 5(2): 189-208.
28. Sezgin A and Dagtoros K. Complementary soft binary piecewise symmetric difference operation: a novel soft set operation, Scientific Journal of Mehmet Akif Ersoy University, 2023; 6(2): 31-45.

29. Çağman N, Çitak F and Aktaş H. Soft int-group and its applications to group theory, Neural Comput Appl, 2012; 2: 151–158.
30. Sezer AS, Çağman N, Atagün AO, Ali MI and Türkmen E. Soft intersection semigroups, ideals and bi-ideals; a new application on semigroup theory I, Filomat, 2015; 29(5): 917-946.
31. Sezer AS, Çağman N and Atagün AO. Soft intersection interior ideals, quasi-ideals and generalized bi-ideals; a new approach to semigroup theory II, J Mult.-Valued Log. Soft Comput, 2014; 23(1-2): 161-207.
32. Sezgin A and Orbay M. Analysis of semigroups with soft intersection ideals, Acta Univ Sapientiae Math, 2022; 14(1): 166-210.
33. Sezgin A. A new approach to semigroup theory I: Soft union semigroups, ideals and bi-ideals, Algebra Lett, 2016; 2016(3): 1–46.
34. Jana C, Pal M, Karaaslan F and Sezgin A. (α, β)-soft intersectional rings and ideals with their applications, New Math Nat Comput, 2019; 15(2): 333–350.
35. Muştuoğlu E, Sezgin A and Türk ZK. Some characterizations on soft uni-groups and normal soft uni-groups, Int J Comput Appl, 2016; 155(10): 1-8.
36. Sezer AS, Çağman N and Atagün AO. Uni-soft substructures of groups, Ann Fuzzy Math Inform, 2015; 9(2): 235–246.
37. Sezer AS. Certain Characterizations of LA-semigroups by soft sets, J Intell Fuzzy Syst, 2014; 27(2): 1035-1046.
38. Özlü Ş and Sezgin A. Soft covered ideals in semigroups, Acta Univ Sapientiae Math, 2020: 12(2): 317-346.
39. Atagün AO and Sezgin A. Soft subnear-rings, soft ideals and soft n-subgroups of near-rings, Math Sci Letters, 2018; 7(1): 37-42.
40. Sezgin A. A new view on AG-groupoid theory via soft sets for uncertainty modeling, Filomat, 2018; 32(8): 2995–3030.
41. Sezgin A, Çağman N and Atagün AO. A completely new view to soft intersection rings via soft uni-int product, Appl Soft Comput, 2017; 54: 366-392.
42. Sezgin A, Atagün AO and Çağman N and Demir H. On near-rings with soft union ideals and applications, New Math Nat Comput, 2022; 18(2): 495-511.
43. Imai Y and Iseki K. On axiom systems of proposition calculi, Proc Jpn Acad, 1966; 42: 19–22.
44. Pant S, Dagtoros K, Kholil MI and Vivas A. Matrices: Peculiar determinant property, OPS Journal, 2024; 1: 1–7.