Pamukkale Univ Muh Bilim Derg, 30(2), 202-211, 2024

Pamukkale Üniversitesi Mühendislik Bilimleri Dergisi

Pamukkale University Journal of Engineering Sciences

Modeling of first order plus time delay system dynamics with adaptive IIR filters based on gradient descent methods and performance analyses for different time delay cases

Birinci mertebe artı zaman gecikmeli sistem dinamiğinin gradyan iniş yöntemine dayali adaptif IIR filtreler ile modellenmesi ve farkli zaman gecikmeleri durumlari için performans analizleri

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Received/Geliş Tarihi: 17.11.2022 Revision/Düzeltme Tarihi: 04.04.2023 doi: 10.5505/pajes.2023.65748
Accepted/Kabul Tarihi: 03.05.2023 Research Article/Araştırma Makalesi

Abstract

In this study, the modeling of First Order Plus Time Delay (FOPTD) dynamics by using adaptive infinite impulse response (IIR) filter based on Gradient Descent (GD) method, which is frequently used in machine learning applications, has been investigated by the help of the input-output data in the time domain. The First Order Time Delay (FOPTD) dynamic system models are the most basic system model that is used in the modeling of control systems. In the study, the IIR filter coefficients are optimized online by using the GD method for convergence of the IIR filter response to the FOPTD dynamic system model response for the same input signal. The distance of the IIR filter output to the output of the FOPTD dynamic system for the same input is expressed by the instant square error function and, recursive GD solutions of this function are used to minimize output mismatches between FOPTD system model and the proposed adaptive IIR filter. Thus, the convergence of the IIR filter to the input-output dynamics of a FOPTD dynamic system is provided in the time domain by performing recursive filter coefficient solutions that are obtained by the GD method. An application of the adaptive IIR filter solutions in the online modeling of FOPTD systems was carried out in MATLAB-Simulink environment. In the developed simulation environment, the collected signals from the inputs and outputs of the FOPTD dynamic system were used to online optimize the IIR filter coefficients in the GD optimization block. In this simulation environment, the convergence performance of the IIR filter response for the time delay system dynamics of the FOPTD plant model is investigated for different time delay values.

Keywords: System modeling, Nonlinear optimization, Gradient descent method, Adaptive IIR Filter, Dynamic system.

Öz

Bu çalışmada makine öğrenmesi uygulamalarında sıklıkla kullanılan ve popüler bir nümerik optimizasyon yöntemi olan Gradyan İniş (GD) yöntemine dayalı adaptif sonsuz impuls cevabı (IIR) filtresi ile Birinci Mertebe Zaman Gecikmeli (FOPTD) sistem dinamiğinin zaman bölgesinde giriş-çıkış verisi yardımı ile modellemesi incelenmiştir. FOPTD dinamik sistem modelleri kontrol sitemlerinin modellemesinde kullanılan en temel sistem modelidir. Çalışmada, IIR filtre katsayıları, aynı giriş işareti için IIR filtre cevabının FOPTD dinamik sistem modelinin cevabına yakınsaması için GD yöntemi ile online optimize edilmiştir. Aynı giriş için IIR filtre çıkışının, FOPTD dinamik sistemin çıkışına uzaklığı anlık karesel hata fonksiyonu ile ifade edilmiş ve bu fonksiyonun özyinelemeli gradyan iniş çözümleri FOPTD sistem cevabı ile tasarlanan adaptif IIR filtre cevabı arasındaki çıkış uyumsuzluğunu minimize etmek için kullanılmıştır. Böylece, zaman bölgesinde IIR filtrenin bir FOPTD dinamik sistemin giriş-çıkış dinamiğine yakınsaması GD yöntemi ile elde edilen özyinelemeli filtre katsayı çözümleri ile sağlanmıştır. Adaptif IIR filtre çözümlerinin FOPTD sistemlerin online modellemesinde uygulaması MATLAB-Simulink ortamında gerçekleştirilmiş. Geliştirilen simülasyon ortamında FOPTD dinamik sistemin giriş ve çıkışlarından alınan işaretler, GD optimizasyon bloğunda IIR filtre katsayılarının online olarak optimize edilmesinde kullanılmış. Bu simülasyon ortamında IIR filtre cevabının FOPTD plant modelinin zaman gecikmeli sistem dinamiğine yakınsama performansı farklı zaman gecikme değerleri için incelenmiştir.

Anahtar kelimeler: Sistem modelleme, Doğrusal olmayan optimizasyon, Gradyan iniş yöntemi, Adaptif IIR filtre, Dinamik sistem.

1 Introduction

Optimization methods have an important place in solving the engineering problem [1]. System modeling has become one of the main areas of system engineering where optimization methods have been frequently used. Specifically, to represent the real system response, the model parameters have been widely optimized to minimize the square error [2]. Mathematical modeling of real systems is a crucial step for system analysis and design. Therefore, nowadays, mathematical modeling is needed in many fields such as data analysis [3], machine learning [4], dynamic system modeling [5], control system design [6].

The GD method is the widely preferred nonlinear optimization technique [7]. The reason is that the GD method provides easily applicable iterative numerical solutions for parameter optimization problems, and this feature has increased its use in solving engineering problems over time. For example, artificial neural networks and its backpropagation training algorithm benefits from GD method in optimization of weight and bias

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coefficients, and artificial neural networks applied in the solution of pattern recognition problem [8], control applications [9] and big data [10]. The GD method was also improved for specific applications.; for instance, a self-adaptive gradient descent search algorithm was proposed for training of fully-connected neural networks [11]. In the field of system modeling; GD method found applications: In order to minimize the sum of square difference errors in the process of constructing a parallel RC equivalent circuit model for modeling dielectric materials, [12] and an adaptive online parameter identification method with GD optimization has been used for an adaptive online parameter identification in finite-control-set model predictive control of a grid connected converter [13]. GD method was also used in the field of adaptive filter design to determine the optimal coefficients of an adaptive IIR filter for signal processing and filtering [14], [15] and GD can improve the convergence performance of the Wiener spline adaptive filter [16].

Compared to other application domains, application of GD algorithm in control systems seem to be relatively limited. However, direct gradient descent control is a developing topic as a branch of intelligent control. The GD algorithm has been directly used as an optimal controller and tracking control of general nonlinear systems via direct gradient descent control has been discussed [17]. The performance of the control method has been demonstrated in control simulations. In another study, continuous time gradient descent has been used as MIT rule and a fractional order adjustment rule was suggested for fractional order systems [18]. In the study conducted in 2017, a model free adaptive control scheme that includes two GD optimizer processes, one for plant response prediction and the other for optimal control law, was suggested and its control performance was investigated for an experimental rotor control system [19]. This adaptive control scheme contributes to the direct GD control and reduces need for the plant modeling. This control scheme has been implemented for the adaptive control of first-order stable time-delay systems and performance improvements of this adaptive GD control method was shown [20]. In the study, it is assumed the inner-model, which expresses the instantaneous input-output relationship with a time-varying finite impulse response (FIR) filter, and adaptive gradient descent control law were obtained according to the FIR filter response assumption. In a thesis study conducted in 2020, the gradient descent method and its adaptive control applications were examined [21]. Several recent studies addressed several gradient-based control solutions [19], [22]-[23]. In a study, to increase control efficiency, gradient descent algorithms and loss models were used to adjust the maximum efficiency angle for different operating conditions in vector controlled permanent magnet synchronous motors [22]. In another study, an application of the adaptive gradient descent control scheme was performed for adaptive control of nonlinear stable system models [23]. In the study, gradient descent optimizers are used for adaptive control of the nonlinear system by considering a higher order polynomial assumption of the instantaneous input-output relationships of the controlled system.

The main reasons for the limited applications of the GD method in control systems are the difficulties of real-time identification and optimization of the dynamic system response, which is mathematically modeled with differential equations, and the necessity to guarantee the system stability and robust settling point control performance of the control systems. In this regard, stability conditions based on Lyapunov stability have been investigated [24]. There is a need to improve the stability conditions of the GD optimization method. For this reason, although the use of the GD method as a controller in the direct control of processes has not been considered reliable yet, due to system stability concerns, however the GD method can find utilization in real-time model identification [12]-[14],[25] and contribute to the adaptive control systems. Investigation of this potential of the GD method is a motivation for the current study. Main advantages of the GD optimization for online or real-time system modeling are that its coefficient update solutions are quite simple and yet effective to numerically solve nonlinear optimization problems. The GD algorithm has low computational complexity that makes the GD optimization suitable for real-time and embedded system applications.

This study investigates online IIR filter identification to represent FOPTD system dynamics, which is widely used in control systems design. An adaptive IIR filter with GD optimization was used for the online discrete time modeling of FOPTD systems from the input-output signals sampled from FOPTD systems. Performance of the adaptive IIR filter for modeling the FOPTD systems was reported. Thus, a real-time discrete time IIR filter model of the plant can be obtained with the input and output data from the controlled plant, and this IIR model of plant response will be used in the real-time adaptation of the controller in future works. The main problem, addressed in this article, is to what extent the dynamic system with a time delay can be represented by an adaptive IIR filter response. To investigate this point, the simulation of FOPTD systems with different time delays was carried out in the MATLAB Simulink environment, and the adaptive IIR filter model was optimized online with the input-output data obtained from the FOPTD system models within the simulation. The adaptive IIR filter response can converge to the response of FOPTD systems online throughout the simulation. Thus, the online modeling performance of the adaptive IIR filter was evaluated for different time-delay of first order dynamic systems in the simulation environment.

There have been studies in the literature on adaptive FIR and IIR filter designs based on GD optimization methods [14]-[15], [26]. In this study, in line with the approaches developed in previous studies [14]-[15], recursive IIR filter coefficient solutions were derived based on the GD method for discrete time model generation from input and output signals of FOPTD systems in the simulation environment. In particular, the effects of time delays on modeling performance of adaptive IIR filters are examined in this work, and potential of the adaptive IIR filter to model FOPTD plant responses are discussed for possible applications in intelligent control. This work was carried out within the scope of a thesis study [21] and the results of these investigations were discussed in this article.

2 Fundamentals and theoretical background

2.1 First order plus time delay system models

First order, time delay system models are linear models with delayed response. FOPTD models can represent the most basic dynamic response of a capacitive system and they are used for empirical description of many dynamic processes in control. First order dynamic system models can be expressed in the form of first order, constant coefficient, linear ordinary differential equations. Hence, FOPTD models are in the category of the linear time invariant models. Accordingly, their time responses can be delayed in time as much as time delay parameter of the model. They are commonly expressed as transfer function models in s-domain.

The transfer function of FOPTD system ($G(s)$) is commonly expressed as follows:

$$G(s) = \frac{K_{DC}}{\tau s+1} e^{-Ls}, \quad (1)$$

Where the parameter K_{DC} is DC gain, τ is the time constant, L is the time delay of FOPTD system model. The parameter s represents the complex variable in Laplace transform.

2.2 Gradient descent method

The GD method is a gradient-based nonlinear optimization method. The GD optimization follows the gradient of a given differentiable objective function and the GD provides a recursive numerical parameter update solution that iteratively advances the solution towards its local minimum by following the gradient of the objective function. The basic numerical GD method is written as [7].

$$x_i[n+1] = x_i[n] - \gamma_i \frac{\partial E}{\partial x_i} \quad (2)$$

The objective function E appearing in the equation is a differentiable objective function, and the coefficient γ_i is the learning step for i. update of parameters. Commonly, in regression modeling problems, the error function E is taken as the square of the instant error and minimized [12],[14].

$$E = \frac{1}{2} e^2, \quad (3)$$

Where the e is the instant error value and it is usually expressed as the difference between the actual value (d) and the calculated value (y).

$$e = d - y \quad (4)$$

GD solutions with parameter updates for a single error value are called stochastic GD solutions in machine learning [7].

Variants of the GD method have been used in machine learning [7]: Momentum, adagrad, adadelta etc. Convergence conditions are important for the practical applications of the GD method [7],[24],[27].

Continuous time optimization gains importance in control applications and gradient descent dynamics is expressed as [18]-[19]:

$$\frac{dx}{dt} = -\eta_i \frac{\partial E}{\partial x} \quad (5)$$

where, the parameter η_i is the learning coefficient. When this equation is solved in discrete time, the numerical GD formulation (equation (2)) is obtained. Let's show this: To solve this equation according to Euler's method, the forward difference equation with the discrete time index n is used as follows:[24], [28]

$$\frac{dx_i}{dt} \cong -\frac{x_i[n+1] - x_i[n]}{\Delta t} \quad (6)$$

If equation (6) is used in equation (5) and arranged, one obtains

$$x_i[n+1] = x_i[n] - \Delta t \eta_i \frac{\partial E}{\partial x_i} \quad (7)$$

The learning step γ_i parameter can be written as $\gamma_i = \Delta t \eta_i$ so that one obtains equation (2). The learning step adjusts the size of the step in the gradient direction [29].

2.3 Infinite impulse response filters

IIR filters model a discrete-time linear time-invariant system that implements feedback from output. It recursively uses the past data from its own output in addition to the input. They are also known as feedback filters because of involving feedback from its output to its input. Due to these feedback features, it can find use in modeling system dynamics [30].

IIR filters are expressed in the discrete time domain with difference equations as[14]

$$y[n] = -\sum_{i=1}^{L} a_i\, y[n-i] + \sum_{j=0}^{M} b_j\, x[n-j] \quad (8)$$

The coefficients of adaptive IIR filters are optimally determined so that the IIR filter converges to a desired filter response. The GD method has been used to determine the optimal IIR coefficients [14]. This method is summarized as follows:

The filter coefficient vectors θ and the vector $X(n)$ are defined by

$$\theta = [a_1 a_2 \,..\, a_L\ b_0 b_1 \,..\, b_M] \quad (9)$$

$$X(n) = [y(n-1) y(n-2) \,....\, y(n-L)\ x(n-1)\ x(n-2) \,....\, x(n-M)\ ] \quad (10)$$

and the filter output is expressed as

$$y(n) = \theta(n)^T X(n) \quad (11)$$

For the error function $E = \frac{1}{2}e^2 = \frac{1}{2}(d-y)^2$, the filter coefficient updates are written according to the GD method as

$$\theta(n+1) = \theta(n) + \gamma(d-y)\nabla_\theta y(n) \quad (12)$$

where, the gradient operation $\nabla_\theta y(n)$ is calculated as

$$\nabla_\theta y(n) = X(n) + \sum_{i=1}^{L} a_i\, \nabla_\theta y(n) \quad (13)$$

In the next sections, this method is analyzed and implemented in the MATLAB/Simulink 2015 environment for dynamic system modeling for control systems [31]. Tests were conducted on a laptop with an Intel Core i5-CX0026NT processor and 8 GB of RAM.

3 Method

In this section, the GD method is applied for tuning coefficients of the adaptive IIR filter model that is implemented to online approximate to the response of FOPTD systems in discrete time. Let the input of a FOPTD dynamic system model be denoted by $u[n]$ and the output by $d[n]$. In order for the IIR filter function expressed in equation 5 to fully represent a FOPTD system, the difference of the two system outputs is expected to decrease zero value in time.

When the following condition is met, the IIR filter output $y[n]$ converges to the dynamic system output $d[n]$ in case of the same $u[n]$ input.

$$\lim_{n\to\infty}(d[n]-y[n]) \to 0 \tag{14}$$

In order to implement this condition iteratively, the objective function can be expressed in the form of equation (3) as

$$E_m = \frac{1}{2}(d[n]-y[n])^2 = \frac{1}{2}(d[n]+\sum_{i=1}^{L} a_i\, y[n-i] - \sum_{j=0}^{M} b_j\, u[n-j])^2 \tag{15}$$

In order for converge of the filter response to the FOPTD system response, the a_i and b_j coefficients are calculated to minimize the modeling error by using the GD method. Since the FOPTD system is a continuous time system, gradient descent dynamics (equation 5) is considered in this system structure, and discrete time coefficient update equations are derived by discretizing the gradient descent dynamics. Thus, an iterative solution of the coefficients of the discrete time IIR filter, which can converge to the continuous time FOPTD system response, is obtained. When applying continuous time GD dynamics for a_i coefficients, one obtains [21].

$$\frac{da_i}{dt} = -\gamma_a \frac{\partial E_m}{\partial a_i} \tag{16}$$

The sensitivity derivative can be written in the discrete form as

$$\frac{\partial E_m}{\partial a_i[n]} = y[n-i](d[n] + \sum_{i=1}^{L} a_i[n]\, y[n-i] - \sum_{j=0}^{M} b_j[n]\, u[n-j]) \tag{17}$$

If the GD dynamics is applied for the b_j coefficient, one obtains [21].

$$\frac{db_j}{dt} = -\gamma_b \frac{\partial E_m}{\partial b_j} \tag{18}$$

The sensitivity derivative can be written in discrete form as

$$\frac{\partial E_m}{\partial b_j[n]} = -u[n-j](d[n] + \sum_{i=1}^{L} a_i[n]\, y[n-i] - \sum_{j=0}^{M} b_j\, [n] u[n-j]) \tag{19}$$

When equations (17) and (19) are used in equations (16) and (18), respectively, they can be rearranged as [21]

$$\frac{da_i}{dt} = -\gamma_a y[n-i](d[n] + \sum_{i=1}^{L} a_i[n]\, y[n-i] - \sum_{j=0}^{M} b_j[n]\, u[n-j]) \tag{20}$$

$$\frac{db_j}{dt} = \gamma_b u[n-i]((d[n] + \sum_{i=1}^{L} a_i\, [n] y[n-i] - \sum_{j=0}^{M} b_j\, [n] u[n-j]) \tag{21}$$

The derivative terms in this equation can be discretized in the form of the forward difference [24].

$$\frac{da_i}{dt} \cong \frac{a_i[n+1]-a_i[n]}{\Delta t} \tag{22}$$

$$\frac{db_j}{dt} \cong \frac{b_j[n+1]-b_j[n]}{\Delta t} \tag{23}$$

Thus, the coefficients a_i and b_j are sampled with $t = n\Delta t$, and discrete time $a_i[n]$ and $b_j[n]$ solutions are obtained.

If equations (22) and (23) are substituted in equations (20) and (21), respectively, the numerical GD solutions for filter coefficient update are expressed as

$$a_i[n+1] = a_i[n] - \lambda_a y[n-i](d[n] + \sum_{i=1}^{L} a_i\, [n] y[n-i] - \sum_{j=0}^{M} b_j[n]\, u[n-j]) \tag{24}$$

$$b_j[n+1] = b_j[n] + \lambda_b u[n-i](d[n] + \sum_{i=1}^{L} a_i[n]\, y[n-i] - \sum_{j=0}^{M} b_j[n]\, u[n-j]) \tag{25}$$

Where learning coefficients can be obtained as $\lambda_a = \Delta t \gamma_a$ and $\lambda_b = \Delta t \gamma_b$ [21]. These update equations are used in Simulink simulation.

4 Numerical work

4.1 Simulations for square wave input

In this section, the adaptive IIR filter solution is used to converge response of the FOPTD system dynamics that is expressed by equation (1). For this purpose, the system diagram in Figure 1 is designed in the MATLAB/Simulink environment [31]. The developed Simulink simulation model is presented in Figure 2.

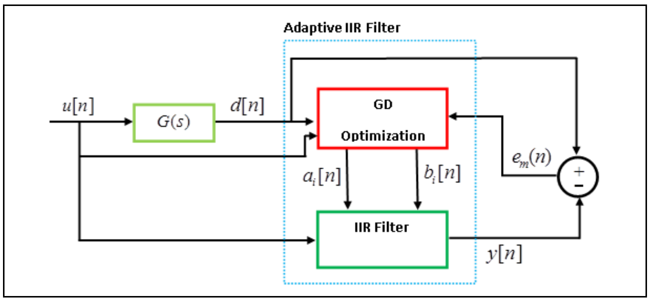


Figure 1. Block diagram of the system developed for dynamic system modeling [21].

In general, IIR filters can represent dynamic systems [30]. However, the time delay component e^{-Ls} in FOPTD system models causes delaying the system response in time, and this effect becomes a factor that can negatively affect the modeling performance of discrete time IIR filter functions to represent FOPTD system.

In this study, how the increase of time delay parameter affects the modeling performance of the adaptive IIR filter with GD optimization are investigated. In order to observe effects of the time delay on the convergence performance of the proposed IIR filter, multiple simulations for FOPTD system functions in Table 1 were performed, and the results were evaluated.

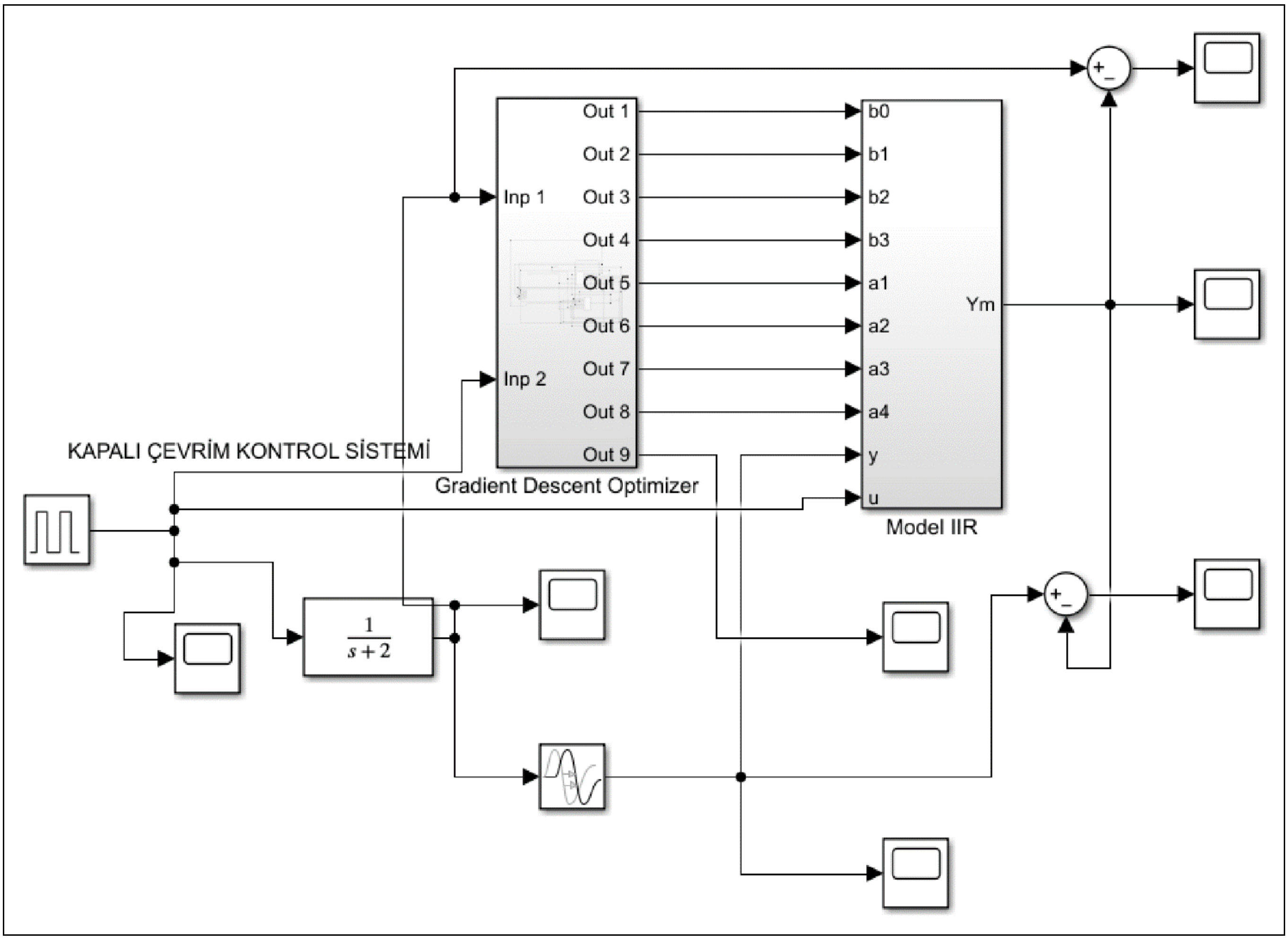


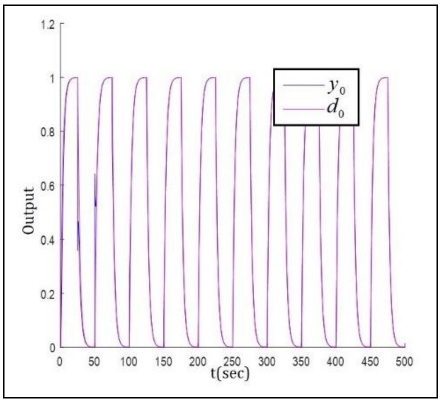
Figure 2. Simulink simulation environment developed for FOPTD system modeling [21].

Table 1. FOPTD system models with different time delays used in the tests.

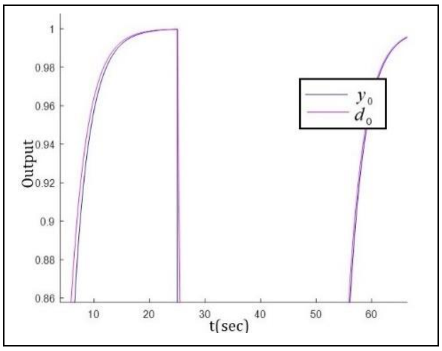
FOPTD System Functions	Time Delay
$G_0(s) = \frac{1}{3s+1}e^{0}$	0 sec (Without Delay)
$G_1(s) = \frac{1}{3s+1}e^{-1}$	1 sec
$G_{10}(s) = \frac{1}{3s+1}e^{-10}$	10 sec
$G_{20}(s) = \frac{1}{3s+1}e^{-20}$	20 sec

The square wave input is applied to inputs of $G_0(s)$, $G_1(s)$, $G_{10}(s)$ and $G_{20}(s)$ separately. For the square wave input, and the convergence of proposed adaptive IIR filter solutions to these models was shown by comparing outputs of the IIR filters and FOPTD in figures. In these figures, obtained filter outputs y_0, y_1, y_{10} and y_{20} are presented with FOPTD system outputs d_0, d_1, d_{10} and d_{20}. The subscripts represent the time delay.

Figure 3 shows the simulation results that demonstrate the online modeling of $G_0(s)$ response via adaptive IIR filter in case of square wave input. In the figure, the convergence of adaptive filter output y_0 to FOPTD system output d_0 is shown. Figure 3(b) presents a comparison of system responses in the interval of 0-90 seconds at the beginning of the simulation. At the second rising edge, it is seen that the filter output starts to converge to the dynamic system output. Figure 3(c) shows the results for the 400-480 seconds interval towards the end of the simulation. Here, it is clearly seen that the difference between the adaptive filter response and the $G_0(s)$ system response considerably decreases compared to the initial period, and the filter response converges to response of the $G_0(s)$ model. In Figure 4, the evolution of E_{m0} modeling error (square error) with time is given. It is seen that the model error E_{m0} decreases with time and confirms the convergence of the system response as seen in Figure 3.



(a)



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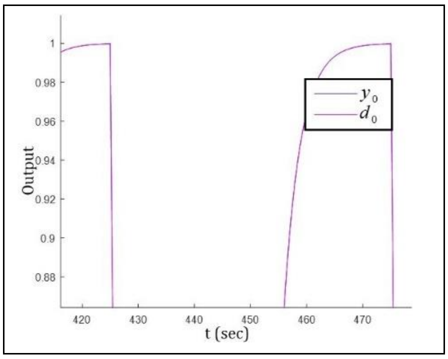


(c)

Figure 3. Adaptive IIR filter output and $G_0(s)$ function output for square wave input signal. (a): Full simulation result. (b): close-up view in the range of 0-90 sec. (c): close-up view in the range of 400-480 sec.

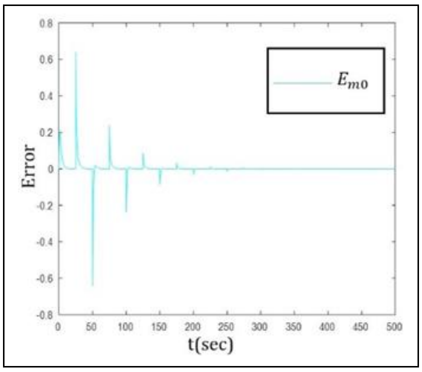
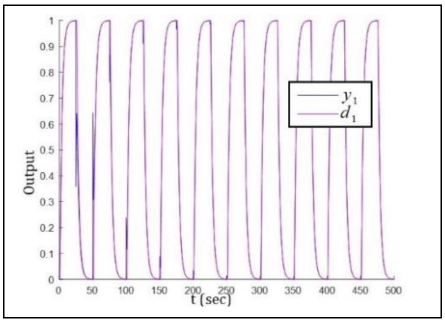
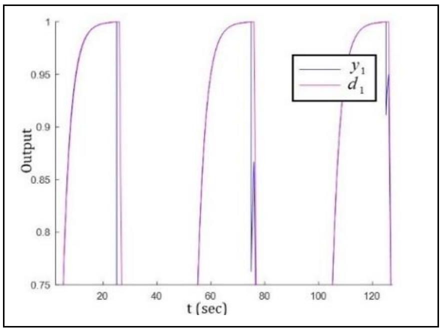


Figure 4. Temporal evolution of model error for function $G_0(s)$.

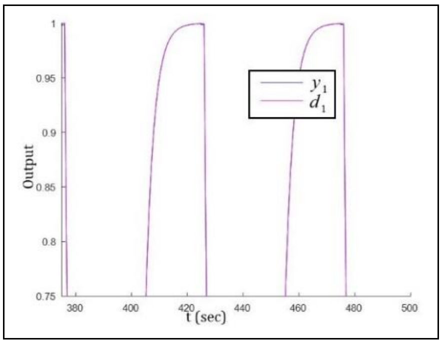
Figure 5 shows the online modeling simulation results of the adaptive IIR filter for $G_1(s)$ model response. Here, the dynamic system model $G_1(s)$ has a time delay of 1 second. Figure 5(b) shows convergence of adaptive filter output y_1 to $G_1(s)$ model output d_1 in the 0-90 seconds interval of the simulation time. When Figure 5(b) and (c) are compared, it is seen that the IIR filter output can coverage to output of FOPTD model $G_1(s)$ between 350-500 seconds. This is because the adaptive IIR filter coefficients were not optimal at the beginning of the simulation, and towards the end of the simulation, the GD method optimizes the filter coefficients, and the filter response can converge to the dynamic system response.



(a)



(b)



(c)

Figure 5. Adaptive IIR filter output and $G_1(s)$ function output for square wave input signal. (a): Full simulation result. (b): Close view in the range of 0-120 sec. (c): close-up view in the range of 350-500 sec.

The decrease of the model error in Figure 6 supports this observation. This result showed that the adaptive filter response began to adapt over time to the dynamic system response of $G_1(s)$ that has a time delay of 1 second.

Figure 7 shows the simulation results for online modeling of FOPTD dynamic system $G_{10}(s)$ via adaptive IIR filter model response for square wave input. The dynamic system model

$G_{10}(s)$ has a 10 seconds time delay. Figures reveal that increasing time delay makes it difficult for the adaptive filter's response to converge to the dynamic system response. By comparing Figure 7(b) and (c), it can be observed that the differences between y_{10} and d_{10} in the 0-120 seconds interval is more than differences in the 380-500 seconds interval.

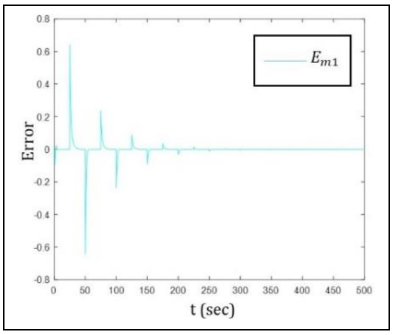
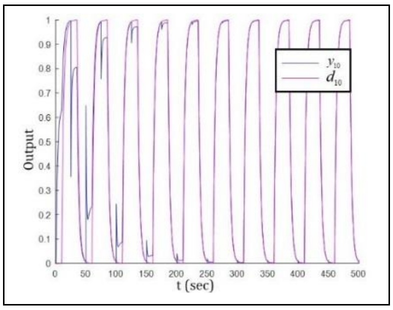
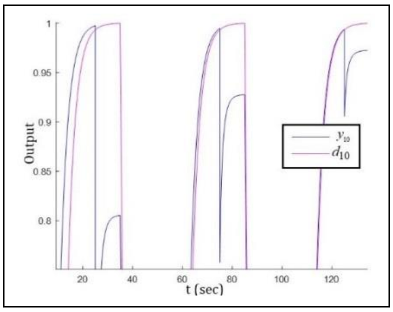


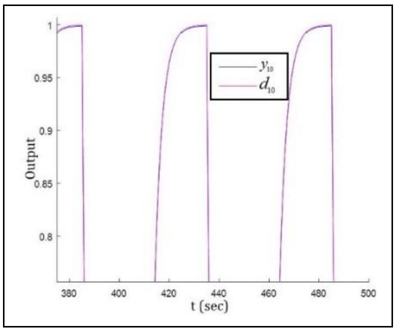
Figure 6. Temporal evolution of model error for function $G_1(s)$.



(a)



(b)



(c)

Figure 7. Adaptive IIR filter output and $G_{10}(s)$ function output for square wave input signal. (a): Full simulation result. (b): Close view in the range of 0-120 sec. (c): Close-up view in the range of 380-500 sec.

The negative effect of the increasing time delay is observed as the adaptive filter prefers to converge to the rising edge and the filter response at the falling edge moves away from the dynamic system response. This situation explains the increase in the model error parameters as time delay increases in Table 2.

Table 2. MAE, MSE and MRE performances.

FOPTD System Functions	MAE	MSE	MRE
$G_0(s)$	0.0037	0.0004	0.0794
$G_1(s)$	0.0092	0.0016	29.464
$G_{10}(s)$	0.0709	0.0158	338.89
$G_{20}(s)$	0.1399	0.0319	689.49

The decrease of the model error in Figure 8 indicates that the adaptive filter output y_{10} can still approximate to the dynamic system output d_{10} in time.

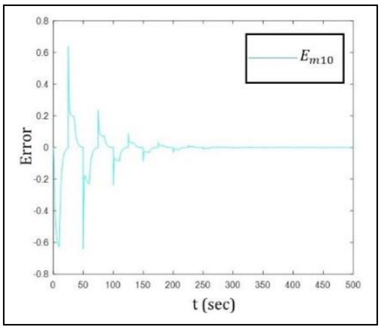
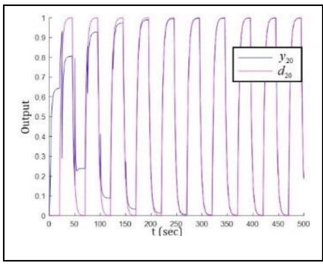
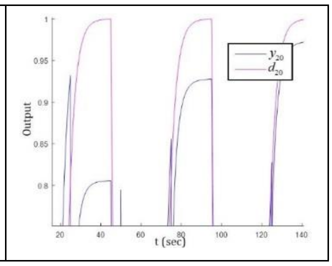


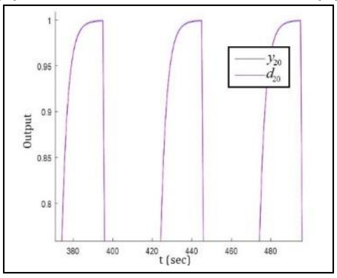
Figure 8. Temporal evolution of model error for function $G_{10}(s)$.

Figure 9 shows the simulation results of online modeling of the $G_{20}(s)$ response via the adaptive IIR filter response for square wave input.





(a) (b)



(c)

Figure 9. Adaptive IIR filter output and $G_{20}(s)$ function output for square wave input signal. (a): Full simulation result. (b): Close view in the range of 0-140 sec. (c): Close-up view in the range of 380-500 sec.

The time delay of the model $G_{20}(s)$ is 20 seconds. Increasing time delay makes it difficult for the adaptive filter response to converge to the dynamic system response. It is clear that increasing time delay causes the filter response to diverge from the dynamic system response at the falling edges. However, in Figure 10, the decrease of the model error with time shows that the IIR filter response can converge to the system response.

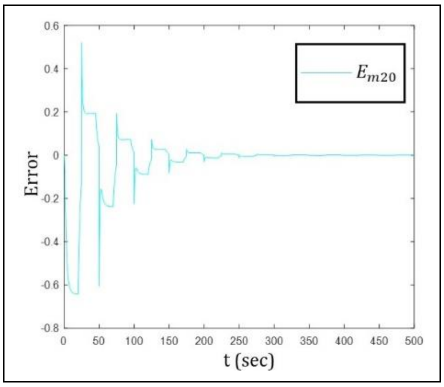


Figure 10. Temporal evolution of model error for function $G_{20}(s)$.

In Table 2, the convergence performance of the adaptive IIR filter response to the FOPTD system response with different time delays of $G_0(s)$, $G_1(s)$, $G_{10}(s)$ and $G_{20}(s)$ is presented. Three different error performance measures were used.

Mean Absolute Error (MAE) performance measure is written as,

$$MAE = \frac{1}{p}\sum_{n=1}^{p}|d_L[n] - y_L[n]| \tag{27}$$

Mean Square Error (MSE) performance measure is written as,

$$MSE = \frac{1}{p}\sum_{n=1}^{p}(d_L[n] - y_L[n])^2 \tag{28}$$

Mean Relative Error (MRE) performance measure is written as,

$$MRE = \frac{1}{p}\sum_{n=1}^{p}\frac{|d_L[n] - y_L[n]|}{d_L[n]} \tag{29}$$

Where the y_L denotes output of the adaptive IIR filter and the d_L denotes FOPTD system. Subscript L is the time delay of the FOPTD system.

4.2 Simulations for sinusoidal wave input

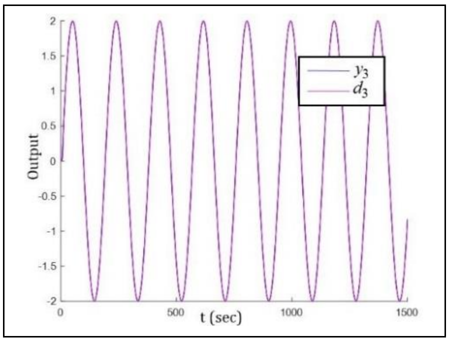
In this section, the simulation model in Figure 2 is used to perform a simulation for a sine wave signal with amplitude 1 and frequency 0.0333. The FOPTD system function seen in Table 3 is implemented in this simulation for modeling with the IIR filter and the results were evaluated.

Table 3. FOPTD system model with 3 s time delay.

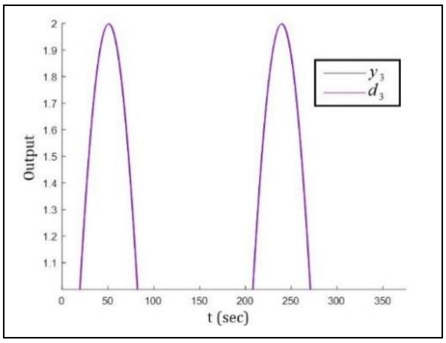
FOPTD System Function	*Time Delay*
$G_3(s) = \frac{2}{s+1}e^{-3s}$	3 sec

For this purpose, the model $G_3(s)$ is simulated for sine wave input and convergence of the adaptive IIR filter solutions is provided. The filter output y_3 and FOPTD system output d_3 are compared. The subscript expresses the time delay of 3 sec.

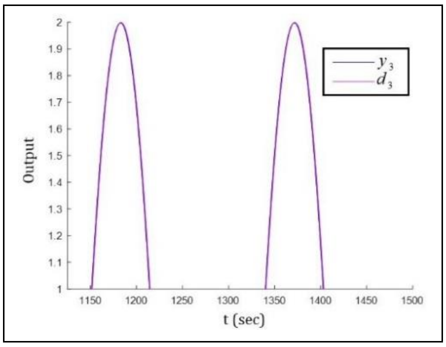
Figure 11 shows the online modeling simulation results of $G_3(s)$ dynamic system model response and adaptive IIR filter response for sine wave input. The dynamic system model $G_3(s)$ has a time delay of 3 s. When Figure 11(b) and (c) are compared, it is observed that the convergence performance is very close to each other in the results obtained between 0-350 seconds and 1150-1500 seconds of the simulation.



(a)



(b)



(c)

Figure 11. Adaptive IIR filter output and $G_3(s)$ function output for square wave input signal. (a): Full simulation result. (b): Close-up view in the range of 0-350 sec. (c): Close-up view in the range of 1150-1500 sec.

The main reason for this is that the filter converges to the sinusoidal wave quite quickly and continues with this performance throughout the simulation. The small and periodic value of the model error in Figure 12 supports this conclusion. In the figure, it is seen that the instant error value decreases very quickly at the beginning and stays within a convergence interval [-0.06, 0.06] in the following periods. Since the square wave is a broadband signal, its convergence could take more time compared to the single frequency sinusoidal wave input.

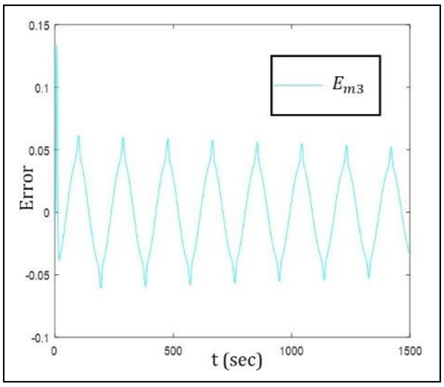


Figure 12. Temporal evolution of model error for function $G_3(s)$.

5 Conclusion

In this study, the convergence performance of the proposed GD based adaptive IIR filters to the FOPTD dynamic system response was investigated. The convergence of the adaptive IIR filter response to the response of dynamic systems with GD optimization will be beneficial in online system identification for intelligent control applications. Two main problems have been studied here. Investigation of convergence performance of the proposed IIR filter on continuous time dynamic system response and examining the effects of time delay e^{-Ls} component of dynamic systems on modeling performance of adaptive IIR filter with GD optimizer. In the simulation studies, it has been observed that the discrete-time IIR filters can converge to the response of dynamic systems recursively (iteratively) with the help of GD optimization for very-low time delays. When the simulations were evaluated, the performance indices in the test with 1 second time delay were $MAE = 0.0092$, $MSE = 0.0016$ and $MRE = 29.464$, while the performance indices in the test with 20 seconds time delay were found to be $MAE = 0.1399$, $MSE = 0.0319$ and $MRE = 689.49$. In the test without time delay, $MAE = 0.0037$, $MSE = 0.0004$ and $MRE = 0.0797$. It has been observed that the increase in the time delay of the FOPTD dynamic system rapidly decreases the modeling performance of the adaptive IIR filter. In future studies, it will be beneficial for intelligent control applications to find solutions for more successful representation of time delay with the help of discrete IIR filters.

6 Author contribution statements

In the study, Nagihan YAGMUR contributed to methodology, software, formal analysis, writing-original draft; Baris Baykant ALAGOZ contributed to conceptualization, methodology, writing-review & editing.

7 Ethics committee approval and conflict of interest statement

“Ethics committee approval is not required for the prepared article”.

“There is no conflict of interest with any person/institution in the prepared article”.

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